

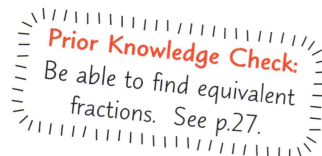
5.1 Percentages

'Per cent' means 'out of 100' — so writing an amount as a percentage means writing it as a number out of 100. Percentages are written using the % symbol and are often used to show proportions.

Writing One Number as a Percentage of Another

Learning Objectives — Spec Ref R9:

- Be able to write one number as a percentage of another without a calculator.
- Be able to write one number as a percentage of another using a calculator.



Writing percentages is a good way to **compare** proportions of different amounts.

E.g. 2 out of 10 and 20 out of 100 are both represented by the **same percentage** — it's 20%.

A good method for writing one number as a percentage of another is to turn it into a fraction — the first number over the second. Then you need to find an **equivalent fraction** that has a **denominator of 100**. Once you've done this, the **numerator** of the equivalent fraction tells you the percentage.

Example 1

Express 15 as a percentage of 50.

- Write '15 out of 50' as a fraction.
- Write an equivalent fraction which is 'out of 100' by multiplying or dividing the top and bottom by the same number.
- The numerator of the equivalent fraction tells you the percentage.

$$\frac{15}{50} = \frac{30}{100}$$

$\begin{array}{c} \times 2 \\ \curvearrowright \\ \times 2 \end{array}$

So 15 is **30%** of 50.

Tip: Sometimes the bigger number will be on top. If that's the case, you'll get a percentage greater than 100%.

Exercise 1



- Q1 A chess club has 25 members. 12 of these members are female. Express the number of female members of the club as a percentage.
- Q2 There are 300 counters in a bag, 45 of which are green. Express the amount of green counters as a percentage.
- Q3 Write each of the following amounts as a percentage.
- | | | |
|-------------------|-----------------|-------------------|
| a) 11 out of 25 | b) 33 out of 50 | c) 3 out of 20 |
| d) 100 out of 400 | e) 48 out of 32 | f) 200 out of 160 |
- Q4 Out of 24 pupils in a class, 18 walk to school. What percentage of the class:
- | | |
|--------------------|---------------------------|
| a) walk to school? | b) do not walk to school? |
|--------------------|---------------------------|
- Q5 39 out of 65 people in a book club have blonde hair. What percentage do not have blonde hair?

If you've got a **calculator** then you don't need to find the equivalent fraction — just **divide** the numerator by the denominator then **multiply by 100%**.

Example 2

Express 333 as a percentage of 360.

1. Write '333 out of 360' as a fraction. $\frac{333}{360}$
2. Divide the top number by the bottom number. $333 \div 360 = 0.925$
3. Multiply by 100% to write as a percentage. $0.925 \times 100\% = \mathbf{92.5\%}$

Tip: In this method we're converting from a fraction to a decimal, then to a percentage — see p.62.

Exercise 2

- Q1 Write each of the following amounts as a percentage.
- | | | |
|-------------------|-------------------|-------------------|
| a) 15 out of 24 | b) 221 out of 260 | c) 661 out of 500 |
| d) 258 out of 375 | e) 323 out of 850 | f) 301 out of 250 |
- Q2 A school has 875 pupils. 525 are boys. What is this as a percentage?
- Q3 171 out of 180 raffle tickets were sold for a summer fete. What percentage of the tickets were sold?
- Q4 Write each of the following amounts as a percentage.
- | | | |
|---------------------|--------------------|---------------------|
| a) 116.6 out of 212 | b) 53.5 out of 428 | c) 226.8 out of 210 |
|---------------------|--------------------|---------------------|
- Q5 The jackpot for a lottery was £10 250. John won £1896.25. What percentage of the total jackpot did he win?

Finding a Percentage of an Amount

Learning Objectives — Spec Ref N12:

- Be able to find a percentage of an amount without a calculator.
- Be able to find a percentage of an amount using a calculator.

You can find some percentages without a calculator using the following rules.

- $50\% = \frac{1}{2}$, so find 50% of something by **dividing by 2** (which is the same as multiplying by $\frac{1}{2}$).
- $25\% = \frac{1}{4}$, so find 25% of something by **dividing by 4** (which is the same as multiplying by $\frac{1}{4}$).
- $10\% = \frac{1}{10}$, so find 10% of something by **dividing by 10** (which is the same as multiplying by $\frac{1}{10}$).
- $5\% = \frac{1}{20}$, so find 5% of something by **dividing by 20** (or by dividing 10% of something by 2).
- $1\% = \frac{1}{100}$, so find 1% of something by **dividing by 100** (which is the same as multiplying by $\frac{1}{100}$).

To find other percentages, add up combinations of the percentages above, e.g. $65\% = 50\% + 10\% + 5\%$.

Example 3

Find 75% of 44.

1. First find 25% by dividing by 4. $25\% \text{ of } 44 = 44 \div 4 = 11$
2. $75\% = 3 \times 25\%$, so multiply by 3. So $75\% \text{ of } 44 = 3 \times 11 = 33$

Tip: You could also find 75% by adding together 50%, 20%, and 5%.

Exercise 3



Q1 Find each of the following.

- | | | |
|---------------|---------------|---------------|
| a) 50% of 24 | b) 25% of 36 | c) 10% of 270 |
| d) 75% of 20 | e) 5% of 140 | f) 35% of 300 |
| g) 65% of 120 | h) 130% of 90 | i) 180% of 70 |

Q2 Find each of the following.

- | | | |
|---------------|--------------|---------------|
| a) 21% of 200 | b) 3% of 260 | c) 62% of 500 |
|---------------|--------------|---------------|

Q3 A wooden plank is 9 m long. 55% of the plank is cut off. What length of wood has been cut off?

To find a percentage of an amount using a calculator, change the percentage into a decimal (by dividing by 100) then **multiply** by the amount.

Example 4

Find 67% of 138.

1. Change 67% into a decimal by dividing by 100. $67\% = 67 \div 100 = 0.67$
2. Multiply the decimal by 138. $67\% \text{ of } 138 = 0.67 \times 138 = 92.46$

Exercise 4

Q1 Find each of the following.

- | | | |
|----------------|---------------|---------------|
| a) 17% of 200 | b) 109% of 11 | c) 68% of 320 |
| d) 221% of 370 | e) 79% of 615 | f) 96% of 911 |

Q2 What is 12% of 68 kg?

Q3 Jeff is on a journey of 385 km. So far, he has completed 31% of his journey. How far has he travelled?

Q4 Which is larger, 22% of £57 or 161% of £8? By how much?

Q5 A jug can hold 2.4 litres of water. It is 34% full. How much more water will fit in the jug?

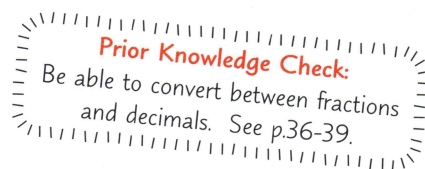
5.2 Percentages, Fractions and Decimals

Percentages, fractions and decimals are three different ways of showing a proportion of something. Being able to convert between them is crucial to understanding how closely related they are.

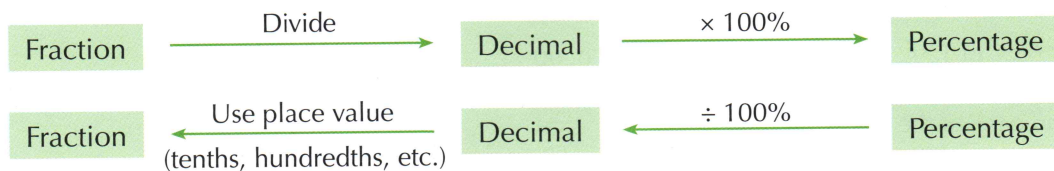
Converting between Percentages, Fractions and Decimals

Learning Objective — Spec Ref R9:

Be able to convert between percentages, fractions and decimals.



You can switch between percentages, fractions and decimals in the following ways.



You can convert directly from a fraction to a percentage by finding an **equivalent fraction** with a **denominator of 100** — then the **numerator** gives the percentage.

You can also convert directly from a percentage to a fraction by writing the **percentage as the numerator** and **100 as the denominator** — then **simplify** the fraction.

Example 1

Write: a) 56% as a decimal

b) 0.24 as a fraction in its simplest terms.

1. Divide by 100% to write it as a decimal.
2. The final digit of 0.24 (the '4') is in the hundredths column, so write 0.24 as 24 hundredths, and then simplify.

$$\text{a) } 56\% \div 100\% = 0.56$$

$$\text{b) } 0.24 = \frac{24}{100} = \frac{6}{25}$$

Example 2

The probability of it raining tomorrow is 0.8. Write this probability as a percentage.

Multiply by 100% to turn the decimal into a percentage.

$$0.8 \times 100\% = 80\%$$

Exercise 1



Q1 a) Find the fraction equivalent to $\frac{3}{20}$ which has 100 as the denominator.

b) Write $\frac{3}{20}$ as: (i) a percentage (ii) a decimal.

Q2 Write each of the following percentages as (i) a decimal, and (ii) a fraction in its simplest terms.

a) 75%

b) 130%

c) 34%

d) 6%

- Q3 Write each of the following fractions as (i) a decimal, and (ii) a percentage.
 a) $\frac{3}{10}$ b) $\frac{7}{8}$ c) $\frac{1}{3}$ d) $\frac{7}{16}$
- Q4 Write each of the following decimals as (i) a percentage, and (ii) a fraction in its simplest terms.
 a) 0.35 b) 1.7 c) 0.6 d) 2.68
- Q5 Raj answers 86% of the questions in a test correctly. Write this as a decimal.
- Q6 $\frac{3}{5}$ of the pupils in a class are right-handed. What percentage of the class are right-handed?
- Q7 Raphael eats 36% of a cake. Write this percentage as a fraction in its lowest terms.

Comparing Percentages, Fractions and Decimals

Learning Objective — Spec Ref R9:

Be able to compare and order percentages, fractions and decimals.

As percentages, fractions and decimals are all used to represent proportions it's useful to be able to **compare** them — you'll have to **convert** them all into the **same form** first.

Example 3

Put $\frac{7}{20}$, 33% and 0.3 in order, from smallest to largest.

Write the amounts in the same form. (Here, I've chosen to write them all as percentages.)

- | | |
|---|--|
| 1. Write $\frac{7}{20}$ as a percentage by first writing it as a fraction out of 100. | $\frac{7}{20} = \frac{35}{100} = 35\%$ |
| 2. Multiply 0.3 by 100% to turn it into a percentage. | $0.3 \times 100\% = 30\%$ |
| 3. Put the percentages in order, from smallest to largest. | 30%, 33%, 35% |
| 4. Rewrite in their original forms. | 0.3, 33%, $\frac{7}{20}$ |

Exercise 2



- Q1 For each of the following pairs, write down which is larger.
 a) 0.35, 32% b) 0.58, 68% c) 0.4, 4% d) 0.09, 90%
 e) $0.2, \frac{21}{100}$ f) $0.6, \frac{7}{10}$ g) $0.7, \frac{3}{4}$ h) $0.55, \frac{3}{5}$
- Q2 Put the numbers in each of the following lists in order, from smallest to largest.
 a) 0.42, 25%, $\frac{2}{5}$ b) 0.505, 45%, $\frac{1}{2}$ c) 0.37, 38%, $\frac{3}{8}$ d) 0.2, 22%, $\frac{2}{9}$
 e) 0.13, 12.5%, $\frac{3}{20}$ f) 0.25, 23%, $\frac{9}{40}$ g) 0.4, 2.5%, $\frac{1}{25}$ h) 0.006, 0.06%, $\frac{3}{50}$
- Q3 In a season, Team X won 14 out of the 20 matches they played. Team Y won 60% of their matches. Which team had the highest proportion of wins?
- Q4 Oliver and Jen each try flicking a set of counters into a box. Oliver gets 65% of the counters into the box. Jen gets $\frac{11}{20}$ of the counters into the box. Who got more counters into the box?

Percentage, Fraction and Decimal Problems

Learning Objective — Spec Ref R9:

Be able to solve problems involving percentages, fractions and decimals.

You might need to convert between percentages, fractions and decimals before you're able to solve a problem. The first step will usually be **converting** them to the **same form**. Once you've converted them there's still work to do — don't forget to answer the question.

Example 4

$\frac{1}{4}$ of pupils in a school bring a packed lunch, 65% have school dinners, and the rest go home for lunch. What percentage of pupils go home for lunch?

1. Write $\frac{1}{4}$ as a percentage by writing it as a fraction out of 100. $\frac{1}{4} = \frac{25}{100} = 25\%$
2. Find the percentage that don't go home for lunch by adding the percentages for 'packed lunches' and 'school dinners'. $25\% + 65\% = 90\%$
3. Subtract this from 100% to find the percentage who do go home for lunch. $100\% - 90\% = 10\%$

Tip: Think about your final answer when deciding how to convert. If you need to find a percentage, it's usually best to convert everything to percentages.

Exercise 3



- Q1 In a school survey of food preferences, 50% of pupils said they prefer pizza, $\frac{1}{5}$ said they prefer shepherd's pie and the rest said they prefer roast chicken. Find the percentage that prefer roast chicken.
- Q2 In a car park, $\frac{2}{5}$ of the cars are red, 0.12 of the total number are white, 15% are blue, and the rest are black. Find the percentage of cars in the car park that are black.
- Q3 $\frac{3}{4}$ of the people at a concert arrived by train, 5% walked, and the rest came by car. What percentage came by car?
- Q4 $\frac{1}{4}$ of Hattie's jackets are leather, $\frac{1}{5}$ are denim, 30% are suede, and the rest are corduroy. Find the percentage of Hattie's jackets that are corduroy.
- Q5 Ainslie is keeping a record of the birds in his garden. Of the birds he has seen this month, $\frac{3}{8}$ were sparrows, 41.5% were blackbirds, and the rest were robins. What percentage were robins?

- Q6 In a school, the ratio of girls to boys is 3 : 7. 25% of the girls and $\frac{1}{5}$ of the boys are part of a sports club. In total what percentage of the school pupils are part of a sports club?



5.3 Percentage Increase and Decrease

Percentages are often used to describe the change in an amount. The change could be an increase or a decrease so make sure you know which one you're trying to find before tackling a question.

Calculating Amounts after a Percentage Increase or Decrease

Learning Objective — Spec Ref R9:

Be able to find amounts after a percentage increase or decrease.

Prior Knowledge Check:

Be able to find the percentage of an amount. See p.60-61.

One method for increasing or decreasing by a percentage is to just work out the percentage of the amount and then:





- **add it** to the original amount if it's a **percentage increase**.
- **subtract it** from the original amount if it's a **percentage decrease**.

Example 1

Increase 450 by 15% without using your calculator.

- Find 10% and 5% of 450.
 $10\% \text{ of } 450 = 450 \div 10 = 45$
 $5\% \text{ of } 450 = 45 \div 2 = 22.5$
- Add these to find 15% of 450.
So $15\% \text{ of } 450 = 45 + 22.5 = 67.5$
- Add this to the original amount.
 $450 + 67.5 = \mathbf{517.5}$

Exercise 1

- Q1 Increase each of the following amounts by the percentage given. 
- a) 90 by 10% b) 60 by 25% c) 80 by 75% d) 270 by 20%
- Q2 Increase each of the following amounts by the percentage given.
- a) 110 by 60% b) 480 by 115% c) 140 by 45% d) 100 by 185%
- Q3 Decrease each of the following amounts by the percentage given. 
- a) 55 by 10% b) 48 by 75% c) 25 by 30% d) 120 by 60%
- Q4 Decrease each of the following amounts by the percentage given.
- a) 125 by 40% b) 11 by 70% c) 150 by 55% d) 520 by 15%
- Q5 A farmer has 380 acres of land. He sells 35% of his land. How much does he have left? 
- Q6 A population decreases by 15% from 2400. What is the new population?
- Q7 A TV costs £485, plus 20% VAT. Find the total cost of the TV after the VAT is added.
- Q8 Roberto's weekly wage of £400 is decreased by 5%. Mary's weekly wage of £350 is increased by 10%. Who now earns more? By how much? 

You can also calculate a percentage increase or decrease in one go using a **multiplier**. To find the multiplier, turn the percentage into a **decimal** and **add it to 1** for an **increase** or **subtract it from 1** for a **decrease**. Then just **multiply** the original amount by your multiplier to get the answer.

Example 2

Fabian deposits £150 into an account which pays 7% interest per year. How much will be in the account after one year?

1. Turn the percentage into a decimal. $7\% = 7 \div 100 = 0.07$
2. It's a percentage increase, so add it to 1. $\text{Multiplier} = 1 + 0.07 = 1.07$
3. Multiply the amount by the multiplier. $£150 \times 1.07 = \text{£160.50}$

Tip: A percentage increase has a multiplier greater than 1. A percentage decrease has a multiplier less than 1.

Example 3

A medium box of frosted flakes contains 500 g of cereal. A small box of frosted flakes contains 45% less cereal than the medium box. How many grams of cereal are in a small box?

1. Turn the percentage into a decimal. $45\% = 45 \div 100 = 0.45$
2. It's a percentage decrease, so subtract it from 1. $\text{Multiplier} = 1 - 0.45 = 0.55$
3. Multiply the amount by the multiplier. $500 \text{ g} \times 0.55 = \text{275 g}$

Exercise 2

Use the multiplier method for the following questions.

Q1 Increase each of the following amounts by the percentage given.

- | | | | |
|---------------|----------------|---------------|---------------|
| a) 490 by 11% | b) 101 by 16% | c) 55 by 137% | d) 89 by 61% |
| e) 139 by 28% | f) 426 by 134% | g) 854 by 89% | h) 761 by 77% |

Q2 Decrease each of the following amounts by the percentage given.

- | | | | |
|---------------|---------------|---------------|---------------|
| a) 77 by 8% | b) 36 by 21% | c) 82 by 13% | d) 101 by 43% |
| e) 189 by 38% | f) 313 by 62% | g) 645 by 69% | h) 843 by 91% |

Q3 David's height increased by 20% between the ages of 6 and 10. He was 50 inches tall at age 6. How tall was he at age 10?

Q4 2 years ago, Alison earned £31 000 per year. Last year, she got a pay rise of 3%. This year, she got a pay cut of 2%. How much does she now earn per year?

Q5 A year ago, University A had 24 500 students and University B had 22 500 students. Over the last year, the number of students increased by 2% at University A, and by 9% at University B. Which University has more students now? By how much?

Q6 The population of Barton is 152 243, and increases by 12% each year. The population of Meristock is 210 059, and decreases by 8% each year. Which town has the larger population after one year? By how much? Give your answer to the nearest whole number.

Simple Interest

Learning Objective — Spec Ref R9:

Be able to solve problems involving simple interest.

Interest is a percentage of money that is added on to an initial figure over a period of time. Simple interest is when a certain percentage of the **original amount** is paid at regular intervals (usually once per year). Because the interest paid is always based on the original amount, and the original amount **never changes**, the amount of interest is the **same** every time it's paid.

Example 4

Kim invests £220 into an account which pays 1% simple interest per year. How much will be in the account after seven years?

- | | |
|---|--|
| 1. Start off by finding 1% of £220. | $1\% \text{ of } 220 = 220 \div 100 = £2.20$ |
| 2. Multiply the interest each year by the number of years. | $£2.20 \times 7 = £15.40$ |
| 3. Add the total amount of interest to the original amount. | $£220 + £15.40 = \text{£235.40}$ |

Exercise 3

- Q1 How much money would be in a savings account that pays simple interest if:
- a) £500 was invested for 2 years in an account which pays 10% interest each year.
 - b) £900 was invested for 6 years in an account which pays 5% interest each year.



- Q2 Gerry puts £5500 into an account that pays 6% simple interest each year. How much will be in his account after: a) 3 years? b) 10 years?

- Q3 Raj invests £2300 into a bank account which pays simple interest. After 4 years there was £2852 in the account. What was the yearly interest rate on Raj's savings account?



- Q4 Padma has forgotten how much money she invested into her savings account which pays simple interest. She knows that after 5 years there was £1440 and after 9 years there was £1632. How much did she originally invest into the account?



Finding the Original Value

Learning Objective — Spec Ref R9:

Be able to find the original value after a percentage increase or decrease.

You can find the **original value** after a percentage increase or decrease by following these steps:

1. Write the new amount as a percentage of the original value.
2. Divide the new amount by the percentage to find 1% of the original amount.
3. Multiply by 100 to give the original value (100%).

Example 5

Penny donates 25% of her comic books to charity. She has 63 comic books left. How many comic books did she have before she donated any to charity?

1. Write 63 as a percentage of the original amount. $100\% - 25\% = 75\%$
So 63 = 75% of the original amount
2. Divide 63 by 75 to work out 1% of the original amount. $1\% \text{ of the original amount} = 63 \div 75 = 0.84$
3. Multiply by 100 to work out 100% of the original amount. $100\% \text{ of the original amount} = 0.84 \times 100 = \mathbf{84 \text{ comic books}}$

Exercise 4

- Q1 A fridge costs £200 after a 50% reduction. Calculate the original price of the fridge.
- Q2 Andy buys a top hat that has been reduced in a sale by 35%. If the sale price is £13.00, find the original price.
- Q3 Find the original value of y if:
- a) it is increased by 20% to give 24
 - b) it is decreased by 40% to give 30
 - c) it is decreased by 70% to give 99
 - d) it is increased by 180% to give 84
- Q4 In the past year, the number of frogs living in a pond has increased by 10% to 528, and the number of newts living there has increased by 15% to 621. How many frogs and how many newts lived in the pond a year ago?
- Q5 Monib is a stamp collector. The number of stamps in his collection increased by 5% in January and by 20% in February. How many stamps did Monib have at the beginning of January if he had 2268 at the beginning of March?



Finding a Change as a Percentage

Learning Objective — Spec Ref R9:

Be able to express the change in an amount as a percentage.

On p.65, you saw how to calculate a new amount after a percentage increase or decrease. Now we'll cover how to find the actual **percentage change** when you know the **original amount** and the **new amount**. To find the change in an amount as a percentage:

1. Calculate the **difference** between the new amount and the original amount.
2. Divide the difference by the **original amount** and multiply by 100%.

You can use this formula to help:

$$\text{Percentage Change} = \frac{\text{Difference in amounts}}{\text{Original amount}} \times 100\%$$

Tip: Percentage changes can come in different forms. You might see them given as percentage profits, discounts, errors, etc.

Example 6

A house price increases from £145 000 to £187 050. Find the percentage increase.

1. Calculate the difference. $187\,050 - 145\,000 = 42\,050$
2. Divide the difference by the original amount. $42\,050 \div 145\,000 = 0.29$
3. Multiply by 100% to write as a percentage. $0.29 \times 100\% = 29\%$
So it is a **29%** increase.

Tip: Always remember to divide by the original amount, not the new amount.

Example 7

In an experiment, the mass of a chemical drops from 75 g to 69 g. Find the percentage decrease.

1. Calculate the difference. $75 - 69 = 6\text{ g}$
2. Divide the difference by the original amount. $6 \div 75 = 0.08$
3. Multiply by 100% to write as a percentage. $0.08 \times 100\% = 8\%$
So it is an **8%** decrease.

Exercise 5

- Q1 Find the percentage increase when:
- a) a price of £10 is increased to £12.
 - b) a price of £20 is increased to £52.
- Q2 Find the percentage decrease when:
- a) a price of £10 is decreased to £8.
 - b) a price of £25 is decreased to £22.
- Q3 The number of people working for a company increases from 45 to 72. Find the percentage increase in the number of people working for the company.
- Q4 The price of a local newspaper increases from 80p to £1. Find the percentage increase.
- Q5 Percy is on a healthy eating plan. His weight drops from 80 kg to 68 kg. Find the percentage decrease in Percy's weight.
- Q6 In a sale, the price of a toaster is discounted from £50 to £30. Find the percentage discount.
- Q7 What is the percentage error when 250 is rounded to the nearest 100?
- Q8 Izzy buys an antique lamp for £450 and sells it for £315. She also buys an antique wardrobe for £980 and sells it for £1127.
- a) What was her percentage loss on the antique lamp?
 - b) What was her percentage profit on the antique wardrobe?
 - c) Overall Izzy made a profit on the two items. Calculate the percentage profit, giving your answer to 2 decimal places.



5.4 Compound Percentage Change

Compound percentage changes involve repeating a percentage increase or decrease. Each time you calculate the percentage change you apply it to the current amount, rather than the original amount.

Compound Growth

Learning Objective — Spec Ref R16:

Be able to solve compound growth problems.

Compound growth is when a quantity gets larger over time due to **successive percentage increases**. The percentage increases always use the **new value**, e.g. if £1000 is increased by 10% each year:

After 1 year: $£1000 \times 1.1 = £1100$

After 2 years: $£1100 \times 1.1 = £1210$ (which is the same as $£1000 \times 1.1^2$)

After 3 years: $£1210 \times 1.1 = £1331$ (which is the same as $£1000 \times 1.1^3$)

So the formula for **compound growth** is:
$$P_n = P_0 \times \left(1 + \frac{r}{100}\right)^n$$

P_n = amount after n periods, P_0 = initial amount,
 n = number of periods, r = percentage rate of change

When applied to money, compound growth is known as **compound interest**.

Tip: The bit of the formula in brackets is just the percentage multiplier — see p.66.

Example 1

The number of bacteria in a sample increases at a rate of 60% each week. At the start of an experiment, a scientist placed 400 bacteria in a jar. How many bacteria will there be after 6 weeks?

1. Use the formula for compound growth:
$$P_n = P_0 \times \left(1 + \frac{r}{100}\right)^n$$
2. Plug in the numbers...
$$P_0 = 400, r = 60, n = 6$$
$$P_n = 400 \times \left(1 + \frac{60}{100}\right)^6$$
$$= 400 \times 16.777\ldots$$
$$= 6710.88\ldots = \mathbf{6711 \text{ bacteria}}$$
3. Round the answer to an appropriate degree of accuracy.

Tip: Always check the context before giving your final answer. Here it makes sense to round it to a whole number.

Exercise 1

- Q1 Calculate how much money you will have after 4 years if you invest £680 at 2.5% annual compound interest.
- Q2 The population of the UK in 2017 was 66 000 000. The population is increasing by 0.6% every year. Assuming that this rate of growth continues, how many people will reside in the UK by 2025? Give your answer to two significant figures.
- Q3 Find the value of a bar of gold, initially valued at £750, after seven months when gold prices are rising at a rate of 0.1% per month.
- Q4 A colony of ants has set up home in Mr Murphy's shed. On Monday there were 250 ants. If the colony of ants grows at a rate of 5% per day, how many ants will there be on Saturday?

- Q5 Mrs Honeybun is expecting twins. Every month her waistline increases by 5%. If her waist measurement started at 70 cm, what is her waistline, to the nearest cm, after nine months?
- Q6 In 2018 a group called 'Smiley Faced People' decide to spread a little happiness around the world. The original group has 20 members. If their numbers increase by 70% every year as more and more people decide to join their happy band, how many people to the nearest 100 will be in the group in 2028?
- Q7 Use the formula for compound growth to calculate the interest earned by the following investments:
a) £750 for 5 years at an annual rate of 3%. b) £50 for 7 years at an annual rate of 5.5%.
- Q8 During 2013 and 2014 a bank paid interest at a compound rate of 2% per annum. During 2015, 2016 and 2017 this rate rose to 3%. Calculate the total interest paid over five years if £650 was invested at the beginning of the year 2013.

Compound Decay

Learning Objective — Spec Ref R16:

Be able to solve compound decay problems.

Compound decay is the opposite of compound growth — it's when an amount gets smaller over time due to successive percentage decreases. When applied to money, compound decay is known as **depreciation**.

The formula for **compound decay** is the same as for growth except that there's a minus sign in the bracket so that the multiplier is less than 1:

$$P_n = P_0 \times \left(1 - \frac{r}{100}\right)^n$$

Example 2

A car was bought for £8000. Its value depreciates by 20% each year. What is the value of the car after 10 years to 2 significant figures?

- Use the formula for compound decay: $P_n = P_0 \times \left(1 - \frac{r}{100}\right)^n$
- Plug in the numbers... $P_0 = £8000, r = 20, n = 10$
- Round the answer to 2 significant figures. $P_n = 8000 \times \left(1 - \frac{20}{100}\right)^{10}$
 $= 8000 \times 0.1073... = 858.9934... = \text{£}860$ (2 s.f.)

Exercise 2

- Q1 Find, to the nearest £100, the value of a car that originally cost £10 000 after five years of depreciation at 15%.
- Q2 Find, to the nearest £, the depreciation on £550 after 3 years at a depreciation rate of 3% per annum.
- Q3 Mr Butterworth is on a diet and is losing weight at a rate of 2% of his total body weight every week. He weighed 110 kg when he started his diet. What is his weight, to the nearest kg, after 8 weeks?
- Q4 Calculate the value of a car after seven years if it was bought for £6000 and depreciation is at a rate of 7.5% per annum. Give your answer to 2 significant figures.
- Q5 The activity of a radioactive source decreases by 6% every hour. If the initial activity is 1200 Bq, calculate the activity after 10 hours to the nearest whole number.

For Questions 6–9, give your answers to 3 significant figures.

- Q6 Calculate the depreciation of a house after five years if it was bought for £650 000 and house prices are falling at a rate of 2.5% per annum.
- Q7 Claire the Evil Genius buys a laser for £68 000. It depreciates at 20% per annum for the first two years and at 15% per annum for the next three years. What is the value of the laser after 5 years?
- Q8 Find the final value of an investment of £3500 that grows by 0.75% per annum for two years but then depreciates at a rate of 1.25% per annum for the next seven years
- Q9 Find the final value of an investment of £1000 that grows by 5% per annum for the first two years but then depreciates at a rate of 2% per annum for the next three years.

More Compound Change Problems

Learning Objective — Spec Ref R16:

Find other unknowns in compound change scenarios such as the time period.

In some compound growth or decay questions, it isn't the amount after n periods that you're trying to find. You might have to **rearrange** the formulas or use **trial and improvement** to find a different value.

Example 3

A motorbike valued at £2500 depreciates at a rate of 5% per month.
After how many whole months will the value of the motorbike be less than £1150?



1. Using the compound decay formula, you know the values of P_n , P_0 and r , and you want to find the value of n :

$$P_n = P_0 \times \left(1 - \frac{r}{100}\right)^n \quad P_n = £1150, P_0 = £2500, r = 5$$

$$1150 = 2500 \times \left(1 - \frac{5}{100}\right)^n = 2500 \times 0.95^n$$

2. Use trial and improvement to find the value of n .

Try $n = 10$: $2500 \times 0.95^{10} = 1496.84...$ n is too small

Try $n = 20$: $2500 \times 0.95^{20} = 896.21...$ n is too big

Try $n = 15$: $2500 \times 0.95^{15} = 1158.22...$ n is too small

Try $n = 16$: $2500 \times 0.95^{16} = 1100.31...$ correct







$n = 16$ months

Exercise 3



- Q1 A beehive population is decreasing at a rate of 4.9% per year. If there are initially 500 bees in the hive, after how many whole years will the beehive population have halved?
- Q2 Mr Quasar is visiting Las Vegas and has taken \$1000 to gamble in the casino. He loses money at a rate of 15% a day. After how many whole days will Mr Quasar have less than \$100 left?
- Q3 The number of foxes in an area is falling by 12% per year. If there are 300 foxes in 2018, after how many whole years will the number have fallen below 200?
- Q4 A biologist has a colony of 500 bacteria. When the population of bacteria grows over 1000, the biologist can start doing some tests on the colony. If the growth in the number of bacteria is at a rate of 15% per hour, after how many whole hours can the biologist start the tests?

Review Exercise

- Q1** Write each of the following amounts as a percentage. 
- a) 13 out of 50 b) 26 out of 40 c) 48 out of 120
- Q2** Sandra gets paid £1385 per month. In December she gets a Christmas bonus of £83.10. What percentage of her monthly salary is this bonus?
- Q3** Sarah has won 65% of her last 60 badminton games. How many games has she won? 
- Q4** Find each of the following:
- a) 3% of 210 b) 41% of 180 c) 4.5% of 900
- Q5** For each of the following lists, write down which amount is not equal to the others. 
- a) 0.5, 20%, $\frac{1}{2}$ b) 0.125, 1.25%, $\frac{1}{8}$ c) 0.22, 44%, $\frac{22}{50}$
- Q6** Kelly and Nasir both had maths tests last week. Kelly scored $\frac{47}{68}$ and Nasir scored $\frac{35}{52}$. Who got the higher percentage score?
- Q7** The insurance for a car normally costs £356. With a no-claims discount, the cost is reduced by 27%. What is the reduced cost of the insurance?
- Q8** House A is valued at £420 000 and House B is valued at £340 000. After 5 years the value of House A has decreased by 15% and the value of House B has increased by 5%. What is the difference in value of the two houses after 5 years? 
- Q9** Ami deposits £650 into a savings account that pays 4% simple interest each year. How much will be in her savings account after 7 years?
- Q10** Dave loves a bargain and buys a feather boa which has been reduced in price by 70%. If the sale price is £2.85, what was the original price of the boa?
- Q11** A cafe sells 270 ice cream sundaes in April and 464 in May. Find the percentage increase in ice cream sales. 
- Q12** Murray invests £575 in a bank account that pays annual compound interest of 7.5%. How much money will he have in the bank after 3 years?
- Q13** Find the final value of an investment of £2500 that earns 0.5% per month for the first six months but then depreciates at a rate of 0.25% per month for the next 9 months.
- Q14** A balloon is deflating. Its volume decreases at a rate of 20% per hour. The balloon initially had a volume of 1500 cm³. How many whole hours will it be before the balloon's volume is less than 400 cm³? 

Exam-Style Questions

- Q1** A school has 1200 pupils. 50% of the pupils get the bus to school, 20% walk and 5% get to school by bicycle. The remaining pupils are driven to school by their parents. How many pupils are driven to school by their parents?



[2 marks]

- Q2** In a sofa store 30% of the sofas are leather. 40% of the leather sofas are black. What percentage of the total number of sofas are made from black leather?



[2 marks]

- Q3** Inflation is the percentage by which prices increase. One year, the rate of inflation in England was 3.2%.

- a) An item was worth £50 at the start of the year. If its value increased at the rate of inflation, how much was it worth at the end of the year?

[2 marks]

Venezuela is a South American country whose currency is the bolivar (Bs). In that same year, the rate of inflation in Venezuela was 108%.

- b) An item was worth 50Bs at the start of the year. If its value increased at the rate of inflation, how much was it worth at the end of the year?

[2 marks]

- Q4** Gareth won the lottery and invested it all into a savings account at the start of 2014. There was £4 862 025 in the account at the start of 2017. If compound interest is added to his account at a rate of 5% per annum, how much did he have in his account at the start of 2014?

[2 marks]

- Q5** A type of new car loses 20% of its value in its first year. It loses 15% per year in every year after the first. Graham buys one of these new cars. Work out the percentage of its value that it has lost after five years. Give your answer to the nearest whole number.



[4 marks]