Section 8 Formulas

8.1 Writing Formulas

A formula is a mathematical set of instructions that you can use for working something out. For example, $s = 4t + 3$ is a formula for s — it tells you how to find s, given the value of t. You can write formulas to help solve real-life problems mathematically.

Learning Objective — Spec Ref A3/A21:
Use given information to write a formula.

Prior Knowledge Check:
Be able to simplify algebraic expressions. See p.75-76.

To write a **formula**, you turn **information** about how different **quantities** relate to each other into **mathematical operations**. **Letters** represent quantities — they're known as **variables**. If a variable is **multiplied** by a number or another variable, you can write them without the × sign, e.g. $4 \times a = 4a$. For example, to find a formula for the amount someone is **paid**, P, if you know they're paid **£9 an hour** and work for h **hours**, **multiply** the number of hours they work by £9. So as a formula, this is $P = 9h$.

Example 1

The cost (C) of hiring a bike is £5 per hour plus a fixed cost of £25. Write a formula for the cost of hiring a bike for h hours.

1. Multiply the number of hours (h) by the cost per hour.
   Cost (in £) for h hours $= 5h$
2. Add on the fixed cost.
   So $C = 5h + 25$

Tip: The fixed cost doesn't depend on h so it can just be added on at the end. Make sure the units are consistent.

Exercise 1

Q1 Claudia owns f films. Barry owns twice as many films as Claudia.
a) How many films does Barry own?
b) How many films do Claudia and Barry own in total?
c) How many films would they own in total if they each gave away 3 of their films?

Q2 Alf has £18 in the bank. He gets a job and is paid £8 for every hour he works. Assuming he spends nothing, write a formula for the amount of money (£M) Alf will have after he has worked for h hours.

Q3 The instructions for cooking a goose are to cook for 50 minutes per kg, plus 25 minutes. Write a formula to find the time taken (t minutes) to cook a goose weighing n kg.

Q4 Write a formula for the cost (£C) of having t trees cut down if it costs p pounds per tree plus a fixed amount of £30.

Q5 A sequence of shapes is made out of matchsticks.
The first shape in the sequence is made from 4 matchsticks.
Each subsequent shape in the sequence is made by adding 3 matchsticks to the previous shape.
Write a formula for the number of matchsticks (M) needed to make the nth shape in the sequence.

PROBLEM SOLVING

8.2 Substituting into a Formula

Once you have the formula for a problem, you can substitute in values to find a solution.

Learning Objective — Spec Ref A2:
Substitute values into a given formula.

Prior Knowledge Check:
Be able to use BODMAS. See p.2.

You can **evaluate** a formula by replacing the **letters** in the formula with actual **values**. This is called **substitution**. Here's the method to follow:

- Write the formula out.
- Write it out again, substituting numbers for letters.
- Work out the calculation — using the correct order of operations (BODMAS).

Example 1

Use the formula $v = u + at$ to find v if $u = 2.6$, $a = -18.3$ and $t = 4.9$.

1. Write the formula out. $v = u + at$
2. Replace each letter with its value. $v = 2.6 + (-18.3 \times 4.9)$
3. Work out the calculation step by step — you do multiplication before addition. $v = 2.6 + (-89.67)$, $v = \mathbf{-87.07}$

Tip: This is the formula for the final velocity (v) of an object, where u is the initial velocity, a is its constant acceleration and t is time.

Exercise 1

Q1 If $x = 4$ and $y = 3$, find z when:

a) $z = x + 2$ b) $z = y - 1$ c) $z = x + y$

d) $z = 3y$ e) $z = 3y - 2$ f) $z = 6x - y$

Q2 If $a = -4$ and $b = -3$, find c when:

a) $c = a - 4$ b) $c = 4b$ c) $c = 6b - a$

d) $c = b^3$ e) $c = -\frac{4b}{2a}$ f) $c = 5a - b^2$

Q3 If $r = \frac{3}{4}$ and $s = -\frac{1}{3}$, find q when:

a) $q = 4r$ b) $q = -2s$ c) $q = rs$

d) $q = \frac{s}{r}$ e) $q = r + s$ f) $q = 4r + s$

In Questions 4 and 5, give all rounded answers to 2 decimal places.

Q4 Use the formula $v = u + at$ to find v if:

a) $u = 3$, $a = 7$ and $t = 5$ b) $u = 12$, $a = 17$ and $t = 15$

c) $u = 2.3$, $a = 4.1$ and $t = 3.4$ d) $u = 5.25$, $a = 9.81$ and $t = 4.39$

e) $u = 3$, $a = -10$ and $t = 5.6$ f) $u = -34$, $a = -1.37$ and $t = 63.25$

Q5 If $x = 12$, $y = 2.5$ and $z = -0.25$, find w if:

a) $w = x + 2y - 4z$ b) $w = -3x + y^3 - (2z)^2$ c) $w = 0.5x - yz$

d) $w = -2x^3 + y^2z$ e) $w = -\frac{12}{x} + \frac{y}{z}$ f) $w = \frac{x^2 + 3y - 8z}{2y^2}$

Wordy problems work in the same way — you write out the formula and substitute in the values. Be careful with **units** — check if the units in the **question** match the units used in the **formula**, and **convert** them if needed (see Section 22).

Example 2

Theo decides to play a game of crazy golf.

The cost of hiring crazy golf equipment, £C, is a fixed price of £3 plus 8p for every minute of use. For g minutes of crazy golf this can be written as the formula $C = 0.08g + 3$.

Theo plays for 2 hours and 30 minutes. How much will hiring the equipment cost him?

1. The formula is for time in minutes, so convert the time in hours to minutes.
   2 hours = 2 × 60 = 120 minutes
   120 + 30 = 150 minutes
   So g = 150 minutes
2. Write out the formula.
   $C = 0.08g + 3$
3. Replace each letter with its value.
   $C = 0.08(150) + 3$
4. Work out the calculation.
   $C = 12 + 3 = 15$
   So it will cost Theo **£15** to hire the equipment.

Tip: Make sure you give the final answer in the correct units and link it back to the context.

Exercise 2

Q1 The formula for working out the average speed (s, in metres per second) of a moving object is $s = \frac{d}{t}$, where d is the distance travelled (in metres) and t is the time taken (in seconds). Find the speed (in metres per second, to 2 d.p.) of each of the following:

a) a runner who travels 800 metres in 110 seconds

b) a cheetah that travels 400 metres in 14 seconds

c) a car that travels 1 km in 1 minute

d) a plane that travels 640 km in 1 hour

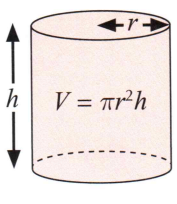
Q2 Use the formula $c = \frac{5}{9}(f - 32)$ to convert the following temperatures in degrees Fahrenheit (f) to degrees Celsius (c).

a) 212 °F b) 68 °F c) −40 °F d) 98.6 °F

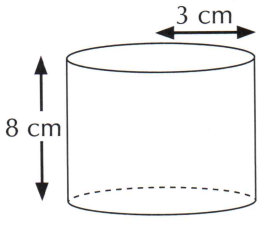
Q3 The sum (S) of the numbers $1 + 2 + 3 + ... + n$ is given by the formula $S = \frac{1}{2}n(n + 1)$. Work out the sum for each of the following.

a) $1 + 2 + 3 + ... + 10$ b) $1 + 2 + 3 + ... + 100$ c) $1 + 2 + 3 + ... + 1000$

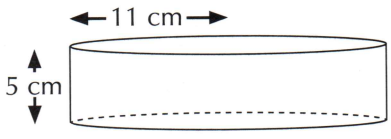
Q4 Find the volumes (V cm^3) of the cylinders below to 2 decimal places, using the formula on the left.



a)



b)



8.3 Rearranging Formulas

Rearranging formulas means making a different letter the subject, e.g. getting '$y = \frac{x}{2}$' from '$x = 2y$'.

Learning Objective — Spec Ref A5:
Rearrange formulas to change the subject.

Prior Knowledge Check:
Be able to simplify expressions (see p.75-76), expand brackets (see p.77) and factorise expressions (see p.80-81).

To **rearrange** a formula to make a **different letter** the **subject**, perform **inverse operations** one by one until that letter is **on its own** on one side of the '=' sign. You might have to **expand brackets**, **factorise** and **collect like terms** together to get the subject on its own.

You're aiming to end up with something in the form '**$Ax = B$**' (where x is the **subject term** and A and B are **numbers**, **letters** or a mix of both). You then **divide** both sides by A to get '$x = ...$'.

If you ended up with '$Ax^2 = B$', you'd need to take **square roots** after dividing by A. If you do this, remember that there's a **negative root** as well as a **positive root**, so you'll need a ± sign.

Tip: An inverse operation does the opposite of (or 'undoes') the original operation — e.g. the inverse of +8 is –8.

Example 1

Make x the subject of the formula $y = 4 + dx$.

1. You need to make x the subject, which means you need to get x on its own. $y = 4 + dx$
2. To get the dx term on its own, subtract 4 from both sides. $y - 4 = dx$
3. You now have the form '$Ax = B$', where $A = d$ and $B = y - 4$. To get x on its own, divide both sides by d. $\frac{y-4}{d} = x$
4. Write as '$x =$ '. $\boldsymbol{x = \frac{y-4}{d}}$

Tip: Subtraction is the inverse of addition, and division is the inverse of multiplication.

Example 2

Make y the subject of the formula $w = \frac{1-y}{2}$.

1. To make y the subject, you need to get it on its own. $w = \frac{1-y}{2}$
2. Multiply both sides by 2 (the denominator of the fraction) to get rid of the fraction. $2w = 1 - y$
3. Add y to both sides (so it's positive). $2w + y = 1$
4. Subtract $2w$ from each side to get y on its own. $\boldsymbol{y = 1 - 2w}$

Example 3

Make r the subject of $V = \frac{4}{3}\pi r^3$.

1. To make r the subject, you need to get it on its own. $V = \frac{4}{3}\pi r^3$
2. Multiply both sides by 3 to get rid of the fraction. $3V = 4\pi r^3$
3. Divide both sides by 4π. $\frac{3V}{4\pi} = r^3$
4. Take the cube root of each side and write as '$r =$ '. $\sqrt[3]{\frac{3V}{4\pi}} = r \Rightarrow \boldsymbol{r = \sqrt[3]{\frac{3V}{4\pi}}}$

Tip: This is the formula for the volume of a sphere of radius r — see p.368.

If the subject appears **more than once**, you're going to have to do some **factorising** (see p.80).

Example 4

Make a the subject of the formula $x(a + 1) = 3(1 - 2a)$.

1. Multiply out the brackets. $x(a + 1) = 3(1 - 2a)$
   $ax + x = 3 - 6a$
2. Collect all the a terms on one side and the non-a terms on the other. $ax + 6a = 3 - x$
3. Factorise the left-hand side to get it into the form to 'Aa = B'. $a(x + 6) = 3 - x$
4. Divide by $(x + 6)$ to get a on its own. $\boldsymbol{a = \frac{3-x}{x+6}}$

Exercise 1

Q1 Make x the subject of each of the following formulas.

a) $y = x + 2$ b) $2z = 3r + x$ c) $y = 4x$

d) $k = 2(1 + 2x)$ e) $v = \frac{2}{3}x - 2$ f) $y + 1 = \frac{x-1}{3}$

Q2 Consider the formula $w = \frac{1}{1+y}$.

a) Multiply both sides of the formula by $1 + y$. b) Hence make y the subject of the formula.

Q3 Make y the subject of the following formulas.

a) $w = \frac{3}{2y}$ b) $z + 2 = \frac{2}{1-y}$ c) $uv = \frac{1}{1-2y}$ d) $a + b = \frac{2}{4-3y}$

Q4 Consider the formula $2k = 12 - \sqrt{w-2}$.

a) Make $\sqrt{w-2}$ the subject of the formula.

b) By first squaring both sides of your answer to part a), make w the subject of the formula.

Q5 Make w the subject of the following formulas.

a) $a = \sqrt{w}$ b) $x = 1 + \sqrt{w}$ c) $y = \sqrt{w-2}$

d) $f - 3 = 2\sqrt{w}$ e) $j = \sqrt{3 + 4w}$ f) $a = \sqrt{1-2w}$

Q6 Consider the formula $t = 1 - 3(z + 1)^2$.

a) Make $(z + 1)^2$ the subject of the formula.

b) By first square rooting both sides of your answer to part a), make z the subject of the formula.

Q7 Make z the subject of the following formulas.

a) $x = 1 + z^2$ b) $2t = 3 - z^2$ c) $xy = 1 - 4z^2$

d) $t + 2 = 3(z - 2)^2$ e) $g = 4 - (2z + 3)^2$ f) $r = 4 - 2(5 - 3z)^2$

Q8 Make a the subject of the following formulas.

a) $x(a + b) = a - 1$ b) $x - ab = c - ad$ c) $c = \frac{1+a}{1-2a}$ d) $2e = \frac{2+3a}{a}$

Review Exercise

Q1 Melanie has half as many sweets as Jane.

a) If Jane has j sweets, write a formula to calculate the number of sweets (m) that Melanie has.

b) Find m when $j = 24$.

Q2 To book a swimming pool for a party, there is a fixed charge of £30 plus a fee of £1.25 for each person who attends.

a) Write a formula to calculate the hire cost (£C) for n people.

b) Calculate C when $n = 32$.

Q3 To hire skates at the park there is a fixed charge of £5, plus a charge of £1.70 for each half-hour.

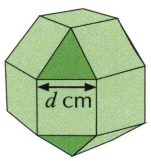
a) Write a formula to calculate the cost (£C) for h half-hour periods.

b) Calculate the cost of hiring skates for two and a half hours.

c) Rearrange your formula to make h the subject.

d) Asher spends £15.20 on hiring skates. How long was he skating for?

Q4 The surface area (A cm^2) of the shape on the right is given approximately by the formula $A = 21.5d^2$.



a) Rearrange the formula to make d the subject.

b) Find d if $A = 55$ cm^2. Give your answer to 2 s.f.

Q5 The time in minutes (T) taken to cook a joint of beef is given by $T = 35w + 25$, where w is the weight of the joint in kg.

a) How long would it take to cook a 1.5 kg joint?

b) Make w the subject of the formula.

c) What weight of beef needs to be cooked for 207 minutes?

Q6 For each of the following formulas, (i) make x the subject, and (ii) find x when $y = -1$.

a) $-2 + y = \dfrac{3}{4-x}$

b) $y = \dfrac{1}{\sqrt{1-x}}$

c) $2(1-x) = y(3+x)$

d) $y = \dfrac{2-3x}{1+2x}$

e) $y = 8 - \dfrac{1}{\sqrt{x}}$

f) $2y - 1 = 3\sqrt{2-x}$

Q7 Consider the formula $s = \left(\dfrac{u+v}{2}\right)t$. By rearranging the formula where necessary, find the value of:

a) s when $u = 2.3$, $v = 1.7$ and $t = 4$.

b) t when $s = 3.3$, $u = 1$ and $v = 2$.

c) u when $s = 4.5$, $t = 6$ and $v = 7$.

d) v when $s = 0.5$, $t = 0.25$ and $u = 3$.

Q8 Consider the formula $x = \dfrac{1+\sqrt{y+3}}{2-z}$.

a) Find the value of x when $y = 1$ and $z = -1$.

b) By first rearranging the formula, find the value of z when $y = 6$ and $x = -2$.

Exam-Style Questions

Q1 Olivia is on holiday in Las Vegas. She sees a TV weather forecast which reports that today's maximum temperature will be 104° Fahrenheit. The formula $C = \frac{5}{9}(F - 32)$ can be used to convert temperatures in Fahrenheit (F) to Celsius (C).

Work out today's forecast maximum temperature in degrees Celsius.

[2 marks]

Q2 Florence has some matchsticks. The number of matchsticks (m) needed to make a pattern of h hexagons is given by the formula $m = 5h + 1$.

a) How many matchsticks will Florence need to make a pattern of 6 hexagons?

[1 mark]

b) (i) Rearrange the formula to make h the subject.

[2 marks]

(ii) How many hexagons will be in the pattern made with 36 matchsticks?

[1 mark]

Q3 The minimum velocity required for a rocket to leave a planet can be found using the formula $V = \sqrt{\frac{2GM}{r}}$. Make M the subject of this formula. Show your working.

[3 marks]

Q4 You are given the formula $g = \frac{8}{5}h + 17$.

a) Rearrange the formula to make h the subject.

[2 marks]

b) Find h if:

(i) $g = 209$

[1 mark]

(ii) $g = -15$

[1 mark]

Q5 A quarterly gas bill has a fixed charge of £7.50 plus 8p for every unit of gas used.

a) Write the formula to calculate £C, the cost for n units of gas.

[2 marks]

b) José uses 760 units of gas. How much will he have to pay?

[1 mark]

c) Rearrange your formula from part a) to make n the subject.

[2 marks]

d) Anna's gas bill is £39.50. How many units of gas did she use?

[1 mark]