Section One — Number

Page 3: Types of Number and BODMAS

1 $\frac{197.8}{\sqrt{0.01+0.23}} = \frac{197.8}{\sqrt{0.24}} = \frac{197.8}{0.489897948...}$
$= 403.757559... = 403.76$ (2 d.p.)
[2 marks available — 2 marks for answer correct to 2 decimal places, otherwise 1 mark for unrounded answer]

2 $\sqrt{\frac{12.71+137.936}{\cos 50° \times 13.2^2}} = \sqrt{\frac{150.646}{0.642787609... \times 174.24}}$
$= \sqrt{1.34506182...}$
$= 1.1597680...$
$= 1.16$ (2 d.p.)
[2 marks available — 2 marks for answer correct to 2 decimal places, otherwise 1 mark for unrounded answer]

3 E.g. $6 = \sqrt{3x+2y}$, so $36 = 3x + 2y$
Try different values of x and see what y-value each one gives:
$x = 2$: $3x = 6$, so $2y = 36 - 6 = 30$, so $y = 15$
$x = 4$: $3x = 12$, so $2y = 36 - 12 = 24$, so $y = 12$
[2 marks available — 1 mark for each correct pair of x and y values]
The only other possible solution is x = 6, y = 9 (you'll get full marks if you got this solution instead).

4 Irrational numbers: π, 0.6π, and $\sqrt{3}$.
[2 marks available — 2 marks for all three correct irrational numbers, lose 1 mark for one missing or a rational number included]
Don't be fooled — $\sqrt{16} = 4$, which is rational.

Page 4: Multiples, Factors and Prime Factors

1 a) $2 \times 3 \times 5 \times 7$
[2 marks available — 1 mark for correct method, 1 mark for all prime factors correct.]
b) $3^2 \times 5^2 \times 7^2$
[2 marks available — 1 mark for a correct method, 1 mark for all prime factors correct]

2 E.g. 4 (even) has three factors (1, 2 and 4).
81 (odd) has five factors (1, 3, 9, 27 and 81).
[2 marks available — 1 mark for each correct example of odd and even square numbers with a suitable number of factors.]
These aren't the only square numbers that would work here — any pair where the odd number has more factors than the even number would get you the marks.

3 Common multiples of 6 and 7:
42, 84, 126, 168, 210, 252, ... ***[1 mark]***
Factors of 252: 1, 2, 3, 4, 6, 7, 9, 12, 14, 18, 21, 28, 36, 42, 63, 84, 126, 252
Factors of 420: 1, 2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20, 21, 28, 30, 35, 42, 60, 70, 84, 105, 140, 210, 420 ***[1 mark for both sets of factors]***
Common factors of 252 and 420:
1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84 ***[1 mark]***
So $x = 84$ ***[1 mark]***
[4 marks available in total — as above]

Page 5: LCM and HCF

1 a) LCM $= 3^7 \times 7^3 \times 11^2$ ***[1 mark]***
b) HCF $= 3^4 \times 11$ ***[1 mark]***

2 a) LCM $= 2^8 \times 5^3 \times 7$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for a common multiple of all three numbers]
b) HCF $= 2^5$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for a common factor of all three numbers]

3 Prime factorisation of $A = A$
Prime factorisation of $B = B$ (as A and B are prime)
So LCM $= A \times B$ (or AB)
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for stating that the prime factorisations of A and B are A and B]

Pages 6-7: Fractions

1 $\frac{5}{6} = \frac{100}{120}$ $\frac{3}{4} = \frac{90}{120}$ $\frac{7}{8} = \frac{105}{120}$ $\frac{4}{5} = \frac{96}{120}$
All these fractions are less than one, and the largest is $\frac{105}{120}$,
so the fraction closest to 1 is $\frac{7}{8}$ ***[1 mark]***

2 a) $3\frac{1}{2} + 2\frac{3}{5} = \frac{7}{2} + \frac{13}{5} = \frac{35}{10} + \frac{26}{10} = \frac{35+26}{10} = \frac{61}{10}$ or $6\frac{1}{10}$
[3 marks available — 1 mark for writing as improper fractions, 1 mark for writing over a common denominator, 1 mark for the correct answer]
b) $3\frac{3}{4} - 2\frac{1}{3} = \frac{15}{4} - \frac{7}{3} = \frac{45}{12} - \frac{28}{12} = \frac{45-28}{12} = \frac{17}{12}$ or $1\frac{5}{12}$
[3 marks available — 1 mark for writing as improper fractions, 1 mark for writing over a common denominator, 1 mark for the correct answer]
If you've used a different method in Q2, but still shown your working, and ended up with the same final answer, then you still get full marks.

3 $1 - \frac{2}{15} - \frac{5}{12} = 1 - \frac{8}{60} - \frac{25}{60} = \frac{27}{60} = \frac{9}{20}$
[3 marks available — 1 mark for writing over a common denominator, 1 mark for doing the subtraction correctly and 1 mark for simplifying to find the correct answer]

4 $\frac{2}{5} + \left(1 - \frac{6}{7}\right) = \frac{2}{5} + \frac{1}{7} = \frac{14}{35} + \frac{5}{35} = \frac{19}{35}$
[3 marks available — 1 mark for finding the unshaded region of shape Y, 1 mark for writing over a common denominator and 1 mark for the correct answer]

5 $a = \frac{3}{4}, b = \frac{5}{2}$, so $\frac{1}{a} + \frac{1}{b} = \frac{4}{3} + \frac{2}{5} = \frac{20}{15} + \frac{6}{15} = \frac{26}{15}$ or $1\frac{11}{15}$
[3 marks available — 1 mark for reciprocal fractions, 1 mark for rewriting over a common denominator, 1 mark for the correct answer]

6 a) $1\frac{2}{3} \times \frac{9}{10} = \frac{5}{3} \times \frac{9}{10} = \frac{45}{30} = \frac{3}{2} = 1\frac{1}{2}$
[3 marks available — 1 mark for multiplying the two fractions together, 1 mark for an equivalent fraction, 1 mark for the correct final answer]
b) $3\frac{1}{2} \div 1\frac{2}{5} = \frac{7}{2} \div \frac{7}{5} = \frac{7}{2} \times \frac{5}{7} = \frac{35}{14} = \frac{5}{2} = 2\frac{1}{2}$
[3 marks available — 1 mark for taking the reciprocal and multiplying the two fractions together, 1 mark for an equivalent fraction, 1 mark for the correct final answer]

7 a) $17\frac{1}{2} \times \frac{1}{5} = \frac{35}{2} \times \frac{1}{5} = \frac{35}{10} = \frac{7}{2}$ ***[1 mark]*** tonnes of flour used to make cheese scones.
Then $\frac{7}{2}$ out of $25 = \frac{7}{2} \div 25 = \frac{7}{50}$ ***[1 mark]***.
[2 marks available in total — as above]
b) $\frac{7}{50} = \frac{14}{100} = 14\%$ ***[1 mark]***

Page 8: Fractions and Recurring Decimals

1 $10 \div 11 = 0.\dot{9}\dot{0}$ ***[1 mark]***

2 Convert to an equivalent fraction with all nines on the bottom
$\frac{7}{33} = \frac{21}{99}$ ***[1 mark]***
Then the number on the top tells you the recurring part,
so $\frac{7}{33} = 0.\dot{2}\dot{1}$ ***[1 mark]***
[2 marks available in total — as above]
The first mark could also be obtained by using a division method.

3 a) Let $r = 0.\dot{7}$, so $10r = 7.\dot{7}$ ***[1 mark]***
$10r - r = 7.\dot{7} - 0.\dot{7}$
$9r = 7$
$r = \frac{7}{9}$ ***[1 mark]***
[2 marks available in total — as above]

b) Let $r = 0.\dot{2}\dot{6}$, so $100r = 26.\dot{2}\dot{6}$ ***[1 mark]***
$100r - r = 26.\dot{2}\dot{6} - 0.\dot{2}\dot{6}$
$99r = 26$
$r = \frac{26}{99}$ ***[1 mark]***
[2 marks available in total — as above]

c) Let $r = 1.\dot{3}\dot{6}$, so $100r = 136.\dot{3}\dot{6}$ ***[1 mark]***
$100r - r = 136.\dot{3}\dot{6} - 1.\dot{3}\dot{6}$
$99r = 135$ ***[1 mark]***
$r = \frac{135}{99}$
$r = \frac{15}{11}$ or $1\frac{4}{11}$ ***[1 mark]***
[3 marks available in total — as above]

4 Let $10r = 5.\dot{9}\dot{0}$, so $1000r = 590.\dot{9}\dot{0}$ ***[1 mark]***
$990r = 585$ ***[1 mark]***
$r = \frac{585}{990} = \frac{13}{22}$ ***[1 mark]***
[3 marks available in total — as above]

Page 9: Rounding Numbers and Estimating

1 a) $\frac{215.7 \times 44.8}{460} \approx \frac{200 \times 40}{500} = \frac{8000}{500} = 16$
[3 marks available — 1 mark for correctly rounding 1 number to 1 significant figure, 1 mark for correctly rounding the other 2 numbers to 1 significant figure, 1 mark for correct answer]

b) The answer to a) will be smaller than the exact answer, because in the rounded fraction the numerator is smaller and denominator is larger compared to the exact calculation.
[2 marks available — 1 mark for 'smaller than the exact answer', 1 mark for correct reasoning]

2 $\sqrt{\frac{2321}{19.673 \times 3.81}} \approx \sqrt{\frac{2000}{20 \times 4}}$
[1 mark for rounding sensibly.]
$= \sqrt{\frac{100}{4}} = \sqrt{25}$ ***[1 mark for either expression]***
$= 5$ ***[1 mark]***
[3 marks available in total — as above]
You might have a different answer if you've rounded differently — as long as your rounding is sensible, you'll get the marks.

3 a) $V = \frac{1}{3}\pi(10)^2 \times 24 \approx \frac{1}{3} \times 3 \times 10^2 \times 20 = 100 \times 20 = 2000 \text{ cm}^3$
[2 marks available — 1 mark for rounding numbers sensibly, 1 mark for a suitable answer using rounded numbers]

b) Surface area $= (\pi \times 10 \times 26) + (\pi \times 10^2)$
$\approx (3 \times 10 \times 30) + (3 \times 10^2) = (30 \times 30) + (3 \times 100)$
$= 900 + 300 = 1200 \text{ cm}^2$
[2 marks available — 1 mark for rounding numbers sensibly, 1 mark for a suitable answer using rounded numbers]

Pages 10-11: Bounds

1 a) 54.05 cm ***[1 mark]***

b) lower bound for the width of the paper = 23.55 cm ***[1 mark]***
lower bound for the perimeter
= (54.05 cm × 2) + (23.55 cm × 2) = 155.2 cm [***1 mark]***
[2 marks available in total — as above]

2 upper bound for x = 57.5 mm ***[1 mark]***
upper bound for y = 32.5 mm ***[1 mark]***
upper bound for area = 57.5 mm × 32.5 mm = 1868.75 mm²
= 1870 mm² to 3 s.f. ***[1 mark]***
[3 marks available in total — as above]

3 Upper bound of $x = 2.25$ ***[1 mark]***
So upper bound of $4x + 3 = 4 \times 2.25 + 3 = 12$
Lower bound of $x = 2.15$ ***[1 mark]***
So lower bound of $4x + 3 = 4 \times 2.15 + 3 = 11.6$
Written as an interval, this is $11.6 \leq 4x + 3 < 12$
[2 marks — 1 for both bounds correct, 1 mark for expressing as an inequality correctly]
[4 marks available in total — as above]

4 Lower bound of difference = 13.65 – 8.35 ***[1 mark]***
= 5.3 litres ***[1 mark]***
[2 marks available in total — as above]

5 Upper bound of area = 5.25 cm² ***[1 mark]***
Lower bound of height = 3.15 cm ***[1 mark]***
2 × (5.25 ÷ 3.15) = 3.33 to 2 d.p. ***[1 mark]***
[3 marks available — as above]

6 lower bound for distance = 99.5 m
upper bound for time = 12.55 s ***[1 mark for both]***
lower bound for speed $= \frac{99.5}{12.55}$ m/s = 7.928... m/s ***[1 mark]***
lower bound for speed to 2 s.f. = 7.9 m/s
lower bound for speed to 1 s.f. = 8 m/s
upper bound for distance = 100.5 m
lower bound for time = 12.45 s ***[1 mark for both]***
upper bound for speed $= \frac{100.5}{12.45}$ m/s = 8.072... m/s ***[1 mark]***
upper bound for speed to 2 s.f. = 8.1 m/s
upper bound for speed to 1 s.f. = 8 m/s
The lower bound to 2 s.f. does not equal the upper bound to 2 s.f., but the lower bound to 1 s.f. does equal the upper bound to 1 s.f. So Dan's speed is 8 m/s to 1 significant figure.
[1 mark for comparing bounds to reach correct answer to 1 s.f.]
[5 marks available in total — as above]

7 lower bound for volume = 0.935 × 0.605 × 0.205 = 0.11596... m³
upper bound for volume = 0.945 × 0.615 × 0.215 = 0.12495... m³
Both the upper bound and lower bound round to 0.12 m³ to 2 d.p. (or 2 s.f.) so the volume to 2 d.p. is 0.12 m³.
[4 marks available — 1 mark for the correct upper and lower bounds for the dimensions, 1 mark for the correct lower bound for volume, 1 mark for the correct upper bound for volume, 1 mark for rounding to a suitable number of decimal places (or significant figures) to obtain the final answer]

Pages 12-13: Standard Form

1 a) $A = 4.834 \times 10^9 = 4\,834\,000\,000$ ***[1 mark]***

b) $B \times C = (2.7 \times 10^5) \times (5.8 \times 10^3) = (2.7 \times 5.8) \times (10^5 \times 10^3)$
$= 15.66 \times 10^8$ ***[1 mark]***
$= 1.566 \times 10^9$ ***[1 mark]***
[2 marks available in total — as above]

c) C, B, A $(5.8 \times 10^3, 2.7 \times 10^5, 4.834 \times 10^9)$ ***[1 mark]***

2 time (s) = distance (miles) ÷ speed (miles/s)
$= (9.3 \times 10^7) \div (1.86 \times 10^5)$ seconds ***[1 mark]***
$= 5 \times 10^2$ seconds ***[1 mark]***
[2 marks available in total — as above]

3 $A = (5 \times 10^5) + (5 \times 10^3) + (5 \times 10^2) + (5 \times 10^{-2})$
= 500 000 + 5000 + 500 + 0.05 ***[1 mark]***
= 505 500.05 ***[1 mark]***
[2 marks available in total — as above]

4 $(4.5 \times 10^9) \div (1.5 \times 10^8) = (4.5 \div 1.5) \times (10^9 \div 10^8)$ ***[1 mark]***
$= 3 \times 10^1 = 30$ ***[1 mark]***. So the ratio is 1 : 30 ***[1 mark]***
[3 marks available in total — as above]

5 a) number of tablets = dose (grams) ÷ dose per tablet (grams)
$= (4 \times 10^{-4}) \div (8 \times 10^{-5})$ ***[1 mark]***
$= (4 \div 8) \times (10^{-4} \div 10^{-5})$
$= 0.5 \times 10^1$ ***[1 mark]***
$= 5$ ***[1 mark]***
[3 marks available in total — as above]

b) new dose = 4×10^{-4} grams + 6×10^{-5} grams ***[1 mark]***
= 4×10^{-4} grams + 0.6×10^{-4} grams ***[1 mark]***
= $(4 + 0.6) \times 10^{-4}$ grams
= 4.6×10^{-4} grams per day ***[1 mark]***
[3 marks available in total — as above]
You could have done this one by turning 4×10^{-4} into 40×10^{-5} and adding it to 6×10^{-5} instead.

6 Total weight of ship and passengers = $7.59 \times 10^7 + 2.1 \times 10^5$
= 7.611×10^7 kg ***[1 mark]***
$(2.1 \times 10^5) \div (7.611 \times 10^7) = 0.002759...$ ***[1 mark]***
$0.002759 \times 100 = 0.28$ % (to 2 d.p.) ***[1 mark]***
[3 marks available in total — as above]

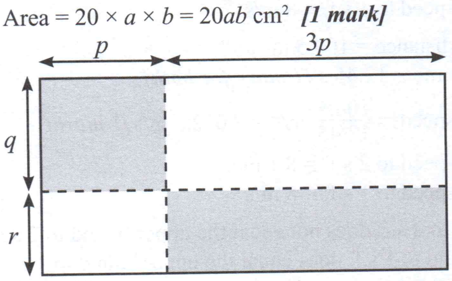
7 $\frac{3^2}{2^{122} \times 5^{120}} = \frac{9}{2^2(2^{120} \times 5^{120})} = \frac{9}{2^2 \times 10^{120}} = \frac{9}{4} \times \frac{1}{10^{120}} = 2.25 \times 10^{-120}$
[2 marks available —1 mark for writing the denominator as a multiple of a power of 10, 1 mark for the correct answer]

Section Two — Algebra

Page 14: Algebra Basics

1 Area = $20 \times a \times b = 20ab$ cm² ***[1 mark]***

2



[1 mark]

3 Height = $7 \times (f + g) + 9 \times (h - g) + 5 \times 2h$ ***[1 mark]***
= $7f + 7g + 9h - 9g + 10h$
= $7f - 2g + 19h$ cm ***[1 mark]***
[2 marks available in total — as above]

4 Perimeter of rectangle = $4x + 3 + 4x + 3 + 5x - 9 + 5x - 9$
= $18x - 12$ cm ***[1 mark]***.
So perimeter of hexagon = $18x - 12$ cm.
Hexagon side length = $(18x - 12) \div 6$ ***[1 mark]***
= $3x - 2$ cm ***[1 mark]***.
[3 marks available in total — as above]

Page 15: Powers and Roots

1 $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ ***[1 mark]***

2 $8^{\frac{4}{3}} = (8^{\frac{1}{3}})^4 = (2)^4 = 16$
[2 marks available — 1 mark for correct working, 1 mark for the correct final answer.]

3 $y^{-3} = \frac{1}{y^3}$, $y^1 = y$, $y^0 = 1$, $y^{\frac{1}{3}} = \sqrt[3]{y}$,
so the correct order is... y^{-3} y^0 $y^{\frac{1}{3}}$ y^1 y^3
[2 marks available — 2 marks for all 5 in the correct order, otherwise 1 mark for any 4 in the correct relative order.]
If you can't identify which term is the smallest just by looking at them, try substituting a value for y into all the expressions and working out the answer. Then it'll be easy to tell which is the smallest.

4 a) $8^2 = 64$ and $9^2 = 81$, so $x = \sqrt{70} \approx 8.4$
[2 marks available — 2 marks for 8.3, 8.4 or 8.5, otherwise 1 mark for any answer between 8 and 9 with 1 d.p.]
b) $3^2 = 9$ and $3^3 = 27$, so $3^{2.7} \approx 20$, $y \approx 2.7$
[2 marks available — 2 marks for 2.6, 2.7 or 2.8, otherwise 1 mark for any answer between 2 and 3 with 1 d.p.]

5 $(9a^4)^{\frac{1}{2}} = \sqrt{9a^4} = 3a^2$ ***[1 mark]***
$\frac{2ab^2}{6a^3b} = \frac{2}{6} \times \frac{a}{a^3} \times \frac{b^2}{b} = \frac{1}{3} \times \frac{1}{a^2} \times b = \frac{b}{3a^2}$ ***[1 mark]***
so $(9a^4)^{\frac{1}{2}} \times \frac{2ab^2}{6a^3b} = 3a^2 \times \frac{b}{3a^2} = b$ ***[1 mark]***
[3 marks available in total — as above]

Page 16: Multiplying Out Brackets

1 a) $5p(6 - 2p) = 30p - 10p^2$
[2 marks available — 1 mark for each term]
b) $(2t - 5)(3t + 4) = (2t \times 3t) + (2t \times 4) + (-5 \times 3t) + (-5 \times 4)$
$= 6t^2 + 8t - 15t - 20$ ***[1 mark]***
$= 6t^2 - 7t - 20$ ***[1 mark]***
[2 marks available in total — as above]

2 $4(5x - 7) + 6(4 - 2x) = 20x - 28 + 24 - 12x$
$= 8x - 4 = 4(2x - 1)$
So $a = 4$, $b = 2$ and $c = -1$
[3 marks available — 1 mark for each correct value]

3 Area = ½ × base × height
= ½$(3x + 5y)(2x - 4y)$ ***[1 mark]***
= ½ × $(6x^2 - 2xy - 20y^2)$ ***[1 mark]***
= $3x^2 - xy - 10y^2$ ***[1 mark]***
[3 marks available in total — as above]
You could have instead multiplied $(2x - 4y)$ by ½ first of all. The area would then just be $(3x + 5y)(x - 2y)$, which is a bit simpler to multiply out.

4 $(x - 1)(2x + 3)(2x - 3) = (x - 1)(4x^2 - 6x + 6x - 9)$
$= (x - 1)(4x^2 - 9)$
$= 4x^3 - 4x^2 - 9x + 9$
[3 marks available — 3 marks for the correct answer, otherwise 1 mark for correctly multiplying two sets of brackets together, 1 mark for attempting to multiply this product by the third set of brackets]
The trick here is spotting that the second pair of brackets multiply out to give just two terms (a difference of two squares), which makes the second multiplication much easier.

Page 17: Factorising

1 a) $7y - 21y^2 = 7(y - 3y^2) = 7y(1 - 3y)$
[2 marks available — 1 mark for each correct factor]
b) $2v^3w + 8v^2w^2 = 2(v^3w + 4v^2w^2) = 2v^2w(v + 4w)$
[2 marks available — 1 mark for each correct factor]

2 a) $x^2 - 16 = x^2 - 4^2 = (x + 4)(x - 4)$ ***[1 mark]***
b) $9n^2 - 4m^2 = (3n)^2 - (2m)^2 = (3n + 2m)(3n - 2m)$
[2 marks available — 1 mark for each correct factor]
c) $3y^2 - 15 = 3(y^2 - 5) = 3[y^2 - (\sqrt{5})^2] = 3(y + \sqrt{5})(y - \sqrt{5})$
[2 marks available — 1 mark for each correct factor]

3 $x^3 - 25x = x(x^2 - 25) = x(x + 5)(x - 5)$
[3 marks available — 1 mark for each correct factor]

Page 18: Manipulating Surds

1 $(2 + \sqrt{3})(5 - \sqrt{3}) = (2 \times 5) + (2 \times -\sqrt{3}) + (\sqrt{3} \times 5) + (\sqrt{3} \times -\sqrt{3})$
$= 10 - 2\sqrt{3} + 5\sqrt{3} - 3$
$= 7 + 3\sqrt{3}$
[2 marks available — 1 mark for correct working, 1 mark for the correct answer]

2 $2\sqrt{50} = 2\sqrt{25 \times 2} = 2 \times 5\sqrt{2} = 10\sqrt{2}$
$(\sqrt{2})^3 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = (\sqrt{2})^2 \times \sqrt{2} = 2\sqrt{2}$
So $2\sqrt{50} - (\sqrt{2})^3 = 10\sqrt{2} - 2\sqrt{2} = 8\sqrt{2}$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correctly simplifying either surd]

3 $\sqrt{396} = \sqrt{36 \times 11} = 6\sqrt{11}$ ***[1 mark]***
$\frac{22}{\sqrt{11}} = \frac{22\sqrt{11}}{11} = 2\sqrt{11}$ ***[1 mark]***
$\sqrt{44} = \sqrt{4 \times 11} = 2\sqrt{11}$
So $\frac{220}{\sqrt{44}} = \frac{220}{2\sqrt{11}} = \frac{220\sqrt{11}}{22} = 10\sqrt{11}$ ***[1 mark]***
So $\sqrt{396} + \frac{22}{\sqrt{11}} - \frac{220}{\sqrt{44}} = 6\sqrt{11} + 2\sqrt{11} - 10\sqrt{11} = -2\sqrt{11}$
[1 mark]
[4 marks available — as above]

4 $\dfrac{1+\sqrt{7}}{3-\sqrt{7}} = \dfrac{(1+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$ ***[1 mark]***

$= \dfrac{3+\sqrt{7}+3\sqrt{7}+7}{9-7}$ ***[1 mark]***

$= \dfrac{10+4\sqrt{7}}{2}$ ***[1 mark]***

$= 5+2\sqrt{7}$ ***[1 mark]***

[4 marks available in total — as above]

Pages 19-20: Solving Equations

1 Let the number of tickets Felix sells be x.
Then Poppy sells $2x$ tickets and Alexi sells $(2x + 25)$ tickets, so $x + 2x + (2x + 25) = 700$
$5x + 25 = 700$
$5x = 675$ and $x = 135$
So Felix sells 135 tickets, Poppy sells $2 \times 135 = 270$ tickets and Alexi sells $(2 \times 135) + 25$ tickets $= 295$ tickets.
[5 marks available — 1 mark for finding expressions for the number of tickets each person sells, 1 mark for forming an equation to solve, 1 mark for solving to find the number of tickets Felix sells, 1 mark for the number of tickets Poppy sells, 1 mark for the number of tickets Alexi sells]

2 $2x + 6 = 5(x - 3)$ ***[1 mark]***
$2x + 6 = 5x - 15$
$21 = 3x$ ***[1 mark]***
$x = 7$ cm ***[1 mark]***
So one side of the triangle measures $2(7) + 6 = 20$ cm ***[1 mark]***
[4 marks available in total — as above]

3 $\frac{5}{4}(2c - 1) = 3c - 2$
$5(2c - 1) = 4(3c - 2)$ ***[1 mark]***
$(5 \times 2c) + (5 \times -1) = (4 \times 3c) + (4 \times -2)$
$10c - 5 = 12c - 8$
$12c - 10c = 8 - 5$ ***[1 mark]***
$2c = 3$ so $c = \frac{3}{2}$ or 1.5 ***[1 mark]***
[3 marks available in total — as above]

4 If Neil worked h hours, Liam worked $(h + 30)$ hours.
$360 \div 4.5 = 80$
$80 = h + (h + 30) = 2h + 30$
$50 = 2h$, so $h = 25$
Neil worked 25 hours and Liam worked $(25 + 30) = 55$ hours.
[3 marks available — 1 mark for forming an equation for the total number of hours, 1 mark for solving the equation to find h, 1 mark for finding the number of hours each boy worked]

5 a) $5x^2 = 180$
$x^2 = 36$ ***[1 mark]***
$x = \pm 6$ ***[1 mark]***
[2 marks available in total — as above]

b) $\dfrac{8-2x}{3} + \dfrac{2x+4}{9} = 12$
$\dfrac{9(8-2x)}{3} + \dfrac{9(2x+4)}{9} = 9 \times 12$
$3(8 - 2x) + (2x + 4) = 108$
$24 - 6x + 2x + 4 = 108$
$6x - 2x = 24 + 4 - 108$
$4x = -80$ so $x = -20$
[4 marks available — 2 marks for rearranging to remove the fractions, 1 mark for rearranging to get all x-terms on one side, 1 mark for correct answer]

6 If one number is x, the other number is $3x$.
$3x^2 = 147$ ***[1 mark]***, so $x^2 = 49$, which means that $x = 7$ (as $x > 0$) ***[1 mark]*** and $3x = 21$ ***[1 mark]***, so Hassan is thinking of 7 and 21.
[3 marks available in total — as above]

Pages 21-22: Formulas

1 a) $F = \frac{9}{5}C + 32$, so $\frac{9}{5}C = F - 32$ and $C = \frac{5}{9}(F - 32)$
[2 marks available — 1 mark for subtracting 32 from each side, 1 mark for the correct answer.]

b) When $F = 41$, $C = \frac{5}{9}(41 - 32) = \frac{5}{9}(9) = 5$ °C
[2 marks available — 1 mark for correct substitution, 1 mark for the correct answer.]

2 a) $P = \dfrac{V^2}{R} = \dfrac{12^2}{16} = \dfrac{144}{16} = 9$ W ***[1 mark]***

b) $P = \dfrac{V^2}{R}$, so $25 = \dfrac{20^2}{R}$ ***[1 mark]***. $R = \dfrac{20^2}{25} = \dfrac{400}{25}$, $= 16\ \Omega$ ***[1 mark]***
[2 marks available in total — as above]

3 $s = \frac{1}{2}gt^2$, so $gt^2 = 2s$ ***[1 mark]***, $t^2 = \dfrac{2s}{g}$ ***[1 mark]***,
$t = \sqrt{\dfrac{2s}{g}}$ ***[1 mark]***
[3 marks available in total — as above]

4 a) $a + y = \dfrac{b-y}{a}$, so...
$a(a + y) = b - y$ ***[1 mark]***, $a^2 + ay = b - y$,
$ay + y = b - a^2$ ***[1 mark]***, $y(a + 1) = b - a^2$ ***[1 mark]***,
$y = \dfrac{b-a^2}{a+1}$ ***[1 mark]***
[4 marks available in total — as above]

b) When $a = 3$ and $b = 6$, $y = \dfrac{6-3^2}{3+1} = -\dfrac{3}{4}$ or -0.75 ***[1 mark]***

5 $x = \sqrt{\dfrac{(1+n)}{(1-n)}}$, so $x^2 = \dfrac{(1+n)}{(1-n)}$ ***[1 mark]***, $x^2(1 - n) = 1 + n$,
$x^2 - x^2n = 1 + n$ ***[1 mark]***, $x^2 - 1 = n + x^2n$ ***[1 mark]***,
$x^2 - 1 = n(1 + x^2)$ ***[1 mark]***,
$n = \dfrac{x^2-1}{1+x^2}$ ***[1 mark]***
[5 marks available in total — as above]

6 Rearrange to make y the subject:
$x = \dfrac{1-y}{x}$
$x^2 = 1 - y$
$y = 1 - x^2$
$x > 1$, so $x^2 > 1$. Then $1 - x^2 < 0$, so y is always negative.
[3 marks available — 1 mark for rearranging the formula to make y the subject, 1 mark for showing that $x^2 > 1$ or $1 - x^2 < 0$, 1 mark for stating that y is negative]

7 $b = \sqrt{2a-1}$ so $b^2 = 2a - 1$ and $b^4 = (2a - 1)^2 = 4a^2 - 4a + 1$
$c = 2b^4 + 4b^2 = 2(4a^2 - 4a + 1) + 4(2a - 1)$
$= 8a^2 - 8a + 2 + 8a - 4 = 8a^2 - 2$
Rearrange to make a the subject:
$c = 8a^2 - 2$
$a^2 = \dfrac{c+2}{8}$
$a = \pm\sqrt{\dfrac{c+2}{8}}$
[4 marks available — 1 mark for writing b^2 and b^4 in terms of a, 1 mark for writing c in terms of a, 1 mark for rearranging to make a^2 the subject, 1 mark for the correct answer]

Page 23: Factorising Quadratics

1 Let the two consecutive even numbers be $2n$ and $2n + 2$.
$2n(2n + 2) = 288$ ***[1 mark]***
$4n^2 + 4n = 288$
$4n^2 + 4n - 288 = 0$
$n^2 + n - 72 = 0$
$(n + 9)(n - 8) = 0$ ***[1 mark]***, so $n = -9$ or $n = 8$.
The numbers are positive, so $n = 8$ ***[1 mark]***.
The larger of the two numbers is $2(8) + 2 = 18$ ***[1 mark]***.
[4 marks available in total — as above]

2 a) $(5x - 9)(x - 2)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

b) Replacing x with $(x - 1)$ in the factorised expression from a)...
$5(x - 1)^2 - 19(x - 1) + 18 = (5(x - 1) - 9)((x - 1) - 2)$ *[1 mark]*
$= (5x - 5 - 9)(x - 1 - 2)$
$= (5x - 14)(x - 3)$ *[1 mark]*
[2 marks available in total — as above]

3 The area of the square is $(x + 3)(x + 3) = x^2 + 6x + 9$. *[1 mark]*
The area of the triangle is $\frac{1}{2}(2x + 2)(x + 3)$
$= \frac{1}{2}(2x^2 + 6x + 2x + 6) = \frac{1}{2}(2x^2 + 8x + 6)$
$= x^2 + 4x + 3$ *[1 mark]*
So the area of the whole shape is $x^2 + 6x + 9 + x^2 + 4x + 3$
$= 2x^2 + 10x + 12$ *[1 mark]*
$2x^2 + 10x + 12 = 60$, so $2x^2 + 10x - 48 = 0$ *[1 mark]*
So $x^2 + 5x - 24 = 0$
$(x - 3)(x + 8) = 0$ *[1 mark]*
$x - 3 = 0$ or $x + 8 = 0$
$x = 3$ or $x = -8$
[1 mark for both solutions]
A length can't have a negative value so the answer must be $x = 3$ *[1 mark]*
[7 marks available in total — as above]

Page 24: The Quadratic Formula

1 a = 1, b = 5 and c = 3

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 3}}{2 \times 1} = \frac{-5 \pm \sqrt{13}}{2}$$

$x = -0.70$ or $x = -4.30$
[3 marks available — 1 mark for correct substitution, 1 mark for each correct solution]

2 a = 2, b = –7 and c = 2

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times 2}}{2 \times 2} = \frac{7 \pm \sqrt{33}}{4}$$

$x = 3.19$ or $x = 0.31$
[3 marks available — 1 mark for correct substitution, 1 mark for each correct solution]

3 a = 3, b = –2 and c = –4

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times -4}}{2 \times 3} = \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm 2\sqrt{13}}{6}$$

$$x = \frac{1 + \sqrt{13}}{3} \text{ or } x = \frac{1 - \sqrt{13}}{3}$$

[3 marks available — 1 mark for correct substitution, 1 mark for each correct solution. Lose a mark if answers aren't simplified]

4 $(x + 3)(3x + 3) = 30$
$3x^2 + 12x + 9 = 30$
$3x^2 + 12x - 21 = 0$
$x^2 + 4x - 7 = 0$
a = 1, b = 4 and c = –7

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times -7}}{2 \times 1} = \frac{-4 \pm \sqrt{44}}{2}$$

$$= \frac{-4 \pm 2\sqrt{11}}{2} = -2 \pm \sqrt{11}$$

Lengths cannot be negative, so $x = -2 + \sqrt{11}$.
So the longer side is $3(-2 + \sqrt{11}) + 3 = -3 + 3\sqrt{11}$ cm
[5 marks available — 1 mark for setting up the quadratic equation, 1 mark for the correct substitution, 1 mark for solving the quadratic equation, 1 mark for choosing the correct value of x, 1 mark for the correct answer]

Page 25: Completing the Square

1 $(x + 2)^2 - 9 = x^2 + 4x + 4 - 9$ *[1 mark]* $= x^2 + 4x - 5$
$a = 4$ and $b = -5$ *[1 mark]*
[2 marks available in total — as above]

2 a) $-10 \div 2 = -5$, so $p = -5$ and the bit in brackets is $(x - 5)^2$. *[1 mark]*
Expanding the brackets: $(x - 5)^2 = x^2 - 10x + 25$. *[1 mark]*
To complete the square: $-5 - 25 = -30$, so $q = -30$. *[1 mark]*
$p = -5$ and $q = -30$
[3 marks available in total — as above]

b) $(x - 5)^2 - 30 = 0$, so $(x - 5)^2 = 30$ and $x - 5 = \pm\sqrt{30}$
So $x = 5 + \sqrt{30}$ or $x = 5 - \sqrt{30}$
[2 marks available — 1 mark for each correct solution]

3 a) $2(x^2 - 4x) + 19$
$4 \div 2 = 2$, so the first bit is $2[(x - 2)^2]$
Expanding the brackets: $2(x^2 - 4x + 4) = 2x^2 - 8x + 8$
To complete the square: $19 - 8 = 11$
So $2x^2 - 8x + 19 = 2(x - 2)^2 + 11$
[4 marks available — 1 mark for dividing the first two terms by 2, 1 mark dividing the x-term by 2 to find the value of b, 1 mark for finding the value of c, 1 mark for the full correct answer]

b) Minimum value = 11, which occurs at $x = 2$, so the coordinates of the minimum point are (2, 11) *[1 mark]*

c) This quadratic is u-shaped and its minimum value is 11, so it's always greater than 0. This means it never crosses the x-axis. *[1 mark]*

Page 26: Algebraic Fractions

1 $$\frac{4x^2 + 10x - 6}{16x^2 - 4} = \frac{(4x - 2)(x + 3)}{(4x - 2)(4x + 2)} = \frac{x + 3}{4x + 2}$$

[3 marks available — 1 mark for correctly factorising the denominator, 1 mark for correctly factorising the numerator, 1 mark for the correct answer]

2 $$\frac{2a - 8}{a^2 - 9} \div \frac{a^2 - 2a - 8}{a^2 + 5a + 6} \times (2a^2 - a - 15)$$

$$= \frac{2a - 8}{a^2 - 9} \times \frac{a^2 + 5a + 6}{a^2 - 2a - 8} \times (2a^2 - a - 15)$$

$$= \frac{2(a - 4)}{(a + 3)(a - 3)} \times \frac{(a + 3)(a + 2)}{(a + 2)(a - 4)} \times (2a + 5)(a - 3)$$

$= 2(2a + 5)$ (or $4a + 10$)
[5 marks available — 1 mark for converting to a multiplication, 1 mark for factorising the first fraction, 1 mark for factorising the second fraction, 1 mark for factorising the quadratic term, 1 mark for cancelling to reach correct answer]

3 $$\frac{2}{3} + \frac{m - 2n}{m + 3n} = \frac{2(m + 3n)}{3(m + 3n)} + \frac{3(m - 2n)}{3(m + 3n)} = \frac{2(m + 3n) + 3(m - 2n)}{3(m + 3n)}$$

$$= \frac{2m + 6n + 3m - 6n}{3(m + 3n)} = \frac{5m}{3(m + 3n)}$$

[3 marks available — 1 mark for finding the common denominator, 1 mark for a correct method for addition, 1 mark for the correct final answer]

4 $$\frac{1}{x - 5} + \frac{2}{x - 2} = \frac{x - 2}{(x - 5)(x - 2)} + \frac{2(x - 5)}{(x - 5)(x - 2)}$$

$$= \frac{(x - 2) + 2(x - 5)}{(x - 5)(x - 2)} = \frac{x - 2 + 2x - 10}{(x - 5)(x - 2)} = \frac{3x - 12}{(x - 5)(x - 2)}$$

[3 marks available — 1 mark for finding the common denominator, 1 mark for a correct method for addition, 1 mark for the correct final answer]

Pages 27-29: Sequences

1 a) Common difference = 5, so $5n$ is in the formula.
To get from $5n$ to each term, you have to subtract 2, so the expression for the nth term is $5n - 2$.
[2 marks available — 2 marks for correct expression, otherwise 1 mark for finding 5n.]

b) The nth term is $5n - 2$ and the $(n + 1)$th term is $5(n + 1) - 2 = 5n + 5 - 2 = 5n + 3$
Product of the nth and $(n + 1)$th terms:
$(5n - 2)(5n + 3) = 25n^2 + 15n - 10n - 6 = 25n^2 + 5n - 6$
[2 marks available — 1 mark for finding an expression for the nth and (n + 1)th terms, 1 mark for multiplying to find the correct product]

2 a) Common difference = 4 so $4n$ is in the formula.
To get from $4n$ to each term, you have to subtract 1, so the expression for the nth term is $4n - 1$.
[2 marks available — 2 marks for correct expression, otherwise 1 mark for finding 4n]

b) All multiples of 4 are even numbers, and an even number minus 1 is always an odd number. So all the terms in this sequence will be odd numbers. ***[1 mark]***
502 is an even number, so 502 cannot be in the sequence. ***[1 mark]***
[2 marks available in total — as above]

3 a) To get from one term to the next, you have to multiply by $\sqrt{2}$, so the next term is $4\sqrt{2}$ and the next one is $4\sqrt{2} \times \sqrt{2} = 8$.
[2 marks available — 1 mark for each correct term]

b) $(\sqrt{2})^n$ ***[1 mark]***

4 a) $u_1 = 0.5$
$u_2 = 2(0.5) + 1 = 1 + 1 = 2$ ***[1 mark]***
$u_3 = 2(2) + 1 = 4 + 1 = 5$ ***[1 mark]***
[2 marks available in total — as above]

b) $u_1 = 1.5$
$u_2 = 2(1.5) + 1 = 3 + 1 = 4$ ***[1 mark]***
$u_3 = 2(4) + 1 = 8 + 1 = 9$ ***[1 mark]***
$u_4 = 2(9) + 1 = 18 + 1 = 19$ ***[1 mark]***
[3 marks available in total — as above]

c) If $u_1 = -1$, all terms in the sequence are also -1 ***[1 mark]***

5 a) The difference between the terms is 4, 6, 8, ... so to find the next term, add 10 onto 20: 20 + 10 = 30.
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for finding the differences between the terms]

b) Sequence: 2 6 12 20
First difference: 4 6 8
Second difference: 2 2 ***[1 mark]***
Coefficient of $n^2 = 2 \div 2 = 1$.
Actual sequence – n^2 sequence:
1 2 3 4
Difference: 1 1 1
So this is a linear sequence with *n*th term *n* ***[1 mark]***.
So the *n*th term is $n^2 + n$ ***[1 mark]***.
[3 marks available in total — as above]

6 a) $u_1 = 2$
$u_2 = \frac{-1}{2(2)} = -0.25$
$u_3 = \frac{-1}{2(-0.25)} = 2$
$u_4 = \frac{-1}{2(2)} = -0.25$
[2 marks available — 2 marks for all three values correct, otherwise 1 mark for one or two values correct]

b) $u_{50} = -0.25$ ***[1 mark]***

7 a) Number of grey squares as a sequence: 1, 5, 9, 13, ...
Common difference = 4 so 4*n* is in the formula.
To get from 4*n* to each term, you have to subtract 3, so the expression for the *n*th term is 4*n* – 3.
[2 marks available — 2 marks for correct expression, otherwise 1 mark for finding 4n]

b) Assume Giles makes the *n*th and (*n* + 1)th patterns.
He uses 4*n* – 3 grey squares in the *n*th pattern and $4(n + 1) - 3 = 4n + 4 - 3 = 4n + 1$ grey squares in the (*n* + 1)th pattern ***[1 mark]***. He uses 414 grey squares in total, so
$(4n - 3) + (4n + 1) = 414$ ***[1 mark]***
$8n - 2 = 414$
$8n = 416$
$n = 52$
So Giles has made the 52nd and 53rd patterns ***[1 mark]***.
[3 marks available in total — as above]

c) Total number of squares:
1 7 17 31
First difference: 6 10 14
Second difference: 4 4 ***[1 mark]***
The second differences are constant so the sequence is quadratic. Coefficient of $n^2 = 4 \div 2 = 2$ ***[1 mark]***.
Actual sequence – $2n^2$ sequence:
–1 –1 –1 –1
So the *n*th term of the sequence is $2n^2 - 1$ ***[1 mark]***.
[3 marks available in total — as above]

Page 30: Inequalities

1 $-4 \leq 2x < 8$ ***[1 mark]***

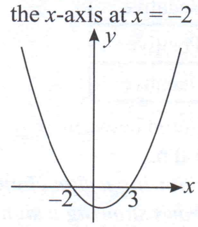
2 $4x + 1 > x - 5$, so $3x > -6$ ***[1 mark]*** and $x > -2$ ***[1 mark]***
[2 marks available in total — as above]

3 $5n - 3 \leq 17$, so $5n \leq 20$, so $n \leq 4$ ***[1 mark]***
$2n + 6 > 8$, so $2n > 2$, so $n > 1$ ***[1 mark]***
Putting these together gives $1 < n \leq 4$, so $n = \{2, 3, 4\}$ ***[1 mark]***
[3 marks available in total — as above]
Don't forget to give your answer in set notation here.

4 $2n + (2n + 2) + (2n + 4) < 1000$ ***[1 mark]***
$6n + 6 < 1000$
$n < 165.666...$ ***[1 mark]***
So the largest possible values of the numbers are obtained when *n* = 165, which gives 330, 332 and 334 ***[1 mark]***.
[3 marks available in total — as above]

5 a) $5x^2 < 80$, so $x^2 < 16$. The solutions of $x^2 = 16$ are $x = 4$ and $x = -4$, so $x^2 < 16$ when $-4 < x < 4$.
[3 marks available — 1 mark for finding the square roots of 16, 1 mark for –4 < x, 1 mark for x < 4]

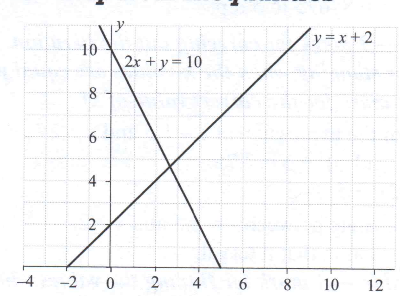
b) $x^2 + 1 = x + 7$ rearranges to give $x^2 - x - 6 = 0$.
$x^2 - x - 6 = 0$ factorises to give $(x + 2)(x - 3) = 0$.
The graph of $y = x^2 - x - 6$ is a u-shaped quadratic that crosses the *x*-axis at $x = -2$ and $x = 3$:



The graph is below 0 when *x* is greater than –2 and less than 3. So $-2 < x < 3$.
[3 marks available — 1 mark for rearranging and factorising the quadratic to find the solutions, 1 mark for –2 < x, 1 mark for x < 3]

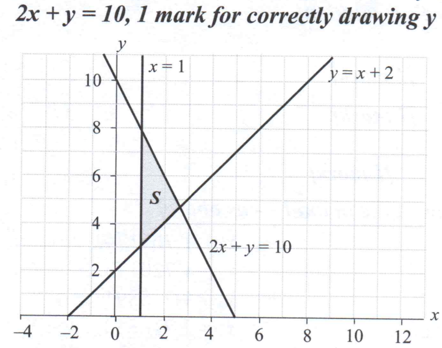
Page 31: Graphical Inequalities

1 a)



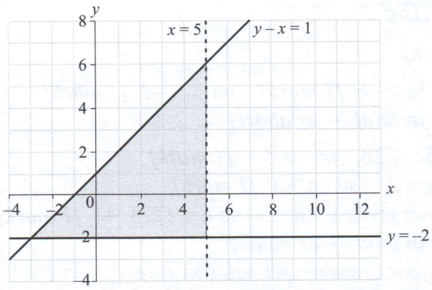
[2 marks available — 1 mark for correctly drawing 2x + y = 10, 1 mark for correctly drawing y = x + 2]

b)



[2 marks available — 2 marks for correctly shaded area, lose 1 mark if the wrong side of one line is shaded]

2



[4 marks available — 1 mark for drawing each line correctly, 1 mark for shading the correct area]

3 $y \geq 2$ ***[1 mark]***, $x + y \leq 8$ ***[1 mark]*** *and* $y \leq x$ ***[1 mark]***
[3 marks available in total — as above]

Pages 32-33: Iterative Methods

1

x	$x^3 - 4x + 2$	
−2	2	Positive
−3	−13	Negative
−2.1	1.139	Positive
−2.2	0.152	Positive
−2.3	−0.967	Negative
−2.21	0.046139	Positive
−2.22	−0.061048	Negative

The sign change shows that there's a solution between $x = -2.22$ and $x = -2.21$, so $x = -2.2$ to 1 d.p.
[4 marks available — 1 mark for rows showing a sign change between x = −2 and x = −3, 1 mark for rows showing a sign change between x = −2.2 and x = −2.3, 1 mark for rows showing a sign change between x = −2.21 and x = −2.22, 1 mark for the correct value of x]

2 $x_0 = 1$ $x_1 = 0.888888...$
$x_2 = 0.876881...$ $x_3 = 0.876750...$
$x_4 = 0.876750...$
$x_4 = x_3$ to 5 d.p. so $x = 0.87675$ to 5 d.p.
[3 marks available — 1 mark for carrying out the iteration correctly, 1 mark for stopping when the x-values are equal when rounded to 5 d.p., 1 mark for the correct value of x]

3 a) Find the value of the expression at $x = 1.5$ and $x = 2$:
$x = 1.5$: $3(1.5) - 2(1.5)^3 + 5 = 2.75$
$x = 2$: $3(2) - 2(2)^3 + 5 = -5$
There is a sign change between $x = 1.5$ and $x = 2$, so there is a solution in that interval.
[2 marks available — 1 mark for finding the values when x = 1.5 and x = 2, 1 mark for stating that a sign change means there's a solution]

b) $3x - 2x^3 + 5 = 0$
$2x^3 = 3x + 5$ ***[1 mark]***
$x^3 = \frac{3x+5}{2}$ ***[1 mark]***
$x = \sqrt[3]{\frac{3x+5}{2}}$ ***[1 mark]***
[3 marks available in total — as above]

c) $x_0 = 2$ $x_1 = 1.765174...$
$x_2 = 1.726657...$ $x_3 = 1.720173...$
$x_4 = 1.719076...$ $x_5 = 1.718891...$
$x_6 = 1.718860...$ $x_7 = 1.718854...$
$x_8 = 1.718853...$
$x_7 = x_8$ to 5 d.p. so $x = 1.71885$ to 5 d.p.
[3 marks available — 1 mark for carrying out the iteration correctly, 1 mark for stopping when the x-values are equal to 5 d.p., 1 mark for the correct value of x]

Pages 34-35: Simultaneous Equations

1 $x + 3y = 11 \xrightarrow{\times 3} 3x + 9y = 33$ ***[1 mark]***

$3x + 9y = 33$
$\underline{3x + y = 9 -}$
$8y = 24$
$y = 3$ ***[1 mark]***

$x + 3y = 11$
$x + (3 \times 3) = 11$
$x = 11 - 9$
$x = 2$ ***[1 mark]***

[3 marks available in total — as above]
For all the simultaneous equation questions, you could have eliminated the other variable and/or substituted into the other equation — you'd get the marks either way.

2 $2x + 3y = 12 \xrightarrow{\times 5} 10x + 15y = 60$ ***[1 mark]***
$5x + 4y = 9 \xrightarrow{\times 2} 10x + 8y = 18$ ***[1 mark]***

$10x + 15y = 60$
$\underline{10x + 8y = 18 -}$
$7y = 42$
$y = 6$ ***[1 mark]***

$2x + 3y = 12$
$2x = 12 - (3 \times 6)$
$2x = -6$
$x = -3$ ***[1 mark]***

[4 marks available in total — as above]

3 Let f be the number of chocolate frogs and m be the number of sugar mice.
$4f + 3m = £3.69$ and $6f + 2m = £3.96$ ***[1 mark]***
$4f + 3m = £3.69 \xrightarrow{\times 2} 8f + 6m = £7.38$
$6f + 2m = £3.96 \xrightarrow{\times 3} 18f + 6m = £11.88$

$18f + 6m = £11.88$
$\underline{8f + 6m = £7.38 -}$
$10f = £4.50$
$f = £0.45$ ***[1 mark]***

$4f + 3m = £3.69$
$3m = £3.69 - (4 \times 0.45)$
$3m = £1.89$
$m = £0.63$ ***[1 mark]***

So a bag with 2 chocolate frogs and 5 sugar mice would cost $(2 \times 0.45) + (5 \times 0.63) = £4.05$ ***[1 mark]***
[4 marks available in total — as above]

4 $x^2 + y = 4$, so $y = 4 - x^2$
$4x - 1 = 4 - x^2$ ***[1 mark]***
$x^2 + 4x - 5 = 0$ ***[1 mark]***
$(x + 5)(x - 1) = 0$ ***[1 mark]***
$x = -5$ or $x = 1$ ***[1 mark]***
When $x = 1$, $y = (4 \times 1) - 1 = 3$
When $x = -5$, $y = (4 \times -5) - 1 = -21$
So the solutions are $x = 1, y = 3$ and $x = -5, y = -21$ ***[1 mark]***
[5 marks available in total — as above]

5 $y = x + 6$, so $2x^2 + (x + 6)^2 = 51$ ***[1 mark]***
$2x^2 + x^2 + 12x + 36 = 51$
$3x^2 + 12x - 15 = 0$ ***[1 mark]***
$(3x - 3)(x + 5) = 0$ ***[1 mark]***
$x = 1$ or $x = -5$ ***[1 mark]***
When $x = 1$, $y = 1 + 6 = 7$
When $x = -5$, $y = -5 + 6 = 1$
So the solutions are $x = 1, y = 7$ and $x = -5, y = 1$ ***[1 mark]***
[5 marks available in total — as above]

6 $y = x^2 + 3x - 1$ and $y = 2x + 5$ so $x^2 + 3x - 1 = 2x + 5$ ***[1 mark]***
$x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$ ***[1 mark]***
$x = -3$ or $x = 2$ ***[1 mark]***
When $x = -3$, $y = (2 \times -3) + 5 = -6 + 5 = -1$
When $x = 2$, $y = (2 \times 2) + 5 = 9$
So the lines intersect at $(-3, -1)$ and $(2, 9)$ ***[1 mark]***
Change in $x = 2 - (-3) = 5$
Change in $y = 9 - (-1) = 10$ ***[1 mark for both]***
$\sqrt{10^2 + 5^2} = \sqrt{125} = 5\sqrt{5}$, so $k = 5$ ***[1 mark]***
[6 marks available in total — as above]

Page 36: Proof

1 $(3n + 2)^2 - (n + 2)^2 = (3n + 2)(3n + 2) - (n + 2)(n + 2)$
$= (9n^2 + 12n + 4) - (n^2 + 4n + 4)$
$= 9n^2 + 12n + 4 - n^2 - 4n - 4$ ***[1 mark]***
$= 8n^2 + 8n$
$= 8n(n + 1)$ ***[1 mark]***
[2 marks available in total — as above]

2 E.g. When $a = 1$, $b = 2$, $c = 3$ and $d = 10$ then
$a < b < c < d$
$\frac{a}{b} = \frac{1}{2}$ and $\frac{c}{d} = \frac{3}{10}$ but $\frac{1}{2} > \frac{3}{10}$ so $\frac{a}{b} > \frac{c}{d}$
which contradicts what Jake says so Jake is not correct.
[3 marks available — 1 mark for finding values of a, b, c and d, 1 mark for showing that these values form a counterexample, 1 mark for stating that Jake is wrong]

3 n is an integer. $2n$ represents any even number, so the difference between the squares of two consecutive even numbers will be given by $(2n + 2)^2 - (2n)^2$.
$(2n + 2)^2 - (2n)^2 = (4n^2 + 8n + 4) - 4n^2$
$= 8n + 4 = 4(2n + 1)$
$= 4x$ (where x is an integer given by $x = 2n + 1$)
Any integer multiplied by 4 is a multiple of 4, so $4x$ must be a multiple of 4 and therefore the difference between the squares of two consecutive even numbers will always be a multiple of 4.
[3 marks available — 1 mark for finding an expression for the difference between the squares, 1 mark for rearranging into the form 4(2x + 1), 1 mark for conclusion]

4 If $2^{64} - 1$ is prime then it's only factors are 1 and itself
$2^{64} - 1 = (2^{32})^2 - 1^2 = (2^{32} + 1)(2^{32} - 1)$ ***[1 mark]***
So $(2^{32} + 1)$ and $(2^{32} - 1)$ are factors of $2^{64} - 1$ ***[1 mark]***
But neither $(2^{32} + 1)$ or $(2^{32} - 1)$ are equal to 1 or $2^{64} - 1$
so $2^{64} - 1$ cannot be prime. ***[1 mark]***
[3 marks available in total — as above]

Page 37: Functions

1 a) $f(7.5) = \frac{3}{2(7.5)+5} = \frac{3}{20} = 0.15$ ***[1 mark]***

b) Write out $x = f(y)$, $x = \frac{3}{2y+5}$ ***[1 mark]***
Rearrange to make y the subject:
$2y + 5 = \frac{3}{x}$
$2y = \frac{3}{x} - 5$ ***[1 mark]***
$y = \frac{3}{2x} - \frac{5}{2}$ so $f^{-1}(x) = \frac{3}{2x} - \frac{5}{2}$ ***[1 mark]***
[3 marks available in total — as above]

c) $ff^{-1}(x) = \frac{3}{2\left(\frac{3}{2x} - \frac{5}{2}\right) + 5}$ ***[1 mark]***
$= \frac{3}{\left(\frac{3}{x} - 5 + 5\right)} = \frac{3}{\left(\frac{3}{x}\right)}$ ***[1 mark]***
$= 3 \times \frac{x}{3} = x$ ***[1 mark]***
[3 marks available in total — as above]

2 a) $g(21) = \sqrt{2 \times 21 - 6} = \sqrt{36} = 6$ ***[1 mark]***

b) $gf(x) = g(f(x)) = \sqrt{2(2x^2 + 3) - 6}$ ***[1 mark]***
$= \sqrt{4x^2 + 6 - 6}$
$= \sqrt{4x^2}$
$= 2x$ ***[1 mark]***
[2 marks available in total — as above]

c) $fg(a) = f(g(a)) = 2(\sqrt{2a - 6})^2 + 3$ ***[1 mark]***
$= 2(2a - 6) + 3$
$= 4a - 9$ ***[1 mark]***
So when $fg(a) = 7$, $4a - 9 = 7$
$a = 4$ ***[1 mark]***
[3 marks available in total — as above]

Section Three — Graphs

Pages 38-39: Straight Line Graphs

1 a) Using $y = mx + c$, where m is the gradient, and c is the y-intercept:
$m = \frac{(7 - (-3))}{(5 - 0)}$
$m = 2$ ***[1 mark]***
When $x = 0$, $y = -3$, so $c = -3$ ***[1 mark]***
So, $y = 2x - 3$ ***[1 mark]***
[3 marks available in total — as above]

b) Using gradient from part a), $m = 2$
When $x = 2$, $y = 10$, so
$10 = (2 \times 2) + c$
i.e. $c = 6$ ***[1 mark]***
So, $y = 2x + 6$ ***[1 mark]***
[2 marks available in total — as above]

c) Difference in x-coordinate from A to B: $5 - 0 = 5$
Difference in y-coordinate from A to B: $7 - (-3) = 10$
So the x-coordinate of P: $0 + \frac{2}{5} \times 5 = 2$
and the y-coordinate of P: $-3 + \frac{2}{5} \times 10 = 1$
The coordinates of P are (2, 1).
[3 marks available — 1 mark for correct difference in x and y-coordinates, 1 mark for correctly multiplying by $\frac{2}{5}$, 1 mark for correct coordinates]

2 $3x + 4 = 2x + 6$ ***[1 mark]***
$x = 2$ ***[1 mark]***
so $y = 3(2) + 4 = 10$ and point M is (2, 10) ***[1 mark]***
Gradient of line **N** $= \frac{1}{2}$ (as it's parallel),
so $y = \frac{1}{2}x + c$ ***[1 mark]***
$10 = \frac{1}{2} \times 2 + c$, so $c = 10 - 1 = 9$
$y = \frac{1}{2}x + 9$ ***[1 mark]***
[5 marks available in total — as above]

3 a) $2a + 4 = 2c$, so $a + 2 = c$
Substitute values $a + 2 = c$ and $b - 6 = d$ into point (c, d):
$(c, d) = (a + 2, b - 6)$
Gradient of **S** $= \frac{b - 6 - b}{a + 2 - a} = \frac{-6}{2} = -3$
[3 marks available — 1 mark for correctly substituting values into a point, 1 mark for finding change in y over change in x, 1 mark for correct answer]

b) Gradient $= \frac{1}{3}$ ***[1 mark]***
So $y = \frac{1}{3}x + c$.
Substitute (6, 3) into the equation:
$3 = \frac{1}{3} \times 6 + c$
$c = 1$
Line **R**: $y = \frac{1}{3}x + 1$ ***[1 mark]***
[2 marks available in total — as above]

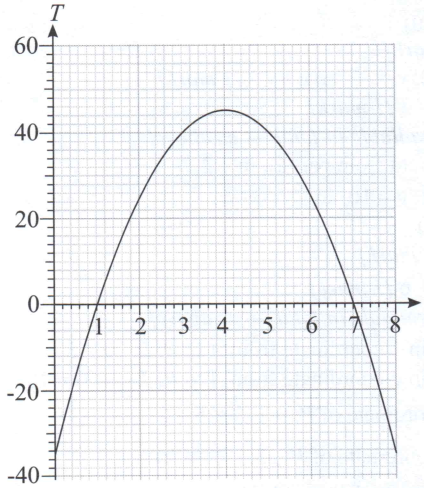
4 Midpoint of line AB: $\left(\frac{5+1}{2}, \frac{7-1}{2}\right) = (3, 3)$
Midpoint of line CD: $\left(\frac{13+3}{2}, \frac{4-2}{2}\right) = (8, 1)$
Gradient of line AB: $\frac{7-(-1)}{5-1} = \frac{8}{4} = 2$
Gradient of the line joining the midpoints of AB and CD:
Gradient $= \frac{1-3}{8-3} = \frac{-2}{5}$
$\frac{-1}{2} \neq \frac{-2}{5}$, therefore James is incorrect.
[4 marks available — 1 mark for saying James is wrong, 1 mark for finding both midpoints, 1 mark for finding both gradients, 1 mark for comparing gradients to show that the lines aren't perpendicular]

Pages 40-41: Quadratic Graphs

1 a) (1.5, –0.25) ***[1 mark]***
If your y-coordinate is between –0.23 and –0.28 you'll get the marks.

b) $a = 2$ ***[1 mark]***

2 a)

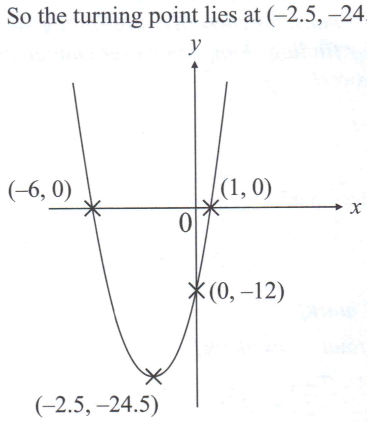


[3 marks available — 1 mark for plotting correct values, 1 mark for drawing a smooth curve through these points, 1 mark for correct intersections and turning point]

b) $t = 4$ ***[1 mark]***

c) $t = 1.8$, $t = 6.2$
[2 marks available — 1 mark for each solution (allow answers ±0.2)]

3 To find intersections with the x-axis, solve $2x^2 + 10x - 12 = 0$:
$x^2 + 5x - 6 = 0$
$(x + 6)(x - 1) = 0$ so $x = -6, x = 1$
So the x-intercepts are (–6, 0) and (1, 0)
To find where the graph crosses the y-axis, substitute $x = 0$ into the equation: $y = 0 + 0 - 12 = -12$
So the y-intercept is (0, –12)
Use symmetry and the x-intercepts to find the turning point of the curve: $x = \frac{1 + (-6)}{2} = -2.5$
$y = 2(-2.5)^2 + 10(-2.5) - 12 = -24.5$
So the turning point lies at (–2.5, –24.5)



[4 marks available in total — 1 mark for correct shaped curve, 1 mark for correctly labelled turning point, 1 mark for correctly labelled y-intercept and 1 mark for correctly labelled x-intercepts]
You can also find the turning point by completing the square.

4 Complete the square: $(x - 2)^2 = x^2 - 4x + 4$ ***[1 mark]***
$(x - 2)^2 + 2 = x^2 - 4x + 6$
So the completed square is $f(x) = (x - 2)^2 + 2$ ***[1 mark]***
So the turning point is (2, 2) ***[1 mark for each coordinate]***
[4 marks available in total — as above]

Pages 42-43: Harder Graphs

1 a) B ***[1 mark]***

b) C ***[1 mark]***

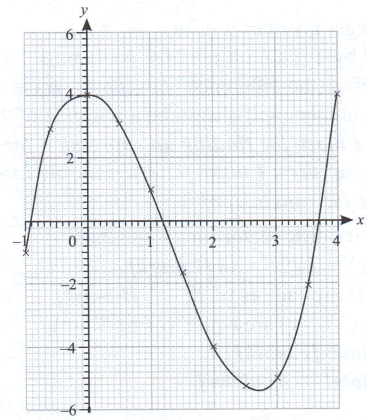
c) A ***[1 mark]***

2 a)

x	2.5	3	3.5	4
y	–5.375	–5	–2.125	4

[2 marks available — 2 marks for all answers correct, otherwise 1 mark for two correct answers]

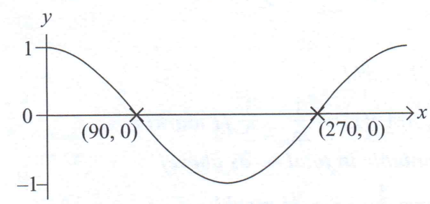
b)



[2 marks available — 1 mark for plotting correct points, 1 mark for joining them with a smooth curve]

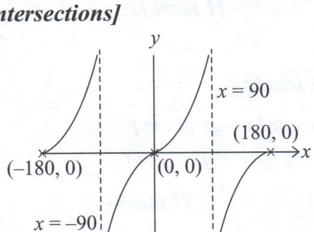
c) Reading off the graph, where the curve intersects the x-axis, $x = -0.9$, $x = 1.2$ and $x = 3.7$ ***[1 mark]***
You'll get the mark if your answers are within 0.1 of the answer.

3 a)



[2 marks available — 1 mark for curve fluctuating between 1 and –1 on the y-axis, 1 mark for correctly labelled intersections]

b)



[3 marks available — 1 mark for correct shaped curve, 1 mark for correctly labelled intersections, 1 mark for showing tan x is undefined at x = –90 and x = 90]

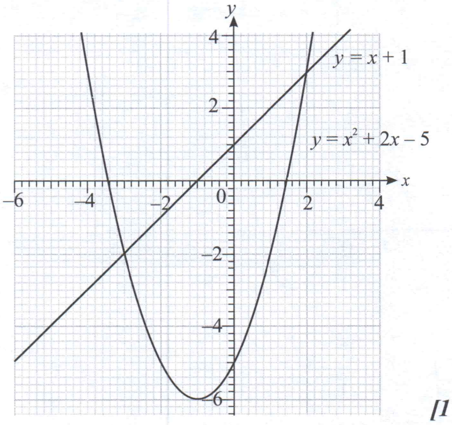
4 a) No, the curve will not pass through the origin.
It is a circle equation, centred at (0, 0) with radius 4.
[2 marks available — 1 mark for correct answer, 1 mark for suitable explanation or graph]

b) $x = -4$, $x = 4$ ***[1 mark]***

Pages 44: Solving Equations from Graphs

1 The graphs cross at (0.5, –4) and (4, 3), so the solutions are $x = 0.5$, $y = -4$ ***[1 mark]***
and $x = 4$, $y = 3$ ***[1 mark]***
[2 marks available in total — as above]

2 Find the equation of the line that should be drawn:
$x^2 + x = 6$
$x^2 + x - 5 = 1$
$x^2 + 2x - 5 = x + 1$
So draw the line $y = x + 1$ to find the solutions ***[1 mark]***

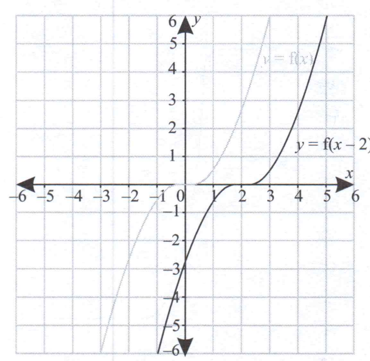


[1 mark]

The solutions to $x^2 + x = 6$ are:
$x = -3$ ***[1 mark]***
$x = 2$ ***[1 mark]***
[4 marks available in total — as above]

Pages 45-46: Graph Transformations

1 a)



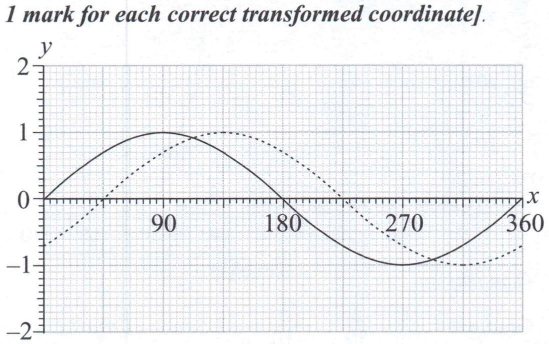
[2 marks available — 2 marks for the correct curve shifted 2 units to the right, otherwise 1 mark for an incorrect curve shifted 2 units to the right]

b) (2, 0) ***[1 mark]***

2 a) $y = f(x + 1) - 2$ is the graph $y = f(x)$ shifted left by 1 unit and down by 2 units. The transformed graph crosses the x-axis twice, so yes, $y = f(x + 1) - 2$ has real roots.
The roots are found at (–4, 0) and (–1, 0).
[2 marks available — 1 mark for each root]

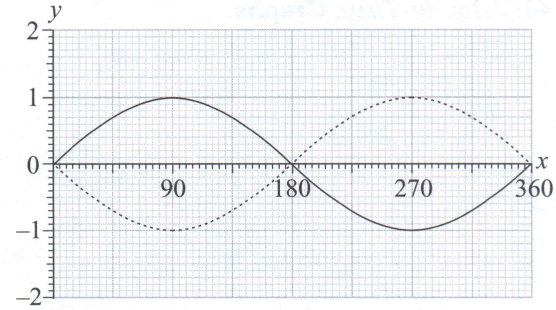
b) The minimum point of f(x) is approximately (–1.5, –0.25)
To find the minimum point of $y = f(x - 4) + 1$ do
(–1.5 + 4, –0.25 + 1)
So the minimum point of $y = f(x - 4) + 1$ is (2.5, 0.75)
[3 marks available — 1 mark for finding a minimum point, 1 mark for each correct transformed coordinate].

3 a)



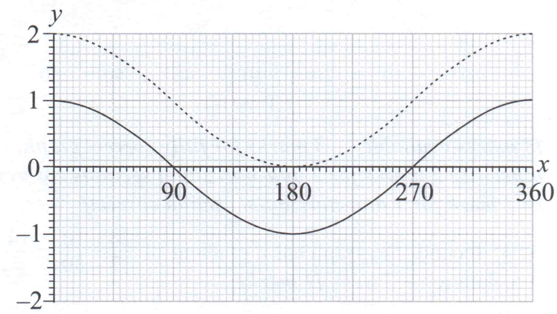
[1 mark]

b)



[1 mark]

4 a)



[2 marks available — 1 mark for correct graph shape, 1 mark for correct position]

b) $x = 90° - 30° = 60°$ and $x = 270° - 30° = 240°$
[2 marks available — 1 mark for each correct answer]
cos x crosses the x-axis at x = 90° and x = 270°.
The reflection doesn't change the x-intercepts, but the translation shifts the graph 30° left (or –30° right).

Page 47: Real-life Graphs

1 a) Plan A: £25 ***[1 mark]***
Plan B: £28 ***[1 mark]***
[2 marks available in total — as above]

b) Mr Barker should use Plan A because it is cheaper. Using 85 units with Plan A would cost £26.50. 85 units with Plan B would cost £34.
[2 marks available — 1 mark for correctly stating which plan, 1 mark for giving a reason]

2 Graph A and 2, Graph B and 3
Graph C and 4, Graph D and 1
[2 marks available — 2 marks for all four correct pairs, otherwise 1 mark for two correct pairs]

Page 48: Distance-Time Graphs

1 a) 1 hour ***[1 mark]***

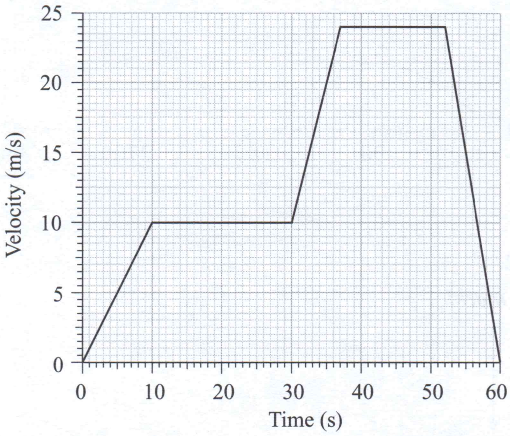
b) Tyrone. He reaches 30 km after 5 hours whereas Selby reaches 30 km after 6 hours. ***[1 mark]***

c) Gradient $= \frac{\text{change in } y}{\text{change in } x} = \frac{25 - 15}{3 - 1.5} = \frac{10}{1.5} = 6.67$ km/h (2 d.p)
[2 marks available — 2 marks for correct answer, otherwise 1 mark for choosing correct x and y values]

d) E.g. Selby is the most likely to have been injured.
The gradient of Selby's line decreases towards the end of the race, whereas Tyrone's gets much steeper. This means Selby was moving much more slowly than Tyrone towards the end of the race. ***[2 marks available — 1 mark for stating Selby is the injured runner, 1 mark for a correct explanation referring to gradients or steepness of lines]***

Page 49: Velocity-Time Graphs

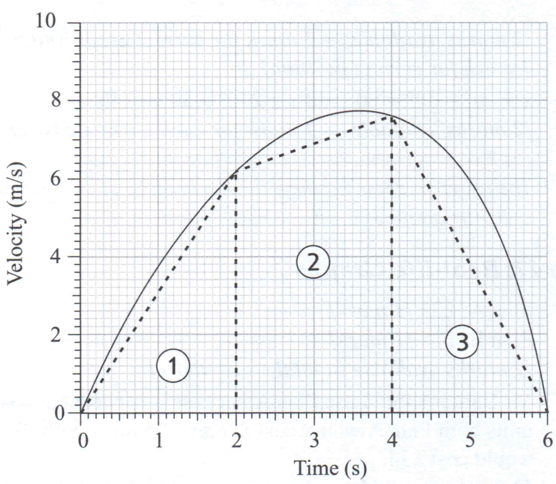
1 a)



[3 marks available — 3 marks for fully correct graph, otherwise 2 marks for 2 non-horizontal sections correct, or 1 mark for 1 non-horizontal section correct]

b) At 35 seconds the graph has a gradient of $\frac{24-10}{37-30} = \frac{14}{7} = 2$.
Acceleration = 2 m/s². ***[1 mark]***

2 E.g.



Area of triangle 1: $\frac{1}{2} \times 6.2 \times 2 = 6.2$ m

Area of trapezium 2: $\frac{1}{2} \times (6.2 + 7.6) \times 2 = 13.8$ m

Area of triangle 3: $\frac{1}{2} \times 7.6 \times 2 = 7.6$ m

Approximate distance travelled = 6.2 + 13.8 + 7.6 = 27.6 m

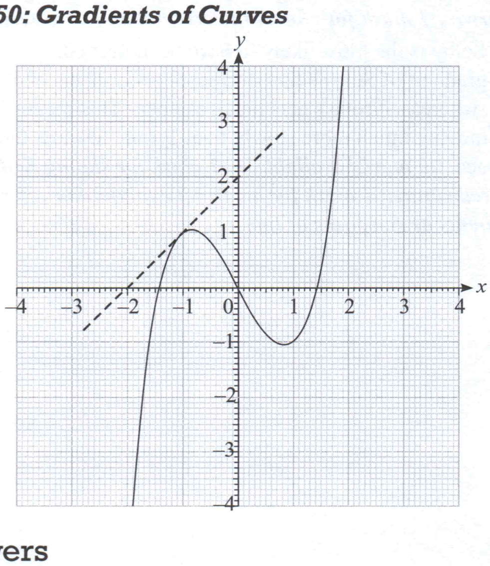
Estimate of average speed = 27.6 ÷ 6 = 5 m/s (1 s.f.)

[4 marks available — 1 mark for attempting to split up the area under the curve, 1 mark for correctly finding the area of 2 or more shapes, 1 mark for using total distance to find the average speed and 1 mark for the correct final answer]

If you've split the shape up differently you'll get a different answer. As long as you've done the correct working for your shapes you'll get the marks.

Page 50: Gradients of Curves

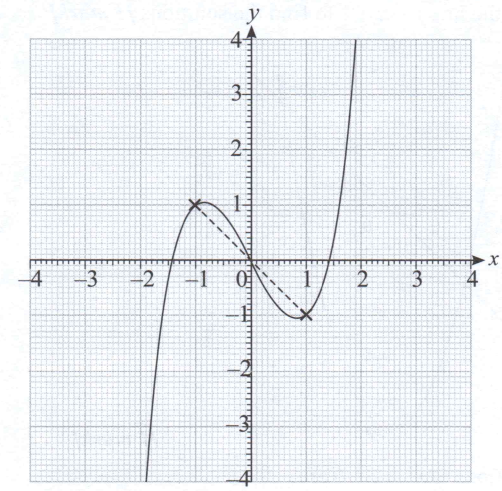
1 a)



Gradient = $\frac{2-0}{0-(-2)} = \frac{2}{2} = 1$

[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correctly drawing the tangent]

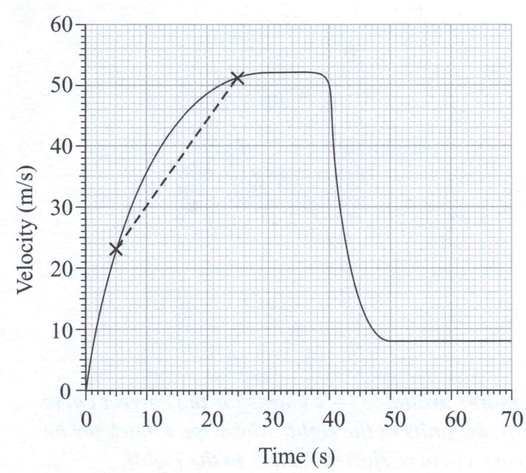
b)



Gradient = $\frac{1-(-1)}{-1-1} = \frac{2}{-2} = -1$

[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correctly drawing a line connecting (-1, 1) and (1, -1)]

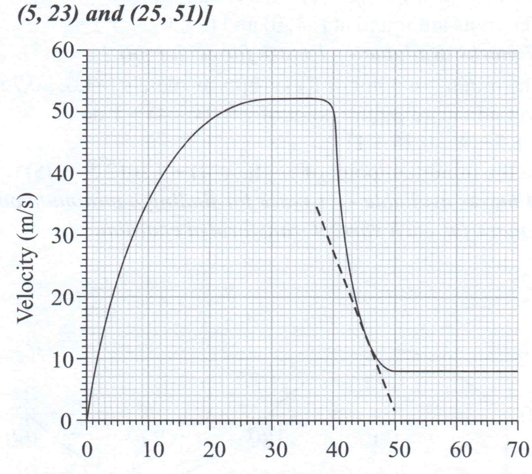
2 a)



Gradient = $\frac{51-23}{25-5} = \frac{28}{20} = 1.4$ m/s²

[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correctly drawing a line connecting (5, 23) and (25, 51)]

b)



[1 mark]

Gradient = $\frac{27-2}{40-50} = \frac{25}{-10} = -2.5$ m/s² ***[1 mark]***

[2 marks available — 2 marks for answer between –2 and –3, otherwise 1 mark for correctly drawing the tangent]

Section Four — Ratio, Proportion and Rates of Change

Pages 51-53: Ratios

1 Longest – shortest = 7 – 5 = 2 parts = 9 cm ***[1 mark]***
1 part = 9 ÷ 2 = 4.5 cm
Original piece of wood is 5 + 6 + 6 + 7 = 24 parts ***[1 mark]***
So, the original piece of wood is 24 × 4.5 = 108 cm ***[1 mark]***
[3 marks available in total — as above]

2 a) $3\frac{3}{4}:1\frac{1}{2} = 4 \times 3\frac{3}{4} : 4 \times 1\frac{1}{2} = 15:6$ ***[1 mark]***
$= 5:2$ ***[1 mark]***
[2 marks available in total — as above]

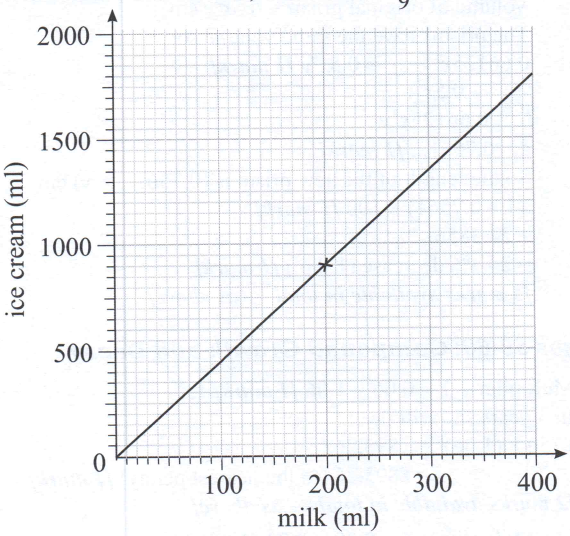
b) 5 + 2 = 7 parts
1 part: 2800 ml ÷ 7 = 400 ml
Yellow paint = 400 ml × 5 = 2000 ml
Blue paint = 400 ml × 2 = 800 ml
[2 marks available — 1 mark for finding the amount of 1 part, 1 mark for finding the correct amounts for both yellow and blue paint]
If your answer to part a) was incorrect, but your answers to part b) were correct for your incorrect ratio, you still get the marks for part b).

3 Edmund, Susan and Peter shared the money in the ratio
$4x + 10 : 2x + 5 : 5x + 3$
$(4x + 10) + (2x + 5) + (5x + 3) = 150$
$11x + 18 = 150$
$11x = 132$
$x = 12$
Edmund: (4 × 12) + 10 = £58
Susan: (2 × 12) + 5 = £29
Peter: (5 × 12) + 3 = £63
[4 marks available — 1 mark for forming equation from ratios, 1 mark for finding the value of x, 1 mark for the correct amount for one person, 1 mark for finding the correct amount for the other two people]

4 a) $\frac{2}{9}$ as much milk is used as ice cream ***[1 mark]***

b) 1 part: 801 ÷ 9 = 89 ml ***[1 mark]***
9 + 2 = 11 parts
89 × 11 = 979 ml ***[1 mark]***
[2 marks available in total — as above]

c) E.g.
Find a point on the graph:
When ice cream = 900 ml, milk = 900 × $\frac{2}{9}$ = 200 ml



[2 marks available — 1 mark for plotting any point on the line correctly, 1 mark for a straight line that passes through that point and (0, 0)]

5 16 kg of Mr Appleseed's Supercompost contains:
$\frac{4}{8} \times 16 = 8$ kg of soil.
$\frac{3}{8} \times 16 = 6$ kg of compost.
$\frac{1}{8} \times 16 = 2$ kg of grit.
Soil costs £8 ÷ 40 = £0.20 per kg.
Compost costs £15 ÷ 25 = £0.60 per kg.
Grit costs £12 ÷ 15 = £0.80 per kg.
16 litres of Mr Appleseed's Supercompost costs:
(8 × 0.2) + (6 × 0.6) + (2 × 0.8) = £6.80
Profit: £10 – £6.80 = £3.20
[5 marks available — 1 mark for the correct mass of one ingredient, 1 mark for the correct masses for the other two ingredients, 1 mark for working out the price per kg for each ingredient, 1 mark for the total cost of 16 kg of Supercompost, 1 mark for the correct answer]

6 $x - 5 : y - 3 = 5:8$ and $x + 5 : y + 7 = 5:7$ ***[1 mark]***
$\frac{x-5}{y-3} = \frac{5}{8}$ and $\frac{x+5}{y+7} = \frac{5}{7}$ ***[1 mark]***
$8(x - 5) = 5(y - 3)$ and $7(x + 5) = 5(y + 7)$
Expand and simplify to give:
$8x - 5y = 25$ [1] and $7x - 5y = 0$ [2] ***[1 mark]***
[1] – [2]: $x = 25$ ***[1 mark]***
Sub into [1]: $(8 \times 25) - 5y = 25$
$5y = 175$, so $y = 35$ ***[1 mark]***
Solution: $x = 25$, $y = 35$
[5 marks available in total — as above]

7 Call the number of black olives b and the number of green olives g.
$b:g = 5:11$ and $b - 3 : g - 1 = 3:7$ ***[1 mark for both]***
$\frac{b}{g} = \frac{5}{11}$ and $\frac{b-3}{g-1} = \frac{3}{7}$ ***[1 mark for both]***
$11b = 5g$ and $7(b - 3) = 3(g - 1)$
$11b - 5g = 0$ [1] and $7b - 3g = 18$ [2] ***[1 mark]***
[1] × 3: $33b - 15g = 0$ [3]
[2] × 5: $35b - 15g = 90$ [4] ***[1 mark for both]***
[4] – [3]: $2b = 90$
$b = 45$ ***[1 mark]***
Sub into [1]: $(11 \times 45) - 5g = 0$
$5g = 495$, so $g = 99$ ***[1 mark]***
Solution: $b = 45$, $g = 99$
[6 marks available in total — as above]
You might have started with $\frac{b}{b+g} = \frac{5}{16}$ *and* $\frac{b-3}{b+g-4} = \frac{3}{10}$ *and used these to form simultaneous equations instead.*

Pages 54-55: Direct and Inverse Proportion

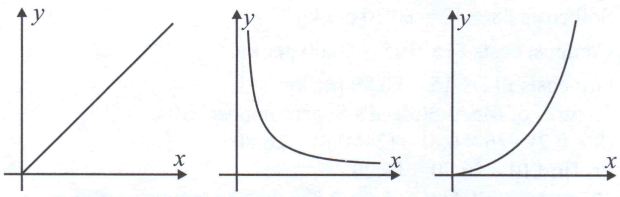
1 1 t-shirt will take: 5 m^2 ÷ 8 = 0.625 m^2 of cotton ***[1 mark]***
85 t-shirts will take: 0.625 m^2 × 85 = 53.125 m^2 of cotton ***[1 mark]***
1 m^2 of cotton costs: £5.50 ÷ 2 = £2.75 ***[1 mark]***
53.125 m^2 of cotton costs £2.75 × 53.125 = £146.09375
= £146.09 ***[1 mark]***
[4 marks available in total — as above]

2 To harvest the same amount as Neil, Sophie will take:
3.5 hours ÷ 2 = 1.75 hours ***[1 mark]***
Sophie needs to harvest three times as much so it will take her:
1.75 × 3 ***[1 mark]*** = 5.25 hours ***[1 mark]***
[3 marks available in total — as above]

3 a) 1 litre of petrol will keep 8 go-karts going for:
20 ÷ 12 = 1.666... minutes ***[1 mark]***
18 litres of petrol will keep 8 go-karts going for:
1.666... × 18 = 30 minutes ***[1 mark]***
18 litres of petrol will keep 1 go-kart going for:
30 × 8 = 240 minutes ***[1 mark]***
18 litres of petrol will keep 6 go-karts going for:
240 ÷ 6 = 40 minutes ***[1 mark]***
[4 marks available in total — as above]
You might have done these steps in a slightly different order — you'd still get all the marks as long as you got the same answer.

b) In 1 minute, 8 go-karts will use $12 \div 20 = 0.6$ litres ***[1 mark]***
In 45 minutes, 8 go-karts will use
$0.6 \times 45 = 27$ litres ***[1 mark]***
27 litres of petrol cost: £1.37 × 27 = £36.99 ***[1 mark]***
[3 marks available in total — as above]

4



[3 marks available — 1 mark for each correct graph]

5 $f \propto \frac{1}{d^2}$, so $f = \frac{k}{d^2}$ ***[1 mark]***
When $d = 100$ and $f = 20$, $20 = \frac{k}{100^2}$,
so $k = 20 \times 100^2 = 200\,000$ ***[1 mark]***
$f = \frac{200\,000}{d^2}$
When $d = 800$, $f = \frac{200\,000}{800^2} = 0.3125$ ***[1 mark]***
[3 marks available in total — as above]

6 a) $h \propto S^2$, so $h = kS^2$ ***[1 mark]***
When $h = 35$ and $S = 50$, $35 = k \times 50^2$,
so $k = 35 \div 50^2 = 0.014$ ***[1 mark]***
So $h = 0.014S^2$
$h = 0.014S^2$ so when $S = 40$, $h = 0.014 \times 40^2 = 22.4$ ***[1 mark]***
[3 marks available in total — as above]

b) If h_0 is the initial height difference, S_0 is the initial speed and h_1 is the changed height difference, then:
$h_0 = k(S_0)^2$
$h_1 = k(1.3S_0)^2$ ***[1 mark]***
$= 1.69k(S_0)^2$ ***[1 mark]*** $= 1.69h_0$ ***[1 mark]***
So the height needs to be increased by 69%. ***[1 mark]***
[4 marks available in total — as above]

Pages 56-58: Percentages

1 $20\% = 20 \div 100 = 0.2$
$0.2 \times$ £927 = £185.40 ***[1 mark]***
£927 + £185.40 ***[1 mark]***
= £1112.40 ***[1 mark]***
[3 marks available in total — as above]

2 For every 2 grapes there are 5 cherries so there are $\frac{2}{5} = 40\%$ as many grapes as cherries. ***[1 mark]***

3 £15 714 = 108%
£15 714 ÷ 108 = £145.50 = 1% ***[1 mark]***
£145.50 × 100 = 100% ***[1 mark]***
= £14 550 ***[1 mark]***
[3 marks available in total — as above]

4 £41 865 – £10 000 = £31 865
$20\% = 20 \div 100 = 0.2$
20% of £31 865 = 0.2 × £31 865 = £6373 ***[1 mark]***
£45 000 – £41 865 = £3135
$40\% = 40 \div 100 = 0.4$
40% of £3135 = 0.4 × £3135 = £1254 ***[1 mark]***
£6373 + £1254 = £7627
(£7627 ÷ £45 000) × 100 ***[1 mark]***
= 16.948... = 16.9% (1 d.p) ***[1 mark]***
[4 marks available in total — as above]

5 a) 40% of 50% = 0.4 × 50% ***[1 mark]***
= 20% ***[1 mark]***
[2 marks available in total — as above]

b) 20% of the animals are black cats so:
100% – 20% = 80% are not black cats ***[1 mark]***
80% of 90 = 90 × 0.8 = 72 ***[1 mark]***
[2 marks available in total — as above]

6 £32 is a 60% profit so £32 = 160% of cost price ***[1 mark]***
1% of cost price = £32 ÷ 160 = £0.20 ***[1 mark]***
He wants an 88% profit = 188% of cost price
188% = £0.20 × 188 = £37.60 ***[1 mark]***
[3 marks available in total — as above]

7 a) Original price = £180, change in price = £5.40
Percentage increase = $\frac{£5.40}{£180} \times 100 = 3\%$
[2 marks available — 1 mark for using the correct formula, 1 mark for the correct answer]

b) Buying tickets in 1 transaction:
(3 × £180) + £5.40 = £545.40
Buying tickets in 3 transactions:
(£180 + £5.40) × 3 = £556.20
Decrease in cost = £556.20 – £545.40 = £10.80 ***[1 mark]***
Percentage decrease = $\frac{£10.80}{£556.20} \times 100$ ***[1 mark]***
= 1.9417... % = 1.94% (2 d.p.) ***[1 mark]***
[3 marks available in total — as above]
Careful here — the percentage saving is actually a percentage change, where the change is the saving and the original amount is the cost of the three separate transactions.

8 A ratio of 3 : 7 means that 3 out of 10 = 30% of the customers were children.
60% of 30% = 0.6 × 30% = 18% were blond-haired children.
100% – 30% = 70% were adults.
20% of 70% = 0.2 × 70% = 14% were blond-haired adults.
So, 18% + 14% = 32% of the customers had blond hair.
[4 marks available — 1 mark for finding blond-haired children %, 1 mark for finding blond-haired adults %, 1 mark for adding together the two percentages, 1 mark for correct answer]

9 a) Original area of triangular face = $\frac{1}{2} \times x \times x = 0.5x^2$ cm²
Area after increase = $\frac{1}{2} \times 1.15x \times 1.15x$
$= 0.66125x^2$ cm² ***[1 mark]***
Change in area = $0.66125x^2 - 0.5x^2 = 0.16125x^2$ cm² ***[1 mark]***
Percentage increase = $\frac{0.16125x^2}{0.5x^2} \times 100$ ***[1 mark]***
= 32.25% ***[1 mark]***
[4 marks available in total — as above]
You could have found the new area by doing 0.5 × 1.15x × 1.15x = 1.15² × 0.5x² = 1.3225 × original area, which means the percentage change is 32.25%.

b) Let ay cm be the length of the new prism.
Volume of new prism = $0.66125x^2 \times ay$ ***[1 mark]***
Volume of original prism = $0.5x^2y$ cm³
The prisms have the same volume so:
$0.66125x^2 \times ay = 0.5x^2y$ ***[1 mark]***
$a = \frac{0.5x^2y}{0.66125x^2y}$
$a = 0.7561...$ ***[1 mark]***
So the length of the new prism is $(0.7561... \times y)$ cm
(1 – 0.7561) × 100 ***[1 mark]***
= 24.3856...%
= 24.4% decrease (1 d.p.) ***[1 mark]***
[5 marks available in total — as above]

Pages 59-60: Compound Growth and Decay

1 Multiplier = 1 + 0.06 = 1.06 ***[1 mark]***
In 3 years she will owe:
£750 × $(1.06)^3$ = £893.262
= £893.26 (to the nearest penny) ***[1 mark]***
[2 marks available in total — as above]

2 a) Multiplier = 1 – 0.08 = 0.92 ***[1 mark]***
Population after 15 years = $2000 \times (0.92)^{15}$
= 572.59...
≈ 573 fish ***[1 mark]***
[2 marks available in total — as above]
It'll take more than 15 years for the population to get down to 572 fish, so you need to round up (the population hasn't dropped to 572 yet so there are still 573).

b) Three quarters of initial population = $2000 \times \frac{3}{4} = 1500$
$2000 \times 0.92 = 1840$
$2000 \times 0.92^2 = 1692.8$
$2000 \times 0.92^3 = 1557.376$
$2000 \times 0.92^4 = 1432.78592 < 1500$
Population is less than $\frac{3}{4}$ of the initial population after 4 years.
[2 marks available — 1 mark for calculating 2000 × 0.92ⁿ for n > 1, 1 mark for correct answer]

3 $5000 \times 0.16 = 800$ trees are planted in 2013 ***[1 mark]***
A maximum of $800 \times 0.75 = 600$ trees are cut down
At the end of 2013 there is a minimum of
$5000 + (800 - 600) = 5200$ pine trees ***[1 mark]***
$5200 \times 0.16 = 832$ trees are planted in 2014 ***[1 mark]***
A maximum of $832 \times 0.75 = 624$ trees are cut down
At the end of 2014 there is a minimum of
$5200 + (832 - 624) = 5408$ pine trees ***[1 mark]***
[4 marks available in total — as above]

4 a) Compound Collectors Account:
Multiplier = $1 + 0.055 = 1.055$ ***[1 mark]***
£10 000 × $(1.055)^5$ = £13 069.60 (2 d.p.) ***[1 mark]***
Simple Savers Account:
6.2% of £10 000 = 0.062 × £10 000 = £620 ***[1 mark]***
5 × £620 = £3100
£10 000 + £3100 = £13 100 so the Simple Savers Account will have the biggest balance after 5 years. ***[1 mark]***
[4 marks available in total — as above]

b) E.g. He might want to deposit more money during the 5 years and he can't with the Simple Savers Account. ***[1 mark]***

5 £2704 = £2500 × $(\text{Multiplier})^2$ ***[1 mark]***
$\frac{£2704}{£2500} = (\text{Multiplier})^2$
Multiplier = $\sqrt{\frac{£2704}{£2500}} = 1.04$ ***[1 mark]***
Interest rate = $1.04 - 1 = 0.04 = 4\%$ ***[1 mark]***
[3 marks available in total — as above]

6 Multiplier = $1 - 0.25 = 0.75$ ***[1 mark]***
$N_0 \times (0.75)^{35-31} = 2\,000\,000$ ***[1 mark]***
$N_0 = 2\,000\,000 \div 0.75^4 = 6\,320\,987.654...$
= £6 300 000 (to the nearest £100 000) ***[1 mark]***
[3 marks available in total — as above]

Page 61: Speed

1 1 hour 15 minutes = 1.25 hours ***[1 mark]***
Distance = speed × time, so distance = 56 × 1.25 = 70 km ***[1 mark]***
[2 marks available in total — as above]

2 a) E.g. 2500 m = 2.5 km. 2.5 km = 2.5 ÷ 1.6 = 1.5625 miles.
102 s ÷ 60 = 1.7 minutes ÷ 60 = 0.02833... hours.
Speed = 1.5625 miles ÷ 0.02833... hours
= 55 mph (to nearest mph)
[3 marks available — 1 mark for converting 2500 metres to miles, 1 mark for converting 102 seconds into hours, 1 mark for the correct final answer]
It doesn't matter whether you do the conversion to miles per hour at the start or the end of the calculation — you could find the speed in m/s, km/s or km/h, and then change it to mph. Whichever way, you should get the same answer.

b) E.g. time = 1.5625 miles ÷ 50 mph = 0.03125 hours
0.03125 hours × 60 × 60 = 113 s (to nearest second)
[2 marks available — 1 mark for dividing the distance by the speed limit, 1 mark for the correct answer]

3 In 2014 he finished with a time of $t - 0.1t = 0.9t$ ***[1 mark]***
$s_1 = \frac{d}{t}$ and $s_2 = \frac{d}{0.9t}$ ***[1 mark]***
So, $s_1 t = 0.9 s_2 t$
$s_2 = \frac{s_1}{0.9} = 1.111... \times s_1$ ***[1 mark]***
So his percentage increase was 11.11% (2 d.p.) ***[1 mark]***
[4 marks available in total — as above]
There are other methods to get to the correct answer — as long as you show full working and get the answer right then you'll get full marks.

Page 62: Density

1 a) Volume = 360 ÷ 1800 ***[1 mark]***
= 0.2 m^3 ***[1 mark]***
[2 marks available in total — as above]

b) Density = 220 ÷ 0.2 ***[1 mark]***
= 1100 kg/m^3 ***[1 mark]***
[2 marks available in total — as above]

2 a) Volume = 4 cm × 4 cm × 4 cm = 64 cm^3 ***[1 mark]***
Mass = 7.9 × 64 ***[1 mark]***
= 505.6 g ***[1 mark]***
[3 marks available in total — as above]

b) 63.2 kg = 63 200 g ***[1 mark]***
Volume of large cube: 63 200 ÷ 7.9 = 8000 cm^3 ***[1 mark]***
Side length of large cube: $\sqrt[3]{8000} = 20$ cm ***[1 mark]***
Ratio of side lengths of the smaller and larger cubes:
4 cm : 20 cm = 1 : 5 ***[1 mark]***
[4 marks available in total — as above]

3 10 cm^3 of brass contains 7 cm^3 of copper and 3 cm^3 of zinc.
7 cm^3 of copper has a mass of 7 × 8.9 = 62.3 g
3 cm^3 of zinc has a mass of 3 × 7.1 = 21.3 g
10 cm^3 of brass has a mass of 62.3 + 21.3 = 83.6 g
Density of brass = 83.6 ÷ 10 = 8.36 g/cm^3
[4 marks available — 1 mark for finding the mass of a stated volume of copper or zinc, 1 mark for finding the total mass of a stated volume of brass, 1 mark for attempting to find density using total mass ÷ total volume and 1 mark for correct final answer]

Page 63: Pressure

1 a) Area of A = 40 cm × 20 cm = 800 cm^2
= 800 ÷ 100 ÷ 100
= 0.08 m^2 ***[1 mark]***
Pressure = 40 N ÷ 0.08 m^2 ***[1 mark]***
= 500 N/m^2 ***[1 mark]***
[3 marks available in total — as above]

b) Three cuboids would have a weight of
3 × 40N = 120 N ***[1 mark]***
Area of B = 3 m × 0.4 m = 1.2 m^2 ***[1 mark]***
Pressure = 120 N ÷ 1.2 m^2 = 100 N/m^2 ***[1 mark]***
[3 marks available in total — as above]

2 a) Area of circular base = $\pi \times (10x)^2 = 100\pi x^2$ cm^2 ***[1 mark]***
$100\pi x^2$ cm^2 = $(100\pi x^2 \div 100 \div 100)$ m^2 = $0.01\pi x^2$ m^2 ***[1 mark]***
Weight = $650 \times 0.01\pi x^2$ ***[1 mark]*** = $6.5\pi x^2$ N ***[1 mark]***
[4 marks available in total — as above]

b) E.g. If the diameter is halved, the area of the circular base becomes: $\pi \times (5x)^2 = 25\pi x^2$ cm^2 = $0.0025\pi x^2$ m^2
Pressure = $6.5\pi x^2 \div 0.0025\pi x^2$ = 2600 N/m^2
2600 N/m^2 ÷ 650 N/m^2 = 4
If the diameter of the circle is halved the pressure increases and is 4 times greater.
[2 marks available — 1 mark for saying that the pressure increases, 1 mark for saying it's 4 times greater]

Section Five — Geometry and Measures

Pages 64-65: Geometry

1 Let a be the third angle in the triangle.
$a = 180° - y - z$ (angles in a triangle add to 180°) ***[1 mark]***
$x = 180° - a$ (angles on a straight line add to 180°) ***[1 mark]***
So $x = 180° - (180° - y - z) = y + z$ ***[1 mark]***
[3 marks available in total — as above]

2 a) Angles on a straight line add up to 180°,
so angle FEC = 180° − 14° = 166° ***[1 mark]***
Angles in a quadrilateral add up to 360°,
so x = 360° − 90 − 62° − 166° = 42° ***[1 mark]***
[2 marks available in total — as above]

b) Angles in a triangle add up to 180° ***[1 mark]***
so y = 180° − 90° − 42° = 48° ***[1 mark]***
[2 marks available in total — as above]

3 Let z be the top-left angle in the parallelogram.
x and z are allied angles, so $z = 180° - x$ ***[1 mark]***
and y and z are allied angles, so $y = 180° - z$ ***[1 mark]***.
So $y = 180° - (180° - x) = x$ ***[1 mark]***
[3 marks available in total — as above]

4 Angles on a straight line add up to 180°,
so angle $ABJ = 180° - 140° = 40°$ ***[1 mark]***
Allied angles add up to 180°,
so angle $JAB = 180° - 150° = 30°$ ***[1 mark]***
Angles in a triangle add up to 180°,
so angle $AJB = 180° - 40° - 30° = 110°$ ***[1 mark]***
Angles on a straight line add up to 180°,
so angle $x = 180° - 110° = 70°$ ***[1 mark]***
[4 marks available in total — as above]

5 Angle BDC = angle $BCD = x$ (triangle BCD is isosceles)
Angle $CBD = 180° - x - x = 180° - 2x$ ***[1 mark]***
(angles in a triangle add to 180°)
Angle BDE = angle $CBD = 180° - 2x$ ***[1 mark]*** (alternate angles)
Angle AED = angle $BDE = 180° - 2x$ ***[1 mark]*** ($ABDE$ is an isosceles trapezium so has a vertical line of symmetry)
$y = 360°$ – angle AED ***[1 mark]*** (angles round a point add to 360°)
So $y = 360° - (180° - 2x) = 180° + 2x$ ***[1 mark]***
[5 marks available in total — as above]
There's more than one way to do the questions above — as long as you show your working and explain each step you'll get the marks.

6 $5x + (4x - 9°) = 180°$ ***[1 mark]***
Rearranging this: $9x = 189°$
Therefore $x = 21°$ ***[1 mark]***
$(4y - 12°) + 2y = 180°$ ***[1 mark]***
Rearranging this: $6y = 192°$
Therefore $y = 32°$ ***[1 mark]***
[4 marks available in total — as above]

Pages 66-67: Polygons

1 Rhombuses have two pairs of equal angles, so one of the other angles must be 62°. ***[1 mark]***
The sum of the angles in a quadrilateral is 360°, so the other angles both equal $(360° - 62° - 62°) \div 2 = 118°$. ***[1 mark]***
[2 marks available in total — as above]

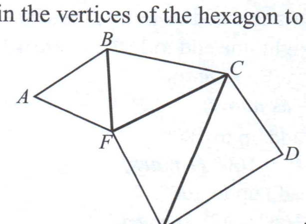
2 Triangle ADX is isosceles, so angle DAX = angle $DXA = 41°$ and angle ADC = angle $ADX = 180° - 41° - 41° = 98°$ (angles in a triangle sum to 180°). ***[1 mark]***
Shape $ABCD$ is a kite, so angle ABC = angle $ADC = 98°$ ***[1 mark]***
Sum of angles in a quadrilateral is 360° so
angle $DAB = 360° - 98° - 53° - 98° = 111°$ ***[1 mark]***
[3 marks available in total — as above]

3 Exterior angle = $180° - 150° = 30°$ ***[1 mark]***
Number of sides = $360° \div 30°$ ***[1 mark]***
= 12 ***[1 mark]***
[3 marks available in total — as above]

4 a) x is the same as an exterior angle, so $x = 360° \div 8$ ***[1 mark]***
$x = 45°$ ***[1 mark]***
[2 marks available in total — as above]

b) $y = (180° - 45°) \div 2$ ***[1 mark]***
$y = 67.5°$ ***[1 mark]***
[2 marks available in total — as above]

5 Join the vertices of the hexagon to split it into triangles:



[1 mark]
The angles in each triangle add up to 180°,
so the sum of the interior angles of the hexagon is:
$4 \times 180°$ ***[1 mark]*** = 720° ***[1 mark]***
[3 marks available in total — as above]

6 Interior angle of regular n-sided polygon = 180° – exterior angle
$= 180° - (360° \div n)$
Interior angle of regular octagon = $180° - (360° \div 8) = 135°$
Interior angle of regular hexagon = $180° - (360° \div 6) = 120°$
Angle CBK = angle ABC – angle $IJK = 135° - 120° = 15°$
[2 marks available in total — 1 mark for using correct method to find interior angle of octagon or hexagon, 1 mark for correct answer]

Pages 68-70: Circle Geometry

1 Angle $DBC = 62°$ ***[1 mark]*** (angles in the same segment are equal)
Angle $ABC = 90°$ ***[1 mark]*** (the angle in a semi-circle is 90°)
Angle $x = 90° - 62° = 28°$ ***[1 mark]***
[3 marks available in total — as above]

2 Angle $DCO = 90°$ ***[1 mark]*** (tangent meets a radius at 90°)
Angle $DOC = 180° - 90° - 24° = 66°$ ***[1 mark]*** (angles in a triangle)
Angle $AOC = 66° \times 2 = 132°$ ***[1 mark]*** (tangents from the same point are the same length, so create two identical triangles)
Angle $ABC = 66°$ ***[1 mark]*** (angle at the centre is twice the angle at the circumference)
Angle $CBE = 180° - 66° = 114°$ ***[1 mark]*** (angles on a straight line)
[5 marks available in total — as above]

3 Angle FBD = angle $BCD = 102°$
(alternate segment theorem) ***[1 mark]***
Angle $CDB = 180° - 147° = 33°$
(angles on a straight line) ***[1 mark]***
Angle $CBD = 180° - 102° - 33° = 45°$
(angles in a triangle) ***[1 mark]***
Angle CAD = angle $CBD = 45°$
(angles in the same segment are equal) ***[1 mark]***
[4 marks available in total — as above]

4 Angles ODE and OBE are both 90° because a tangent always meets a radius at 90°. ***[1 mark]***
Angle $DOB = 100°$ because angles in a quadrilateral add up to 360°. ***[1 mark]***
Angle $DCB = 50°$ because an angle at the centre is twice the angle at the circumference. ***[1 mark]***
Angle $DAB = 130°$ because opposite angles of a cyclic quadrilateral sum to 180°. ***[1 mark]***
[4 marks available in total — as above]

5 Angle $DEG = 53°$ and angle $AEF = 37°$
(alternate segment theorem) ***[1 mark]***
Angle $AED = 180° - 53° - 37° = 90°$
(angles on a straight line)***[1 mark]***
The chord AD must be a diameter of the circle (angle in a semi-circle is 90°), so AD must pass through the centre of the circle. ***[1 mark]***
[3 marks available in total — as above]

6 a) Angle DAB = Angle $BDE = 53°$
(alternate segment theorem) ***[1 mark]***
Angle $DOB = 2 \times$ Angle $DAB = 106°$ (angle at the centre is twice the angle at the circumference) ***[1 mark]***
[2 marks available in total — as above]
You could have done this one by splitting the triangle DOB into two identical right-angles triangles and working out the angles.

b) OB and OD are both radii, so OBD is an isosceles triangle. The radius OC crosses chord BD at right-angles, so it bisects BD ***[1 mark]*** and divides the isosceles triangle OBD in half, which means angle $COB = 0.5 \times$ angle DOB ***[1 mark]***.
[2 marks available in total — as above]

7 Opposite angles in a cyclic quadrilateral add up to 180°, so angle $ADC = 180°$ – angle $ABC = 180° - 119° = 61°$
Angle CDX = angle ADC – angle $ADX = 61° - 31° = 30°$
If X was the centre of the circle, XD and XC would be radii, so triangle CXD would be isosceles and angles CDX and XCD would be equal. Here angle $CDX = 30°$ and angle $XCD = 28°$ so the angles are not equal, and therefore X is not the centre of the circle.
[3 marks available in total — 1 mark for finding angle ADC, 1 mark for finding angle CDX, 1 mark for using "two radii from an isosceles triangle" to explain why X cannot be the centre.]

Page 71: Congruent Shapes

1 F is the midpoint of AC so $AF = FC$ and opposite sides of a parallelogram are equal so $DE = FC$.
Therefore $AF = DE$. ***[1 mark]***
E is the midpoint of CB so $CE = EB$, and opposite sides of a parallelogram are equal so $CE = FD$. Therefore $FD = EB$. ***[1 mark]***
D is the midpoint of AB, so $AD = DB$. ***[1 mark]***
Satisfies condition SSS so triangles are congruent. ***[1 mark]***
[4 marks available in total — as above]

2 E.g. $KP = OL$ (they are diameters of identical circles) ***[1 mark]***
Angle KMP = angle ONL
(angles in a semicircle = 90°) ***[1 mark]***
Angle MKO = angle NLP
(alternate angles in parallel lines are equal) ***[1 mark]***
Satisfies condition AAS so triangles are congruent. ***[1 mark]***
[4 marks available in total — as above]
There are other ways you could have done this one — for example, you could show that the condition RHS holds.

3 Angle CAE = angle EBD
(angles in the same segment are equal) ***[1 mark]***
Angle ACE = angle EDB
(angles in the same segment are equal) ***[1 mark]***
$AC = DB$
Satisfies condition AAS so triangles are congruent. ***[1 mark]***
[3 marks available in total — as above]

Page 72: Similar Shapes

1 a) Scale factor from $EFGH$ to $ABCD = 9 \div 6 = 1.5$ ***[1 mark]***
$EF = 6 \div 1.5 = 4$ cm ***[1 mark]***
[2 marks available in total — as above]

b) $BC = 4 \times 1.5 = 6$ cm ***[1 mark]***

2 63 m = 6300 cm
Scale factor = 6300 ÷ 60 ***[1 mark]***
= 105 ***[1 mark]***
Height of flagpole = 8 cm × 105 = 840 cm = 8.4 m ***[1 mark]***
[3 marks available in total — as above]
The triangles created between James' eyes and his finger and his eyes and the flagpole are similar.

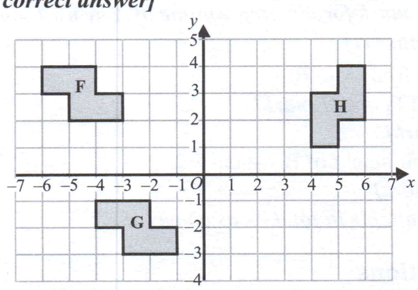
3 Lines AD and EF are parallel (they're both parallel to BCG)
Angles in a rectangle are 90° so angle ABC = angle CEF
Corresponding angles are equal so angle BAC = angle ECF
Corresponding angles are equal so angle ACB = angle CFE
Triangles ABC and CEF have all three angles the same so are similar.
[3 marks available — 1 mark for showing one angle is the same, 1 mark for showing that the rest are the same (the third angle can be implied from two angles the same), 1 mark for stating that the triangles are similar because their angles are the same]

Pages 73-74: The Four Transformations

1 a) $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

[2 marks available — 1 mark for $\begin{pmatrix} \pm 2 \\ \pm 5 \end{pmatrix}$, 1 mark for fully correct answer]

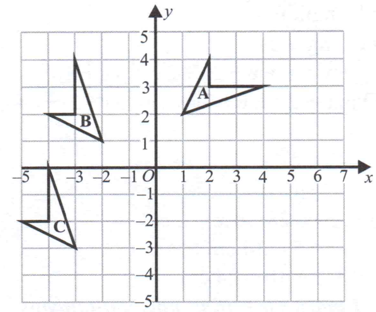
b)



[2 marks available — 1 mark for a rotation of 90° clockwise around any point, 1 mark for correct centre of rotation]

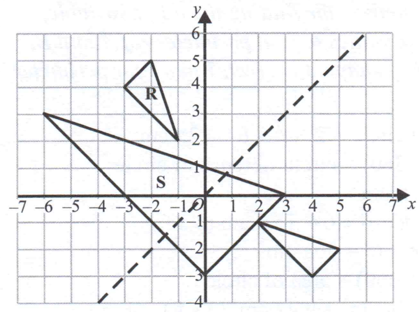
2 a) Rotation 90° anti-clockwise around the point (0, 0)
[3 marks available — 1 mark for rotation, 1 mark for correct angle and direction of rotation, 1 mark for correct centre of rotation]

b)



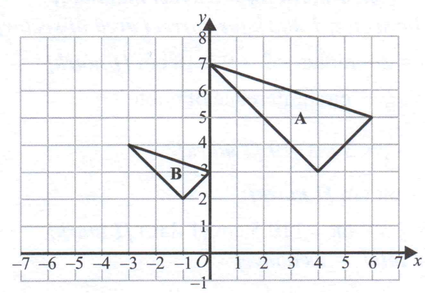
[1 mark for correct translation]

3



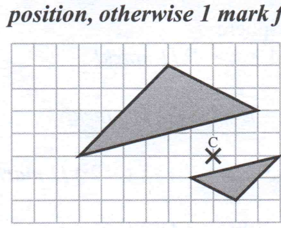
[3 marks available — 3 marks for correct shape S in correct position, otherwise 1 mark for correctly reflected shape, 1 mark for one coordinate of shape S correct]

4



[2 marks available — 2 marks for correct shape in correct position, otherwise 1 mark for one coordinate correct]

5



[2 marks available — 2 marks for correct shape in correct position, otherwise 1 mark for one coordinate correct]

Pages 75-76: Perimeter and Area

1 Lawn area = (30 m × 10 m) – (π × (5 m)^2) = 221.460... m^2
Boxes of seed needed = 221.460... m^2 ÷ 10 m^2 = 22.15
So Lynn must buy 23 boxes. Total cost = 23 × £7 = £161
[3 marks available — 1 mark for correctly calculating the lawn area, 1 mark for dividing the area by 10 m^2 to find the number of boxes, 1 mark for the correct answer]

2 Shorter parallel side of the trapezium
= 52 cm – 16 cm – 16 cm = 20 cm
To see this, split the shape into two triangles and a rectangle. The two triangles are both isosceles, so the base of each triangle is 16 cm long.
Area of trapezium = 0.5 × (52 + 20) × 16 = 576 cm^2
[2 marks available — 1 mark for finding shorter parallel side of trapezium, 1 mark for correct answer]

3 Area of triangle = $0.5 \times 2x \times 2x = 2x^2$ ***[1 mark]***
Area of square = x^2
$2x^2 - x^2 = 9$ cm^2 ***[1 mark]***
$x^2 = 9$ cm^2 so $x = 3$ cm ***[1 mark]***
Perimeter of square = 4 × 3 cm = 12 cm ***[1 mark]***
[4 marks available in total — as above]

4 Let A have width x and length y.
Then B has width x and length $2y$.
The perimeter of C is $y + x + y + x + 2y + 2x = 4x + 4y$
and the perimeter of D is $x + 2y + x + 2y + (y - x) + x + y = 2x + 6y$
So, $4x + 4y = 28$ (1)
$2x + 6y = 34$ (2) $\xrightarrow{\times 2}$ $4x + 12y = 68$ (3)
(3) – (1): $8y = 40$, so $y = 5$ cm
Substitute into (1): $4x + 20 = 28$, so $x = 2$ cm
Perimeter of $A = 2 + 5 + 2 + 5 = 14$ cm
Perimeter of $B = 2 + 10 + 2 + 10 = 24$ cm
[6 marks available — 1 mark for setting up simultaneous equations, 1 mark for a correct method for finding one variable, 1 mark for a correct method for finding the other variable, 1 mark for using these values to find perimeters of A and B, 1 mark for perimeter of shape A correct, 1 mark for perimeter of shape B correct]

5 Circumference of full circle $= 2 \times \pi \times 6 = 12\pi$ cm
Length of arc = $(30 \div 360) \times$ circumference of circle
$= (30 \div 360) \times 12\pi = \pi$ cm
Perimeter of sector $= \pi + 6 + 6 = 15.1$ cm (3 s.f.)
Area of full circle $= \pi \times 6^2 = 36\pi$ cm^2
Area of sector = $(30 \div 360) \times$ area of circle
$= (30 \div 360) \times 36\pi = 3\pi$ cm$^2 = 9.42$ cm^2 (3 s.f.)
[5 marks available — 1 mark for a correct method for calculating the length of the arc, 1 mark for correct arc length, 1 mark for correct perimeter of sector, 1 mark for a correct method for finding the area of the sector, 1 mark for correct area of sector]

6 Each straight section = 2 × radius = $2 \times 9 = 18$ cm ***[1 mark]***
Each curved section = $\frac{1}{3} \times$ circumference of circle
$= \frac{1}{3} \times 2 \times \pi \times 9$ ***[1 mark]***
$= 6\pi$ cm ***[1 mark]***
Total length $= 3 \times 18 + 3 \times 6\pi = 110.5$ cm (1 d.p.) ***[1 mark]***
[4 marks available in total — as above]

Pages 77-79: 3D Shapes — Surface Area and Volume

1 Surface area of curved part of hemisphere =
½ × surface area of a sphere $= \frac{1}{2} \times 4 \times \pi \times 7^2$ ***[1 mark]***
$= 307.876...$ cm^2
Surface area of curved part of cone $= \pi \times 2 \times 12$ ***[1 mark]***
$= 75.398...$ cm^2
Surface area of flat top of hemisphere $= (\pi \times 7^2) - (\pi \times 2^2)$ ***[1 mark]***
$= 141.371...$ cm^2
Total surface area = 307.876... + 75.398... + 141.371...
$= 525$ cm^2 (to 3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

2 Slanting length of cone = 16 cm
Length of arc $= (90 \div 360) \times (2 \times \pi \times 16) = 8\pi$ ***[1 mark]***
The circumference of the base $= 8\pi$, so the diameter of the base is $8\pi \div \pi = 8$. The radius is therefore $8 \div 2 = 4$ cm ***[1 mark]***
Curved surface area of cone $= (\pi \times 4 \times 16)$ ***[1 mark]***
Area of base of cone $= (\pi \times 4^2)$ ***[1 mark]***
Total surface area of cone $= (\pi \times 4 \times 16) + (\pi \times 4^2)$
$= 80\pi$ cm^2 ***[1 mark]***
[5 marks available in total — as above]

3 Volume $= \frac{4}{3}\pi r^3 = 478$ cm^3 ***[1 mark]***
$r = \sqrt[3]{\frac{3 \times 478}{4\pi}} = 4.8504...$ cm ***[1 mark]***
Surface area $= 4\pi r^2 = 4\pi \times (4.8504...)^2$ ***[1 mark]***
$= 295.6$ cm^2 (1 d.p.) ***[1 mark]***
[4 marks available in total — as above]

4 Volume $= \frac{1}{2} \times (\frac{4}{3} \times \pi \times 9^3) - \frac{1}{2} \times (\frac{4}{3} \times \pi \times 8^3)$ ***[1 mark]***
$= 1526.814... - 1072.330...$ ***[1 mark]*** $= 454$ cm^3 (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]
You still get full marks if you simplified the volume before multiplying everything through — e.g. you got $\frac{2}{3}\pi(729 - 512)$.

5 Volume of cone $= \frac{1}{3}(\pi \times 6^2 \times 18) = 216\pi$ cm^3 ***[1 mark]***
So $\frac{4}{3}\pi r^3 = 216\pi$ cm^3 ***[1 mark]***
$r^3 = 162$ ***[1 mark]***
$r = 5.4513...$ cm $= 5.45$ cm (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

6 Let r be the radius of the spheres.
Volume of cuboid $= 2r \times 2r \times 4r = 16r^3$ ***[1 mark]***
Volume of both spheres $= 2 \times \frac{4}{3}\pi r^3 = \frac{8}{3}\pi r^3$ ***[1 mark]***
Percentage of box occupied by spheres $= \frac{\frac{8}{3}\pi r^3}{16r^3} \times 100$ ***[1 mark]***
$= \frac{800\pi}{48} = 52.4\%$ ***[1 mark]***
[4 marks available in total — as above]

7 Cross-sectional area = area of trapezium
$= 0.5 \times (0.7 + 0.4) \times 0.3 = 0.165$ m^2
Volume of trough $= 0.165 \times 1.2 = 0.198$ m^3 = 198 000 cm^3
Rate of flow = 9 litres/minute = 9000 cm^3/minute
Time to fill trough = 198 000 ÷ 9000 = 22 minutes
[4 marks available — 1 mark for calculating volume of trough, 1 mark for converting volume or rate of flow to appropriate units, 1 mark for dividing volume by rate of flow, 1 mark for correct answer]

8 a) Cross-sectional area of pipe: $0.2^2 \times \pi = 0.12566...$ m^2
Cross-sectional area of water: $0.12566... \div 2 = 0.06283...$ m^2
[1 mark]

b) Rate of flow = 2520 litres per minute
= 2520 ÷ 60 litres per second
= 42 litres per second ***[1 mark]***
= 42 000 cm^3/s
= 0.042 m^3/s ***[1 mark]***
Speed = Rate of flow ÷ cross-sectional area of water
= 0.042 m^3/s ÷ 0.06283... m^2 ***[1 mark]***
= 0.66845... m/s = 0.668 m/s (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

Page 80: More Enlargements

1 $1^3 : 7^3 = 1 : 343$ ***[1 mark]***

2 Area of enlarged shape $= 7 \times 3^2$ ***[1 mark]***
$= 63$ cm^2 ***[1 mark]***
[2 marks available in total — as above]

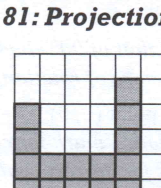
3 Let x be the scale factor for length from cylinder A to cylinder B.
$x^3 = \frac{64}{27}$, so $x = \sqrt[3]{\frac{64}{27}} = \frac{4}{3}$ ***[1 mark]***
Then $x^2 = \frac{4^2}{3^2} = \frac{16}{9}$, so s.a. of cylinder B $= 81\pi \times \frac{16}{9}$ ***[1 mark]***
$= 144\pi$ cm^2 ***[1 mark]***
[3 marks available in total — as above]

4 a) Scale factor from **A** to **C**:
$n^2 = 108\pi \div 12\pi = 9$ so $n = 3$
Volume of **A** $= 135\pi$ cm$^3 \div 3^3 = 5\pi$ cm^3
[4 marks available — 1 mark for finding n^2, 1 mark for finding n, 1 mark for dividing volume of A by n^3, 1 mark for correct answer]

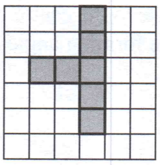
b) Scale factor from **A** to **B**:
$m^2 = 48\pi \div 12\pi = 4$ ***[1 mark]***
$m = 2$ ***[1 mark]***
Perpendicular height of **B** = 4 cm × 2
= 8 cm ***[1 mark]***
[3 marks available in total — as above]

Page 81: Projections

1 a)

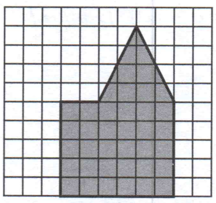


[1 mark]

b) 

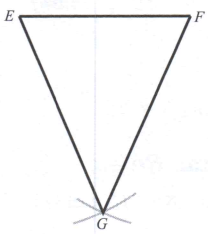
[1 mark]

It doesn't matter which way round you've drawn your plan view — just as long as it's the correct shape.

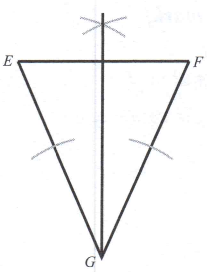
2 

[2 marks available — 1 mark for rectangular part correct, 1 mark for triangular part correct]

Pages 82-83: Loci and Construction

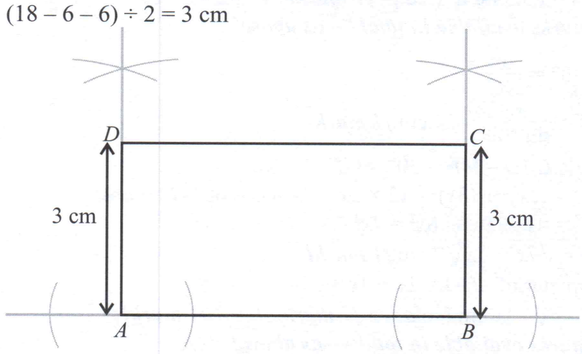
1 a) 

[2 marks available — 1 mark for arcs drawn with a radius of 4.5 cm, 1 mark for completed triangle]

b) 

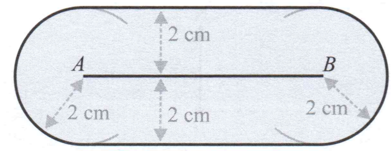
[2 marks available — 1 mark for correct construction arcs, 1 mark for correct bisector]

2 *AB* = 6 cm, so sides perpendicular to *AB* have length $(18 - 6 - 6) \div 2 = 3$ cm



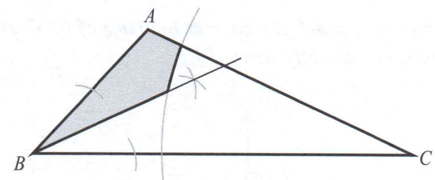
[4 marks available — 1 mark for working out that lines AD and BC are 3 cm, 1 mark for correctly constructing a perpendicular line at A, 1 mark for correctly constructing a perpendicular line at B, 1 mark for a completely correctly constructed rectangle]

The diagram shows just one way of constructing the rectangle. In the exam, you'll get the marks as long as your construction lines are correct and visible.

3 

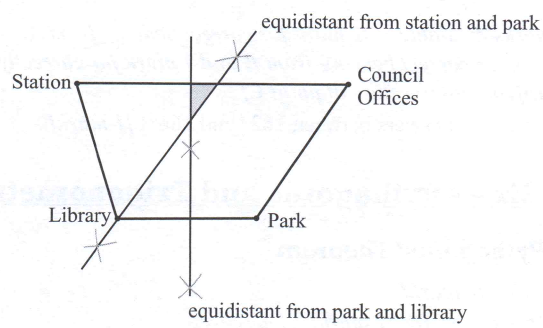
Scale: 1 cm represents 1 m

[2 marks available — 1 mark for correct semicircles, 1 mark for correct shaded area]

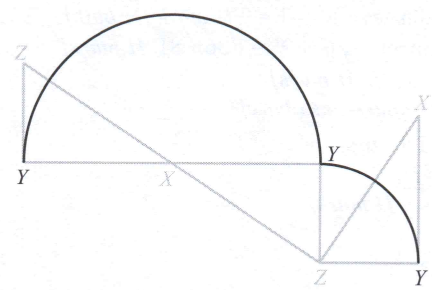
4 

[4 marks available — 1 mark for radius of 6.5 cm with centre at C, 1 mark for construction arcs on AB and BC for angle bisector at ABC, 1 mark for correct angle bisector at ABC, and 1 mark for the correct shading]

Make sure you remember to leave in your construction lines.

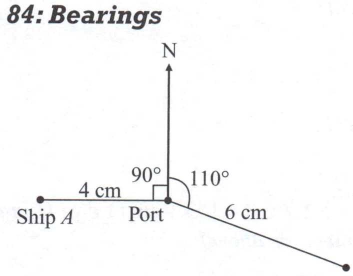
5 

[3 marks available — 1 mark for perpendicular bisector between Library and Park, 1 mark for perpendicular bisector between Station and Park, 1 mark for the correct shaded area]

6 

[3 marks available — 1 mark for constructing either the semicircle or quarter circle correctly, 1 mark for the two parts of the locus being joined together, 1 mark for a completely correct diagram]

Page 84: Bearings

1 a) 

[4 marks available — 1 mark for Ship A 4 cm from the port, 1 mark for correct bearing for ship A, 1 mark for ship B 6 cm from the port, and 1 mark for correct bearing for Ship B]

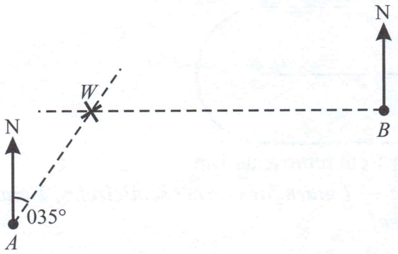
This diagram has been drawn a bit smaller to make it fit — your measurements should match the labels given on the diagram here.

b) 102° (accept answers between 100° and 104°) ***[1 mark]***

c) 180° – 102° = 78°
360° – 78° = 282° (accept answers between 280° and 284°)
[2 marks available — 1 mark for correctly using 102°, 1 mark for correct answer]

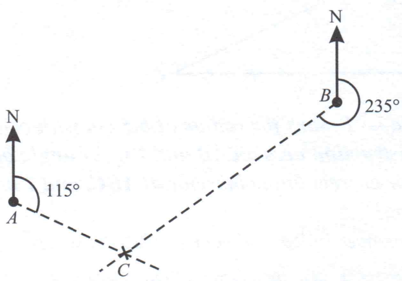
You could also do this by adding 180° to 102°.

2 a)



[2 marks available — 1 mark for correct bearing of 035° and 1 mark for marking W directly west of B]

b)



[3 marks available — 1 mark for correct bearing from A, 1 mark for correct bearing from B and 1 mark for correctly identifying intersection at point C]

c) 164° (accept answers between 162° and 166°) *[1 mark]*

Section Six — Pythagoras and Trigonometry

Page 85: Pythagoras' Theorem

1 $AB^2 = 4^2 + 8^2$ *[1 mark]*
$AB = \sqrt{16+64} = \sqrt{80}$ *[1 mark]*
$AB = 8.94$ cm (2 d.p) *[1 mark]*
[3 marks available in total — as above]

2 Difference in x-coordinates = 8 – 2 = 6
Difference in y-coordinates = 8 – –1 = 9 *[1 mark for both]*
So length of line segment = $\sqrt{6^2+9^2} = \sqrt{36+81}$ *[1 mark]*
$= \sqrt{117} = \sqrt{9\times 13} = 3\sqrt{13}$ *[1 mark]*
[3 marks available in total — as above]

3 Let h be the height of the triangle:
$13^2 = 5^2 + h^2$ *[1 mark]*
$h = \sqrt{169-25} = \sqrt{144}$ *[1 mark]*
$h = 12$ cm *[1 mark]*
Area, $A = \frac{1}{2}\times 10\times 12$
$A = 60$ cm² *[1 mark]*
[4 marks available in total — as above]

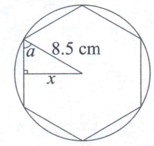
4 Length of EA:
$28.3^2 = 20^2 + EA^2$ *[1 mark]*
$EA = \sqrt{800.89-400}$
$EA = 20.02...$ *[1 mark]*
Length of CE:
$54.3^2 = 20^2 + CE^2$ *[1 mark]*
$CE = \sqrt{2948.49-400}$
$CE = 50.48...$ *[1 mark]*
Perimeter = 28.3 + 54.3 + EA + CE = 153.1 cm (1 d.p) *[1 mark]*
[5 marks available in total — as above]

Page 86: Trigonometry

1 $\sin x = \frac{14}{18}$ *[1 mark]*
$x = \sin^{-1}\left(\frac{14}{18}\right)$ *[1 mark]*
$x = 51.1°$ (1 d.p) *[1 mark]*
[3 marks available in total — as above]

2 $\tan 60° = \frac{4}{y}$ *[1 mark]*
$y = \frac{4}{\tan 60°}$ *[1 mark]*
$y = \frac{4}{\sqrt{3}}$ m *[1 mark]*
[3 marks available in total — as above]

3 Call the distance from the centre of the circle to the centre of an edge x. The radius bisects the interior angle forming angle a.



Sum of the interior angles of a hexagon = 4 × 180° = 720°
Each interior angle of a hexagon = 720° ÷ 6 = 120° *[1 mark]*
$a = 120 \div 2 = 60°$ *[1 mark]*
$\sin 60° = \frac{x}{8.5}$ *[1 mark]*
$x = 8.5 \times \sin 60°$ *[1 mark]*
$x = 7.36$ cm (2 d.p) *[1 mark]*
[5 marks available in total — as above]
You could also use the calculation cos 30° × 8.5 to find the value of x. As long as you make sure you show your working, you'll get full marks if your answer is correct.

4 $\tan 30° + \sin 60° = \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}$ *[1 mark]*
$= \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{6} + \frac{3\sqrt{3}}{6}$ *[1 mark]*
$= \frac{5\sqrt{3}}{6}$ *[1 mark]*
[3 marks available in total — as above]

Pages 87-88: The Sine and Cosine Rules

1 a) $AC^2 = 10^2 + 7^2 - (2\times 10\times 7\times \cos 85°)$ *[1 mark]*
$AC = \sqrt{149 - 140\times\cos 85°}$
$AC = 11.7$ cm (3 s.f) *[1 mark]*
[2 marks available in total — as above.]

b) Area = $\frac{1}{2}\times 10\times 7\times \sin 85°$ *[1 mark]*
Area = 34.9 cm² (3 s.f) *[1 mark]*
[2 marks available in total — as above]

2 First you need to find one angle using the cosine rule.
E.g. use angle CAB.
$\cos A = \frac{14^2+12^2-19^2}{2\times 14\times 12}$ *[1 mark]*
$A = \cos^{-1}\left(\frac{-21}{336}\right)$
$A = 93.58...°$ *[1 mark]*
Area = $\frac{1}{2}\times 14\times 12\times \sin 93.58...°$ *[1 mark]*
Area = 83.84 cm² (2.d.p) *[1 mark]*
[4 marks available in total — as above]

3 $\sin 30° = \frac{x}{AC}$
$AC = \frac{x}{\sin 30°} = 2x$ cm *[1 mark]*
Angle CAD = 90° – 30° = 60°
$CD^2 = (2x)^2 + (3x)^2 - (2\times 2x\times 3x\times\cos 60°)$ *[1 mark]*
$CD^2 = 4x^2 + 9x^2 - 6x^2 = 7x^2$
$CD = \sqrt{7x^2} = x\sqrt{7}$ cm *[1 mark]*
Perimeter of $ACD = 2x + 3x + x\sqrt{7}$
$= (5+\sqrt{7})x$ cm (so $a = 5$ *[1 mark]*, $b = 7$ *[1 mark]*)
[5 marks available in total — as above]

4 $\frac{36}{\sin ABC} = \frac{17}{\sin 26°}$ *[1 mark]*
$\sin ABC = \frac{36\times\sin 26°}{17}$ *[1 mark]*
$ABC = 180 - \sin^{-1}\left(\frac{36\times\sin 26°}{17}\right) = 111.8°$ (1 d.p.) *[1 mark]*
[3 marks available in total — as above]

5 Angle ABD = 180° – 90° – 31° – 12° = 47°
Angle ACB = 180° – 12° – 47° = 121° *[1 mark for both]*
Use the sine rule: $\frac{3.3}{\sin 12°} = \frac{AB}{\sin 121°}$
$AB = \frac{3.3}{\sin 12°}\times\sin 121°$ *[1 mark]*
$AB = 13.6050...$ m *[1 mark]*

Find length BD: $\cos 47° = \frac{BD}{13.6050...}$
$BD = \cos 47° \times 13.6050...$ ***[1 mark]***
$BD = 9.2786... = 9.28$ m (3 s.f.) ***[1 mark]***
[5 marks available in total— as above]
There's more than one way of doing this question. As long as you've used a correct method to get the right answer you'll still get the marks.

6 First, split $ABCD$ into two triangles, ABC and ACD.
$\frac{55}{\sin ACB} = \frac{93}{\sin 116°}$ ***[1 mark]***
$\sin ACB = \frac{\sin 116°}{93} \times 55$
Angle $ACB = \sin^{-1}(0.531...) = 32.109...°$ ***[1 mark]***
Angle $BAC = 180° - 116° - 32.109...°$ so,
Area of $ABC = \frac{1}{2} \times 93 \times 55 \times \sin(180 - 116 - 32.10...)°$ ***[1 mark]***
$= 1351.106...\text{cm}^2$ ***[1 mark]***
Angle $ACD = 78° - 32.109...°$ so
Area of $ACD = \frac{1}{2} \times 93 \times 84 \times \sin(78 - 32.10...)°$
$= 2804.531...\text{cm}^2$ ***[1 mark]***
Area of $ABCD = 1351.106... + 2804.531... = 4155.637...$
$= 4160\text{ cm}^2$ (3 s.f.) ***[1 mark]***
[6 marks available in total — as above]

Page 89: 3D Pythagoras and Trigonometry

1 $BH^2 = 6^2 + 3^2 + 4^2$ ***[1 mark]***
$BH = \sqrt{61}$ ***[1 mark]***
$BH = 7.81$ cm (3 s.f) ***[1 mark]***
[3 marks available in total — as above]

2 $FG = 80 \div 2 \div 8 = 5$ cm
$DF^2 = 8^2 + 2^2 + 5^2$ ***[1 mark]***
$DF = \sqrt{93}$ ***[1 mark]***
You could also calculate the length of either AG, BH, or CE.
The angle between the stick and the place $CDHG$ is the angle between the line DF and the line DG, so
$\sin FDG = \frac{5}{\sqrt{93}}$ ***[1 mark]***
$FDG = \sin^{-1}\left(\frac{5}{\sqrt{93}}\right)$ ***[1 mark]*** $= 31°$ (2 s.f.) ***[1 mark]***
[5 marks available in total — as above]

Pages 90-91: Vectors

1 a) $\begin{pmatrix}-3\\5\end{pmatrix} - \begin{pmatrix}5\\4\end{pmatrix} = \begin{pmatrix}-8\\1\end{pmatrix}$ ***[1 mark]***
b) $4 \times \begin{pmatrix}5\\4\end{pmatrix} - \begin{pmatrix}-4\\-6\end{pmatrix} = \begin{pmatrix}20\\16\end{pmatrix} - \begin{pmatrix}-4\\-6\end{pmatrix} = \begin{pmatrix}24\\22\end{pmatrix}$ ***[1 mark]***
c) $2 \times \begin{pmatrix}-3\\5\end{pmatrix} + \begin{pmatrix}5\\4\end{pmatrix} + 3 \times \begin{pmatrix}-4\\-6\end{pmatrix} = \begin{pmatrix}-6\\10\end{pmatrix} + \begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}-12\\-18\end{pmatrix}$
$= \begin{pmatrix}-13\\-4\end{pmatrix}$ ***[1 mark]***

2 a) $4\mathbf{a} + 3\mathbf{b}$ ***[1 mark]***
b) $4\mathbf{a} + 1.5\mathbf{b}$ ***[1 mark]***

3 a) $\overrightarrow{CD} = -2\mathbf{a}$ ***[1 mark]***
b) $\overrightarrow{AC} = 2\mathbf{d} + 2\mathbf{a}$ ***[1 mark]***
c) $\overrightarrow{BL} = \mathbf{d} - \mathbf{a}$ ***[1 mark]***

4 a) $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$ ***[1 mark]***
$\overrightarrow{AB} = \mathbf{b} - 2\mathbf{a}$ or $-2\mathbf{a} + \mathbf{b}$
$\overrightarrow{OM} = 2\mathbf{a} + \frac{1}{2}(-2\mathbf{a} + \mathbf{b})$ or $\overrightarrow{OM} = 2\mathbf{a} + \frac{1}{2}(\mathbf{b} - 2\mathbf{a})$
$= \mathbf{a} + \frac{1}{2}\mathbf{b}$ ***[1 mark]***
[2 marks available in total — as above]
b) $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$ ***[1 mark]***
As AX:XB = 1:3, AX must be one quarter of AB, so:
$\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{4}\overrightarrow{AB}$
$\overrightarrow{OX} = 2\mathbf{a} + \frac{1}{4}(\mathbf{b} - 2\mathbf{a})$ ***[1 mark]***
$\overrightarrow{OX} = \frac{3}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}$ ***[1 mark]***
[3 marks available in total — as above]

5 a) $\overrightarrow{BX} = \overrightarrow{BC} + \overrightarrow{CX}$
$\overrightarrow{BC} = 6\overrightarrow{BW} = 6\mathbf{b}$ ***[1 mark]***
As AX = 2XC, CX must be one third of CA, so:
$\overrightarrow{CX} = \frac{1}{3}\overrightarrow{CA}$ ***[1 mark]***
$\overrightarrow{CA} = -3\mathbf{a} - 6\mathbf{b}$ ***[1 mark]***
$\overrightarrow{CX} = \frac{1}{3}(-3\mathbf{a} - 6\mathbf{b}) = -\mathbf{a} - 2\mathbf{b}$
$\overrightarrow{BX} = 6\mathbf{b} - \mathbf{a} - 2\mathbf{b} = 4\mathbf{b} - \mathbf{a}$ ***[1 mark]***
[4 marks available in total — as above]
There are other ways of doing this one — you could start by writing $\overrightarrow{BX}$ as $\overrightarrow{BA} + \overrightarrow{AX}$ instead.
b) From part a) $\overrightarrow{BX} = 4\mathbf{b} - \mathbf{a}$:
$ABCD$ is a parallelogram, so:
$\overrightarrow{CD} = \overrightarrow{BA} = -\overrightarrow{AB} = -3\mathbf{a}$ ***[1 mark]***
$\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CD} = -\frac{3}{2}\mathbf{a}$ ***[1 mark]***
$\overrightarrow{BM} = \overrightarrow{BC} + \overrightarrow{CM}$
$= 6\mathbf{b} - \frac{3}{2}\mathbf{a} = \frac{3}{2}(4\mathbf{b} - \mathbf{a})$ ***[1 mark]***
B, X and M must be three points on a straight line because the lines BM and BX are both scalar multiples of the vector $4\mathbf{b} - \mathbf{a}$.
[1 mark]
[4 marks available in total — as above]
There are other ways of doing this bit too — for example, you could show that XM and BX are scalar multiples of each other.

Section Seven — Probability and Statistics

Page 92: Probability Basics

1 a) P(strawberry) $= \frac{2}{2+5} = \frac{2}{7}$ ***[1 mark]***
b) P(banana) $= \frac{5}{7}$
$2 \times$ P(strawberry) $= 2 \times \frac{2}{7} = \frac{4}{7}$ so Amy is wrong,
she is more than twice as likely to pick a banana sweet.
[2 marks available — 1 mark for finding the probability of choosing a banana sweet, 1 mark for saying Amy is wrong with a valid explanation]

2 Number of red counters $= p - n$ ***[1 mark]***
Probability of getting a red counter $= \frac{p-n}{p}$ ***[1 mark]***
[2 marks available in total — as above]

3 P(stripy sock) $= 2y$ ***[1 mark]***
$0.4 + y + 2y = 1$ ***[1 mark]***
$3y = 0.6$
$y = 0.2$ ***[1 mark]***
[3 marks available in total — as above]

Page 93: Counting Outcomes

1

		Dice					
		1	2	3	4	5	6
Cards	2	3	4	5	6	7	8
	4	5	6	7	8	9	10
	6	7	8	9	10	11	12
	8	9	10	11	12	13	14
	10	11	12	13	14	15	16

P(scoring more than 4) $= \frac{\text{number of ways to score more than 4}}{\text{total number of possible outcomes}}$
$= \frac{28}{30} = \frac{14}{15}$
[3 marks available — 1 mark for finding the total number of possible outcomes, 1 mark for finding the number of ways to score more than 4, 1 mark for the correct answer]

2 Combinations of sandwich and drink = 5 × 8 = 40
Combinations of sandwich and snack = 5 × 4 = 20
Combinations of sandwich, snack and drink = 5 × 4 × 8 = 160
Total number of possible combinations = 40 + 20 + 160 = 220
[3 marks available — 1 mark for the correct number of combinations for one meal deal, 1 mark for the correct number of combinations for the other two meal deals, 1 mark for the correct answer]

3 a) Total number of different ways for the spinners to land $= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$ ***[1 mark]***

b) Number of ways of not spinning any 1's $= 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$ ***[1 mark]***

So P(not spinning any 1's) $= \frac{243}{1024}$ ***[1 mark]***
[2 marks available in total — as above]

4 a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$ ***[1 mark]***

b) Number of ways for all lights to be red or blue $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ ***[1 mark]***
P(all lights red or blue) $= \frac{64}{4096} = \frac{1}{64}$ ***[1 mark]***
[2 marks available in total — as above]

Pages 94-95: Probability Experiments

1 a) $50 \times 0.12 = 6$
[2 marks available — 1 mark for correct method, 1 mark for the correct answer]

b) E.g. On a fair dice, the theoretical frequency of each number is 0.166..., so as 1 has a much higher relative frequency and 5 has a much lower relative frequency, the dice is probably not fair.
[2 marks available —1 mark for 'not fair' or similar, 1 mark for an explanation including numbers or relative frequency]

c) E.g. no, each dice roll is random, so in a small number of trials like 50 she is likely to get different results.
[1 mark]

2 a)

Number on counter	1	2	3	4	5
Frequency	23	25	22	21	9
Relative Frequency	**0.23**	**0.25**	**0.22**	**0.21**	**0.09**

[2 marks available — 2 marks for all correct answers, otherwise 1 mark for any frequency ÷ 100]

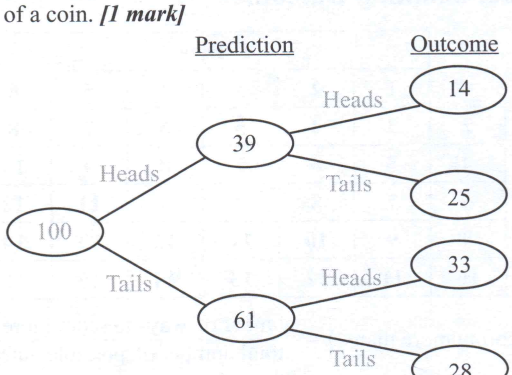
b) Elvin is likely to be wrong. The bag seems to contain fewer counters numbered 5. ***[1 mark]***

c) P(odd number) = 0.23 + 0.22 + 0.09 ***[1 mark]***
= 0.54 ***[1 mark]***
[2 marks available in total — as above]

3 a) i) $\frac{1}{2} \times 8 = 4$ ***[1 mark]***

ii) E.g. Danielle predicted correctly five times, which is close to the number you'd expect her to get correct if she was just guessing, so there is no evidence that she can predict the flip of a coin. ***[1 mark]***

b) i)



[3 marks available — 1 mark for the numbers 39 and 61 in the correct places, 1 mark for the numbers 14 and 25 in the correct places, 1 mark for the numbers 33 and 28 in the correct places]

ii) Relative frequency of predicting the outcome correctly = relative frequency of heads, heads or tails, tails.
$= \frac{14 + 28}{100} = \frac{42}{100}$ or $\frac{21}{50}$ or 0.42
[2 marks available — 1 mark for 14 + 28, 1 mark for the correct answer]

c) E.g. The experiment in part b) has more trials so the results are more reliable. ***[1 mark]***

Page 96: The AND / OR Rules

1 a) P(prime number) = 0.15 + 0.2 + 0.1 = 0.45
P(multiple of 2) = 0.15 + 0.25 = 0.4
P(a prime multiple of 2) = 0.15
P(prime number or multiple of 2) = 0.45 + 0.4 – 0.15 = 0.7
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]
Alternatively you work out which numbers are prime or multiples of two: 2, 3, 4 and 5. Then add the probabilities of these outcomes together 0.15 + 0.2 + 0.25 + 0.1 = 0.7.

b) i) P(spun twice) = P(1st spin isn't 2) × P(2nd spin is 2)
= (1 – 0.15) × 0.15 ***[1 mark]***
= 0.85 × 0.15 = 0.1275 ***[1 mark]***
[2 marks available — as above]

ii) P(spun more than twice) = 1 – P(spun twice or once)
= 1 – (0.1275 + 0.15) ***[1 mark]***
= 1 – 0.2775 = 0.7225 ***[1 mark]***
[2 marks available — as above]

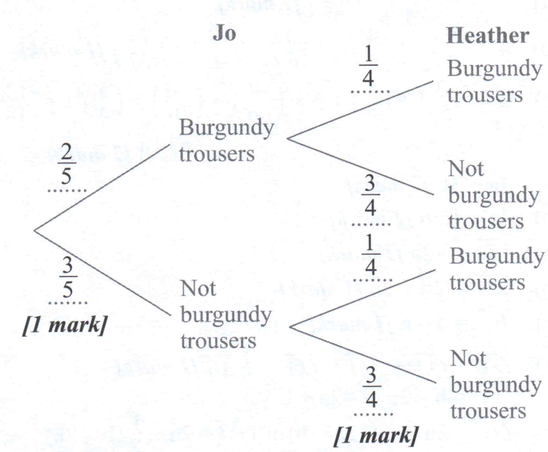
2 a) P(losing) = 1 – 0.3 = 0.7
P(losing 3 games) = 0.7 × 0.7 × 0.7 = 0.343 ***[1 mark]***

b) P(wins at least 1 prize) = 1 – P(doesn't win a prize)
= 1 – (0.7 × 0.7) ***[1 mark]***
= 1 – 0.49 = 0.51 ***[1 mark]***
[2 marks available in total — as above]

c) P(winning 1 prize in 3 games)
= P(win, lose, lose) + P(lose, win, lose) + P(lose, lose, win)
= (0.3 × 0.7 × 0.7) + (0.7 × 0.3 × 0.7) + (0.7 × 0.7 × 0.3)
[1 mark]
= 0.147 + 0.147 + 0.147 = 0.441 ***[1 mark]***
Shaun is wrong, there is only a 44.1% chance of winning exactly once in 3 games. ***[1 mark]***
[3 marks available in total — as above]

Pages 97: Tree Diagrams

1 a)



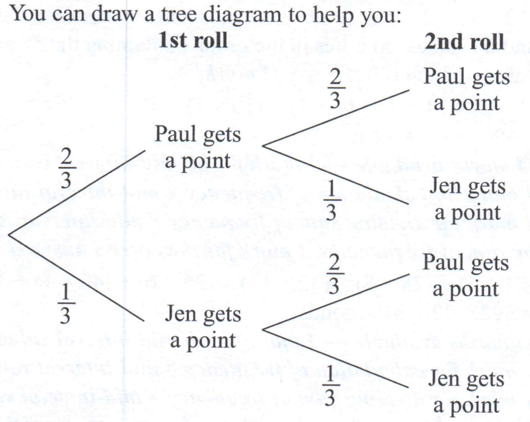
[2 marks available in total — as above]

b) P(neither wear burgundy trousers) $= \frac{3}{5} \times \frac{3}{4}$ ***[1 mark]***
$= \frac{9}{20}$ ***[1 mark]***
[2 marks available in total — as above]

2 a) P(Paul's point) = P(1, 2, 3 or 6) = $\frac{2}{3}$

P(Jen's point) = 1 – $\frac{2}{3}$ = $\frac{1}{3}$ ***[1 mark for both probabilities]***

You can draw a tree diagram to help you:



P(they draw after two rolls) = P(they both get 1 point)

$= \left(\frac{2}{3}\times\frac{1}{3}\right)+\left(\frac{1}{3}\times\frac{2}{3}\right)$ ***[1 mark]***

$= \frac{2}{9}+\frac{2}{9}=\frac{4}{9}$ ***[1 mark]***

[3 marks available in total — as above]

b) P(Paul wins) = P(Paul wins 3-0) + P(Paul wins 2-1)

$= \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right)+\left(\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\right)+\left(\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}\right)+\left(\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}\right)$

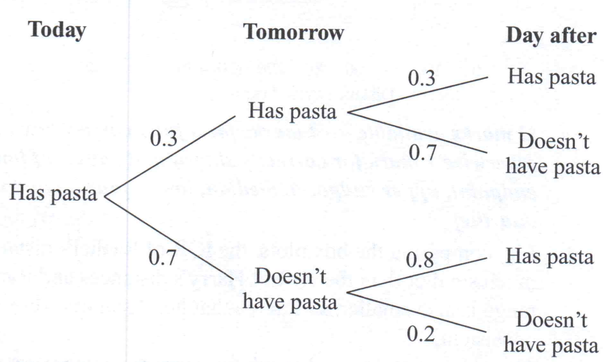
$= \frac{8}{27}+\frac{4}{27}+\frac{4}{27}+\frac{4}{27}=\frac{20}{27}$

[3 marks available — 1 mark for finding the probability of Paul winning 3-0, 1 mark for finding the probabilities of Paul winning 2-1, 1 mark for the correct final answer]

You could draw another tree diagram showing 3 rolls if you're struggling to find the right probabilities.

Page 98: Conditional Probability

1 a)



[2 marks available — 1 mark for the correct probabilities for tomorrow, 1 mark for the correct probabilities for the day after]

b) P(pasta on 1 of the next 2 days) = P(pasta then no pasta) + P(no pasta then pasta)

= (0.3 × 0.7) + (0.7 × 0.8) ***[1 mark]***

= 0.21 + 0.56 = 0.77 ***[1 mark]***

[2 marks available in total — as above]

2 a) P(2nd milk given 1st is milk) = $\frac{6}{11}$ ***[1 mark]***

1 milk chocolate has been taken, so there are 6 milk chocolates left out of 11 remaining chocolates.

b) P(at least one milk) = 1 – P(no milk)

= 1 – P(white then white)

$= 1-\left(\frac{5}{12}\times\frac{4}{11}\right)$ ***[1 mark]***

$= 1-\frac{20}{132}=\frac{112}{132}=\frac{28}{33}$ ***[1 mark]***

[2 marks available in total — as above]

c) P(milk and white) = P(milk then white) + P(white then milk)

$= \left(\frac{7}{12}\times\frac{5}{11}\right)+\left(\frac{5}{12}\times\frac{7}{11}\right)$ ***[1 mark]***

$= \frac{35}{66}$ ***[1 mark]***

[2 marks available in total — as above]

You might find it helpful to draw a tree diagram for parts b) and c).

Page 99-100: Sets and Venn Diagrams

1 a) $(x + 4) + 3x + (2x + 3) + 9 = 40$

$6x + 16 = 40$ ***[1 mark]***

$6x = 24$

$x = 4$ ***[1 mark]***

[2 marks available in total — as above]

b) $n(A \cap B) = 3x = 12$ ***[1 mark]***

$P(A \cap B) = \frac{12}{40}=\frac{3}{10}$ ***[1 mark]***

[2 marks available in total — as above]

2 a) P(neither) = 1 – P(pie or drink)

= 1 – (P(pie) + P(drink) – P(both)) ***[1 mark]***

= 1 – (0.33 + 0.64 – 0.27) = 0.3 ***[1 mark]***

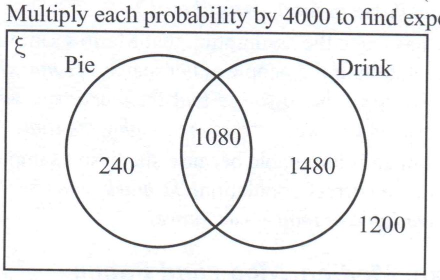
[2 marks available in total — as above]

You could also find the probabilities of only getting a pie and only getting a drink (see below) and then work out 1 – (P(only a pie) + P(only a drink) + P(both)).

b) P(only a pie) = 0.33 – 0.27 = 0.06 P(both) = 0.27

P(only a drink) = 0.64 – 0.27 = 0.37 P(neither) = 0.3

Multiply each probability by 4000 to find expected frequencies:



[3 marks available — 1 mark for P(only a pie) and P(only a drink), 1 mark for multiplying each probability by 4000, 1 mark for a completely correct Venn diagram]

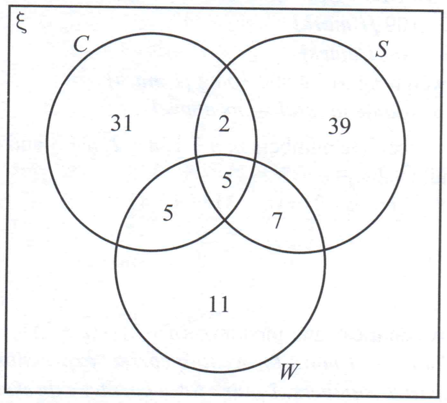
c) P(pie given no drink) = $\frac{240}{1440}=\frac{1}{6}$

[2 marks available — 1 mark for finding the expected number of people who don't get a drink (correct denominator), 1 mark for the correct answer]

3 P(male given singer) = $\frac{21}{54}=\frac{7}{18}$

[2 marks available — 1 mark for finding the percentage of singers (correct denominator), 1 mark for the correct answer]

4 a)



[3 marks available — 3 marks for a completely correct diagram, otherwise 1 mark for at least 2 correct entries or 2 marks for at least 4 correct entries]

b) $\frac{31+5+5+2+7+39}{100}=\frac{89}{100}$ or 0.89 ***[1 mark]***

c) $\frac{7+5}{11+5+5+7}$ ***[1 mark]***

$= \frac{12}{28}=\frac{3}{7}$ ***[1 mark]***

[2 marks available in total — as above]

Page 101: Sampling and Data Collection

1 a) E.g.

Number of chocolate bars	Tally	Frequency
0-2		
3-5		
6-8		
9-11		
12 or more		

[2 marks available — 1 mark for a suitable tally table, 1 mark for non-overlapping classes that cover all possible values]

b) E.g. Faye's results are likely to be biased because she hasn't selected her sample at random from all the teenagers in the UK. Also, her sample is too small to represent the whole population. So Faye can't use her results to draw conclusions about teenagers in the UK.
[2 marks available — 1 mark for a correct comment based on bias or sample size, 1 mark for stating that Faye can't draw conclusions about teenagers in the UK with reasoning]

2 a) Proportion of people in sample who travelled by car
$= 22 \div 50 = 0.44$ ***[1 mark]***
Estimate of number of people at match who travelled by car
$= 0.44 \times 5000$ ***[1 mark]*** $= 2200$ ***[1 mark]***
[3 marks available in total — as above]

b) E.g. Daisy has made the assumption that Mario's sample is a fair representation of the people at her match. ***[1 mark]***
Or you could say she has assumed that the proportions who travelled by car to the two matches are roughly the same.
E.g. her estimate is unreliable because she hasn't sampled people from the correct population. ***[1 mark]***
[2 marks available in total — as above]

Page 102: Mean, Median, Mode and Range

1 a) Yes, the mean number is higher than 17 because the 11th data value is higher than the mean of the original 10 values.
[1 mark]

b) You can't tell if the median number is higher than 15, because you don't know the other data values.
[1 mark]

2 a) 23, 26, 36 (in any order)
range = 13, median = 26
[2 marks available — 1 mark for all three weights correct, 1 mark for both range and median correct]

b) $32 + 23 + 31 + 28 + 36 + 26 = 176$
$4 \times 27.25 = 109$ ***[1 mark]***
$176 - 109 = 67$ ***[1 mark]***
so, goats weighing 31 kg and 36 kg ***[1 mark]***
[3 marks available in total — as above]

3 Call the five consecutive numbers n, $n + 1$, $n + 2$, $n + 3$ and $n + 4$
Median = middle value = $n + 2$

$$\text{Mean} = \frac{n+(n+1)+(n+2)+(n+3)+(n+4)}{5}$$
$$= \frac{5n+10}{5}$$
$$= n + 2$$

Difference between mean and median = $(n + 2) - (n + 2) = 0$
[3 marks available — 1 mark for writing correct expressions for five consecutive numbers, 1 mark for a correct expression for the mean, 1 mark for showing that the difference between the expression for the mean and the expression for the median is zero]

Page 103: Grouped Frequency Tables

1 a) The modal class is the one with the highest frequency, so that's $3 \le x \le 5$ ***[1 mark]***

b) $(10 + 1) \div 2 = 5.5$, so the median is halfway between the 5th and 6th values, so it lies in the group containing the 5th and 6th values, which is $3 \le x \le 5$ ***[1 mark]***

c) $(1 \times 2) + (4 \times 4) + (7 \times 3) + (10 \times 1) \div 10$
$= 49 \div 10 = 4.9$ cm
[4 marks available — 1 mark for all mid-interval values, 1 mark for calculation of frequency × mid-interval value, 1 mark for dividing sum of frequency × mid-interval values by sum of frequencies, 1 mark for the correct answer]

2 a) $(24 \times 4) + (28 \times 8) + (32 \times 13) + (36 \times 6) + (40 \times 1) \div 32$
$= 992 \div 32 = 31$ seconds
[4 marks available — 1 mark for all mid-interval values, 1 mark for calculation of frequency × mid-interval value, 1 mark for dividing sum of frequency × mid-interval values by sum of frequencies, 1 mark for the correct answer]

b) There were 32 pupils and $13 + 6 + 1 = 20$ got a time of more than 30 seconds ***[1 mark]***, $(20 \div 32) \times 100 = 62.5\%$ ***[1 mark]***
[2 marks available in total — as above]

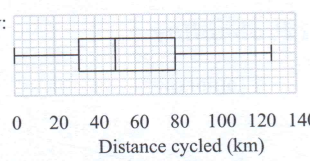
c) E.g. You couldn't use these results because you don't know the ages of the pupils in the sample, or whether any of the times were run by boys, so you can't tell if the results would fairly represent 16-year-old boys. ***[1 mark for a sensible comment]***

Page 104: Box Plots

1 a) IQR = 72 – 52 ***[1 mark]***
= 20 km ***[1 mark]***
[2 marks available in total — as above]

b) The interquartile range doesn't include outliers, so it should be a more reliable measure of the spread. ***[1 mark]***

c) Harry:



[2 marks available — 2 marks for a fully correct box plot, otherwise 1 mark for correctly showing at least 3 of lower endpoint, upper endpoint, median, lower quartile and upper quartile]

d) E.g. comparing the box plots, the IQR of Rachel's distances is much smaller than the IQR of Harry's distances and Rachel's range is also smaller, so I agree that her distances were more consistent.
[2 marks available — 1 mark for correctly comparing the values of the range or IQR, 1 mark for a correct conclusion (supported by a correct comparison)]

2 E.g. the median time taken by the boys is the same as the median time taken by the girls, so on average the boys and girls took the same time. The interquartile range for the boys is smaller than the interquartile range for the girls, so the times taken by the boys were more consistent than the times taken by the girls.
[2 marks available — 1 mark for a correct comparison of the median, 1 mark for a correct comparison of the interquartile range OR range (for both marks, at least one comparison must be given in the context of the data)]
'In the context of the data' means you need to explain what your comparison shows about the times taken by the boys and girls.

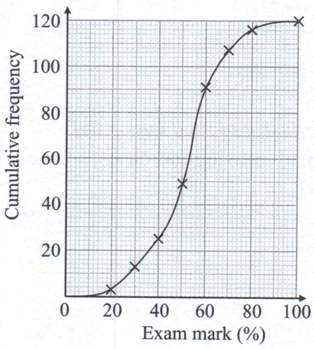
Pages 105-106: Cumulative Frequency

1 a)

Exam mark (%)	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70	≤ 80	≤ 100
Cumulative Frequency	3	13	25	49	91	107	116	120

[1 mark]

b)



[3 marks available — 1 mark for all points plotted at correct class boundaries, 1 mark for all points plotted at correct heights, 1 mark for joining them with a smooth curve or straight lines]

A common mistake in exams is not plotting the points at the top end of the interval. But you wouldn't make that mistake, would you?

c) Median plotted at 60 gives a value of 53%
[1 mark, accept answers ± 1%]

d) Lower quartile at 30 gives a value of 43%
Upper quartile at 90 gives a value of 60%
Inter-quartile range = 60 – 43 = 17%
[2 marks available — 1 mark for correct method, 1 mark for correct answer, accept answers ± 2%]

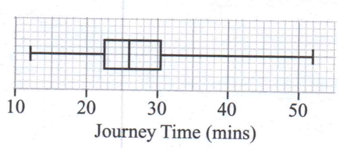
e) $\frac{1}{5}$ of pupils got lower than grade 5,
$\frac{1}{5}$ of 120 = 24 pupils
Reading from the graph at a cumulative frequency of 24 gives 39% , so the mark needed to get a grade 5 was about 39%.
[3 marks available — 1 mark for finding the number of pupils who got lower than grade 5, 1 mark for drawing a line across from 24 on the cumulative frequency axis, 1 mark for an answer in the range 37-41%]

2 a) i) Number of journeys between 27 and 47 mins = 49 – 28 = 21
[2 marks available — 1 mark for reading the cumulative frequencies off at 27 and 47 minutes, 1 mark for correct answer]

ii) 48 journeys took 40 minutes or less, so 2 journeys took longer. Percentage of total number = (2 ÷ 50) × 100 = 4%
[2 marks available — 1 mark for correct method, 1 mark for correct answer]

b) The answers are estimates because they're based on grouped data, rather than the actual data values. ***[1 mark]***

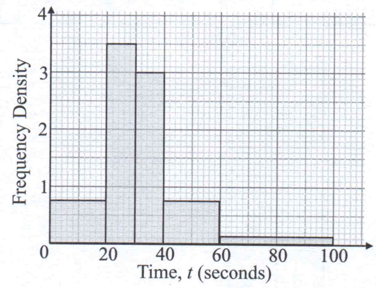
c)



[3 marks available — 1 mark for plotting end points correctly, 1 mark for plotting median correctly (± 0.5) and 1 mark for plotting lower and upper quartiles correctly (± 0.5)]

Pages 107-108: Histograms and Frequency Density

1 To find the scale, find the frequency density of one bar.
Frequency density = frequency ÷ class width = 15 ÷ 20 = 0.75.
So the height of the first bar is 0.75.



Time, t (s)	Frequency
$0 < t \leq 20$	15
$20 < t \leq 30$	35
$30 < t \leq 40$	30
$40 < t \leq 60$	15
$60 < t \leq 100$	5

[3 marks available — 1 mark for the correct scale on the frequency density axis, 1 mark for the correct entry in the table, 1 mark for the correct bar on the histogram]

2 *This question is asking you to estimate the mean amount of time the children watched TV for, then compare that to the mean time for the adults.*

Time, m (minutes)	Frequency (f)	Mid-interval Value (x)	fx
$40 \leq m < 60$	20 × 1 = 20	50	1000
$60 \leq m < 70$	10 × 7 = 70	65	4550
$70 \leq m < 80$	10 × 4 = 40	75	3000
$80 \leq m < 120$	40 × 2 = 80	100	8000
$120 \leq m < 140$	20 × 3 = 60	130	7800
Total	270		24 350

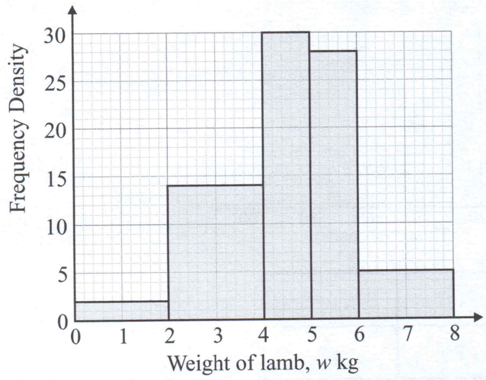
Mean for children = 24 350 ÷ 270 = 90.185...
= 90.2 minutes (to 1 d.p.)

E.g. the data supports the hypothesis since the mean time for the adults is longer than the mean time for the children, and the large samples mean the results should represent the population.
[4 marks available — 1 mark for a correct method to find the frequencies, 1 mark for multiplying the frequencies by the mid-interval values, 1 mark for the correct mean, 1 mark for a correct conclusion based on a comparison of the means]

3 a) Estimate of number of lambs between 3.5 and 4 kg
= 0.5 × 22 ***[1 mark]*** = 11
11 + (1 × 26) + (1 × 16) + (2 × 3) ***[1 mark]***
= 59 out of 100 = 59% ***[1 mark]***
[3 marks available in total — as above]

b)

Weight, w kg	$0 < w \leq 2$	$2 < w \leq 4$	$4 < w \leq 5$	$5 < w \leq 6$	$6 < w \leq 8$
Frequency	4	28	30	28	10
Frequency Density	2	14	30	28	5



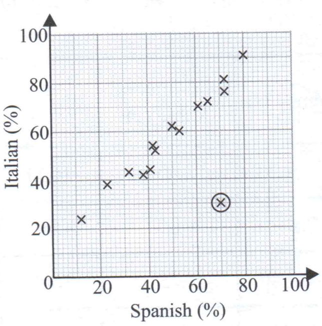
[3 marks available — 1 mark for the correct frequency densities, 1 mark for correctly labelling the axes and drawing 5 bars with no gaps, 1 mark for the correct histogram]

c) E.g. the second histogram shows more lambs with heavier weights and fewer with lighter weights than the first, which suggests there is a difference between the two farms.
[1 mark for a correct comment based on a comparison of the histograms]

Pages 109-110: Other Graphs and Charts

1 The data shows a slight downward trend in the numbers of swallows seen. ***[1 mark]***

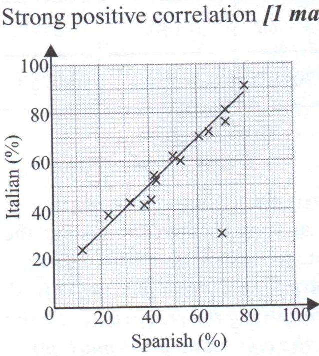
2 a)



[1 mark]

b) Strong positive correlation ***[1 mark]***

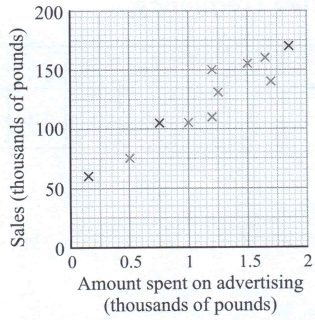
c)



[1 mark for line of best fit passing between (10, 16) & (10, 28) and (80, 82) & (80, 96)]

3 There are 360° in a circle, so
$2x + 3x + 4x + x + 90° = 360°$
$10x = 270°$
$x = 27°$
The sector for leek & potato is $3x = 3 \times 27 = 81°$,
so $\frac{81}{360} \times 80 = 18$ students chose leek & potato soup.
[4 marks available — 1 mark for forming an equation in terms of x, 1 mark for solving to find the value of x, 1 mark for a correct method to find the number of students, 1 mark for the correct answer]

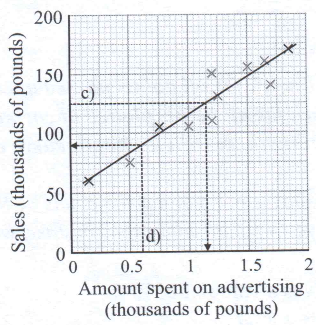
4 a)



[1 mark if all three points are plotted correctly]

b) As the amount spent on advertising increases, so does the value of sales. ***[1 mark]***
Or you could say there's a positive correlation between the amount spent and the value of sales.

c)



See graph — £1150
[2 marks available — 1 mark for drawing a line of best fit, 1 mark for reading off the correct answer, allow answers ± £100]

d) See graph above — £90 000
[1 mark, allow answers ± £10 000]

e) E.g. using the trend to predict sales for values over £3000 might be unreliable because those values are outside their range of data and they don't know whether the same pattern would continue. However, the data shows strong positive correlation, so the trend will probably continue.
[2 marks available — 1 mark for each sensible comment]

How to get answers for the Practice Papers
You can download or print out worked solutions to Practice Papers 1, 2 & 3 by going to
www.cgpbooks.co.uk/gcsemathsanswers

Formulas in the Exams

GCSE Maths uses a lot of formulas — that's no lie. You'll be scuppered if you start trying to answer a question without the proper formula to start you off. Thankfully, CGP is here to explain all things formula-related.

You're Given these Formulas

Fortunately, those lovely, cuddly examiners give you some of the formulas you need to use.

For a sphere radius *r*, or a cone with base radius *r*, slant height *l* and vertical height *h*:

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r l$

And, actually, that's your lot I'm afraid. As for the rest...

Learn All The Other Formulas

Sadly, there are a load of formulas which you're expected to be able to remember straight out of your head. There isn't space to write them all out below, but here are the highlights:

Compound Growth and Decay:

$$N = N_0\left(1 + \frac{r}{100}\right)^n$$

where N = total amount, N_0 = initial amount, r = percentage change and n = number of days/weeks/years etc.

The Quadratic Equation:

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Where $P(A)$ and $P(B)$ are the probabilities of events A and B respectively:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

or: $P(A \text{ or } B) = P(A) + P(B)$ (If A and B are mutually exclusive.)

$P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$

or: $P(A \text{ and } B) = P(A) \times P(B)$ (If A and B are independent.)

Area of trapezium $= \frac{1}{2}(a + b)h_v$

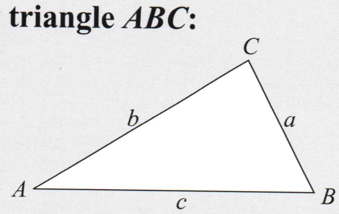
For a right-angled triangle:

Pythagoras' theorem: $a^2 + b^2 = c^2$

Trigonometry ratios:

$\sin x = \frac{\text{O}}{\text{H}}$, $\cos x = \frac{\text{A}}{\text{H}}$, $\tan x = \frac{\text{O}}{\text{A}}$

For any triangle *ABC*:



Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$

Compound Measures:

Speed $= \frac{\text{Distance}}{\text{Time}}$ Density $= \frac{\text{Mass}}{\text{Volume}}$ Pressure $= \frac{\text{Force}}{\text{Area}}$

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