Answers

Section One — Number

Pages 3-4: Fractions

1 a) Ratio of girls to boys in Year 7 is 3 : 2, so fraction of Year 7 that are girls = $\frac{3}{3+2} = \frac{3}{5}$ ***[1 mark]***.
$20\% = \frac{1}{5}$ of girls in Year 7 have blonde hair, so $\frac{1}{5} \times \frac{3}{5} = \frac{3}{25}$ of the pupils in Year 7 are girls with blonde hair ***[1 mark]***.
Fraction of whole school that are Year 7 girls:
$\frac{1}{5} \times \frac{3}{25} = \frac{3}{125}$ ***[1 mark]***
[3 marks available in total — as above]

b) E.g. The answer to part a) would convert to a terminating decimal because the only prime factor of the denominator is 5.
[1 mark for a correct conclusion based on part a)]

2 a) $\frac{a}{11} + \frac{b}{6} = \frac{6a + 11b}{66}$ ***[1 mark]***
$\frac{25}{33} = \frac{50}{66}$, so $\frac{6a+11b}{66} = \frac{50}{66}$ and $6a + 11b = 50$ ***[1 mark]***
Putting in $a = 1$ gives $11b = 50 - 6 = 44$, so $b = 44 \div 11 = 4$
[1 mark for both a and b]
[3 marks available in total — as above]
You might have to use trial and error to find the values of a and b.

b) $\frac{25}{33}$ is equivalent to $\frac{75}{99}$ ***[1 mark]*** $= 0.\dot{7}\dot{5}$ ***[1 mark]***
[2 marks available in total — as above]
You could also do this one by working out 25 ÷ 33.

3 Let $r = 0.14\dot{6}$. Then $100r = 14.\dot{6}$ and $1000r = 146.\dot{6}$
$1000r - 100r = 146.\dot{6} - 14.\dot{6}$, so $900r = 132$
$r = \frac{132}{900}$, so $1.14\dot{6} = 1\frac{132}{900}$
$\frac{96}{180} = \frac{480}{900}$, so $\frac{96}{180} + 1.14\dot{6} = \frac{480}{900} + 1\frac{132}{900} = 1\frac{612}{900} = 1\frac{17}{25}$
[5 marks available — 1 mark for a correct method for converting the recurring decimal to a fraction, 1 mark for a correct fraction, 1 mark for putting both fractions over a common denominator, 1 mark for adding the fractions, 1 mark for simplifying correctly]

4 $0.\dot{2} = \frac{2}{9}$, so $2.\dot{2} = 2\frac{2}{9} = \frac{20}{9}$
Area of one tile = $\left(\frac{20}{9}\right)^2 = \frac{400}{81}$ cm^2
Heather will need $1600 \div \frac{400}{81} = 1600 \times \frac{81}{400} = 324$ tiles
[4 marks available — 1 mark for writing the side length as a fraction, 1 mark for finding the area of the tile, 1 mark for dividing the total area by the area of one tile, 1 mark for the correct answer]

5 Let $r = 0.1\dot{2}\dot{7}$. Then $10r = 1.\dot{2}\dot{7}$ and $1000r = 127.\dot{2}\dot{7}$
$1000r - 10r = 127.\dot{2}\dot{7} - 1.\dot{2}\dot{7}$, so $990r = 126$
$r = \frac{126}{990} = \frac{14}{110} = \frac{7}{55} = \frac{35}{275}$
$\frac{160}{1375} = \frac{32}{275}$
So $\frac{38}{275}$ is the biggest.
[4 marks available — 1 mark for a correct method for converting the recurring decimal to a fraction, 1 mark for a correct fraction, 1 mark for putting all converted fractions over a common denominator, 1 mark for stating which is biggest]

6 Let $r = 0.0\dot{4}$. Then $10r = 0.\dot{4}$ and $100r = 4.\dot{4}$
$100r - 10r = 4.\dot{4} - 0.\dot{4}$, so $90r = 4$
$r = \frac{4}{90} = \frac{2}{45}$, so $\frac{7x-3}{6} = \frac{2}{45}$
$7x - 3 = \frac{12}{45} = \frac{4}{15}$
$7x = 3 + \frac{4}{15} = \frac{49}{15}$, so $x = \frac{7}{15}$
[4 marks available — 1 mark for a correct method for converting the recurring decimal to a fraction, 1 mark for a correct fraction, 1 mark for a correct method for solving the equation, 1 mark for the correct answer]
There are other ways of rearranging the equation for this one.

Pages 5-6: Bounds

1 a) $4z^3 = \frac{(x^{\frac{1}{2}}y^{-3}z)^2}{y^{-5}} = \frac{xy^{-6}z^2}{y^{-5}} = xy^{-1}z^2$
$4z = \frac{x}{y}$, so $z = \frac{x}{4y}$
[3 marks available — 3 marks for the correct answer, otherwise 1 mark for simplifying the powers on the numerator, 1 mark for dividing both sides by z^2]

b) Upper bound for $x = 6.85$, lower bound for $y = 1.15$
Upper bound for $z = \frac{6.85}{4 \times 1.15} = 1.489... = 1.49$ (3 s.f.)
[3 marks available — 1 mark for finding the upper bound of x, 1 mark for finding the lower bound of y, 1 mark for the correct answer]

2 Maximum possible score before the vault:
upper bound = 16.425 + 13.155 + 14.885 = 44.465 ***[1 mark]***
Lowest possible leading score = 60.145
Shannon's minimum score after the vault
= 60.145 + 0.05 = 60.195 ***[1 mark]***
So lowest possible score for the vault
= 60.195 – 44.465 = 15.73 ***[1 mark]***
[3 marks available in total — as above]

3 Lower bound for $c = 11.75$, upper bound for $a = 6.25$ ***[1 mark]***
So lower bound for scale factor = 11.75 ÷ 6.25 = 1.88 ***[1 mark]***
Lower bound for $b = 3.45$ ***[1 mark]***
Lower bound for $d = 3.45 \times 1.88 = 6.486$ cm ***[1 mark]***
[4 marks available in total — as above]

4 Lower bound for $S = 179.15$
Upper bound for $S = 179.25$ ***[1 mark for both]***
Lower bound for $x = 59.5$
Upper bound for $x = 60.5$ ***[1 mark for both]***
$S = \frac{x}{360} \times \pi r^2$, so $r^2 = \frac{360S}{x\pi}$ ***[1 mark]***
Lower bound for r: $r^2 = \frac{360 \times 179.15}{60.5 \times \pi} = 339.323...$,
so $r = 18.420... = 18.42$ (2 d.p.) ***[1 mark]***
Upper bound for r: $r^2 = \frac{360 \times 179.25}{59.5 \times \pi} = 345.219...$,
so $r = 18.580... = 18.58$ (2 d.p.) ***[1 mark]***
[5 marks available in total — as above]

5 Lower bound for $A = 2850$, upper bound for $A = 2950$
Lower bound for $x = 96.95$, upper bound for $x = 97.05$
Lower bound for $y = 78.85$, upper bound for $y = 78.95$
[2 marks for all bounds correct, otherwise 1 mark for four or five bounds correct]
Lower bound for θ: $\sin\theta = \frac{2 \times 2850}{97.05 \times 78.95} = 0.743...$
so $\theta = \sin^{-1}(0.743...) = 48.066...°$ ***[1 mark]***
Upper bound for θ: $\sin\theta = \frac{2 \times 2950}{96.95 \times 78.85} = 0.771...$
so $\theta = \sin^{-1}(0.771...) = 50.515...°$ ***[1 mark]***
Value of θ using the rounded values:
$\sin\theta = \frac{2 \times 2900}{97.0 \times 78.9} = 0.757...$
so $\theta = \sin^{-1}(0.757...) = 49.274...°$ ***[1 mark]***
Difference between lower bound and rounded value
= 49.274... – 48.066... = 1.207...°
Difference between upper bound and rounded value
= 50.515... – 49.274... = 1.241...° ***[1 mark for both]***
So maximum possible error = 1.24° (3 s.f.) ***[1 mark]***
[7 marks available in total — as above]

Pages 7-8: Standard Form

1 $(3 \times 10^{11})^4 = 3^4 \times (10^{11})^4 = 81 \times 10^{44}$ ***[1 mark]*** $= 8.1 \times 10^{45}$ ***[1 mark]***
[2 marks available in total — as above]

2 50% of $6.4 \times 10^5 = 3.2 \times 10^5$ acres,
10% of $6.4 \times 10^5 = 0.64 \times 10^5$ acres,
5% of $6.4 \times 10^5 = 0.32 \times 10^5$ acres, ***[1 mark for suitable method]***
so 65% $= 3.2 \times 10^5 + 0.64 \times 10^5 + 0.32 \times 10^5$
$= (3.2 + 0.64 + 0.35) \times 10^5 = 4.16 \times 10^5$ acres of woodland ***[1 mark]***
If three-quarters are protected, then one quarter is not protected:
$4.16 \times 10^5 \div 4 = 1.04 \times 10^5 = 104\,000$ acres are not protected
[1 mark].
[3 marks available in total — as above]

3 $\frac{25^2 \times 6}{2^2 \times 50^4} = \frac{25 \times 25 \times 6}{2 \times 2 \times 50 \times 50 \times 50 \times 50} = \frac{6}{2 \times 2 \times 2 \times 2 \times 50 \times 50}$
$= \frac{3}{2 \times 2 \times 2 \times 50 \times 50} = \frac{3}{2} \times \frac{1}{2 \times 2 \times 50 \times 50}$
$= \frac{3}{2} \times \frac{1}{10\,000} = 1.5 \times 10^{-4}$
[3 marks available — 1 mark for expanding powers, 1 mark for cancelling common factors, 1 mark for the correct answer in standard form]
There are other ways to simplify this, but you'll get the same answer.

4 Upper bound for weight $= 4.25 \times 10^4 = 42\,500$ N
Lower bound for area $= 25$ m^2
Upper bound for pressure $= 42\,500 \div 25 = 1700$ N/m$^2 > 1600$ N/m^2,
so it is not definitely safe for the shipping container to be transported on the cargo ship.
[3 marks available — 1 mark for finding the upper bound for weight and the lower bound for area, 1 mark for the upper bound for pressure, 1 mark for the conclusion]

5 Volume of salt in the Heron Sea = 12% of 1.4×10^{14}
$= 0.12 \times 1.4 \times 10^{14} = 1.68 \times 10^{13}$ litres ***[1 mark]***
Volume of salt in the Cobalt Sea = 8% of 8.5×10^{12}
$= 0.08 \times 8.5 \times 10^{12} = 6.8 \times 10^{11}$ litres ***[1 mark]***
Percentage decrease: $\frac{1.68 \times 10^{13} - 6.8 \times 10^{11}}{1.68 \times 10^{13}} \times 100$
$= 95.952...\% = 95.95\%$ (2 d.p.) ***[1 mark]***
[3 marks available in total — as above]

6 1.8×10^{12} kg $= 1.8 \times 10^{15}$ g
Number of muffins $= 1.8 \times 10^{15} \div 120 = 1.8 \times 10^{15} \div 1.2 \times 10^2$
$= \frac{1.8}{1.2} \times \frac{10^{15}}{10^2} = 1.5 \times 10^{13}$
Number of muffins eaten per person $= 1.5 \times 10^{13} \div 7.2$ billion
$= 1.5 \times 10^{13} \div 7.2 \times 10^9 = \frac{1.5}{7.2} \times \frac{10^{13}}{10^9} \approx 0.2 \times 10^4 = 2000$
The newspaper is not likely to be correct, as it would mean that last year every person in the world ate over 2000 muffins each (which is more than 5 per day).
[4 marks available — 1 mark for converting either mass, 1 mark for finding the number of muffins, 1 mark for estimating the number of muffins eaten per person, 1 mark for stating that the newspaper's claims are incorrect with a suitable explanation]

7 $a = 2^{10} \times 5^9$
$b = (3 \times 3) \times 10^6 = (3 \times 3) \times (2 \times 5)^6 = 2^6 \times 3^2 \times 5^6$ ***[1 mark]***
$c = 24 \times 10^8 = 2 \times 2 \times 2 \times 3 \times (2 \times 5)^8 = 2^{11} \times 3 \times 5^8$ ***[1 mark]***
LCM $= 2^{11} \times 3^2 \times 5^9$ ***[1 mark]*** $= 2^2 \times 3^2 \times (2 \times 5)^9 = 2^2 \times 3^2 \times 10^9$
$= 36 \times 10^9 = 3.6 \times 10^{10}$ ***[1 mark]***
[4 marks available in total — as above]

Section Two — Algebra

Pages 9-10: Powers

1 $111^{\frac{1}{4}} \times 111^{\frac{1}{4}} = 111^{\frac{1}{4}+\frac{1}{4}} = 111^{\frac{1}{2}} = \sqrt{111}$
$10^2 = 100$ and $11^2 = 121$, so $\sqrt{111}$ lies between 10 and 11.
111 is just closer to 121 that 100, so $\sqrt{111} \approx 10.5$ to 3 s.f.
[2 marks available — 1 mark for using the rules of powers to simplify the expression, 1 mark for a sensible estimation to 3 s.f. (allow 10.4 or 10.6)]
You might have been tempted to estimate the quartic (4th) root of 111 and multiply the results together, but this would be very tricky.

2 $81^{\frac{3}{4}} = (81^{\frac{1}{4}})^3$ ***[1 mark]*** $= (\sqrt[4]{81})^3 = 3^3 = 27$ ***[1 mark]***
[2 marks available in total — as above]

3 $a^7 \times (25a^6b^{10}c^5)^{\frac{1}{2}} = \sqrt{25}\,a^{(6 \div 2)+7}b^{10 \div 2}c^{5 \div 2} = 5a^{10}b^5c^{\frac{5}{2}}$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for at least 2 of 5, a^{10}, b^5 or $c^{\frac{5}{2}}$ correct]

4 $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$ ***[1 mark]*** and $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ ***[1 mark]***
So $125^{\frac{1}{3}} \times 3^{-2} = 5 \times \frac{1}{9} = \frac{5}{9} = 0.555... = 0.\dot{5}$ ***[1 mark]***
[3 marks available in total — as above]

5 $\left(\frac{64}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{64}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$
[2 marks available — 1 mark for inverting the fraction and making the power positive, 1 mark for the correct answer]

6 $\left(\frac{729}{8x}\right)^{\frac{1}{3}} = \frac{9}{4}$, so $8x = 4^3 = 64$ ***[1 mark]***, which means $x = 8$ ***[1 mark]***
[2 marks available in total — as above]

7 $b = (4c+3)^{\frac{1}{3}}$, so $b^3 = (4c+3)^{\frac{1}{3} \times 3} = 4c + 3$ ***[1 mark]***
and $b^6 = (4c + 3)^2 = 16c^2 + 24c + 9$ ***[1 mark]***
So $a = 3b^3 + 2b^6 = 3(4c + 3) + 2(16c^2 + 24c + 9)$
$= 12c + 9 + 32c^2 + 48c + 18 = 32c^2 + 60c + 27$ ***[1 mark]***
[3 marks available in total — as above]

8 $\left(\frac{64}{49}\right)^{-\frac{x}{y}} = \left(\frac{49}{64}\right)^{\frac{x}{y}} = \frac{343}{512}$, so $(\sqrt[y]{49})^x = 343$ and $(\sqrt[y]{64})^x = 512$
$\sqrt{49} = 7$ and $7^3 = 343$, and $\sqrt{64} = 8$ and $8^3 = 512$,
so $x = 3$ and $y = 2$
[3 marks available — 1 mark for inverting the fraction and making the power positive, 1 mark for forming two equations in terms of x and y, 1 mark for finding the values of x and y]
You have to use a bit of trial and error to find the values of x and y.

9 $2\frac{7}{9} = \frac{25}{9}$ and $3\frac{1}{3} = \frac{10}{3}$ ***[1 mark for both]***
$\left(2\frac{7}{9}\right)^{-\frac{1}{2}} = \left(\frac{25}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{25}\right)^{\frac{1}{2}}$ ***[1 mark]*** $= \frac{3}{5}$ ***[1 mark]***
$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ ***[1 mark]***
So $\frac{\left(2\frac{7}{9}\right)^{-\frac{1}{2}} \times 3\frac{1}{3}}{2^{-2}} = \frac{\frac{3}{5} \times \frac{10}{3}}{\frac{1}{4}} = 2 \div \frac{1}{4} = 8$ ***[1 mark]***
[5 marks available in total — as above]

Pages 11-12: Surds

1 $(\sqrt{2})^2 = 2$, $(\sqrt{2})^3 = 2\sqrt{2}$, $(\sqrt{2})^4 = 4$, $(\sqrt{2})^5 = 4\sqrt{2}$
So $\sqrt{2} + (\sqrt{2})^2 + (\sqrt{2})^3 + (\sqrt{2})^4 + (\sqrt{2})^5$
$= \sqrt{2} + 2 + 2\sqrt{2} + 4 + 4\sqrt{2} = 6 + 7\sqrt{2}$
[3 marks available — 3 marks for the correct answer, otherwise 2 marks for expanding all four brackets correctly, otherwise 1 mark for expanding two or three brackets correctly]

2 $\sqrt{343} = \sqrt{49 \times 7} = 7\sqrt{7}$ ***[1 mark]***, $\frac{21}{\sqrt{7}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$ ***[1 mark]***
and $4\sqrt{252} = 4\sqrt{36 \times 7} = 24\sqrt{7}$ ***[1 mark]***
So $\sqrt{343} + \frac{21}{\sqrt{7}} - 4\sqrt{252} = 7\sqrt{7} + 3\sqrt{7} - 24\sqrt{7}$
$= -14\sqrt{7}$ ***[1 mark]***
[4 marks available in total — as above]

3 Using Pythagoras' theorem:
$a^2 = (6 + \sqrt{3})^2 - (3 + 2\sqrt{3})^2$ ***[1 mark]***
$= (36 + 6\sqrt{3} + 6\sqrt{3} + 3) - (9 + 6\sqrt{3} + 6\sqrt{3} + 12)$
$= (39 + 12\sqrt{3}) - (21 + 12\sqrt{3})$ ***[1 mark]***
$= 39 - 21 = 18$ ***[1 mark]***
$a^2 = 18$, so $a = \sqrt{18} = 3\sqrt{2}$ cm ***[1 mark]***
[4 marks available in total — as above]

4 $(\sqrt{5} - 6)^3 = (\sqrt{5} - 6)(\sqrt{5} - 6)(\sqrt{5} - 6)$
$= (5 - 6\sqrt{5} - 6\sqrt{5} + 36)(\sqrt{5} - 6)$
$= (41 - 12\sqrt{5})(\sqrt{5} - 6)$
$= 41\sqrt{5} - 246 - 60 + 72\sqrt{5} = 113\sqrt{5} - 306$
[3 marks available — 1 mark for expanding the first pair of brackets, 1 mark for expanding the new pair of brackets, 1 mark for the correct answer]

5 Volume $= \sqrt{5}(1+\sqrt{5})(2+3\sqrt{5}) = (\sqrt{5}+5)(2+3\sqrt{5})$
$= 2\sqrt{5} + 15 + 10 + 15\sqrt{5} = 25 + 17\sqrt{5}$ cm^3
[4 marks available — 1 mark for multiplying either bracket by $\sqrt{5}$, 1 mark for multiplying the result by the remaining bracket, 1 mark for correct expansion, 1 mark for the correct answer]

6 $\frac{2\sqrt{3}}{3+\sqrt{3}} = \frac{2\sqrt{3}(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})} = \frac{6\sqrt{3}-6}{9-3} = \frac{6\sqrt{3}-6}{6} = \sqrt{3}-1$

$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{7+4\sqrt{3}}{4-3} = \frac{7+4\sqrt{3}}{1} = 7+4\sqrt{3}$

So $\frac{2\sqrt{3}}{3+\sqrt{3}} + \frac{2+\sqrt{3}}{2-\sqrt{3}} = (\sqrt{3}-1)+(7+4\sqrt{3}) = 6+5\sqrt{3}$
[5 marks available — 2 marks for correctly rationalising the denominator of the first fraction (or 1 mark for multiplying by $(3-\sqrt{3})$), 2 marks for correctly rationalising the denominator of the second fraction (or 1 mark for multiplying by $(2+\sqrt{3})$), 1 mark for the correct answer]

7 $(1+2\sqrt{2})^2 = (1+2\sqrt{2})(1+2\sqrt{2}) = 1+2\sqrt{2}+2\sqrt{2}+8$
$= 9+4\sqrt{2}$ ***[1 mark]***

So $\frac{(1+2\sqrt{2})^2}{\sqrt{2}-1} = \frac{9+4\sqrt{2}}{\sqrt{2}-1} = \frac{(9+4\sqrt{2})(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$ ***[1 mark]***

$= \frac{17+13\sqrt{2}}{2-1}$ ***[1 mark]*** $= 17+13\sqrt{2}$ ***[1 mark]***

[4 marks available in total — as above]

Pages 13-14: Quadratic Equations

1 Surface area of a sphere $= 4\pi r^2$
$36\pi x^2 + 48\pi x + 16\pi = 4\pi(9x^2+12x+4) = 4\pi(3x+2)^2$
So $(3x+2)^2 = r^2$, which means the radius is $(3x+2)$ cm
[3 marks available — 1 mark for taking out a factor of 4π, 1 mark for factorising, 1 mark for square rooting to find the answer]

2 a) $2x^2 - 3x - 35 = (2x+7)(x-5)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

b) $2(2x-1)^2 - 3(2x-1) - 35 = 0$
$(2(2x-1)+7)((2x-1)-5) = 0$ ***[1 mark]***
$(4x+5)(2x-6) = 0$ ***[1 mark]***,
so $x = -1.25$ or $x = 3$ ***[1 mark for both]***
[3 marks available in total — as above]

3 $\frac{x}{2x+1} - \frac{x+3}{x-1} = 2$
$x(x-1) - (x+3)(2x+1) = 2(x-1)(2x+1)$
$x^2 - x - 2x^2 - x - 6x - 3 = 2(2x^2 + x - 2x - 1)$
$-x^2 - 8x - 3 = 4x^2 - 2x - 2$
$0 = 5x^2 + 6x + 1 = (5x+1)(x+1)$, so $x = -0.2$ or $x = -1$
[4 marks available — 1 mark for multiplying through by (x – 1)(2x + 1), 1 mark for expanding brackets and rearranging to give a quadratic in the standard format, 1 mark for factorising, 1 mark for both correct answers]

4 a) $3x^2 - 14x - 24 = (3x+4)(x-6)$
[2 marks available — 1 mark for correct numbers in brackets, 1 mark for correct signs]

b) $3x^2 - 14x - 24 = (3x+4)^2$
$(3x+4)(x-6) = (3x+4)^2$
$(3x+4)(x-6) - (3x+4)^2 = 0$ ***[1 mark]***
$(3x+4)[(x-6)-(3x+4)] = 0$
$(3x+4)(-2x-10) = 0$ ***[1 mark]***
So $x = -\frac{4}{3}$ ***[1 mark]*** or $x = -5$ ***[1 mark]***
[4 marks available in total — as above]
Don't be tempted to cancel a factor of (3x + 4) from each side of the equation — you'd only end up with one solution instead of two.

5 Surface area of a cylinder $= 2\pi rh + 2\pi r^2$
So $2\pi r + 2\pi r^2 = 31\pi$ (as $h = 1$) ***[1 mark]***
$2r^2 + 2r - 31 = 0$ ***[1 mark]***

$r = \frac{-2 \pm \sqrt{2^2 - (4\times2\times-31)}}{2\times2} = \frac{-2\pm\sqrt{252}}{4}$ ***[1 mark]***

$= \frac{-2\pm6\sqrt{7}}{4} = \frac{-1\pm3\sqrt{7}}{2}$

The radius must be positive, so $r = \frac{-1+3\sqrt{7}}{2}$ m ***[1 mark]***.
[4 marks available in total — as above]
You're asked for the exact value of r, so leave your answer in surd form.

6 $\frac{1}{x} + \frac{6}{x+2} = 5$, so $x + 2 + 6x = 5x(x+2)$
$7x + 2 = 5x^2 + 10x$
$0 = 5x^2 + 3x - 2 = (5x-2)(x+1)$, so $x = \frac{2}{5}$ or $x = -1$

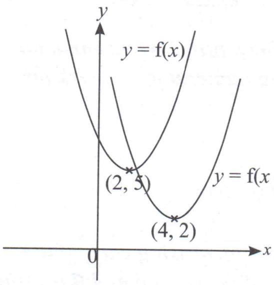
$\frac{1}{1-3x}$ is positive when $x = -1$, so $\frac{1}{1-3(-1)} = \frac{1}{4}$.
[5 marks available — 1 mark for multiplying through by x(x + 2), 1 mark for expanding brackets and rearranging to give a quadratic in the standard form, 1 mark for factorising, 1 mark for both correct values of x, 1 mark for the correct answer]

Pages 15-16: Completing the Square

1 $7 \div 2 = \frac{7}{2}$, so $a = \frac{7}{2}$ and the brackets are $\left(x+\frac{7}{2}\right)^2$ ***[1 mark]***.
Expanding the brackets: $\left(x+\frac{7}{2}\right)^2 = x^2 + 7x + \frac{49}{4}$ ***[1 mark]***
To complete the square: $11 - \frac{49}{4} = -\frac{5}{4}$, so $b = -\frac{5}{4}$ ***[1 mark]***
So $x^2 + 7x + 11 = \left(x+\frac{7}{2}\right)^2 - \frac{5}{4}$.
[3 marks available in total — as above]

2 a) f(x) has a turning point at (2, 5), so the completed square form is $(x-2)^2 + 5$. Expanding the brackets:
$f(x) = (x-2)^2 + 5 = x^2 - 4x + 4 + 5 = x^2 - 4x + 9$
So $p = -4$ and $q = 9$.
[3 marks available — 1 mark for using the turning point to find the completed square form, 1 mark for expanding the brackets, 1 mark for the values of p and q]

b) $f(x-2) - 3 = (x-2-2)^2 + 5 - 3 = (x-4)^2 + 2$, so the turning point on the transformed graph has coordinates (4, 2).
[2 marks available — 1 mark for each correct coordinate]
This is a translation by 2 units right and 3 units down, so you could add/subtract these values from the original turning point.

c)


[2 marks available — 1 mark for x-translation, 1 mark for y-translation]

3 $r(x+4)^2 + t = rx^2 + 8rx + 16r + t$
Equating coefficients with $3x^2 + sx + 29$ gives $3x^2 = rx^2$,
so $r = 3$ ***[1 mark]***. $sx = 8rx$, so $s = 8r = 8 \times 3 = 24$ ***[1 mark]***.
$29 = 16r + t$, so $t = 29 - (16 \times 3) = -19$ ***[1 mark]***.
$y = 3(x+4)^2 - 19$ has a turning point at $(-4, -19)$ ***[1 mark]***.
[4 marks available in total — as above]

4 As the turning point is (4, 7), the equation of the curve can be written as $y = (x-4)^2 + 7$ ***[1 mark]***
Substituting in $x = 11$ gives
$y = (11-4)^2 + 7 = 49 + 7 = 56$ (as required) ***[1 mark]***
[2 marks available in total — as above]

5 a) To find the turning point, complete the square:
$3x - x^2 + 5 = -(x^2 - 3x - 5)$
$-3 \div 2 = -\frac{3}{2}$, so the first bit is $-\left(x-\frac{3}{2}\right)^2$ ***[1 mark]***.
Expanding brackets gives $-x^2 + 3x - \frac{9}{4}$
To complete the square: $5 - -\frac{9}{4} = \frac{29}{4}$ ***[1 mark]***
So the completed square form is $-\left(x-\frac{3}{2}\right)^2 + \frac{29}{4}$
So the turning point has coordinates $\left(\frac{3}{2}, \frac{29}{4}\right)$ ***[1 mark]***.
[3 marks available in total — as above]

b) The turning point will be a maximum as the coefficient of x^2 is negative (so the quadratic is n-shaped) ***[1 mark]***.

6 a) Dividing the first two terms by 5: $5(x^2 + 4x) + 12$
$4 \div 2 = 2$, so the first bit is $5(x + 2)^2$
Expanding brackets gives: $5(x + 2)^2 = 5(x^2 + 4x + 4)$
$= 5x^2 + 20x + 20$
To complete the square: $12 - 20 = -8$
So $5x^2 + 20x + 12 = 5(x + 2)^2 - 8$
[4 marks available — 1 mark for dividing the first two terms by 5, 1 mark for finding the value of v, 1 mark for finding the value of w, 1 mark for the fully correct answer]

b) $5x^2 + 20x + 12 = 0$
So $5(x + 2)^2 - 8 = 0$
$(x + 2)^2 = \frac{8}{5}$
$x + 2 = \pm\sqrt{\frac{8}{5}}$, so $x = -2 \pm\sqrt{\frac{8}{5}}$
$x = -0.735$ (3 s.f.) or $x = -3.26$ (3 s.f.)
[2 marks available — 1 mark for rearranging the completed square, 1 mark for both correct x-values]

Pages 17-18: Algebraic Fractions

1 $\frac{2v^2-18}{v^2+3v} \times \frac{v^2-v}{v^2+8v-9} = \frac{2(v+3)(v-3)}{v(v+3)} \times \frac{v(v-1)}{(v+9)(v-1)}$
$= \frac{2(v-3)}{v+9}$
[5 marks available — 1 mark for factorising each of the numerators and denominators, 1 mark for the correct answer]

2 $\frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times b = \frac{1}{5x^2-80y^2} \times (40y - 10x)$
$= \frac{10(4y-x)}{5(x+4y)(x-4y)} = \frac{-10(x-4y)}{5(x+4y)(x-4y)} = \frac{-2}{x+4y}$
[4 marks available — 1 mark for multiplying by b, 1 mark for factorising a, 1 mark for factorising b, 1 mark for the correct answer]
Here you had to spot that $(4y - x) = -1 \times (x - 4y)$.

3 $\frac{3}{x} + \frac{2x}{x+4} = \frac{3(x+4)+x(2x)}{x(x+4)} = \frac{2x^2+3x+12}{x(x+4)}$
[3 marks available — 1 mark for putting over a common denominator, 1 mark for adding numerators, 1 mark for correctly simplifying]

4 $\frac{x+7}{x^2} \times \frac{x^2+2x}{x^2-49} \times \frac{6x-42}{3x+6}$
$= \frac{x+7}{x^2} \times \frac{x(x+2)}{(x+7)(x-7)} \times \frac{6(x-7)}{3(x+2)} = \frac{2}{x}$
[5 marks available — 1 mark for factorising each of the numerators and denominators of the second and third fractions, 1 mark for the correct answer]

5 $\frac{1}{x^2} + \frac{x+3}{x-2} - \frac{4}{x} = \frac{(x-2)+x^2(x+3)-4x(x-2)}{x^2(x-2)}$
$= \frac{x-2+x^3+3x^2-4x^2+8x}{x^2(x-2)} = \frac{x^3-x^2+9x-2}{x^2(x-2)}$
[4 marks available — 1 mark for multiplying top and bottom of the first fraction by (x – 2), 1 mark for multiplying top and bottom of the second fraction by x², 1 mark for multiplying top and bottom of the third fraction by x(x – 2), 1 mark for simplifying]

6 $\frac{x^2-5}{2x^2-7x-4} \times \frac{2x+1}{x-\sqrt{5}} = \frac{(x+\sqrt{5})(x-\sqrt{5})}{(2x+1)(x-4)} \times \frac{2x+1}{x-\sqrt{5}} = \frac{x+\sqrt{5}}{x-4}$
[3 marks available — 1 mark for correctly factorising x² – 5, 1 mark for factorising the denominator of the first fraction, 1 mark for the correct answer]

7 $\frac{14x-35}{2x^2+x-15} \div \frac{4xy-12y}{2x^2y-18y} = \frac{14x-35}{2x^2+x-15} \times \frac{2x^2y-18y}{4xy-12y}$
$= \frac{7(2x-5)}{(2x-5)(x+3)} \times \frac{2y(x+3)(x-3)}{4y(x-3)}$
$= \frac{7}{2}$, so $\frac{14x-35}{2x^2+x-15} \div \frac{4xy-12y}{2x^2-18y} \equiv k$, where $k = \frac{7}{2}$
[6 marks available — 1 mark for inverting the second fraction and multiplying, 1 mark for factorising each of the numerators and denominators, 1 mark for the value of k]

Pages 19-20: Sequences

1 5th term $= \left(\frac{1}{2}\right)^5 = \frac{1}{2^5} = \frac{1}{32}$
8th term $= \left(\frac{1}{2}\right)^8 = \frac{1}{2^8} = \frac{1}{256}$ ***[1 mark for both]***
Difference $= \frac{1}{32} - \frac{1}{256} = \frac{8}{256} - \frac{1}{256} = \frac{7}{256}$ ***[1 mark]***
[2 marks available in total — as above]

2 a) Sequence: 3 9 21 39
First difference: 6 12 18
Second difference: 6 6 ***[1 mark]***
Coefficient of $n^2 = 6 \div 2 = 3$
Actual sequence – $3n^2$ sequence: 0 –3 –6 –9
Difference: –3 –3 –3
So this is a linear sequence with nth term $-3n + 3$ ***[1 mark]***.
So the nth term of Justin's sequence is $3n^2 - 3n + 3$ ***[1 mark]***.
[3 marks available in total — as above]

b) Set the sequences equal to each other and solve for n:
$93 - 6n = 3n^2 - 3n + 3$ ***[1 mark]***
$0 = 3n^2 + 3n - 90$
$0 = n^2 + n - 30 = (n + 6)(n - 5)$ ***[1 mark]***
So $n = -6$ or $n = 5$. n must be positive, so $n = 5$,
which gives a value of $93 - (6 \times 5) = 93 - 30 = 63$ ***[1 mark]***.
[3 marks available in total — as above]
Check your answer by putting in into Justin's sequence.

3 nth term: $3n^2 - 4n + 1$
$(n + 1)$th term: $3(n + 1)^2 - 4(n + 1) + 1 = 3n^2 + 2n$ ***[1 mark]***
Sum $= 3n^2 - 4n + 1 + 3n^2 + 2n = 6n^2 - 2n + 1$
So $6n^2 - 2n + 1 = 581$ ***[1 mark]***
$6n^2 - 2n - 580 = 0$
$3n^2 - n - 290 = 0$, so $(3n + 29)(n - 10) = 0$ ***[1 mark]***
So $n = -\frac{29}{3}$ or $n = 10$. n must be a positive integer, so $n = 10$.
So the two terms are the 10th and 11th terms, which are
$3(10)^2 - 4(10) + 1 = 261$ ***[1 mark]***
and $3(11)^2 - 4(11) + 1 = 320$ ***[1 mark]***
[5 marks available in total — as above]

4 Difference between the first and second terms:
$(8x - 29) - (6x + 1) = 2x - 30$
Difference between the second and third terms:
$(5x + 6) - (8x - 29) = -3x + 35$
In an arithmetic sequence, the difference between each pair of terms is the same, so $2x - 30 = -3x + 35$
$5x = 65$, which means $x = 13$.
So the first three terms are $6(13) + 1 = 79$, $8(13) - 29 = 75$ and $5(13) + 6 = 71$. The nth term of this sequence is $83 - 4n$, so the 20th term is $83 - 4(20) = 83 - 80 = 3$.
[6 marks available — 1 mark for finding the differences between both pairs of terms, 1 mark for setting these expressions equal to each other, 1 mark for solving to find the value of x, 1 mark for using the value of x to find the first three terms, 1 mark for finding an expression for the nth term, 1 mark for using this expression to find the 20th term]

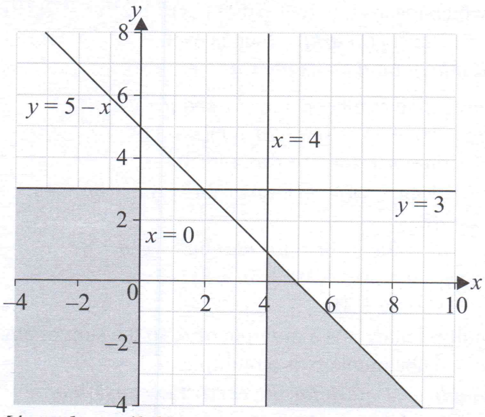
5 Find the value of n for which $50 - \frac{1}{2}n^2 < 0$
$50 - \frac{1}{2}n^2 < 0$, so $100 - n^2 < 0$
$(10 + n)(10 - n) < 0$ ***[1 mark]***
So $n < -10$ or $n > 10$. n must be a positive integer, so $n > 10$ ***[1 mark]***, which means $n = 11$ is the first term that's less than 0.
The 11th term is $50 - \frac{1}{2}(11)^2 = 50 - 60.5 = -10.5$ ***[1 mark]***.
[3 marks available in total — as above]
Quickly sketch the quadratic if you need to.

6 8th term of Kim's sequence $= (\sqrt{3})^8 = 3^4 = 81$ ***[1 mark]***
Alex's sequence is quadratic, and the first two differences are 3 and 6, so the next differences will be 9, 12 and 15.
4th term $= 27 + 9 = 36$ ***[1 mark]***, 5th term $= 36 + 12 = 48$,
6th term $= 48 + 15 = 63$ ***[1 mark]***
Sum of terms $= 81 + 63 = 144 = 12^2$ ***[1 mark]***
[4 marks available in total — as above]

Pages 21-22: Quadratic Inequalities

1 $x^2 + x - 56 = 0$ factorises to give $(x + 8)(x - 7) = 0$. The graph of $y = x^2 + x - 56$ is a u-shaped quadratic that crosses the x-axis at $x = -8$ and $x = 7$, and the graph is below 0 between these points. So $-8 < x < 7$.
[3 marks available — 1 mark for factorising the quadratic to find the solutions, 1 mark for –8 and 7, 1 mark for correct inequality symbols]
Sketching a graph is always handy when solving quadratic inequalities.

2 $4x - x^2 = 0$ factorises to give $x(4 - x) = 0$, so $4x - x^2 \leq 0$ means $x \leq 0$ and $x \geq 4$.



[4 marks available — 1 mark for solving the quadratic inequality, 1 mark for drawing the lines x = 0, x = 4 and y = 3, 1 mark for drawing the line y = 5 – x, 1 mark for shading the correct regions]

3 Area of circle A = πr^2 cm²
Area of circle B = $\pi(3r)^2$ cm² = $9\pi r^2$ cm²
Sum of areas = $\pi r^2 + 9\pi r^2 = 10\pi r^2$ cm², so $10\pi r^2 > 160\pi$
$r^2 > 16$ so $r < -4$ or $r > 4$
r cannot be negative so $r > 4$. The smallest possible integer value of r is 5, so the smallest possible radius of circle A is 5 cm and the smallest possible radius of circle B is $3 \times 5 = 15$ cm.
[3 marks available — 1 mark for finding an expression for the sum of the areas, 1 mark for forming an inequality for the areas and solving to find r, 1 mark for the correct radius of both circles]

4 $3x^2 - x - 90 \geq 5x + 15$, so $3x^2 - 6x - 105 \geq 0$
Solve the quadratic equation $3x^2 - 6x - 105 = 0$
$x^2 - 2x - 35 = 0$
$(x + 5)(x - 7) = 0$
The graph of $y = 3x^2 - 6x - 105$ is a u-shaped quadratic that crosses the x-axis at $x = -5$ and $x = 7$, and the graph is above 0 when $x \leq -5$ and $x \geq 7$.
[4 marks available — 1 mark for rearranging the inequality, 1 mark for factorising the quadratic to find the solutions, 1 mark for –5 and 7, 1 mark for correct inequality symbols]

5 a) The volume of cuboid A is $3x(x + 2) = 3x^2 + 6x$ cm³. ***[1 mark]***
The volume of cuboid B is $4x^2$ cm³. ***[1 mark]***
$3x^2 + 6x < 4x^2$ so $x^2 - 6x > 0$. ***[1 mark]***
[3 marks available in total — as above]

b) $x^2 - 6x = 0$ factorises to give $x(x - 6) = 0$ ***[1 mark]***.
The graph of $y = x^2 - 6x$ is a u-shaped quadratic that crosses the x-axis at $x = 0$ and $x = 6$, and the graph is positive (i.e. > 0) when $x < 0$ and $x > 6$ ***[1 mark]***.
The smallest positive integer solution is $x = 7$ ***[1 mark]***, so the smallest possible volume of cuboid B is $4 \times 7 \times 7 = 196$ cm³. ***[1 mark]***
[4 marks available in total — as above]

6 $x^2 \leq \frac{23x - 45}{2}$, so $2x^2 \leq 23x - 45$, which means $2x^2 - 23x + 45 \leq 0$.
$2x^2 - 23x + 45 = 0$ factorises to give $(2x - 5)(x - 9) = 0$. The graph of $y = 2x^2 - 23x + 45$ is a u-shaped quadratic that crosses the x-axis at $x = 2.5$ and $x = 9$, and the graph is below 0 between these points. So $2.5 \leq x \leq 9$.
[4 marks available — 1 mark for rearranging the inequality, 1 mark for factorising the quadratic to find the solutions, 1 mark for 2.5 and 9, 1 mark for correct inequality symbols]

Pages 23-24: Iterative Methods

1 a) Put $x = 1$ into the equation of each graph:
$(1 - 1)^2(2(1) + 5) = 0$ and $10(1)^2 - 8(1) - 2 = 0$
The y-value is the same on each graph, so the graphs must intersect at that point.
[2 marks available — 1 mark for calculating the y-value for both graphs, 1 mark for stating that the values are equal so the graphs must intersect]

b) At the points of intersection, the graphs cross so they must be equal. So $(x - 1)^2(2x + 5) = 10x^2 - 8x - 2$
$(x^2 - 2x + 1)(2x + 5) = 10x^2 - 8x - 2$
$2x^3 + 5x^2 - 4x^2 - 10x + 2x + 5 = 10x^2 - 8x - 2$
$2x^3 - 9x^2 + 7 = 0$
[3 marks available — 1 mark for expanding two of the brackets on the LHS, 1 mark for multiplying the product by the third bracket, 1 mark for simplifying]

c) Put $x = 4$ and $x = 5$ into the expression from b):
$x = 4$: $2(4)^3 - 9(4)^2 + 7 = 128 - 144 + 7 = -9$
$x = 5$: $2(5)^3 - 9(5)^2 + 7 = 250 - 225 + 7 = 32$
There is a sign change between $x = 4$ and $x = 5$, so there must be a solution in the interval.
[2 marks available — 1 mark for finding the values when x = 4 and x = 5, 1 mark for stating that a sign change means there's a root]

d)

x	$2x^3 - 9x^2 + 7$	
4	–9	Negative
5	32	Positive
4.1	–6.448	Negative
4.2	–3.584	Negative
4.3	–0.396	Negative
4.4	3.128	Positive
4.31	–0.058918	Negative
4.32	0.281536	Positive

So $x = 4.3$ to 1 d.p.
[4 marks available — 3 marks for a correctly filled in table, 1 mark for the correct value of x, otherwise 2 marks for 5 or 6 rows correct or 1 mark for 3 or 4 rows correct]

2 a) Put $x = 1$ and $x = 1.5$ into the expression:
$x = 1$: $(1)^3 + 3(1)^2 - 5 = 1 + 3 - 5 = -1$
$x = 1.5$: $(1.5)^3 + 3(1.5)^2 - 5 = 3.375 + 6.75 - 5 = 5.125$
There is a sign change between $x = 1$ and $x = 1.5$, so there must be a solution in the interval.
[2 marks available — 1 mark for finding the values when x = 1 and x = 1.5, 1 mark for stating that a sign change means there's a root]

b) $x_0 = 1$ $x_1 = 1.111111...$ $x_2 = 1.103835...$
$x_3 = 1.103803...$ $x_4 = 1.103803...$
$x_3 = x_4$ to 5 d.p. so $x = 1.10380$ to 5 d.p.
[3 marks available — 2 marks for carrying out the iteration an appropriate number of times, 1 mark for the correct value of x]

3 a) $x^3 - 3x^2 - 4x + 10 = 0$
$x^3 - 3x^2 = 4x - 10$
$x^2(x - 3) = 4x - 10$ ***[1 mark]***
$x^2 = \frac{4x - 10}{x - 3}$ ***[1 mark]***
$x = \pm\sqrt{\frac{4x - 10}{x - 3}}$, so $a = 4$ and $b = 3$ ***[1 mark]***
[3 marks available in total — as above]

b) $x_0 = 1.5$ $x_1 = 1.6329...$ $x_2 = 1.5927...$
$x_3 = 1.6058...$ $x_4 = 1.6016...$ $x_5 = 1.6030...$
$x_4 = x_5$ to 3 s.f. so $x = 1.60$ to 3 s.f.
[3 marks available — 2 marks for carrying out the iteration an appropriate number of times, 1 mark for the correct value of x]

Pages 25-26: Simultaneous Equations

1 $2x - y = x + 4$, so $x = y + 4$
Then $(y + 4)^2 + 4y^2 = 37$ ***[1 mark]***
$y^2 + 8y + 16 + 4y^2 = 37$
$5y^2 + 8y - 21 = 0$ ***[1 mark]***
$(5y - 7)(y + 3) = 0$ ***[1 mark]***, so $y = 1.4$ or $y = -3$ ***[1 mark]***
When $y = 1.4$, $x = 1.4 + 4 = 5.4$
When $y = -3$, $x = -3 + 4 = 1$
So the solutions are $x = 5.4$, $y = 1.4$ and $x = 1$, $y = -3$ ***[1 mark]***
[5 marks available in total — as above]

2 Area $= (2x \times x) + (y \times 3y) = 2x^2 + 3y^2$
So $2x^2 + 3y^2 = 83$
Base length $= 2x + y$, so $2x + y = 9$, which means $y = 9 - 2x$
So $2x^2 + 3(9 - 2x)^2 = 83$
$2x^2 + 243 - 108x + 12x^2 = 83$
$14x^2 - 108x + 160 = 0$
$7x^2 - 54x + 80 = 0$, so $(7x - 40)(x - 2) = 0$
So $x = 40 \div 7 = 5.714...$ or $x = 2$. x is an integer so $x = 2$.
When $x = 2$, $y = 9 - 2(2) = 9 - 4 = 5$.
[5 marks available — 1 mark for forming equations for the area and base length, 1 mark for substituting an expression for x or y into the quadratic equation, 1 mark for factorising the quadratic equation, 1 mark for the correct value of x, 1 mark for the correct value of y]

3 $7x - y = 25$, so $y = 7x - 25$
Then $x^2 + (7x - 25)^2 = 25$ ***[1 mark]***
$x^2 + 49x^2 - 350x + 625 = 25$
$50x^2 - 350x + 600 = 0$
$x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$ ***[1 mark]***, so $x = 3$ or $x = 4$ ***[1 mark]***
When $x = 3$, $y = (7 \times 3) - 25 = 21 - 25 = -4$
When $x = 4$, $y = (7 \times 4) - 25 = 28 - 25 = 3$
So the line intersects the circle at $(3, -4)$ and $(4, 3)$ ***[1 mark]***
Change in $x = 4 - 3 = 1$, change in $y = 3 - (-4) = 7$ ***[1 mark for both]***
Length $AB = \sqrt{1^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$ ***[1 mark]***
[6 marks available in total — as above]

4 Face A: $P = \frac{F}{A}$, so $10 = \frac{120}{A}$, which means area of face $A = 12$ m^2
So $(x + 1) \times y = 12$ or $xy + y = 12$ (1)
Face B: $P = \frac{F}{A}$, so $7.5 = \frac{120}{A}$, which means area of face $B = 16$ m^2
So $(x + 3) \times y = 16$ or $xy + 3y = 16$ (2)
(2) – (1): $2y = 4$, so $y = 2$
$xy + y = 12$, so $2x + 2 = 12$, which means $2x = 10$ so $x = 5$
Volume $= (5 + 1) \times (5 + 3) \times 2 = 6 \times 8 \times 2 = 96$ m^3
[6 marks available — 1 mark for finding the areas of faces A and B, 1 mark for finding an equation for the area of face A, 1 mark for finding an equation for the area of face B, 1 mark for solving the simultaneous equations to find the value of y, 1 mark for finding the value of x, 1 mark for the correct volume]
There are other ways of doing one — you could start by finding expressions for the pressure exerted by each face in terms of x and y.

5 $2x - 5 = -x^2 + 15x - 41$ ***[1 mark]***
$x^2 - 13x + 36 = 0$
$(x - 4)(x - 9) = 0$ ***[1 mark]***, so $x = 4$ or $x = 9$ ***[1 mark]***
When $x = 4$, $y = 2(4) - 5 = 3$
When $x = 9$, $y = 2(9) - 5 = 13$ ***[1 mark for both y-values]***
$ABDC$ is a trapezium with $CD = 9 - 4 = 5$, $AC = 3$ and $BD = 13$
[1 mark for all lengths correct],
so area $= \frac{1}{2}(3 + 13) \times 5 = 40$ units^2 ***[1 mark]***
[6 marks available in total — as above]

Pages 27-28: Proof

1 LHS: $(2n + 1)^3 - 1 \equiv (4n^2 + 4n + 1)(2n + 1) - 1$
$\equiv 8n^3 + 4n^2 + 8n^2 + 4n + 2n + 1 - 1$
$\equiv 8n^3 + 12n^2 + 6n \equiv 2n(4n^3 + 6n + 3) \equiv$ RHS
[3 marks available — 1 mark for correctly expanding cubed bracket, 1 mark for simplifying, 1 mark for factorising to show that LHS ≡ RHS]

2 Take three rational numbers, $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ (where a, b, c, d, e and f are all integers and b, d and $f \neq 0$).
Their product is $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{ace}{bdf}$ ***[1 mark]***.
The product of integers is also an integer, so ace and bdf are both integers, which means that $\frac{ace}{bdf}$ is rational ***[1 mark]***.
[2 marks available in total — as above]

3 If a and b are both odd, then let $a = 2m + 1$ and $b = 2n + 1$ for integers m and n. Then $a + b = (2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1) = 2x$ (where $x = m + n + 1$) ***[1 mark]***. So $a + b$ is always even, which means that $(a + b)^{40} = (2x)^{40} = 2^{40}x^{40} = 2(2^{39}x^{40}) = 2y$, where $y = (2^{39}x^{40})$ ***[1 mark]***, which is even.
[2 marks available in total — as above]

4 Take three consecutive numbers, n, $n + 1$ and $n + 2$.
Their cubes are n^3, $(n + 1)^3 = (n^2 + 2n + 1)(n + 1)$
$= n^3 + n^2 + 2n^2 + 2n + n + 1 = n^3 + 3n^2 + 3n + 1$
and $(n + 2)^3 = (n^2 + 4n + 4)(n + 2) = n^3 + 2n^2 + 4n^2 + 8n + 4n + 8$
$= n^3 + 6n^2 + 12n + 8$.
Their sum is $n^3 + n^3 + 3n^2 + 3n + 1 + n^3 + 6n^2 + 12n + 8$
$= 3n^3 + 9n^2 + 15n + 9 = 3(n^3 + 3n^2 + 5n + 3) = 3x$
(where $x = n^3 + 3n^2 + 5n + 3$).
Any integer multiplied by 3 is a multiple of 3, so the sum of any three consecutive cube numbers is a multiple of 3.
[4 marks available — 1 mark for the correct expansion of (n + 1)³, 1 mark for the correct expansion of (n + 2)³, 1 mark for adding the terms and simplifying the result, 1 mark for writing as a multiple of 3]

5 Max's number $= 5n + 1$ for some integer n
The square of Max's number is $(5n + 1)^2 = (5n + 1)(5n + 1)$
$= 25n^2 + 10n + 1$
Samira's number $= 5n - 3$ and the square of her number is
$(5n - 3)(5n - 3) = 25n^2 - 30n + 9$
Difference $= (25n^2 + 10n + 1) - (25n^2 - 30n + 9) = 40n - 8$
$= 8(5n - 1) = 8x$ (where $x = 5n - 1$)
Any integer multiplied by 8 is a multiple of 8, so the difference between the squares of their numbers is a multiple of 8.
[5 marks available — 1 mark for finding an expression for Max's number, 1 mark for finding an expression for Samira's number, 1 mark for squaring both numbers, 1 mark for finding the difference between the squares, 1 mark for writing as a multiple of 8]
Here, you could have written the difference as $(5n + 1)^2 - (5n - 3)^2$ and used the difference of two squares to find the expression $40n - 8$.

6 $3^8 - 7^4 = (3^4)^2 - (7^2)^2 = (3^4 + 7^2)(3^4 - 7^2)$
$= (81 + 49)(81 - 49) = 130 \times 32$
$= 13 \times 10 \times 32 = 13x$ (where $x = 10 \times 32$)
Any integer multiplied by 13 is a multiple of 13, so $3^8 - 7^4$ is a multiple of 13.
[3 marks available — 1 mark for factorising using the difference of two squares, 1 mark for finding the value of each factor, 1 mark for writing as a multiple of 13]

7 $15^{12} + 12^{16} = (3 \times 5)^{12} + (3 \times 2^2)^{16}$
$= (3^{12} \times 5^{12}) + (3^{16} \times 2^{32})$ ***[1 mark]***
$= 3^2[(3^{10} \times 5^{12}) + (3^{14} \times 2^{32})]$ ***[1 mark]***
$= 9[(3^{10} \times 5^{12}) + (3^{14} \times 2^{32})]$
$= 9x$ (where $x = (3^{10} \times 5^{12}) + (3^{14} \times 2^{32})$) ***[1 mark]***
Any integer multiplied by 9 is a multiple of 9, so $15^{12} + 12^{16}$ is a multiple of 9.
[3 marks available in total — as above]

Pages 29-30: Functions

1 a) $fg(4) = f(g(4)) = f(4^2 + 4) = f(20) = \sqrt{2(20) - 8} = \sqrt{32} = 4\sqrt{2}$
[2 marks available — 1 mark for finding the value of g(4), 1 mark for using this value to find fg(4), giving the answer as a simplified surd]
You could have found an expression for fg(x) and put in x = 4.

b) $gf(x) = g(f(x)) = g(\sqrt{2x - 8}) = (\sqrt{2x - 8})^2 + 4$ ***[1 mark]***
$= 2x - 8 + 4 = 2x - 4$ ***[1 mark]***
[2 marks available in total — as above]

c) Write out $x = f(y)$: $x = \sqrt{2y-8}$ ***[1 mark]***
Rearrange to make y the subject: $x^2 = 2y - 8$ ***[1 mark]***
$x^2 + 8 = 2y$
$y = \frac{x^2+8}{2}$, so $f^{-1}(x) = \frac{x^2+8}{2}$ ***[1 mark]***
[3 marks available in total — as above]

2 Write out $x = f(y)$: $x = \frac{y+5}{2}$ ***[1 mark]***
Rearrange to make y the subject: $2x = y + 5$
$y = 2x - 5$, so $f^{-1}(x) = 2x - 5$ ***[1 mark]***
Write out $x = g(y)$: $x = 3y - 10$ ***[1 mark]***
Rearrange to make y the subject: $x + 10 = 3y$
$y = \frac{x+10}{3}$, so $g^{-1}(x) = \frac{x+10}{3}$ ***[1 mark]***
When $f^{-1}(x) = g^{-1}(x)$, $2x - 5 = \frac{x+10}{3}$ ***[1 mark]***
$6x - 15 = x + 10$
$5x = 25$ so $x = 5$ ***[1 mark]***
[6 marks available in total — as above]

3 $fg(x) = f(g(x)) = f(\sin x) = 2\sin x - 1$ ***[1 mark]***
$2\sin x - 1 = 0$
$2\sin x = 1$
$\sin x = \frac{1}{2}$ ***[1 mark]***
So $x = 30°$ ***[1 mark]*** and $x = 180° - 30° = 150°$ ***[1 mark]***
[4 marks available in total — as above]
sin 30° = ½ is one of the common trig values you should know. You need to use the symmetry of the graph to find the second value.

4 a) $fgg(x) = f(g(g(x))) = f(g(x + 2))$
$= f((x + 2) + 2) = f(x + 4)$ ***[1 mark]***
$= (x + 4)^2 + 4(x + 4) + 3$ ***[1 mark]***
$= x^2 + 8x + 16 + 4x + 16 + 3$
$= x^2 + 12x + 35$ ***[1 mark]***
[3 marks available in total — as above]

b) $fgg(x) = 0$ means that $x^2 + 12x + 35 = 0$
$(x + 7)(x + 5) = 0$, so $x = -7$ or $x = -5$
[2 marks available — 1 mark for setting expression from part a) equal to 0 and factorising, 1 mark for both correct x-values]

5 $h^{-1}(x) = \sqrt[3]{x}$ ***[1 mark]***
$fgh^{-1}(x) = f(g(h^{-1}(x))) = f(g(\sqrt[3]{x}))$
$= f((\sqrt[3]{x})^2 + 3(\sqrt[3]{x}))$ ***[1 mark]*** $= f(x^{\frac{2}{3}} + 3x^{\frac{1}{3}})$ ***[1 mark]***
$= 3(x^{\frac{2}{3}} + 3x^{\frac{1}{3}}) + 1$
$= 3x^{\frac{2}{3}} + 9x^{\frac{1}{3}} + 1$ ***[1 mark]***
[4 marks available in total — as above]

6 $fg(x) = f(g(x)) = f(2x + 1) = \frac{4(2x+1)}{(2x+1)+9} = \frac{8x+4}{2x+10}$
$= \frac{4x+2}{x+5}$ ***[1 mark]***
So $\frac{4x+2}{x+5} = x$ ***[1 mark]***
$4x + 2 = x(x + 5)$
$4x + 2 = x^2 + 5x$ ***[1 mark]***
$0 = x^2 + x - 2 = (x + 2)(x - 1)$ ***[1 mark]***
So $x = -2$ or $x = 1$ ***[1 mark for both]***
[5 marks available in total — as above]

Section Three — Graphs

Pages 31-32: Coordinates and Ratio

1 a) Difference in x-coordinates from A to M: $4 - -2 = 6$
M is the midpoint of AB, so difference in x-coordinates from M to $B = 6$, so x-coordinate of $B = 4 + 6 = 10$ ***[1 mark]***,
Difference in y-coordinates from A to M: $5 - 2 = 3$,
so difference in y-coordinates from M to $B = 3$,
so y-coordinate of $B = 5 + 3 = 8$ ***[1 mark]***.
So the coordinates of B are (10, 8).
[2 marks available in total — as above]

b) Difference in x-coordinates from M to B: $10 - 4 = 6$
Difference in y-coordinates from M to B: $8 - 5 = 3$
P is $\frac{1}{3}$ of the way along MB,
so the x-coordinate of P is $4 + (\frac{1}{3} \times 6) = 6$
and the y-coordinate of P is $5 + (\frac{1}{3} \times 3) = 6$
So the coordinates of P are (6, 6).
Difference between x-coordinates of A and P: $6 - -2 = 8$
Difference between x-coordinates of A and B: $10 - -2 = 12$
so the ratio $AP:AB = 8:12 = 2:3$
[3 marks available — 1 mark for a correct method to find the coordinates of P, 1 mark for the correct coordinates of P, 1 mark for the correct ratio]
Instead of finding the coordinates of P, you could have found AP and AB in terms of MP and used this to find the ratio.

2 Coordinates of C are $(2 + 4, -1) = (6, -1)$
Coordinates of M are $(2 + 2, -1 + 2) = (4, 1)$
Difference in x-coordinates from E to M: $4 - 0 = 4$
Difference in x-coordinates from M to C: $6 - 4 = 2$
So $EM:MC = 4:2 = 2:1$
[3 marks available — 1 mark for finding the coordinates of C and M, 1 mark for finding the differences between the x- or y-coordinates of E & M and M & C, 1 mark for the correct ratio]

3 Difference in x-coordinates from E to B: $6 - 0 = 6$
Difference in y-coordinates from E to B: $4 - -4 = 8$
P is $\frac{3}{4}$ of the way along EB, so has x-coordinate $0 + \frac{3}{4} \times 6 = 4.5$
and y-coordinate $-4 + \frac{3}{4} \times 8 = 2$, so P has coordinates (4.5, 2).
$PF^2 = (6 - 4.5)^2 + (0 - 2)^2 = 6.25$, so $PF = \sqrt{6.25} = 2.5$.
[4 marks available — 1 mark for correct difference in x- and y-coordinates of E and B, 1 mark for correct coordinates of P, 1 mark for using Pythagoras to find PF², 1 mark for the correct answer]

4 Difference in x-coordinates from A to M: $3 - -2 = 5$
M is the midpoint of AC, so difference in x-coordinates from M to $C = 5$, so x-coordinate of $C = 3 + 5 = 8$
C has the same y-coordinate as D, so the coordinates of C are (8, 2)
Difference in x-coordinates from M to C: $8 - 3 = 5$
Difference in y-coordinates from M to C: $2 - 4.5 = -2.5$
N is $\frac{3}{5}$ of the way along MC,
so the x-coordinate of N is $3 + \frac{3}{5} \times 5 = 6$
and the y-coordinate of N is $4.5 + \frac{3}{5} \times -2.5 = 3$
So the coordinates of N are (6, 3).
B has the same y-coordinate as A, and the difference in x-coordinates of C and D is $8 - -5 = 13$, so the x-coordinate of B is $-2 + 13 = 11$. So B has coordinates (11, 7).
Distance $NB^2 = (11 - 6)^2 + (7 - 3)^2 = 5^2 + 4^2 = 41$
$NB = \sqrt{41}$
[6 marks available — 1 mark for the correct coordinates of C, 1 mark for a correct method to find the coordinates of N, 1 mark for correct coordinates of N, 1 mark for the correct coordinates of B, 1 mark for a correct method to find the distance NB, 1 mark for the correct answer]
There are different ways to find the coordinates of N and C.

Pages 33-34: Perpendicular Lines

1 Gradient of L_1: $\frac{20-6}{11-4} = \frac{14}{7} = 2$,
so gradient of line $L_2 = -1 \div 2 = -\frac{1}{2}$
L_2 passes through (28, 0), so $0 = -\frac{1}{2}(28) + c$
$0 = -14 + c$, so $c = 14$
So the equation of line L_2 is $y = -\frac{1}{2}x + 14$
[3 marks available — 1 mark for finding the gradient of line L_1, 1 mark for finding the gradient of L_2, 1 mark for the correct answer]

2 Gradient of $SQ = 4$
The diagonals of a kite cross at right angles so are perpendicular, so gradient of $PR = -\frac{1}{4}$.
Point R has coordinates (8, 15), so $15 = -\frac{1}{4}(8) + c$
$15 = -2 + c$, so $c = 17$
So the equation of PR is $y = -\frac{1}{4}x + 17$
[3 marks available — 1 mark for stating that the diagonals of a kite are perpendicular, 1 mark for finding the gradient of PR, 1 mark for the correct answer]

3 Equation of L_1 is $x + 5y = 100$, or $y = -\frac{1}{5}x + 20$, so has gradient $-\frac{1}{5}$.
L_2 is perpendicular to L_1 so has gradient $-1 \div -\frac{1}{5} = 5$.
L_2 passes through (2, 4), so $4 = 5(2) + c$
$4 = 10 + c$, so $c = -6$
So the equation of line L_2 is $y = 5x - 6$
At M, $-\frac{1}{5}x + 20 = 5x - 6$
$26 = \frac{26}{5}x$, so $x = 5$
When $x = 5$, $y = 5(5) - 6 = 25 - 6 = 19$
So the coordinates of M are (5, 19).
[5 marks available — 1 mark for finding the gradient of L_1, 1 mark for finding the gradient of L_2, 1 mark for finding the equation of L_2, 1 mark for setting the equations for L_1 and L_2 equal to each other and solving to find x or y, 1 mark for the correct answer]
Check your answer by putting your x-coordinate into the equation of L_1 and checking you get the correct y-coordinate.

4 Equation of L_1 is $2y - x = 14$, or $y = \frac{1}{2}x + 7$ so has gradient $\frac{1}{2}$
L_2 is perpendicular to L_1 so has gradient $-1 \div \frac{1}{2} = -2$.
L_2 passes through point (6, 10) so $10 = -2(6) + c$
$10 = -12 + c$, so $c = 22$. So the equation of L_2 is $y = -2x + 22$
R is the y-intercept of L_2, so the y-coordinate of R is 22.
RQ is horizontal so the y-coordinate of Q is 22
Q lies on L_1 so $22 = \frac{1}{2}x + 7$
$15 = \frac{1}{2}x$ so $x = 30$
So the coordinates of Q are (30, 22).
[5 marks available — 1 mark for finding the gradient of L_1, 1 mark for finding the gradient of L_2, 1 mark for finding the equation of L_2, 1 mark for finding the y-coordinate of Q, 1 mark for the correct answer]

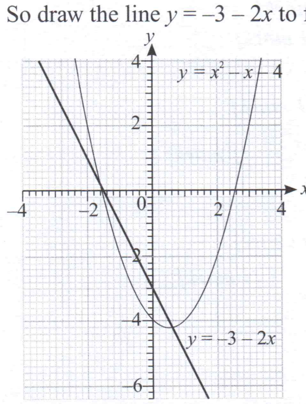
5 Equation of L_1 is $2x + 3y = 12$, or $y = -\frac{2}{3}x + 4$, so has gradient $-\frac{2}{3}$.
L_2 is parallel to L_1, so also has gradient $-\frac{2}{3}$.
L_2 passes through (6, 13), so $13 = -\frac{2}{3}(6) + c_2$
$13 = -4 + c_2$, so $c_2 = 17$. So the equation of L_2 is $y = -\frac{2}{3}x + 17$.
L_3 is perpendicular to L_1 and L_2 so has gradient $\frac{3}{2}$.
It passes through (3, 2) so $2 = \frac{3}{2}(3) + c_3$
$2 = \frac{9}{2} + c_3$, so $c_3 = -\frac{5}{2}$. So the equation of L_3 is $y = \frac{3}{2}x - \frac{5}{2}$
When L_2 and L_3 intersect, $-\frac{2}{3}x + 17 = \frac{3}{2}x - \frac{5}{2}$
$\frac{39}{2} = \frac{13}{6}x$ so $x = 9$.
When $x = 9$, $y = -\frac{2}{3}(9) + 17 = -6 + 17 = 11$
So L_2 and L_3 intersect at (9, 11).
[6 marks available — 1 mark for finding the gradient of L_1, 1 mark for finding the equation of L_2, 1 mark for finding the gradient of L_3, 1 mark for finding the equation of L_3, 1 mark for setting the equations for L_2 and L_3 equal to each other and solving to find x or y, 1 mark for the correct answer]

Pages 35-36: Harder Graphs

1 The two closest points are (3, 3) and (–3, –3).
Use Pythagoras to find the distance between them, d:
$d^2 = (3 - -3)^2 + (3 - -3)^2 = 6^2 + 6^2 = 72$, so $d = \sqrt{72} = 6\sqrt{2}$
[3 marks available — 1 mark for finding the two closest points, 1 mark for using Pythagoras to find the distance, 1 mark for the correct answer]

2 Use Pythagoras to find the radius, r:
$r^2 = 7^2 + 24^2 = 625$, so $r = \sqrt{625} = 25$ ***[1 mark]***
The equation of a circle is given by $x^2 + y^2 = r^2$,
so the equation is $x^2 + y^2 = 625$ ***[1 mark].***
[2 marks available in total — as above]

3 Find the equation of the line that should be drawn:
$x^2 + x = 1$
$x^2 + x - 4 = -3$
$x^2 - x - 4 = -3 - 2x$
So draw the line $y = -3 - 2x$ to find the solutions ***[1 mark]***:



[1 mark]
So solutions to $x^2 + x = 1$ are $x \approx -1.6$ ***[1 mark]*** and $x \approx 0.6$ ***[1 mark]***
[4 marks available in total — as above]

4 The line from the origin to the point (–4, –3) is a radius, so has gradient $\frac{0 - -3}{0 - -4} = \frac{3}{4}$, so the tangent at this point has gradient $-\frac{4}{3}$ as a tangent meets a radius at 90°.
The tangent passes through the point (–4, –3), so $-3 = -\frac{4}{3}(-4) + c$
$-3 = \frac{16}{3} + c$, so $c = -\frac{25}{3}$
So the equation of the tangent at (–4, –3) is $y = -\frac{4}{3}x - \frac{25}{3}$
[3 marks available — 1 mark for finding the gradient of the radius, 1 mark for finding the gradient of the tangent, 1 mark for the correct answer]

5 Substituting the coordinates of A and B into the curve $y = ab^x$ gives $36 = ab^2$ and $81 = ab^4$ ***[1 mark]***
C has coordinates $(2, a(2b)^2) = (2, 4ab^2) = (2, 4 \times 36)$
$= (2, 144)$ ***[1 mark]***
and D has coordinates $(4, a(2b)^4) = (4, 16ab^4) = (4, 16 \times 81)$
$= (4, 1296)$ ***[1 mark]***
Gradient of $CD = \frac{1296 - 144}{4 - 2}$ ***[1 mark]*** $= \frac{1152}{2} = 576$ ***[1 mark]***
[5 marks available in total — as above]

Pages 37-38: Trig Graphs

1 $x = 285°$ ***[1 mark]***

2 Using the symmetry of the graph, $x = 90° - 15° = 75°$ ***[1 mark]***
and $x = 270° - 15° = 255°$ ***[1 mark]***
[2 marks available in total — as above]

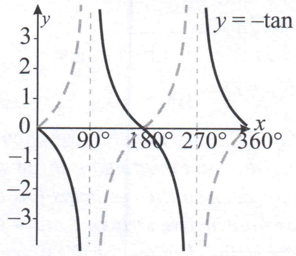
3 $y = \sin 2x + 3 = \sin 2(22.5°) + 3 = \sin 45° + 3 = \frac{1}{\sqrt{2}} + 3 = \frac{\sqrt{2}}{2} + 3$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for attempting to find sin 45°]
You didn't actually need the graph for this one — it's just there to give you an idea of the shape. Don't forget to rationalise the denominator.

4 a) Using the symmetry of the graph,
$x = 180° - 55° = 125°$ ***[1 mark]***
b) Extending the graph, $x = 180° + 55° = 235°$ ***[1 mark]***
c) The cos graph has a line of symmetry at $x = 0$,
so $x = -55°$ ***[1 mark]***

5 The two graphs intersect when $x = 45°$, so $\tan 45° = -\sin 45° + c$,
which means $c = \tan 45° + \sin 45°$ ***[1 mark]***
$\tan 45° = 1$ and $\sin 45° = \frac{1}{\sqrt{2}}$, so $c = 1 + \frac{1}{\sqrt{2}}$ ***[1 mark]***
So $a = -\sin 90° + 1 + \frac{1}{\sqrt{2}}$
$= -1 + 1 + \frac{1}{\sqrt{2}}$ ***[1 mark]*** $= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ***[1 mark]***
[4 marks available in total — as above]

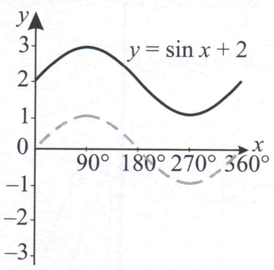
Pages 39-41: Graph Transformations

1 a)



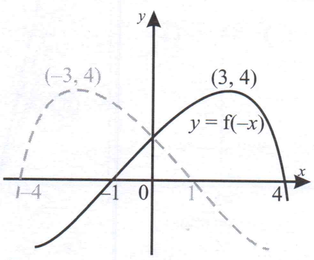
[1 mark]

b)



[1 mark]

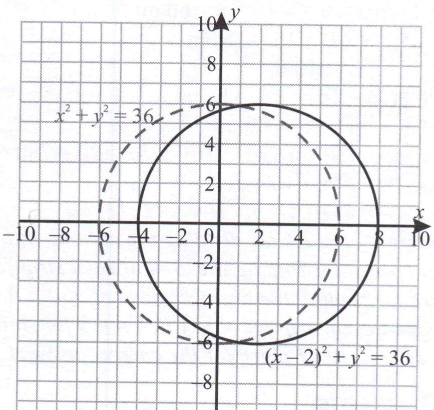
2 a)



[3 marks available — 1 mark for reflecting the graph in the y-axis, 1 mark for the correct turning point, 1 mark for both correct x-intercepts]

b) Turning point = (–3 – 3, 4 + 2) = (–6, 6)
[2 marks available — 1 mark for the correct x-coordinate, 1 mark for the correct y-coordinate]

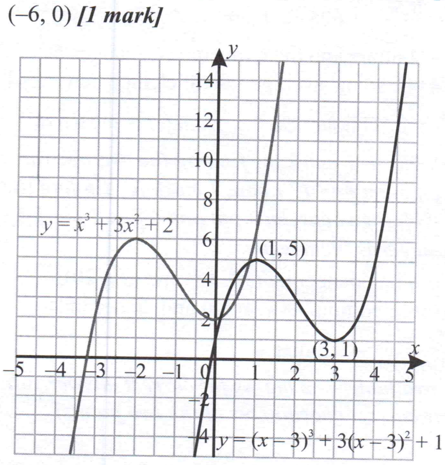
3 a)



[2 marks available — 1 mark for a translation of 2 units to the right, 1 mark for the correct x-intercepts]

b) (–6, 0) ***[1 mark]***

4 a)



[3 marks available — 1 mark for a translation of 3 units to the right, 1 mark for a translation of 1 unit down, 1 mark for both correct turning points]

b) $(x - 3)^3 = (x - 3)(x - 3)(x - 3) = (x^2 - 6x + 9)(x - 3)$ ***[1 mark]***
$= x^3 - 3x^2 - 6x^2 + 18x + 9x - 27$
$= x^3 - 9x^2 + 27x - 27$ ***[1 mark]***
$(x - 3)^3 + 3(x - 3)^2 + 1$
$= x^3 - 9x^2 + 27x - 27 + 3(x^2 - 6x + 9) + 1$ ***[1 mark]***
$= x^3 - 6x^2 + 9x + 1$ ***[1 mark]***
[4 marks available in total — as above]

5 a) $5 \div 2 = \frac{5}{2}$, so the brackets are $(x - \frac{5}{2})^2$ ***[1 mark]***.
Expanding the brackets: $(x - \frac{5}{2})^2 = x^2 - 5x + \frac{25}{4}$
To complete the square: $7 - \frac{25}{4} = \frac{3}{4}$ ***[1 mark]***
So $x^2 + 5x + 7 = (x - \frac{5}{2})^2 + \frac{3}{4}$.
So the coordinates of the turning point are $(\frac{5}{2}, \frac{3}{4})$ ***[1 mark]***
[3 marks available in total — as above]

b) Turning point $= (\frac{5}{2} - 3, \frac{3}{4} - 2) = (-\frac{1}{2}, -\frac{5}{4})$
[2 marks available — 1 mark for each correct coordinate]

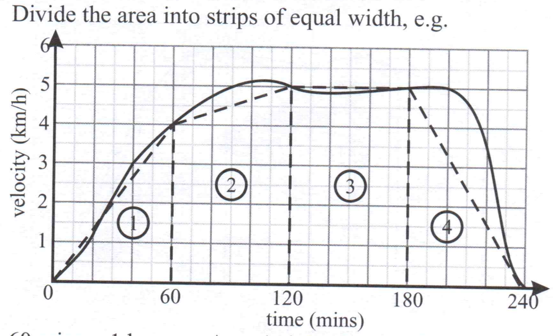
c) x-intercepts are where $f(x + 3) - 2 = 0$
$f(x + 3) - 2 = (x + 3)^2 - 5(x + 3) + 7 - 2 = x^2 + x - 1$ ***[1 mark]***
So $x^2 + x - 1 = 0$
$x = \frac{-1 \pm \sqrt{1^2 - (4 \times 1 \times -1)}}{2 \times 1}$ ***[1 mark]*** $= \frac{-1 \pm \sqrt{5}}{2}$
So $x = \frac{-1 + \sqrt{5}}{2}$ or $x = \frac{-1 - \sqrt{5}}{2}$ ***[1 mark for both solutions]***
[3 marks available in total — as above]
You could use the completed square form to solve the equation.

6 a) $\frac{ab}{x - a} + b = \frac{ab}{x - a} + \frac{b(x - a)}{x - a}$ ***[1 mark]***
$= \frac{ab + b(x - a)}{x - a} = \frac{ab + bx - ab}{x - a} = \frac{bx}{x - a}$ ***[1 mark]***
[2 marks available in total — as above]

b) From part a), $\frac{3x}{x - 2} = \frac{3(2)}{x - 2} + 3 = \frac{6}{x - 2} + 3$ ***[1 mark]***
So the transformation is a translation 2 units to the right ***[1 mark]*** and 3 units up ***[1 mark]***.
[3 marks available in total — as above]

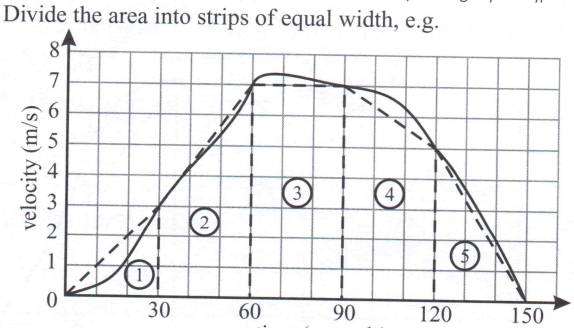
Pages 42-44: Velocity-Time Graphs

1 Divide the area into strips of equal width, e.g.



60 mins = 1 hour, so Area 1: 0.5 × 1 × 4 = 2 km
Area 2: 0.5(4 + 5) × 1 = 4.5 km   Area 3: 5 × 1 = 5 km
Area 4: 0.5 × 1 × 5 = 2.5 km
Total area = distance = 2 + 4.5 + 5 + 2.5 = 14 km
[3 marks available — 1 mark for dividing the area up into strips, 1 mark for working out the area of each strip, 1 mark for a suitable answer]
You might have a different answer if you divided up the graph differently.

2 a) Divide the area into strips of equal width, e.g.



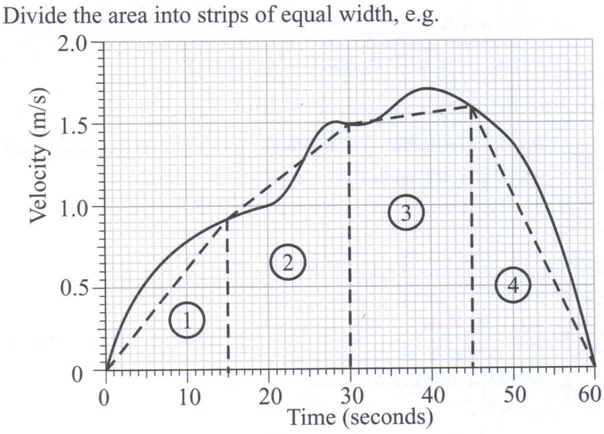
Strip 1: area = 0.5 × 30 × 3 = 45 m
Strip 2: area = 0.5(3 + 7) × 30 = 150 m
Strip 3: area = 7 × 30 = 210 m
Strip 4: area = 0.5(7 + 5) × 30 = 180 m
Strip 5: area = 0.5 × 30 × 5 = 75 m
Total area = distance = 45 + 150 + 210 + 180 + 75 = 660 m

[3 marks available — 1 mark for dividing the area up into strips, 1 mark for working out the area of each strip, 1 mark for a suitable answer]

b) E.g. the answer is likely to be an underestimate, as most of the strips go below the curve, so the strips cover a smaller area.

[2 marks available — 1 mark for a suitable answer, 1 mark for a sensible explanation]

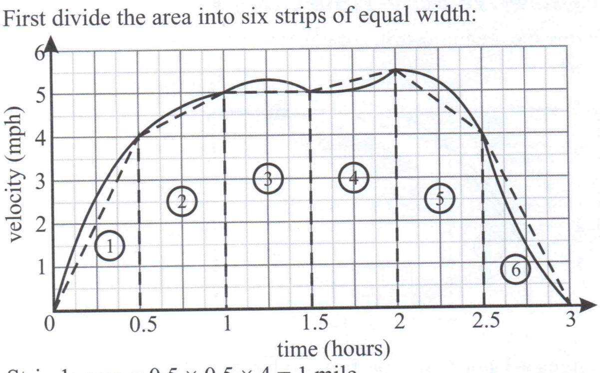
3 Divide the area into strips of equal width, e.g.



Area 1: 0.5 × 15 × 0.9 = 6.75 m
Area 2: 0.5(0.9 + 1.5) × 15 = 18 m
Area 3: 0.5(1.5 + 1.6) × 15 = 23.25 m
Area 4: 0.5 × 15 × 1.6 = 12 m
Total area = distance = 6.75 + 18 + 23.25 + 12 = 60 m
Speed = distance ÷ time = 60 ÷ 60 = 1 m/s

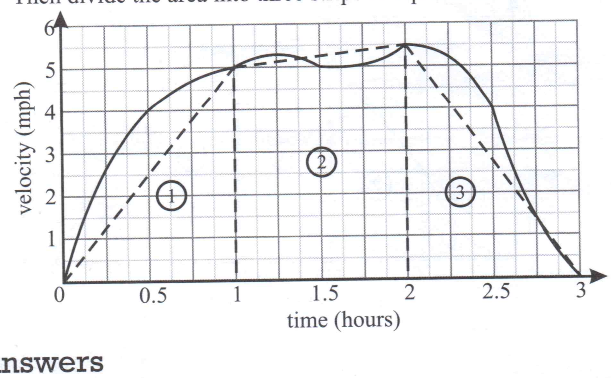
[4 marks available — 1 mark for dividing the area up into strips, 1 mark for working out the area of each strip, 1 mark for a suitable distance, 1 mark for dividing by the total time taken to find the speed]

4 First divide the area into six strips of equal width:



Strip 1: area = 0.5 × 0.5 × 4 = 1 mile
Strip 2: area = 0.5(4 + 5) × 0.5 = 2.25 miles
Strip 3: area = 5 × 0.5 = 2.5 miles
Strip 4: area = 0.5(5 + 5.5) × 0.5 = 2.625 miles
Strip 5: area = 0.5(5.5 + 4) × 0.5 = 2.375 miles
Strip 6: area = 0.5 × 0.5 × 4 = 1 mile
Total area = distance = 1 + 2.25 + 2.5 + 2.625 + 2.375 + 1
= 11.75 miles

Then divide the area into three strips of equal width:

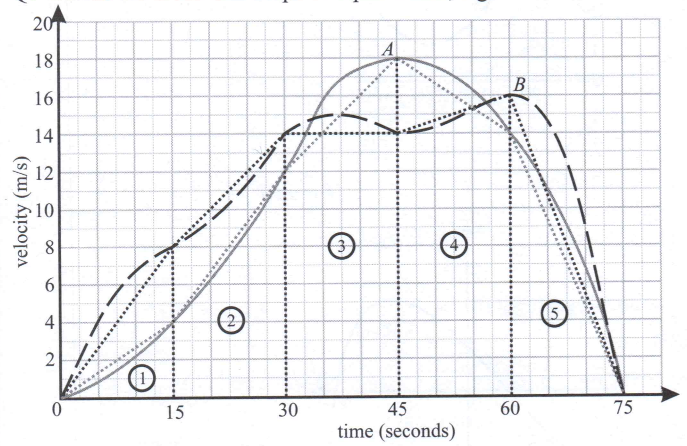


Strip 1: area = 0.5 × 1 × 5 = 2.5 miles
Strip 2: area = 0.5(5 + 5.5) × 1 = 5.25 miles
Strip 3: area = 0.5 × 1 × 5.5 = 2.75 miles
Total area = distance = 2.5 + 5.25 + 2.75 = 10.5 miles

Percentage difference = $\frac{11.75 - 10.5}{11.75} \times 100 = 10.638... = 10.6\%$

[6 marks available — 1 mark for finding the area for each of the three strips, 1 mark for finding the total area using three strips, 2 marks for finding the area for each of the six strips (otherwise 1 mark for finding the area of four or five strips), 1 mark for finding the total area using six strips, 1 mark for the percentage difference]

Q5 Divide the areas into strips of equal width, e.g.



Object *A*: Strip 1: area = 0.5 × 15 × 4 = 30 m
Strip 2: area = 0.5(4 + 12) × 15 = 120 m
Strip 3: area = 0.5(12 + 18) × 15 = 225 m
Strip 4: area = 0.5(18 + 14) × 15 = 240 m
Strip 5: area = 0.5 × 15 × 14 = 105 m
Total area = distance = 30 + 120 + 225 + 240 + 105 = 720 m
Average speed = 720 ÷ 75 = 9.6 m/s
Object *B*: Strip 1: area = 0.5 × 15 × 8 = 60 m
Strip 2: area = 0.5(8 + 14) × 15 = 165 m
Strip 3: area = 14 × 15 = 210 m
Strip 4: area = 0.5(14 + 16) × 15 = 225 m
Strip 5: area = 0.5 × 15 × 16 = 120 m
Total area = distance = 60 + 165 + 210 + 225 + 120 = 780 m
Average speed = 780 ÷ 75 = 10.4 m/s
So ratio of speeds = 9.6 : 10.4 = 96 : 104 = 12 : 13

[6 marks available — 1 mark for dividing the area up into strips of equal width, 1 mark for finding the area of each strip for object A, 1 mark for finding the average speed of object A, 1 mark for find the area of each strip for object B, 1 mark for finding the average speed of object B, 1 mark for the correct answer]

Pages 45-46: Gradients

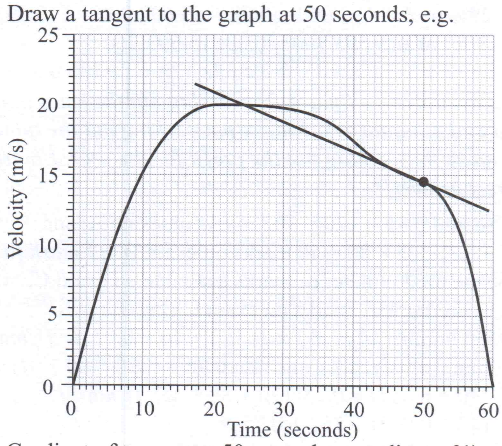
1 0800 to 1200 is 4 hours and the temperature changes by 29 – 22.5 = 6.5 °C, so the average rate of change between 8 am and 12 noon is $\frac{6.5}{4}$ = 1.625 °C/hour
1600 to 1800 is 2 hours and the temperature changes by 26.5 – 22.5 = 4 °C, so the average rate of change between 4 pm and 6 pm is $\frac{4}{2}$ = 2 °C/hour. So the manager is incorrect.

[3 marks available — 1 mark for finding the average rate of change between 8 am and 12 noon, 1 mark for finding the average rate of change between 4 pm and 6 pm, 1 mark for saying the manager is incorrect]

2 a) Gradient of line connecting (10, 15) and (20, 20)
= average acceleration between 10 and 20 seconds
$= \frac{20 - 15}{20 - 10} = \frac{5}{10} = 0.5$ m/s^2

[2 marks available — 2 marks for correct answer, otherwise 1 mark for correct coordinates of the two points]

b) Draw a tangent to the graph at 50 seconds, e.g.



Gradient of tangent at 50 seconds = gradient of line connecting (29, 19) and (57, 13):

$\frac{13-19}{57-29} = \frac{-6}{28} = -0.21428... = -0.214 \text{ m/s}^2$ (3 s.f.)

[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correctly drawing the tangent]

It can be a bit tricky to draw the tangent exactly — as long as you got a similar answer, you'll be fine.

3 Plant A: gradient of line connecting (0, 0) and (7, 6)

= average rate of growth during the first week: $\frac{6-0}{7-0} = \frac{6}{7}$ cm/day

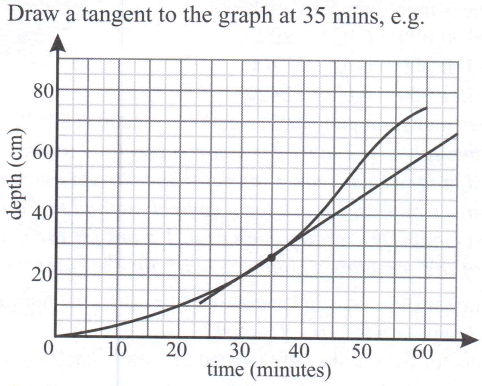
Plant B: gradient of line connecting (7, 4) and (14, 7)

= average rate of growth during the first week: $\frac{7-4}{14-7} = \frac{3}{7}$ cm/day

Ratio = $\frac{6}{7} : \frac{3}{7} = 6:3 = 2:1$

[3 marks available in total — 1 mark for finding the average rate of growth for plant A, 1 mark for finding the average rate of growth for plant B, 1 mark for the correct ratio]

4 a) Draw a tangent to the graph at 35 mins, e.g.



Gradient of tangent at 35 mins = gradient of line connecting

(45, 40) and (60, 60): $\frac{60-40}{60-45} = \frac{20}{15} = \frac{4}{3}$ cm/min

[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correctly drawing the tangent]

b) After 60 mins, the water is 75 cm deep, so average rate = gradient of line connecting (0, 0) and (60, 75):

$= \frac{75-0}{60-0} = \frac{75}{60} = 1.25$ cm/min

[2 marks available — 2 marks for the correct answer, otherwise 1 mark for correct coordinates of the two points]

Section 4 — Ratio, Proportion and Rates of Change

Pages 47-48: Ratios

1 Speed = Distance ÷ Time

Speed of rowing boat = 18 ÷ 2.25 = 8 km/h

Speed of canoe = 15 ÷ 1.5 = 10 mph ≈ 10 × 1.6 = 16 km/h

So the ratio of the speeds is 8 km/h : 16 km/h = 1 : 2

[4 marks available — 1 mark for finding the speed of the boat, 1 mark for finding the speed of the canoe, 1 mark for a ratio with both parts in the same units, 1 mark for the correct answer]

You could have converted the rowing boat's speed into mph — just make sure both speeds have the same units before you simplify your ratio.

2 Let triangle A have height x cm and base length y cm.

Then triangle B has height $x + 1$ cm and base length $3y$ cm.

The area of triangle B is 45 cm², so the area of triangle A is $(45 \div 9) \times 2 = 10$ cm².

Triangle A: $\frac{1}{2} \times x \times y = 10$ so $xy = 20$ [1]

Triangle B: $\frac{1}{2} \times (x+1) \times 3y = 45$, rearranging gives $xy + y = 30$ [2]

Substituting [1] into [2] gives: $20 + y = 30$ so $y = 10$

Put this value back into [1] to give $x = 2$.

Height of triangle A = 2 cm and height of triangle B = 2 + 1 = 3 cm.

So the ratio of vertical heights is 2 : 3.

[5 marks available — 1 mark for setting up an equation for the area of triangle A, 1 mark for setting up an equation for the area of triangle B, 1 mark for a correct method to solve the equations simultaneously, 1 mark for the correct vertical heights of each triangle, 1 mark for the correct answer]

3 E.g. Say that Jenna has completed x% and Harvey has completed y%. Then $x : y = 3 : 4$ and $x + 7 : y + 7 = 7 : 9$.

$\frac{x}{y} = \frac{3}{4}$ and $\frac{x+7}{y+7} = \frac{7}{9}$

$4x = 3y$ and $9(x + 7) = 7(y + 7)$

$4x - 3y = 0$ [1] and $9x - 7y = -14$ [2]

[1] × 7: $28x - 21y = 0$ [3], and [2] × 3: $27x - 21y = -42$ [4]

[3] – [4]: $x = 42$, so she has completed 42% of the game.

[4 marks available — 1 mark for setting up two ratios in terms of x and y and converting them to fractions, 1 mark for multiplying out the fractions to form a pair of simultaneous equations, 1 mark for a correct method to solve the simultaneous equations, 1 mark for the correct answer]

4 $x - 10 : y - 10 = 2 : 5$ and $x + 8 : y + 8 = 1 : 2$.

$\frac{x-10}{y-10} = \frac{2}{5}$ and $\frac{x+8}{y+8} = \frac{1}{2}$

$5(x - 10) = 2(y - 10)$ and $2(x + 8) = y + 8$

$5x - 2y = 30$ [1] and $2x - y = -8$ [2]

[2] × 2: $4x - 2y = -16$ [4]

[1] – [3]: $x = 46$

Substitute into [2] to find y: $(2 \times 46) - y = -8$ so $y = 100$

So x as a percentage of y is: $\frac{46}{100} \times 100 = 46\%$

[5 marks available — 1 mark for setting up two ratios in terms of x and y and converting them to fractions, 1 mark for multiplying out the fractions to form a pair of simultaneous equations, 1 mark for a correct method to solve the simultaneous equations, 1 mark for the correct values of x and y, 1 mark for the correct answer]

5 Let the original number of red and green sweets in the bag be x.

Let the number of yellow sweets be y.

The numbers of red, yellow and green sweets remaining are $x - 5$, $y - 15$ and $x - 25$ respectively.

red : yellow = 2 : 3 so $x - 5 : y - 15 = 2 : 3$

$\frac{x-5}{y-15} = \frac{2}{3}$

$3(x - 5) = 2(y - 15)$, so $3x - 2y = -15$ [1]

yellow : green = 3 : 1 so $y - 15 : x - 25 = 3 : 1$

$\frac{y-15}{x-25} = 3$

$y - 15 = 3(x - 25)$, so $3x - y = 60$ [2]

[2] – [1]: $y = 75$

Substituting back into [2] gives $3x - 75 = 60$, so $x = 45$

The total number of sweets originally in the bag is 45 + 75 + 45 = 165

The fraction that are yellow is $\frac{75}{165} = \frac{5}{11}$

[6 marks available — 1 mark for giving the number of red, yellow and green sweet in terms of x and y, 1 mark for setting up two ratios in terms of x and y and converting them to fractions, 1 mark for multiplying out the fractions to form a pair of simultaneous equations, 1 mark for a correct method to solve the simultaneous equations, 1 mark for the correct values of x and y, 1 mark for the correct answer]

Pages 49-50: Direct and Inverse Proportion

1 a) $x \propto M$, so $x = kM$ ***[1 mark]***

When $M = 40$, $x = 2$

$2 = k \times 40$, so $k = 2 \div 40 = \frac{1}{20}$ ***[1 mark]***

So $x = \frac{1}{20}M$ or $M = 20x$ ***[1 mark]***

[3 marks available in total — as above]

b) $x = \frac{1}{20}M$ so when $M = 55$, $x = \frac{1}{20} \times 55 = 2.75$ cm ***[1 mark]***

2 a) $w \propto r^3$, so $w = kr^3$ ***[1 mark]***

When $r = 6$, $w = 1080$, so $1080 = k \times 6^3$, so $k = 5$ ***[1 mark]***

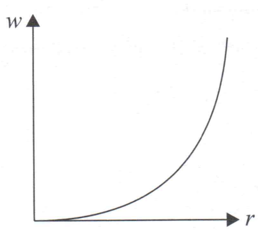
So $w = 5r^3$

When $w = 8640$, $5r^3 = 8640$ ***[1 mark]***

$r^3 = 1728$, so $r = 12$ ***[1 mark]***

[4 marks available in total — as above]

b)



[1 mark]

3 $p \propto q$ and $q \propto \frac{1}{r}$ so $p = Aq$ and $q = \frac{B}{r}$ ***[1 mark for both]***

When $p = 8$, $q = 25$ so $A = 8 \div 25 = 0.32$ ***[1 mark]***

When $q = 25$, $r = 16$ so $B = 25 \times 16 = 400$ ***[1 mark]***

So when $r = 2$, $q = \frac{400}{2} = 200$ ***[1 mark]***

And so $p = 0.32 \times 200 = 64$ ***[1 mark]***

[5 marks available in total — as above]

4 $a \propto \frac{1}{b}$ and $a \propto \frac{1}{c^2}$ so $a = \frac{M}{b}$ and $a = \frac{N}{c^2}$

Substituting for a gives: $\frac{M}{b} = \frac{N}{c^2}$ and so

$b = \frac{M}{N}c^2$ where $\frac{M}{N}$ is just a constant so $b \propto c^2$.

[3 marks available — 1 mark for writing both proportions as equations, 1 mark for substituting for a to equate the two fractions, 1 mark for rearranging into the form b = kc² where k is a constant and explaining that this is the equation of a direct proportion]

5 Let the object have density $= d_1$ and volume $= v$.

Call the object's density after the increase d_2.

Volume of the object after the increase $= 1.4v$.

$d \propto \frac{1}{v}$ so $d_1 = \frac{k}{v}$ and $d_2 = \frac{k}{1.4v}$ ***[1 mark]***

k is the same in both cases as the mass of the object doesn't change.

$d_1v = k$ and $1.4d_2v = k$

$d_1v = 1.4d_2v$

$d_1 = 1.4d_2$ ***[1 mark]***

So $d_2 = \frac{1}{1.4}d_1 = 0.71428...\ d_1$ ***[1 mark]***

d_2 is 71.428...% of d_1, so the percentage decrease is

100 – 71.428... = 28.571... = 28.6% (1 d.p.) ***[1 mark]***

[4 marks available in total — as above]

6 $y \propto \frac{1}{x^2}$ and $y \propto z^3$ so $y = \frac{A}{x^2}$ and $y = Bz^3$ ***[1 mark for both]***

From the graph, when $x = 3$, $y = 80$ so $A = 80 \times 3^2 = 720$ ***[1 mark]***

So when $x = 2$, $y = \frac{720}{2^2} = 180$ ***[1 mark]***

When $y = 180$, $z = 5$, so $B = 180 \div 5^3 = 1.44$ ***[1 mark]***

So when $z = 15$, $y = 1.44 \times 15^3 = 4860$ ***[1 mark]***

[5 marks available in total — as above]

Pages 51-52: Percentages

1 1st quarter sales: £12 000 000

2nd quarter sales: £12 000 000 × 1.1 = £13 200 000

3rd quarter sales: £13 200 000 × 1.1 = £14 520 000

4th quarter sales: £14 520 000 × 1.1 = £15 972 000

Total sales = £12 000 000 + £13 200 000 + £14 520 000 + £15 972 000 = £55 692 000

Profit = 28% of £55 692 000 = 0.28 × 55 692 000 = £15 593 760

Bonus = 30% of £15 593 760 = 0.3 × 15 593 760 = £4 678 128 = £4 700 000 (2 s.f.)

[4 marks available — 1 mark for working out the sales in the 2nd, 3rd and 4th quarters, 1 mark for working out the total yearly sales, 1 mark for working out the profit for the year, 1 mark for the correct answer]

2 Simone scored 0.85 × 120 = 102 marks on exam A, and 0.5 × 80 = 40 marks on exam B. ***[1 mark for both]***

Let M be the total number of marks available on exam C, then she scored $0.95M$ marks on exam C.

So her total number of marks was $102 + 40 + 0.95M$. ***[1 mark]***

Her total number of marks was also $0.75 \times (120 + 80 + M)$

So $102 + 40 + 0.95M = 0.75(120 + 80 + M)$ ***[1 mark]***

$142 + 0.95M = 150 + 0.75M$

$0.2M = 8$, so $M = 40$ ***[1 mark]***

[4 marks available in total — as above]

3 Call the side length of cube $A = s$, then side length of cube $B = 1.2s$ ***[1 mark]***

Let P_A be the pressure of cube A, P_B be the pressure of cube B and w be the weight of each cube.

Pressure $= \frac{\text{Force}}{\text{Area}}$, so $P_A = \frac{w}{s^2}$ so $w = P_As^2$ ***[1 mark]***

$P_B = \frac{w}{(1.2s)^2}$ so $w = 1.44P_Bs^2$ ***[1 mark]***

Equating values of w gives: $P_As^2 = 1.44P_Bs^2$, so $P_A = 1.44P_B$

So P_A is 144% of P_B ***[1 mark]***

[4 marks available in total — as above]

4 Each year, 10% is added to the customer's money and the investment company takes 12 – 10 = 2%

The customer's money at the start of each year is:

Year 1 = £100 000, Year 2 = £100 000 × 1.1 = £110 000,

Year 3 = £110 000 × 1.1 = £121 000

And the investment company makes:

Year 1 = £100 000 × 0.02 = £2000,

Year 2 = £110 000 × 0.02 = £2200,

Year 3 = £121 000 × 0.02 = £2420

So the investment company makes £2000 + £2200 + £2420 = £6620

[5 marks available — 1 mark for a correct method to find the customer's money each year, 1 mark for the correct amounts of the customer's money, 1 mark for a correct method to find the investment company's money each year, 1 mark for the correct amounts of the company's money, 1 mark for the correct answer]

5 Let the number of houses in Liverstone be L, then the number of houses in Ashmouth is $1.7L$. ***[1 mark]***

Expressions for the number of terraced houses in each village are:

Liverstone: $0.28L$ Ashmouth: $0.2 \times 1.7L$

So $(0.2 \times 1.7L) - 0.28L = 480$ ***[1 mark]***

$0.34L - 0.28L = 480$

$0.06L = 480$, so $L = 8000$ ***[1 mark]***

So there are 8000 houses in Liverstone, and 8000 × 1.7 = 13 600 houses in Ashmouth. ***[1 mark]***

There are 8000 × 0.38 = 3040 semi-detached houses in Liverstone and 13 600 × 0.4 = 5440 semi-detached houses in Ashmouth.

The difference in the number of semi-detached houses is 5440 – 3040 = 2400 ***[1 mark]***

[5 marks available in total — as above]

Section Five — Geometry and Measures

Pages 53-55: Circle Geometry

1 E.g. By the alternate segment theorem, angle $ABC = x$ ***[1 mark]***

As $AB = AC$, angle $ACB = x$ (isosceles triangle) ***[1 mark]***

AB is parallel to DE, so angle $BAC = x$ (alternate angles) ***[1 mark]***

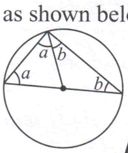
All three angles inside the triangle are equal, so ABC is an equilateral triangle. ***[1 mark]***

[4 marks available in total — as above]

There's often more than one way to answer circle theorem questions — as long as you show clearly what you're doing, you'll get the marks.

2 E.g. Triangle *AOD* is isosceles,
so angle $ODA = (180° - 128°) \div 2 = 26°$ ***[1 mark]***
Triangle *BOC* is also isosceles, so angle $OBC = 41°$ ***[1 mark]***
ABCD is a cyclic quadrilateral, so angle ABC + angle $CDA = 180°$
Angle $CDA = 180° - (59° + 41°) = 80°$ ***[1 mark]***
Angle CDO = angle CDA – angle $ODA = 80° - 26° = 54°$ ***[1 mark]***
[4 marks available in total — as above]

3 E.g. Divide the triangle into two isosceles triangles, as shown below, and label the angles *a* and *b*:



[1 mark]. Then $a + a + b + b = 180°$ ***[1 mark]*** (as the angles in a triangle add up to 180°). So $2a + 2b = 180°$, which means $a + b = 90°$ ***[1 mark]***, so the angle is a right angle.
[3 marks available in total — as above]

4 E.g. Angle $CDE = 44°$ (the angle at the centre is double the angle at the circumference) ***[1 mark]***
Angle $FBC = 76°$ (angles in the same segment are equal) ***[1 mark]***
Angle $BFE = 180° - 44° - 76° = 60°$ (angles in a triangle add up to 180°, and *BDF* is a triangle) ***[1 mark]***.
[3 marks available in total — as above]

5 E.g. By the alternate segment theorem, angle $DBC = 54°$ ***[1 mark]***
DB is a straight line, so angle $DXC = 180° - 94° = 86°$ ***[1 mark]***
If *X* was the centre of the circle, then angle $DXC = 2 \times$ angle DBC ***[1 mark]***. But $2 \times$ angle $DBC = 2 \times 54° = 108° \neq 86°$ ***[1 mark]***, so *X* is not the centre of the circle.
[4 marks available in total — as above]

6 E.g. Angle *AFB* = angle *ABF* (tangents from the same point are the same length, so triangle *ABF* is isosceles)
Angle AFB = angle $ABF = (180° - 36°) \div 2 = 72°$ ***[1 mark]***
Angle $AFO = 90°$ (tangent meets a radius at 90°),
so angle BFO = angle $FBO = 90° - 72° = 18°$ ***[1 mark]***
Angle $FEC = 180° - 56° - 18° = 106°$ ***[1 mark]***
(opposite angles in a cyclic quadrilateral add up to 180°)
Angle $DCE = 180° - 112° = 68°$ and angle $DEC = 180° - 106° = 74°$ ***[1 mark for DCE and DEC]*** (angles on a straight line)
Angle $CDE = 180° - 68° - 74° = 38°$ ***[1 mark]***
[5 marks available in total — as above]

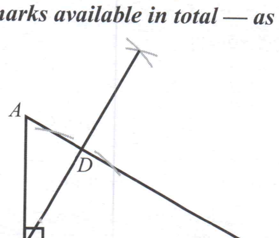
7 E.g. Angle $ACB = x°$ (by the alternate segment theorem) ***[1 mark]***
Angle $OCF = 90°$ (tangent meets a radius at 90°) ***[1 mark]***
So angle $OCA = 90 - y°$ ***[1 mark]***
a = angle ACB – angle $OCA = x - (90° - y) = x + y - 90°$ ***[1 mark]***
[4 marks available in total — as above]

Pages 56-57: Similarity and Congruence

1 Angle *BAF* = angle *BCF* ***[1 mark]***
(opposite angles in a parallelogram are equal)
angle *AEB* = angle *DBC* ***[1 mark]*** (alternate angles)
angle *ABE* = angle *BDC* (as the other two pairs of angles match up)
All three angles match up, so the triangles are similar ***[1 mark]***.
[3 marks available in total — as above]

2 The trapezium is isosceles, so side *AD* = side *BC* ***[1 mark]***.
Angles in the same segment are equal, so angle *DAX* = angle *CBX* and angle *ADX* = angle *BCX* ***[1 mark for both angle pairs]***.
The condition AAS holds, so triangles *AXD* and *BXC* are congruent ***[1 mark]***.
[3 marks available in total — as above]

3 a)



[2 marks available — 1 mark for a correct method, 1 mark for the correct line drawn]

b) Angle *ADB* = angle *CDB* = 90° (as *BD* is perpendicular to *AC*)
angle *DCB* + angle *CBD* = 90° (angles in a triangle),
angle *ABD* + angle *DAB* = 90° (angles in a triangle)
and angle *ABD* + angle *CBD* = 90° (angles in a right angle)
So angle *DCB* = angle *ABD* and angle *DAB* = angle *CBD*
All three angles match up, so the triangles are similar.
[3 marks available — 1 mark for stating that angle ADB = angle CDB, 1 mark for showing that another pair of angles are equal, 1 mark for showing that the final pair of angles are equal and stating that this means the triangles are similar]

4 Opposite angles in a parallelogram are equal,
so angle *MAQ* = angle *NCP* ***[1 mark]***.
Opposite sides of a parallelogram are equal, and *M* and *P* are the midpoints of *AB* and *CD* respectively,
so side *AM* = side *CP* ***[1 mark]***.
Opposite sides of a parallelogram are equal, and *N* and *Q* are the midpoints of *BC* and *DA* respectively,
so side *AQ* = side *NC* ***[1 mark]***.
The condition SAS holds, so triangles *AMP* and *PNC* are congruent ***[1 mark]***.
[4 marks available in total — as above]

5 The *y*-axis and the line L_3 are parallel, so angle *RQP* = angle *NMP* ***[1 mark]*** and angle *QRP* = angle *MNP* ***[1 mark]*** (alternate angles).
Find the lengths of *NP* and *PR*:
Putting $x = 0$ and $y = 0$ into the equation for line L_2 gives the coordinates $N(0, 5)$ and $R(4, 0)$ ***[1 mark for both points]***.
Find the lengths of sides *NP* and *PR*:
$NP^2 = (2 - 0)^2 + (2.5 - 5)^2 = 4 + 6.25 = 10.25$
So $NP = \sqrt{10.25}$ ***[1 mark]***
$PR^2 = (4 - 2)^2 + (2.5 - 0)^2 = 4 + 6.25 = 10.25$
So $PR = \sqrt{10.25}$
So $NP = PR$ ***[1 mark]***
The condition AAS holds, so triangles *MNP* and *QRP* are congruent ***[1 mark]***.
[6 marks available in total — as above]

6 Prove that triangles *AOM* and *BOM* are congruent:
OM is common to both triangles.
Angles $OMA = OMB = 90°$ and $OB = OA$ (both are a radius)
The condition RHS holds, so triangles *AOM* and *BOM* are congruent. Therefore $AM = MB$ so *OP* bisects the chord.
[4 marks available — 1 mark for any statement of congruence, 1 mark for the other two statements of congruence, 1 mark for RHS, 1 mark for stating that AM = MB]

Pages 58-59: Arcs, Sectors and Segments

1 Area $= \frac{x}{360} \times 12^2 \times \pi$
$88\pi = \frac{144\pi x}{360} = \frac{2\pi x}{5}$
$x = 88 \times \frac{5}{2} = 220°$
[3 marks available — 1 mark for a correct formula for the area of a sector, 1 mark for substituting in the numbers correctly, 1 mark for the correct value of x]

2 a) Area of sector $= \frac{70}{360} \times 6^2 \times \pi = 7\pi$ cm²
Area of triangle $= ½ \times 6^2 \times \sin 70° = 16.914...$ cm²
Area of segment $= 7\pi - 16.914... = 5.076... = 5.08$ cm² (3 s.f.)
[3 marks available — 1 mark for finding the area of the sector, 1 mark for finding the area of the triangle, 1 mark for the correct answer]

b) Perimeter of segment = arc + chord
Arc length $= \frac{70}{360} \times 2 \times \pi \times 6 = \frac{7}{3}\pi$ cm
To find the chord length, use the cosine rule:
$a^2 = 6^2 + 6^2 - (2 \times 6 \times 6 \times \cos 70°) = 47.374...$
$a = 6.882...$ cm
Perimeter $= \frac{7}{3}\pi + 6.882... = 14.213... = 14.2$ cm (3 s.f.)
[4 marks available — 1 mark for the arc length, 1 mark for putting the numbers into the cosine rule formula correctly, 1 mark for the chord length, 1 mark for the correct answer]

3 Arc length = 21.6 – 8 – 8 = 5.6 cm ***[1 mark]***
Circumference = 2 × 8 × π = 50.265... cm ***[1 mark]***
50.265... ÷ 5.6 = 8.975... ≈ 9 sectors in total ***[1 mark]***
[3 marks available in total — as above]
Don't forget to round your answer — there'll be a whole number of sectors (and the original perimeter was rounded as well).

4 a) Area of segment with radius 8 cm = $(\frac{45}{360} \times 8^2 \times \pi)$
$= 8\pi$ cm² ***[1 mark]***
Area of segment with radius 5 cm = $(\frac{45}{360} \times 5^2 \times \pi)$
$= \frac{25}{8}\pi$ cm² ***[1 mark]***
Area of white icing = $8\pi - \frac{25}{8}\pi = 15.315...$
= 15.3 cm² (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]
It's always a good idea to leave your working in terms of π for as long as possible — it means you don't lose any accuracy later on.

b) Area of top of slice = $\frac{45}{360} \times 10^2 \times \pi = 12.5\pi$ cm² ***[1 mark]***
Volume of slice = 12.5π × 8 = 100π = 314.159...
= 314 cm³ (3 s.f.) ***[1 mark]***
[2 marks available in total — as above]

5 a) Angle DFC = (360° – 140°) ÷ 2 = 110°
and angle DCF = 60° ÷ 2 = 30° ***[1 mark for both]***
Using the sine rule, $\frac{DC}{\sin 110°} = \frac{1.6}{\sin 30°}$ ***[1 mark]***
So $DC = \sin 110° \times \frac{1.6}{\sin 30°} = 3.0070...$
= 3.01 cm (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]
There are other ways of answering this question — but you should still find that DC = 3.01 cm to 3 s.f.

b) From a), AD = 3.007... cm and AC = 6.014... cm
(as D is the midpoint of AC).
The diagram has a vertical line of symmetry, so $BE = AD$.
Arc length $AB = \frac{60}{360} \times 2 \times \pi \times 6.014... = 6.297...$ cm
Arc length $DE = \frac{140}{360} \times 2 \times \pi \times 1.6 = 3.909...$ cm
Perimeter = 3.007... + 3.007... + 6.297... + 3.909...
= 16.221... = 16.2 cm (3 s.f.)
[5 marks available — 1 mark for finding AC, 1 mark for the correct lengths of AD and BE, 1 mark for the correct arc length AB, 1 mark for the correct arc length DE, 1 mark for the correct answer]

Pages 60-62: 3D Shapes — Surface Area and Volume

1 Area of cross-section = ½$ab \sin C$ = ½ × 4 × 2 × sin 67°
= 3.682... cm²
Volume of prism = 3.682... × 9 = 33.138... = 33.1 cm³ (3 s.f.)
[3 marks available — 1 mark for putting the numbers into the area formula correctly, 1 mark for the correct cross-sectional area, 1 mark for the correct answer]

2 Volume = $\frac{1}{3}\pi r^2 h$, so $(3.2 \times 10^{26})\pi = \frac{1}{3} \times \pi \times (4 \times 10^8)^2 \times x$
$(3.2 \times 10^{26})\pi = \frac{1}{3} \times \pi \times 16 \times 10^{16} \times x$
$9.6 \times 10^{26} = (1.6 \times 10^{17})x$
$\frac{9.6}{1.6} \times \frac{10^{26}}{10^{17}} = x$, so $x = 6 \times 10^9$ m
[3 marks available — 1 mark for squaring (4 × 10⁸) correctly, 1 mark for a correct method for solving the equation, 1 mark for the correct answer]

3 Let V_A = volume of sphere A, V_B = volume of sphere B and R_B = radius of sphere B. Then
$V_A = \frac{4}{3} \times \pi \times 6^3 = 288\pi$ cm³ ***[1 mark]***
$V_B = 1.6 \times V_A = 1.6 \times 288\pi = 460.8\pi$ cm³ ***[1 mark]***
$R_B^3 = 460.8\pi \div \frac{4}{3} \div \pi = 345.6$ ***[1 mark]***
So $R_B = \sqrt[3]{345.6} = 7.017... = 7.02$ cm (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

4 a) The removed cone is similar and is one-third of the height of the original cone, so has radius 5 cm and height 12 cm ***[1 mark]***
Volume of frustum
$= (\frac{1}{3} \times \pi \times 15^2 \times 36) - (\frac{1}{3} \times \pi \times 5^2 \times 12)$ ***[1 mark]***
= 2700π – 100π = 2600π cm³ ***[1 mark]***
[3 marks available in total — as above]

b) Use Pythagoras to find the slant height, l, of the original cone:
$l^2 = 36^2 + 15^2 = 1521$, so $l = \sqrt{1521} = 39$ cm ***[1 mark]***
Slant height of removed cone = 39 ÷ 3 = 13 cm.
Surface area of frustum = curved area of original cone – curved area of removed cone + both circular faces
= (π × 15 × 39) – (π × 5 × 13) + (π × 15²) + (π × 5²) ***[1 mark]***
= 585π – 65π + 225π + 25π = 770π cm² ***[1 mark]***
[3 marks available in total — as above]

5 Volume of cylinder = π × 10² × 21 = 2100π cm³ ***[1 mark]***
Volume of cone = $\frac{1}{3} \times \pi \times 10^2 \times 21 = 700\pi$ cm³ ***[1 mark]***
Volume not taken up by the cone = 2100π – 700π
= 1400π cm³ ***[1 mark]*** = 0.0014π m³ ***[1 mark]***
Density = mass ÷ volume, so mass = density × volume
mass = 0.52 × 0.0014π = 0.002287... kg ***[1 mark]***
= 2.287... g = 2.29 g (3 s.f.) ***[1 mark]***
[6 marks available in total — as above]
Be careful with the units here — you're given the dimensions of the shape in cm, but the density in kg/m³. It doesn't matter when you do the unit conversions, as long as you end up with an answer in g.

6 Surface area of cylinder = $2\pi rh + 2\pi r^2 = 12\pi r + 2\pi r^2$ ***[1 mark]***
Surface area of cone = $\pi rl + \pi r^2 = 7\pi r + \pi r^2$ ***[1 mark]***
Combined surface area = $12\pi r + 2\pi r^2 + 7\pi r + \pi r^2$
$= 19\pi r + 3\pi r^2$ ***[1 mark]***
So $110\pi = 19\pi r + 3\pi r^2$, so $3r^2 + 19r - 110 = 0$ ***[1 mark]***
$(3r - 11)(r + 10) = 0$ ***[1 mark]***, so $r = \frac{11}{3}$ or $r = -10$
The radius must be positive, so $r = \frac{11}{3}$ cm ***[1 mark]***
[6 marks available in total — as above]

7 Weight of one piece = 5000 ÷ 8 = 625 N ***[1 mark]***
Area of flat face = $\pi r^2 \div 4 = \pi \times 1.4^2 \div 4 = 0.49\pi$ m² ***[1 mark]***
Pressure = force ÷ area = 625 ÷ 0.49π ***[1 mark]*** = 406.007...
= 406 N/m² (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]

8 The slant length, l, of the cone is:
$\sqrt{(3k)^2 + (4k)^2} = \sqrt{9k^2 + 16k^2} = \sqrt{25k^2} = 5k$ cm ***[1 mark]***
Curved surface area of cone = $\pi \times 3k \times 5k = 15k^2\pi$ ***[1 mark]***
Curved surface area of hemisphere = $0.5 \times 4\pi(3k)^2 = 18k^2\pi$ ***[1 mark]***
The total surface area of the object is
$15k^2\pi + 18k^2\pi = 33k^2\pi$ ***[1 mark]***
So $33k^2\pi = 3993\pi$ ***[1 mark]***, so $k^2 = 121$, so $k = 11$ ***[1 mark]***
[6 marks available in total — as above]

9 Volume of one sphere = (30% of 1200) ÷ 2 = 180 cm³
$\frac{4}{3}\pi r^3 = 180$ ***[1 mark]*** so $r^3 = 42.971...$, so $r = 3.502...$ cm ***[1 mark]***
Volume of one cone = (1200 ÷ 3) ÷ 4 = 100 cm³, so $\frac{1}{3}\pi r^2 h = 100$
$r = 3.502...$ cm, so $\frac{1}{3}\pi(3.502...)^2 h = 100$ ***[1 mark]***
$h = 7.783...$ cm ***[1 mark]***
Volume of cube = (7.783...)³ = 471.570... cm³ ***[1 mark]***
Amount of steel left = 1200 – 360 – 400 = 440 cm³
so she does not have enough steel left ***[1 mark]***
[6 marks available in total — as above]

Pages 63-64: Rates of Flow

1 Volume of tank = π × 1.6² × 4.5 = 11.52π m³ ***[1 mark]***
= 11.52π × 1000 = 11 520π litres ***[1 mark]***
Rate of flow = volume ÷ time = 11 520π ÷ 25 ***[1 mark]***
= 460.8π = 1447.645... = 1450 litres per minute (3 s.f.) ***[1 mark]***
[4 marks available in total — as above]
To convert m³ to litres, you can first convert it to cm³ (by multiplying by 100 × 100 × 100) then divide it by 1000 (as 1 litre = 1000 cm³), or you can multiply by 1000 to do it all in one go.

2 a) The volume of liquid when it is 2 cm deep is the volume of a frustum formed by removing the top 6 cm of the cone.
$6 \div 8 = 0.75$, so the removed cone has a radius of
$2 \times 0.75 = 1.5$ cm ***[1 mark]***.
Volume of frustum
$= (\frac{1}{3}\pi \times 2^2 \times 8) - (\frac{1}{3}\pi \times 1.5^2 \times 6) = 19.373...$ cm^3 ***[1 mark]***
Time taken to fill = volume ÷ rate of flow = 19.373... ÷ 0.5
= 38.746... = 38.7 s (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

b) Volume of cone $= \frac{1}{3}\pi \times 2^2 \times 8 = 33.510...$ cm^3
So 89.5% of total volume = 33.510... × 0.895 = 29.991... cm^3
Time taken to fill = volume ÷ rate of flow = 29.991... ÷ 0.5
= 59.9733... = 60 s (2 s.f.)
[2 marks available — 1 mark for finding 89.5% of the volume, 1 mark for finding the time taken to fill this volume]

3 a) Cross-sectional area = ½$ab \sin C$ = ½ × $2\sqrt{3}$ × 4 × sin 60°
= ½ × $2\sqrt{3}$ × 4 × $\frac{\sqrt{3}}{2}$ = 6 m^2
[2 marks available — 1 mark for putting the numbers into the area formula correctly, 1 mark for the correct answer]

b) Rate of flow = 90 000 litres per minute
= 90 000 ÷ 60 = 1500 litres per second ***[1 mark]***
= 1 500 000 cm^3/s = 1.5 m^3/s ***[1 mark]***
Speed = rate of flow ÷ cross-sectional area of water
= 1.5 ÷ 6 ***[1 mark]*** = 0.25 m/s ***[1 mark]***
[4 marks available in total — as above]

4 Volume of cone $= \frac{1}{3}\pi \times 30^2 \times 70 = 21\,000\pi$ cm^3 ***[1 mark]***
Volume of cylinder $= \pi \times 30^2 \times 60 = 54\,000\pi$ cm^3 ***[1 mark]***
Total volume $= 21\,000\pi + 54\,000\pi = 75\,000\pi$ cm^3
$= 75\pi$ litres ***[1 mark]***

2 minutes = 120 seconds, and time taken = volume ÷ rate of flow, so time taken to empty – time taken to fill = 120 s:
$\frac{75\pi}{(x-2)\pi} - \frac{75\pi}{x\pi} = 120$ ***[1 mark]***
$75x - 75(x-2) = 120x(x-2)$
$120x^2 - 240x - 150 = 0$ ***[1 mark]***
$4x^2 - 8x - 5 = 0$
$(2x+1)(2x-5) = 0$ ***[1 mark]***
So $x = -0.5$ or $x = 2.5$, so $x = 2.5$ ***[1 mark]*** (as it must be positive).
[7 marks available in total — as above]

Pages 65-67: Enlargement

1 a) Ratio of surface areas = $1^2 : 3^2 : 5^2 = 1 : 9 : 25$ ***[1 mark]***

b) Height of large cylinder = 6 × 5 = 30 cm
Find the radius of the large cylinder: $6750\pi = \pi \times r^2 \times 30$
So $r^2 = 6750 \div 30 = 225$, which means $r = 15$ cm
Radius of small cylinder = 15 ÷ 5 = 3 cm,
so radius of middle cylinder = 3 × 3 = 9 cm
[3 marks available — 1 mark for finding the radius of the large cylinder, 1 mark for finding the radius of the small cylinder, 1 mark for the correct answer]

2 a) (Scale factor)$^3 = \frac{160}{2.5 \times 10^6}$ ***[1 mark]*** $= \frac{1}{15\,625}$,
so scale factor $= \frac{1}{25}$ ***[1 mark]***
[2 marks available in total — as above]

b) Surface area of original = 98 × 25^2 = 61 250 = 6.125 × 10^4 m^2
[2 marks available — 1 mark for correct working, 1 mark for the correct answer in standard form]

3 a) Scale factor $= \sqrt[3]{\frac{1536\pi}{648\pi}} = \sqrt[3]{\frac{64}{27}} = \frac{4}{3}$ ***[1 mark]***
Surface area of cone B $= 198\pi \times \left(\frac{4}{3}\right)^2 = 352\pi$ cm^2 ***[1 mark]***
[2 marks available in total — as above]

b) 3 : 4 ***[1 mark]***

4 a) Volume of cuboid A = 1.5 × 2.5 × 5 = 18.75 cm^3
Volume of cuboid B $= 18.75 \times \left(\frac{8}{5}\right)^3 = 76.8$ cm^3
[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]
You could have worked out the side lengths of cuboid B and multiplied them together to find the volume.

b) Density = mass ÷ volume and 0.06 kg = 60 g
Cuboid A: density = 60 ÷ 18.75 = 3.2 g/cm^3
Cuboid B: density = 60 ÷ 76.8 = 0.78125 g/cm^3
% decrease $= \frac{3.2 - 0.78125}{3.2} \times 100 = 75.585... = 75.6\%$ (3 s.f.)
[3 marks available — 1 mark for finding the density of cuboid A, 1 mark for finding the density of cuboid B, 1 mark for the correct answer]

5 Height: scale factor from small vase to medium vase
$= \sqrt{\frac{160}{90}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$ ***[1 mark]***
So height of small vase $= 20 \div \frac{4}{3} = 15$ cm ***[1 mark]***
Volume: scale factor from small vase to large vase
$= \sqrt{\frac{1440}{90}} = \sqrt{16} = 4$ ***[1 mark]***
Volume of large vase = 0.016 m^3 = 16 000 cm^3
So volume of small vase = 16 000 ÷ 4^3 = 250 cm^3 ***[1 mark]***
[4 marks available in total — as above]

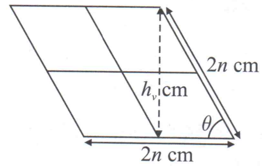
6 Scale factor $= \sqrt[3]{8.1 \div 2.4} = \sqrt[3]{3.375} = 1.5$ ***[1 mark]***
Time taken to decorate large bead = 8 × 1.5^2 = 18 mins ***[1 mark]***
Time taken to decorate 5 small beads and 4 large beads
= (5 × 8) + (4 × 18) = 112 mins ***[1 mark]***
$1\frac{3}{4}$ hours = 105 mins so Anna does not have enough time to decorate all the beads ***[1 mark]***.
[4 marks available in total — as above]

Section Six — Pythagoras and Trigonometry

Pages 68-69: Trigonometry

1 The missing angle in triangle P is 180° – 90° – 30° = 60°
The missing angle in triangle Q is 180° – 90° – 60° = 30°
[1 mark for both missing angles]
So both triangles have angles of 60°, 30° and 90° ***[1 mark]***
Hypotenuse of P = 9.7 cm
Hypotenuse of $Q = \frac{4.85}{\cos 60°}$ ***[1 mark]*** $= \frac{4.85}{0.5} = 9.7$ cm.
Condition AAS holds so triangles P and Q are congruent ***[1 mark]***.
[4 marks available in total — as above]
You can use any of the conditions SSS, AAS, SAS or RHS to prove congruence — if you show full working, you'll get full marks.

2 The large rhombus has the following dimensions:



$h_v = 2n \times \sin\theta = 2n\sin\theta$ cm
Area of rhombus $= 2n \times 2n\sin\theta = 4n^2\sin\theta$ cm^2
[3 marks available — 1 mark for a correct method to work out the height of the large or small rhombus, 1 mark for the correct height, 1 mark for multiplying height by base length to get $4n^2\sin\theta$ cm^2]
You could also work out the area of one small rhombus then multiply it by 4 to get the area of the large rhombus.

3 Diagonals of a kite meet at right angles. So FMG = 90°.
$FM = \frac{9}{\tan 58°} = 5.623...$ cm ***[1 mark]***
Angle $FEM = \tan^{-1}\left(\frac{5.623...}{5}\right) = 48.360...°$ ***[1 mark]***
Angle HEM = angle FEM = 48.360...°
So angle FEH = 48.360...° + 48.360...° = 96.721...°
= 96.7° (1 d.p.) ***[1 mark]***
[3 marks available in total — as above]

4 $BC^2 = 11^2 - 7^2 = 72$ so $BC = \sqrt{72} = 6\sqrt{2}$ ***[1 mark]***
$BD = \frac{6\sqrt{2}}{\cos 30°} = \frac{6\sqrt{2}}{(\sqrt{3}/2)}$ ***[1 mark]***
$= \frac{2 \times 6\sqrt{2}}{\sqrt{3}} = \frac{12\sqrt{2}}{\sqrt{3}} = \frac{12\sqrt{2}\sqrt{3}}{3} = 4\sqrt{6}$ ***[1 mark]***

So the radius of the semicircle = $4\sqrt{6} \div 2 = 2\sqrt{6}$ cm ***[1 mark]***

So the area of the semicircle = $\frac{\pi \times (2\sqrt{6})^2}{2}$

$= \frac{24\pi}{2} = 12\pi$ cm^2 ***[1 mark]***

[5 marks available in total — as above]

5 a) Angle $ECA = 90°$ as a tangent meets a radius at 90°.
$AC = 4 \times \tan 58° = 6.401...$ cm ***[1 mark]***
So the radius of the circle is $6.401... \div 2 = 3.200...$ cm ***[1 mark]***
Area of the circle = $\pi \times 3.200...^2 = 32.183...$
$= 32.18$ cm^2 (2 d.p.) ***[1 mark]***
[3 marks available in total — as above]

b) Angle $ABC = 90°$ and angle $CAB = 180° - 90° - 58° = 32°$
$AB = 6.401... \times \cos 32° = 5.428...$ cm
$BC = 6.401... \times \sin 32° = 3.392...$ cm ***[1 mark for both]***
Area of triangle $ABC = 0.5 \times 5.428... \times 3.392...$
$= 9.207...$ cm^2 ***[1 mark]***

Angle $ADC = 90°$
$AD = 6.401... \times \cos 53° = 3.852...$ cm
$DC = 6.401... \times \sin 53° = 5.112...$ cm ***[1 mark for both]***
Area of triangle $ADC = 0.5 \times 3.852... \times 5.112...$
$= 9.847...$ cm^2 ***[1 mark]***

Shaded area = $32.183... - 9.207... - 9.847...$
$= 13.128... = 13.13$ cm^2 (2 d.p.) ***[1 mark]***
[5 marks available in total — as above]
There are other angles you can use to get the side lengths of the triangles but they will all give the same answer.

Pages 70-72: The Sine and Cosine Rules

1 Using the cosine rule,

$$\cos x = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(4\sqrt{2})^2 + (\sqrt{6})^2 - (\sqrt{14})^2}{2 \times 4\sqrt{2} \times \sqrt{6}} = \frac{32 + 6 - 14}{8\sqrt{12}}$$

$$= \frac{24}{8\sqrt{12}} = \frac{3}{\sqrt{12}} = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

So $\cos x = \frac{\sqrt{3}}{2}$, which means that $x = 30°$

[4 marks available — 1 mark for substituting the numbers into the cosine rule formula correctly, 1 mark for simplifying the expression for cos x, 1 mark for rationalising denominator, 1 mark for showing that x = 30°]

2 Using the sine rule: $\frac{BD}{\sin 30°} = \frac{8}{\sin 70°}$ ***[1 mark]***

$BD = \frac{8}{\sin 70°} \times \sin 30° = 4.256...$ m ***[1 mark]***

$\frac{4}{\sin BDC} = \frac{4.256...}{\sin 60°}$ ***[1 mark]***

$\sin BDC = \frac{\sin 60°}{4.256...} \times 4 = 0.813...$ ***[1 mark]***

Angle $BDC = \sin^{-1}(0.813...) = 54.468...° = 54.5°$ (3 s.f.) ***[1 mark]***
[5 marks available in total — as above]

3 Use the cosine rule to find the missing side of triangle DEF:
$EF^2 = (3\sqrt{2})^2 + 9^2 - (2 \times 3\sqrt{2} \times 9 \times \cos 45°)$ ***[1 mark]***
$= 18 + 81 - (54\sqrt{2} \times \frac{1}{\sqrt{2}}) = 45$ ***[1 mark]***
$EF = \sqrt{45} = 3\sqrt{5}$ cm ***[1 mark]***
The sides are proportional — all the sides in triangle ABC are $\sqrt{5}$ times longer than the equivalent sides in triangle FDE (e.g. $9 \times \sqrt{5} = 9\sqrt{5}$) so they are similar. ***[1 mark]***
[4 marks available in total — as above]

4 Start by working out the radius, r, of the circle:
angle ODC = angle $OCD = (180° - 74°) \div 2 = 53°$

$\frac{r}{\sin 53°} = \frac{18}{\sin 74°}$, so $r = \frac{18}{\sin 74°} \times \sin 53° = 14.954...$ cm

Area $X = \frac{26}{360} \times \pi \times 14.954...^2 = 50.743...$ cm^2

Area of triangle $ODC = \frac{1}{2} \times 14.954... \times 14.954... \times \sin 74°$
$= 107.490...$ cm^2

Area $Y = \frac{74}{360} \times \pi \times 14.954...^2 - 107.490... = 36.932...$ cm^2

So area X is bigger.

[6 marks available — 1 mark for working out angle ODC or OCD, 1 mark for substituting the numbers into the sine rule correctly, 1 mark for the correct radius of the circle, 1 mark for working out area X, 1 mark for working out the area of triangle ODC, 1 mark for working out area Y with a correct answer]

5 Use the cosine rule to find BD and EB
$BD^2 = 15^2 + 12^2 - 2 \times 15 \times 12 \times \cos 77° = 288.017...$
So $BD = 16.971...$ cm
$EB^2 = 14^2 + 11^2 - 2 \times 14 \times 11 \times \cos 73° = 226.949...$
So $EB = 15.064...$ cm
Use the sine rule to find angle DBC and angle ABE:

$\frac{16.971...}{\sin 77°} = \frac{12}{\sin DBC}$, so $\sin DBC = \frac{12 \times \sin 77°}{16.971...} = 0.688...$

So angle $DBC = \sin^{-1}(0.688...) = 43.548...°$

$\frac{15.064...}{\sin 73°} = \frac{14}{\sin ABE}$, so $\sin ABE = \frac{14 \times \sin 73°}{15.064...} = 0.888...$

So angle $ABE = \sin^{-1}(0.888...) = 62.711...°$
Angle $EBD = 129° - 43.548...° - 62.711...° = 22.740...°$

Area $BED = \frac{1}{2} \times 16.971... \times 15.064... \times \sin 22.740...°$
$= 49.414... = 49.4$ cm^2 (3 s.f.)

[5 marks available — 1 mark for finding BD, 1 mark for finding EB, 1 mark for finding angle DBC, 1 mark for finding angle ABE, 1 mark for the correct answer]
You could have found angles DBC and ABE using the cosine rule.

6 $AB = x$ cm and $AB : BC = 1 : 2$, so $BC = 2x$ cm
So $\frac{1}{2} \times x \times 2x \times \sin 50° = 38$ ***[1 mark]***
$x^2 = \frac{38}{\sin 50°} = 49.605...$, so $x = 7.043...$ ***[1 mark]***
So $AB = 7.043...$ cm and $BC = 7.043 \times 2 = 14.086...$ cm
Use the cosine rule to find AC:
$AC^2 = 7.043...^2 + 14.086...^2 - 2 \times 7.043... \times 14.086... \times \cos 50°$
$= 120.484...$, so $AC = 10.976... = 11.0$ cm (3 s.f.)
[4 marks available — 1 mark for setting up an equation for the area of the triangle, 1 mark for finding the length of AB, 1 mark for a correct method to find side AC, 1 mark for the correct answer]

7 Angle CAO = angle $CBO = 90°$
So angle $ACB = 360° - 118° - 90° - 90° = 62°$
Angle CAB = angle $CBA = (180° - 62°) \div 2 = 59°$
Using the cosine rule to find BA:
$BA^2 = 8.5^2 + 8.5^2 - 2 \times 8.5 \times 8.5 \times \cos 118° = 212.338...$
So $BA = 14.571...$ cm

Use the sine rule to find CA: $\frac{CA}{\sin 59°} = \frac{14.571...}{\sin 62°}$

$CA = \sin 59° \times \frac{14.571...}{\sin 62°} = 14.146...$ cm

So area of triangle $ABC = \frac{1}{2} \times 14.571... \times 14.146... \times \sin 59°$
$= 88.347...$ cm^2

Area of triangle $OAB = \frac{1}{2} \times 8.5 \times 8.5 \times \sin 118° = 31.896...$ cm^2

So area of minor segment $= \left(\frac{118}{360} \times \pi \times 8.5^2\right) - 31.896...$
$= 42.502...$ cm^2

Subtract the area of the minor segment from the area of triangle ABC: $88.347... - 42.502... = 45.845... = 45.8$ cm^2 (3 s.f.)
[7 marks available — 1 mark for finding angles ACB and CAB/CBA, 1 mark for finding the length of BA, 1 mark for finding CA or CB, 1 mark for finding the area of triangle ABC, 1 mark for working out the area of triangle OAB, 1 mark for finding the area of the minor segment, 1 mark for the correct answer]

Pages 73-75: 3D Pythagoras and Trigonometry

1 The vertical height of each cone is $6 \text{ m} \div 2 = 3$ m
Let the radius of the cones be r, then, using Pythagoras:
$r^2 = 4.2^2 - 3^2 = 8.64$ so $r = 2.939...$ m ***[1 mark]***

Volume of one cone = $\frac{1}{3} \times \pi \times 2.939...^2 \times 3$
$= 27.143...$ m^3 ***[1 mark]***

Volume of the whole solid = $27.143... \times 2$
$= 54.286... = 54.3$ m^3 (3 s.f.) ***[1 mark]***
[3 marks available in total — as above]

2 $BA = 2\sqrt{3} \times \tan 30° = 2\sqrt{3} \times \frac{1}{\sqrt{3}} = 2$ cm ***[1 mark]***

So the area of the triangular face $= \frac{1}{2} \times 2\sqrt{3} \times 2$

$= 2\sqrt{3}$ ***[1 mark]***

$BC = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$ cm ***[1 mark]***

So the length of the prism is $32 \div 4 = 8$ cm

Volume of the prism $= 2\sqrt{3} \times 8 = 16\sqrt{3}$ cm^3 ***[1 mark]***

[4 marks available — 1 mark for a correct method to find the area of the triangular face, 1 mark for the correct area of the triangular face, 1 mark for the correct length of BC, 1 mark for the correct answer]

3 Using Pythagoras' theorem on triangle AXV:

$AX^2 = 8.9^2 - 7.2^2 = 27.37$, so $AX = \sqrt{27.37}$ ***[1 mark]***

and $AC = 2\sqrt{27.37}$ ***[1 mark]***

Now using Pythagoras' theorem on triangle ABC:

$AB^2 = (2\sqrt{27.37})^2 - 4.2^2 = 91.84$ ***[1 mark]***,

so $AB = \sqrt{91.84} = 9.583... = 9.58$ cm (3 s.f.) ***[1 mark]***

[4 marks available in total — as above]

4 $XE = 8$ cm as the hexagon is made from equilateral triangles.

Let Y be the midpoint of ED, then EXY is a right-angled triangle.

$XY^2 = 8^2 - 4^2 = 48$, so $XY = \sqrt{48}$ cm ***[1 mark]***

The angle between planes VED and $ABCDEF$ is angle VYX.

$\tan VYX = \frac{15}{\sqrt{48}}$ ***[1 mark]***, so $VYX = \tan^{-1}\left(\frac{15}{\sqrt{48}}\right)$

$VYX = 65.208... = 65.2°$ (1 d.p.) ***[1 mark]***

[3 marks available in total — as above]

5 a) The required angle is angle HBE

$BE^2 = 5^2 + 5^2 = 50$, so $BE = \sqrt{50}$ ***[1 mark]***

$\tan HBE = \frac{7}{\sqrt{50}}$ ***[1 mark]***

So $HBE = \tan^{-1}\left(\frac{7}{\sqrt{50}}\right) = 44.710...° = 44.7°$ (3 s.f.) ***[1 mark]***

[3 marks available in total — as above]

You could have found length HB instead of BE.

b) $MN^2 = 7^2 + 5^2 = 74$, so $MN = \sqrt{74}$ ***[1 mark]***

Then BMN is a right-angled triangle.

$\tan BMN = \frac{2.5}{\sqrt{74}}$ ***[1 mark]***

So $BMN = \tan^{-1}\left(\frac{2.5}{\sqrt{74}}\right) = 16.204...°$

So angle $BMF = 16.204... \times 2 = 32.4°$ (3 s.f.) ***[1 mark]***

[3 marks available in total — as above]

There are other methods you could use here — e.g. you could find MB instead of MN and use that to find the angle.

6 Let X be the point 4 cm from D on the line DA.

Then AXB is a right angled triangle.

$BX^2 = 5^2 - (7-4)^2 = 16$ so $BX = 4$ cm $= DC$ ***[1 mark]***

Now use this to work out all the sides of triangle DEG:

$DG^2 = 11^2 + 4^2 = 137$ so $DG = \sqrt{137}$ cm ***[1 mark]***

$GE^2 = 4^2 + 7^2 = 65$ so $GE = \sqrt{65}$ cm ***[1 mark]***

$DE^2 = 11^2 + 7^2 = 170$ so $DE = \sqrt{170}$ cm ***[1 mark]***

Now use the cosine rule to find angle DEG:

$\cos DEG = \frac{\sqrt{170}^2 + \sqrt{65}^2 - \sqrt{137}^2}{2 \times \sqrt{170} \times \sqrt{65}} = 0.466...$

Angle $DEG = \cos^{-1}(0.466...) = 62.216...° = 62.2°$ (3 s.f.) ***[1 mark]***

[5 marks available in total — as above]

7 $AN = 12 \times \frac{3}{4} = 9$ cm and $ND = 12 - 9 = 3$ cm ***[1 mark for both]***

In triangle CNF: $CF = 12$ cm

$CN^2 = 8^2 + 9^2 + 4^2 = 161$ so $CN = \sqrt{161}$ cm ***[1 mark]***

$NF^2 = 3^2 + 8^2 + 4^2 = 89$ so $NF = \sqrt{89}$ cm ***[1 mark]***

Now use the cosine rule to find angle CNF:

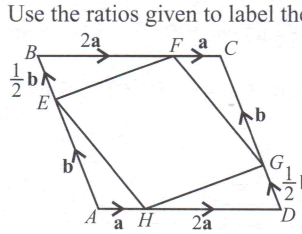
$\cos CNF = \frac{\sqrt{161}^2 + \sqrt{89}^2 - 12^2}{2 \times \sqrt{161} \times \sqrt{89}}$ ***[1 mark]*** $= 0.442...$

Angle $CNF = \cos^{-1}(0.442...) = 63.719...° = 63.7°$ (1 d.p.) ***[1 mark]***

[5 marks available in total — as above]

Pages 76-77: Vectors

1 Use the ratios given to label the shape:



First work out $\overrightarrow{HG}$ and $\overrightarrow{EF}$:

$\overrightarrow{HG} = \overrightarrow{HD} + \overrightarrow{DG} = 2\mathbf{a} + \frac{1}{2}\mathbf{b}$ and $\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF} = \frac{1}{2}\mathbf{b} + 2\mathbf{a}$

As $\overrightarrow{HG}$ and $\overrightarrow{EF}$ are the same vector, HG and EF are parallel lines.

Then work out $\overrightarrow{HE}$ and $\overrightarrow{GF}$:

$\overrightarrow{HE} = \overrightarrow{HA} + \overrightarrow{AE} = -\mathbf{a} + \mathbf{b}$ and $\overrightarrow{GF} = \overrightarrow{GC} + \overrightarrow{CF} = \mathbf{b} - \mathbf{a}$

As $\overrightarrow{HE}$ and $\overrightarrow{GF}$ are the same vector, HE and GF are parallel lines.

$EFGH$ has two pairs of parallel sides, so is a parallelogram.

[3 marks available — 1 mark for working out $\overrightarrow{HG}$ and $\overrightarrow{EF}$, 1 mark for working out $\overrightarrow{HE}$ and $\overrightarrow{GF}$, 1 mark for an explanation that the shape has two pairs of parallel lines so is a parallelogram]

2 a) PQD and ACD are similar, and $AD:PD = 5:3$,

so $\overrightarrow{CA} = \frac{5}{3} \times \overrightarrow{QP} = \frac{5}{3}\mathbf{b}$ ***[1 mark]***

b) $\overrightarrow{PR} = \overrightarrow{PA} + \overrightarrow{AR}$

$AD:PD = 5:3$ so $AP:PD = 2:3$ and $3\overrightarrow{PA} = 2\overrightarrow{DP}$

$\overrightarrow{PA} = \frac{2}{3}\overrightarrow{DP} = \frac{2}{3}(-\mathbf{a} + \mathbf{b}) = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ ***[1 mark]***

$\overrightarrow{AR} = \frac{2}{5}\overrightarrow{AC} = -\frac{2}{5}\overrightarrow{CA} = -\frac{2}{5} \times \frac{5}{3}\mathbf{b} = -\frac{2}{3}\mathbf{b}$ ***[1 mark]***

$\overrightarrow{PR} = -\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} - \frac{2}{3}\mathbf{b} = -\frac{2}{3}\mathbf{a}$

$\overrightarrow{DQ} = -\mathbf{a}$ so $k = \frac{2}{3}$ ***[1 mark]***

[3 marks available in total — as above]

3 E.g. $\overrightarrow{AC}$ and $\overrightarrow{AE}$ are parallel vectors as ACE is a straight line.

$\overrightarrow{AC} = k\mathbf{b} + 6\mathbf{a}$ ***[1 mark]***

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = 6\mathbf{a} + (4\mathbf{a} + 25\mathbf{b}) = 10\mathbf{a} + 25\mathbf{b}$ ***[1 mark]***

Comparing the coefficients of $\mathbf{a}$, $\overrightarrow{AE} = \frac{10}{6}\overrightarrow{AC}$ ***[1 mark]***

So $k = 25 \div \frac{10}{6} = 15$ ***[1 mark]***

[4 marks available in total — as above]

There are other ways to solve this question — if you got the answer and showed your working then you'll get full marks.

4 E.g. $AB:BC:CD = 4:3:4$

$\overrightarrow{AB} = 3\mathbf{a}$, $\overrightarrow{BC} = 3\mathbf{a} \times \frac{3}{4} = \frac{9}{4}\mathbf{a}$ and $\overrightarrow{CD} = 3\mathbf{a}$ ***[1 mark]***

$\overrightarrow{FE} = \overrightarrow{CD} = 3\mathbf{a}$

$\overrightarrow{AE} = \overrightarrow{AF} + \overrightarrow{FE} = (\frac{15}{4}\mathbf{a} + 2\mathbf{b}) + 3\mathbf{a} = \frac{27}{4}\mathbf{a} + 2\mathbf{b}$ ***[1 mark]***

$\overrightarrow{FC} = \overrightarrow{FA} + \overrightarrow{AC} = -\left(\frac{15}{4}\mathbf{a} + 2\mathbf{b}\right) + 3\mathbf{a} + \frac{9}{4}\mathbf{a} = \frac{3}{2}\mathbf{a} - 2\mathbf{b}$ ***[1 mark]***

$4\overrightarrow{FM} = \overrightarrow{MC}$ so $\overrightarrow{FM} = \frac{1}{5}\overrightarrow{FC} = \frac{1}{5}(\frac{3}{2}\mathbf{a} - 2\mathbf{b}) = \frac{3}{10}\mathbf{a} - \frac{2}{5}\mathbf{b}$

$\overrightarrow{AM} = \overrightarrow{AF} + \overrightarrow{FM} = (\frac{15}{4}\mathbf{a} + 2\mathbf{b}) + (\frac{3}{10}\mathbf{a} - \frac{2}{5}\mathbf{b})$

$= \frac{81}{20}\mathbf{a} + \frac{8}{5}\mathbf{b}$ ***[1 mark]***

$\overrightarrow{AM}$ is not a scalar multiple of $\overrightarrow{AE}$ so they are not parallel and so AME is not a straight line. ***[1 mark]***

[5 marks available in total — as above]

You could have shown that $\overrightarrow{AM}$ and $\overrightarrow{ME}$ aren't parallel instead.

Section 7 — Probability and Statistics

Pages 78-80: Probability

1 a) Frequency of odd number $= 54 + 38 + 61 = 153$

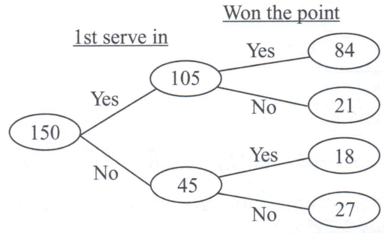
Relative frequency of an odd number $= \frac{153}{200} = 0.765$

[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

b) E.g. the relative frequency of getting a 5 is $\frac{61}{200} = 0.305$ which is less than 0.5, so he is actually unlikely to get a 5.

[1 mark for any valid explanation]

2 a) His first serve was in on 150 × 0.7 = 105 points.
So his first serve wasn't in on 150 – 105 = 45 points.
He won 105 × 0.8 = 84 points and lost 105 – 84 = 21 points when his first serve was in.
He won 45 × 0.4 = 18 points and lost 45 – 18 = 27 points when his first serve wasn't in.



[3 marks available — 3 marks for a fully correct frequency tree, otherwise 2 marks for at least 5 values correct or 1 mark for at least 3 values correct]

b) He won 84 + 18 = 102 points so the probability that he won a randomly chosen point is $\frac{102}{150} = \frac{17}{25} = 0.68$

[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

3 For 1 scoop there are 16 combinations.
For 2 scoops there are 16 × 16 = 256 combinations ***[1 mark]***.
For 3 scoops there are 16 × 16 × 16 = 4096 combinations ***[1 mark]***.
4096 + 256 + 16 = 4368 so his statement is not correct ***[1 mark]***.
[3 marks available in total — as above]

4 a) Let x = P(3). Then P(1) = $3x$, so P(odd) = $x + 3x = 4x$
P(even) = 2 × P(odd) = $2 \times 4x = 8x$
P(even) + P(odd) = 1, so $4x + 8x = 12x = 1$
$x = \frac{1}{12}$, so P(3) = $\frac{1}{12}$

[2 marks available — 1 mark for finding expressions for the probabilities, 1 mark for the correct answer]

b) P(1 even and 1 odd) = P(even, odd) + P(odd, even)
$= \left(\frac{2}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{2}{3}\right) = \frac{4}{9}$

[2 marks available — 1 mark for a correct method, 1 mark for the correct answer]

5 a) There are 20 possibilities for each number so there are 20 × 20 × 20 = 8000 combinations ***[1 mark]***.

b) There are 8 prime numbers, 10 odd numbers and 4 square numbers on the lock.
So there are 8 × 10 × 4 = 320 possible combinations.
Which is 8000 – 320 = 7680 fewer combinations.
The percentage decrease is $\frac{7680}{8000} \times 100 = 96\%$.

[4 marks available — 1 mark for working out the number choices for each number, 1 mark for working out the number of possible combinations, 1 mark for the correct method to find the percentage decrease, 1 mark for the correct answer]

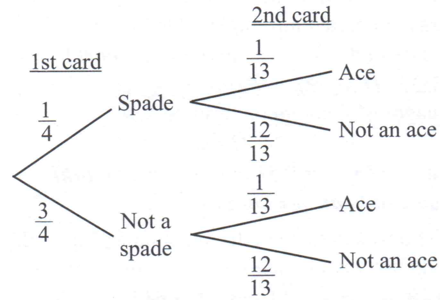
6 $G_AG_B = 0.24$ [1]
$G_AB_B = 0.56$ and $B_B = 1 - G_B$
So, $G_A(1 - G_B) = 0.56$, which means $G_A - G_AG_B = 0.56$ [2]
Substitute [1] into [2]: $G_A - 0.24 = 0.56$, so $G_A = 0.8$
Putting this value back into [1]: $0.8G_B = 0.24$, so $G_B = 0.3$
So in Year 9 there are (30 × 0.8) + (30 × 0.3) = 33 girls

[4 marks available — 1 mark for forming the two simultaneous equations, 1 mark for finding G_A, 1 mark for finding G_B, 1 mark for the correct answer]

7 At the start, Bag A: P(red) = $\frac{n}{n+14}$ and Bag B: P(red) = $\frac{30}{30+n}$
When 2 blue balls are moved,
Bag A contains 12 blue and n red so P(red) = $\frac{n}{n+12}$ ***[1 mark]***
Bag B contains 30 red and $n + 2$ blue, so P(red) = $\frac{30}{n+32}$ ***[1 mark]***
The probabilities are equal so: $\frac{n}{n+12} = \frac{30}{n+32}$
$n(n + 32) = 30(n + 12)$ ***[1 mark]***
$n^2 + 32n = 30n + 360$
$n^2 + 2n - 360 = 0$ ***[1 mark]***
$(n - 18)(n + 20) = 0$, so $n = 18$ (as n must be positive) ***[1 mark]***
The original probability of picking a red ball from Bag B is
$\frac{30}{30+18} = \frac{30}{48} = \frac{5}{8}$ ***[1 mark]***
[6 marks available in total — as above]

Pages 81-82: Tree Diagrams

1 a)



[2 marks available in total — 2 marks for a fully correct tree diagram, otherwise 1 mark for at least 3 correct probabilities]

b) P(not a spade and not an ace) = $\frac{3}{4} \times \frac{12}{13} = \frac{36}{52} = \frac{9}{13}$ ***[1 mark]***

For questions 2-4, you can draw tree diagrams to help you if you want.

2 a) There are 5 prime numbers between 1 and 12 so P(prime) = $\frac{5}{12}$
P(3 prime numbers) = P(prime) × P(prime) × P(prime)
$= \frac{5}{12} \times \frac{5}{12} \times \frac{5}{12} = \frac{125}{1728}$

[2 marks available — 1 mark for the correct probability of a prime number, 1 mark for the correct answer]

b) P(roll < 5) = $\frac{1}{3}$ and P(roll ≥ 5) = $\frac{2}{3}$
P(one roll < 5) = P(1st number is < 5, other 2 are ≥ 5)
+ P(2nd number is < 5, other 2 are ≥ 5)
+ P(3rd number is < 5, other 2 are ≥ 5)
P (one roll < 5) = $\left(\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{1}{3} \times \frac{2}{3}\right) + \left(\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3}\right)$
$= \frac{12}{27} = \frac{4}{9}$

[3 marks available — 1 mark for the correct probabilities for less than a 5 and 5 or more, 1 mark for a correct method for finding the answer, 1 mark for the correct answer]

3 P(less than 5 fish) = 1 – P(5 or more fish)
P(5 or more fish) = P(1, 2, 2) + P(2, 1, 2) + P(2, 2, 1) + P(2, 2, 2)
= (0.5 × 0.3 × 0.3) + (0.3 × 0.5 × 0.3) + (0.3 × 0.3 × 0.5)
+ (0.3 × 0.3 × 0.3) = 0.045 + 0.045 + 0.045 + 0.027 = 0.162
P(less than 5 fish) = 1 – 0.162 = 0.838
[4 marks available in total — 1 mark for finding the outcomes that will give 5 or more fish, 1 mark for calculating the probability of each of the outcomes, 1 mark for adding the probabilities together, 1 mark for the correct answer]

4 a) There are 6 × 6 = 36 possible outcomes and there are three ways to win the game (rolling a 5 and 6, 6 and 5, or 6 and 6).
So the probability of winning is $\frac{3}{36} = \frac{1}{12}$.
So you'd estimate that a prize will be won every 12 games.
If there are 20 prizes, you'd estimate that the stall will run out of prizes after 12 × 20 = 240 games.
It takes £2 each go, so expected takings are 240 × £2 = £480.
[3 marks available — 1 mark for finding the number of winning outcomes, 1 mark for the correct probability of winning, 1 mark for explaining why this means you'd estimate 240 games will be played and therefore £480 will be taken]

b) P(wins at least one prize) = 1 – P(no prizes)
P(no prizes) = $\frac{11}{12} \times \frac{11}{12} = \frac{121}{144}$ ***[1 mark]***
P(wins at least one prize) = $1 - \frac{121}{144}$ ***[1 mark]*** = $\frac{23}{144}$ ***[1 mark]***
[3 marks available in total — as above]

Pages 83-84: Conditional Probability

1 P(both the same) = P(chocolate, chocolate) + P(plain, plain)

$= \left(\frac{8}{14} \times \frac{7}{13}\right) + \left(\frac{6}{14} \times \frac{5}{13}\right) = \frac{43}{91}$

[3 marks available — 1 mark for finding the outcomes where both biscuits are the same, 1 mark for working out the probabilities of these outcomes, 1 mark for the correct answer]

2 a) P(one C and one S) = P(C, S) + P(S, C)

$= \left(\frac{5}{12} \times \frac{3}{11}\right) + \left(\frac{3}{12} \times \frac{5}{11}\right) = \frac{5}{22}$

[3 marks available — 1 mark for finding the outcomes where one tub is chocolate and one tub is strawberry, 1 mark for working out the probabilities of these outcomes, 1 mark for the correct answer]

b) P(at least one V) = 1 – P(no V's)

P(no V's) = $\left(\frac{8}{12} \times \frac{7}{11}\right) = \frac{56}{132} = \frac{14}{33}$ ***[1 mark]***

P(at least one V) = $1 - \frac{14}{33}$ ***[1 mark]*** $= \frac{19}{33}$ ***[1 mark]***

[3 marks available in total — as above]

3 P(aerobics) = 0.4 , P(not aerobics) = 1 – 0.4 = 0.6 and
P(run given not aerobics) = 0.7,
P(not run given not aerobics) = 1 – 0.7 = 0.3
P(not run and not aerobics)
= P(not aerobics) × P(not run given not aerobics) = 0.6 × 0.3
= 0.18.

[3 marks available — 1 mark for finding the missing probabilities, 1 mark for putting the numbers into the formula correctly, 1 mark for the correct answer]

You could have done this one using a tree diagram instead.

4 a) P(at least one prize) = 1 – P(no prizes)

20 tickets end in a 0 or 5 so the probability of picking a winning ticket is $\frac{20}{100}$ and a losing ticket is $\frac{80}{100}$. ***[1 mark]***

P(no prizes) = $\frac{80}{100} \times \frac{79}{99} = \frac{316}{495}$ ***[1 mark]***

P(at least one prize) = $1 - \frac{316}{495}$ ***[1 mark]*** $= \frac{179}{495}$ ***[1 mark]***

[4 marks available in total — as above]

b) There are 100 – 40 = 60 tickets left and 20 – 5 = 15 of them are winning tickets so the probability of picking a winning ticket is $\frac{15}{60}$ and a losing ticket is $\frac{45}{60}$.

P(no prizes) = $\frac{45}{60} \times \frac{44}{59} = \frac{33}{59}$

P(at least one prize) = $1 - \frac{33}{59} = \frac{26}{59} = 0.440...$

$\frac{179}{495} = 0.361...$

0.440... > 0.361... so her chances of winning are better than Amy's.

[3 marks available in total — 1 mark for working out the probability of picking a losing ticket, 1 mark for calculating the probability of Carla winning at least one prize, 1 mark for saying she has a better chance to win by comparing the fractions]

5 P(picking a green counter) = $\frac{n}{n+(n+1)} = \frac{n}{2n+1}$ ***[1 mark]***

P(picking a blue counter) = $\frac{n+1}{n+(n+1)} = \frac{n+1}{2n+1}$ ***[1 mark]***

P(picking 2 green counters) = $\frac{n}{2n+1} \times \frac{n-1}{2n} = \frac{n-1}{4n+2}$ ***[1 mark]***

P(picking 2 blue counters) = $\frac{n+1}{2n+1} \times \frac{n}{2n} = \frac{n+1}{4n+2}$ ***[1 mark]***

P(both counters are the same) = $\frac{n-1}{4n+2} + \frac{n+1}{4n+2}$

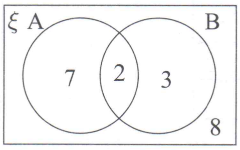
$= \frac{2n}{4n+2} = \frac{n}{2n+1}$ ***[1 mark]***

[5 marks available in total — as above]

Pages 85-86: Venn Diagrams

1 a) $2n + 1$ generates set A = {3, 5, 7, 9, 11, 13, 15, 17, 19}

$\frac{n(n+1)}{2}$ generates set B = {1, 3, 6, 10, 15}



[3 marks available — 3 marks for a completely correct Venn diagram, otherwise 1 mark for listing or finding the number of elements in set A and 1 mark for listing or finding the number of elements in set B]

b) $\frac{2}{20} = \frac{1}{10}$ ***[1 mark]***

2 a) 106 + 19 = 125 students attend the disco
106 + 19 + 33 + 42 = 200 students in total.
125 : 200 ***[1 mark]*** = 5 : 8 ***[1 mark]***

[2 marks available in total — as above]

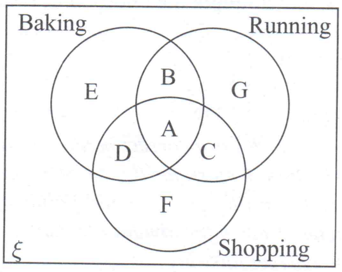
b) (i) $\frac{19}{125}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

(ii) $\frac{106}{139}$

[2 marks available — 1 mark for the correct numerator, 1 mark for the correct denominator]

3 a)



A: 10% of 80 = 0.1 × 80 = 8 ***[1 mark]***
B: 22 – 8 = 14 C: 18 – 8 = 10 ***[1 mark for both]***
Half the people only liked 1 activity so the other half liked 2 or 3 activities: 0.5 × 80 = 40.
D: 40 – (14 + 10 + 8) = 8 ***[1 mark]***
E: 43 – (14 + 8 + 8) = 13
F: 35 – (8 + 8 + 10) = 9 ***[1 mark for both E and F]***
G: 80 – (8 + 14 + 10 + 8 + 13 + 9) = 18 ***[1 mark]***

[5 marks available in total — as above]

b) $\frac{14+8+8}{14+8+8+10} = \frac{30}{40} = \frac{3}{4}$

[2 marks available — 1 mark for correct calculation, 1 mark for the correct answer]

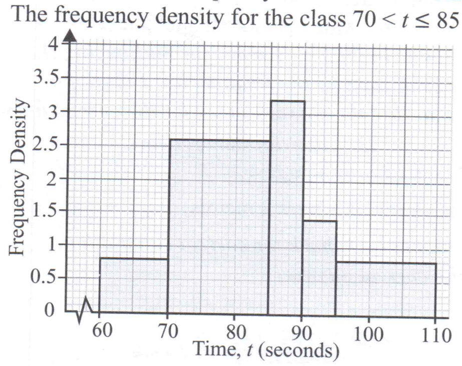
4 There are 50 people in the choir, so
$n(n+2) + n + (2n-3) + (8-n) = 50$ ***[1 mark]***
$n^2 + 4n + 5 = 50$, so $n^2 + 4n - 45 = 0$
$(n+9)(n-5) = 0$, so $n = 5$ (as n has to be positive) ***[1 mark]***
The number of people who play the piano is
$n(n+2) + n = (5 \times 7) + 5 = 40$ ***[1 mark]***

P(both play piano) = $\frac{40}{50} \times \frac{39}{49}$ ***[1 mark]*** $= \frac{156}{245}$ ***[1 mark]***

[5 marks available in total — as above]

Pages 87-88: Histograms

1 $70 < t \leq 85$ has a frequency of 39. Class width = 85 – 70 = 15.
The frequency density for the class $70 < t \leq 85$ is 39 ÷ 15 = 2.6, so



Time t (seconds)	Frequency
$60 < t \le 70$	$10 \times 0.8 = 8$
$70 < t \le 85$	$15 \times 2.6 = 39$
$85 < t \le 90$	$5 \times 3.2 = 16$
$90 < t \le 95$	$5 \times 1.4 = 7$
$95 < t \le 110$	$15 \times 0.8 = 12$

Total number of members = 8 + 39 + 16 + 7 + 12 = 82
[4 marks available in total — 1 mark for finding the frequency density of the 70 < t ≤ 85 class, 1 mark for using this to find the frequency densities of the other classes, 1 mark for finding the frequencies of each class, 1 mark for the correct answer]

2 a) Start by working out the frequency of bouncy balls in each class: The whole histogram represents 600 bouncy balls and there are 30 big squares in the histogram so each big square represents 600 ÷ 30 = 20 bouncy balls.

Weight w (grams)	Frequency
$40 < w \le 41$	$20 \times 4 = 80$
$41 < w \le 41.5$	$20 \times 4 = 80$
$41.5 < w \le 42.5$	$20 \times 6 = 120$
$42.5 < w \le 43.5$	$20 \times 12 = 240$
$43.5 < w \le 44$	$20 \times 4 = 80$

To estimate the mean, first multiply the mid-point of each class by the frequency:
(40.5 × 80) + (41.25 × 80) + (42 × 120) + (43 × 240) + (43.75 × 80) = 25 400
So mean = 25 400 ÷ 600 = 42.3333...= 42.3 g (to 1 d.p.)
[5 marks available — 1 mark for a correct method to work out the frequency of each class, 1 mark for the correct frequencies for each class, 1 mark for multiplying the mid-points of each class by the frequencies, 1 mark for dividing by 600 to find the mean, 1 mark for the correct answer]

b) E.g. The median weight is between the 300th and 301st ball, which are both in the class $42.5 < w \le 43.5$ so Sumi is correct.
[2 marks available — 1 mark for saying she is correct, 1 mark for a correct explanation]

3 a) The frequencies are given by the area of each bar.

Height (metres)	Frequency
1 to 2	1 × 12 = 12
2 to 2.5	0.5 × 32 = 16
2.5 to 2.75	0.25 × 88 = 22
2.75 to 3	0.25 × 72 = 18
3 to 4	1 × 12 = 12

[1 mark]

Total = 12 + 16 + 22 + 18 + 12 = 80
Estimate of number of statues between 1.75 m to 2.75 m = ((2 – 1.75) × 12) + 16 + 22 = 41 ***[1 mark]***
Percentage = (41 ÷ 80) × 100 = 51.25% ***[1 mark]***
[3 marks available in total — as above]

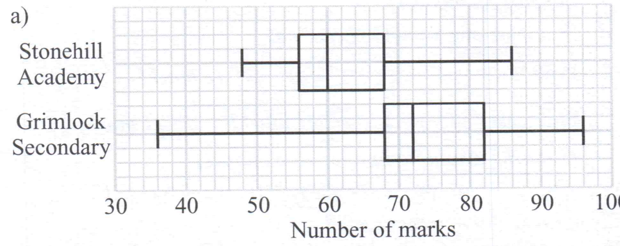
b) To estimate the mean height, first multiply the midpoint of each interval by the frequency:
(1.5 × 12) + (2.25 × 16) + (2.625 × 22) + (2.875 × 18) + (3.5 × 12)
= 18 + 36 + 57.75 + 51.75 + 42 = 205.5
Estimate of mean = 205.5 ÷ 80
= 2.57 metres (2 d.p.) which is more than 2.5 metres.
[3 marks available in total — 1 mark for multiplying the mid-points of each class by the frequencies, 1 mark for dividing by 80 to find the mean, 1 mark for the correct answer]

c) There are 16 + 22 + 18 + 12 = 68 statues over 2 metres.
There are 12 statues over 3 metres.
P(over 3 metres given it is over 2 metres) = $\frac{12}{68} = \frac{3}{17}$.
[2 marks available — 1 mark for correct calculation, 1 mark for the correct answer]

Pages 89-90: Comparing Data Sets

1 The tickets took 30 minutes to sell out in 2013 but only 24 minutes in 2014 so they sold out quicker in 2014.
The median in 2013 was 12.5 minutes. So the first half of the tickets sold out quicker in 2013 then slowed down, whereas in 2014, sales started slowly then sped up.
The interquartile range in 2013 was 15.5 – 10 = 5.5 minutes and in 2014 it was 3 minutes. Ticket sales were more concentrated around the median time in 2014 as the interquartile range is smaller.
[4 marks available — 1 mark for comparing the times it took the tickets to sell out, 1 mark for finding and comparing the medians, 1 mark for working out the interquartile range in 2013, 1 mark for comparing the interquartile ranges]

2 a)



[2 marks available — 2 marks for a fully correct box plot, otherwise 1 mark for correctly showing at least 3 of lower endpoint, upper endpoint, median, lower quartile and upper quartile]

b) The median is higher at Grimlock Secondary so on average their GCSE maths pupils scored higher. The range and interquartile range are both smaller at Stonehill Academy so their GCSE maths pupils scored more consistent marks.
[2 marks available — 1 mark for comparing the medians with a valid explanation, 1 mark for comparing the ranges or interquartile ranges with a correct explanation]

3 a) (i) E.g. The maximum range of house prices in town A is £360 000 – £200 000 = £160 000.
The minimum range of house prices in town B is £340 000 – £180 000 = £160 000.
So the statement is incorrect the range of house prices in town A cannot be greater than the range for town B.
[2 marks available — 1 mark for working out the range of house prices in each town, 1 mark for a correct comparison]

(ii) E.g. It is impossible to tell — the histogram doesn't have a scale on the frequency density axis. You can only tell the proportion of houses in certain price brackets, not the number of houses.
[2 marks available — 1 mark for saying you can't tell this from the histogram, 1 mark for a correct explanation]

b) Work out the proportion of houses between £280 000 and £320 000 in each town by counting the number of squares of the histogram they take up.
In town A: 8 big squares are between £280 000 and £320 000. There are 24 squares in the histogram. The proportion of houses between £280 000 and £320 000 is $\frac{8}{24} = \frac{1}{3}$.
In town B: 10 big squares are between £280 000 and £320 000. There are 40 squares in the histogram. The proportion of houses between £280 000 and £320 000 is $\frac{10}{40} = \frac{1}{4}$.
$\frac{1}{3} > \frac{1}{4}$ so there is a higher proportion of houses between £280 000 and £320 000 in town A.
[4 marks available in total — 1 mark for the correct method to work out the proportions, 1 mark for the correct proportion for town A, 1 mark for the correct proportion for town B, 1 mark for the correct answer]

Formulas in the Exams

GCSE Maths uses a lot of formulas — that's no lie. You'll be scuppered if you start trying to answer a question without the proper formula to start you off. Thankfully, CGP is here to explain all things formula-related.

You're Given these Formulas

Fortunately, those lovely, cuddly examiners give you some of the formulas you need to use.

For a sphere radius r, or a cone with base radius r, slant height l and vertical height h:

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r l$

And, actually, that's your lot I'm afraid. As for the rest...

Learn All The Other Formulas

Sadly, there are a load of formulas which you're expected to be able to remember straight out of your head. There isn't space to write them all out below, but here are the highlights:

Compound Growth and Decay:

$$N = N_0\left(1 + \frac{r}{100}\right)^n$$

where N = total amount, N_0 = initial amount, r = percentage change and n = number of days/weeks/years etc.

The Quadratic Equation:

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Where P(A) and P(B) are the probabilities of events A and B respectively:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

or: $P(A \text{ or } B) = P(A) + P(B)$ (If A and B are mutually exclusive.)

$P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$

or: $P(A \text{ and } B) = P(A) \times P(B)$ (If A and B are independent.)

Area of trapezium $= \frac{1}{2}(a + b)h_v$

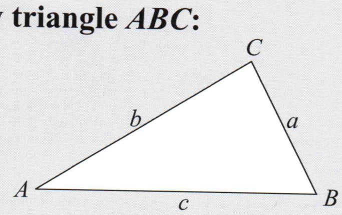
For a right-angled triangle:

Pythagoras' theorem: $a^2 + b^2 = c^2$

Trigonometry ratios:

$\sin x = \frac{\text{O}}{\text{H}}$, $\cos x = \frac{\text{A}}{\text{H}}$, $\tan x = \frac{\text{O}}{\text{A}}$

For any triangle ABC:



Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle $= \frac{1}{2}ab\sin C$

Compound Measures:

Speed $= \frac{\text{Distance}}{\text{Time}}$ Density $= \frac{\text{Mass}}{\text{Volume}}$ Pressure $= \frac{\text{Force}}{\text{Area}}$

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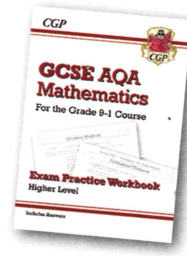

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