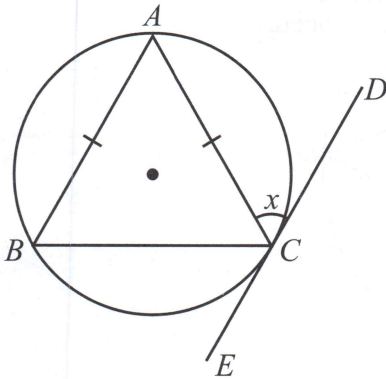


Circle Geometry

- 1 The diagram shows triangle ABC , where A , B and C are points on the circumference of a circle. $AB = AC$, and DE is a tangent to the circle at C .



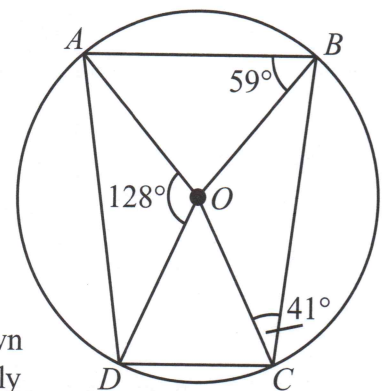
Angle $DCA = x$. DE is parallel to AB .
Prove that ABC is an equilateral triangle.

Give geometrical reasons to support the statements you make.

[Total 4 marks]

- 2 The diagram below shows the circle with centre O .
 A , B , C and D are points on the circumference of the circle.

Find the size of angle CDO .

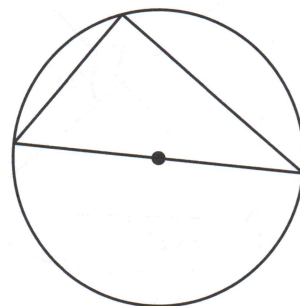


Not drawn
accurately

$CDO = \dots\dots\dots^\circ$
[Total 4 marks]

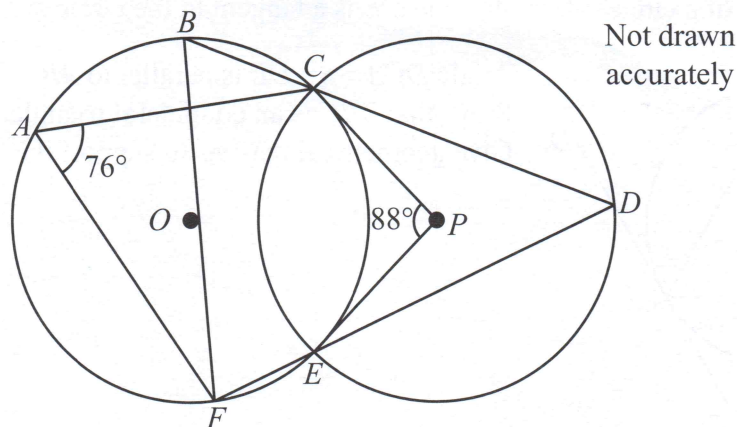
- 3 Prove that the angle formed at the circumference when a triangle is drawn from both ends of a diameter is 90° .

Start by splitting the triangle into two isosceles triangles.



[Total 3 marks]

- 4 The diagram below shows two intersecting circles with centres O and P . The circles intersect at C and E . A , B and F are points on the circumference with centre O , and D is a point on the circumference of the circle with centre P . BD and DF are straight lines.

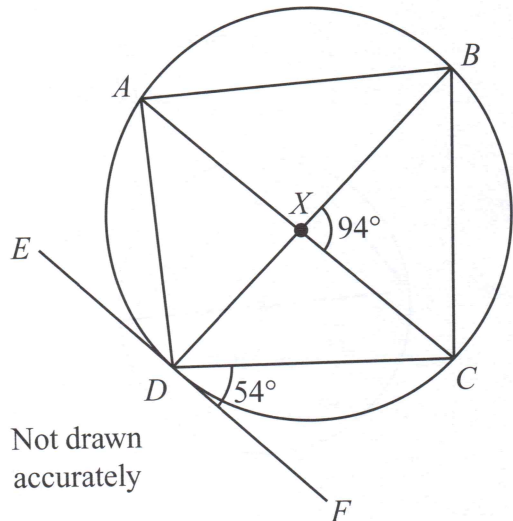


Find the size of angle BFE . Give reasons for each step of your working.

$BFE = \dots\dots\dots^\circ$
[Total 3 marks]

- 5 Points A , B , C and D are points on the circumference of the circle below. EF is a tangent that meets the circle at D , and AC and BD are straight lines.

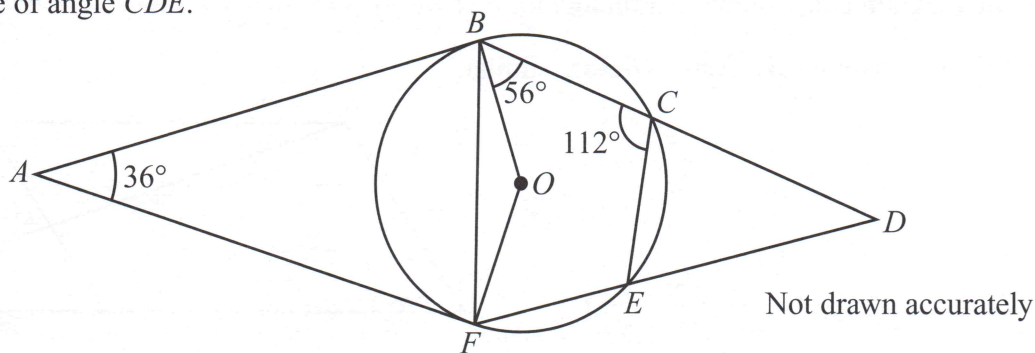
Show that X is NOT the centre of the circle.



[Total 4 marks]

- 6 The diagram shows a circle with centre O . B , C , E and F are points on the circumference of the circle. AB and AF are tangents to the circle, and BD and DF are straight lines.

Find the size of angle CDE .

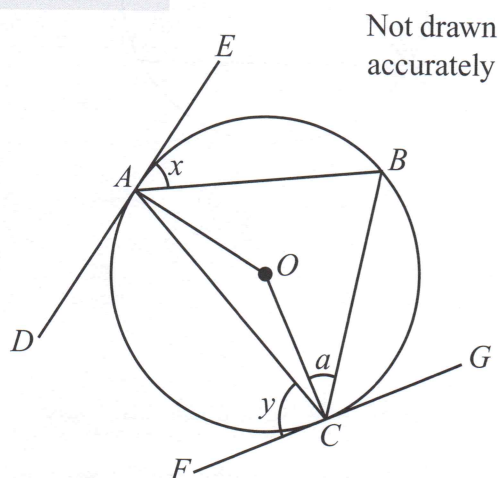


$CDE = \dots\dots\dots^\circ$
[Total 5 marks]

- 7 The diagram shows a circle with centre O .
 DE is a tangent to the circle at A and FG is a tangent to the circle at C .

Prove that $a = x + y - 90^\circ$.

State any circle theorems that you use.



[Total 4 marks]

Exam Practice Tip

It's not always easy to spot which circle theorems you need to use — so just go through them one by one until you find one that works. There are usually one or two you can discount straight away (for example, if there are no tangents on the diagram, you probably aren't going to need the theorems that involve tangents).

Score

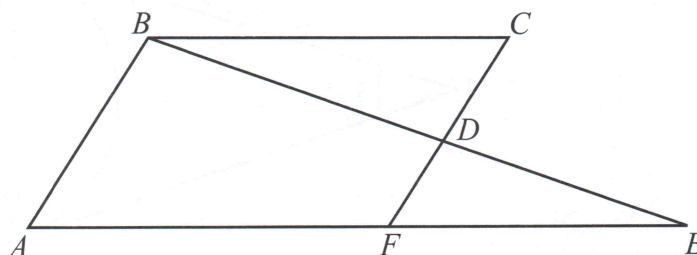
27



Similarity and Congruence

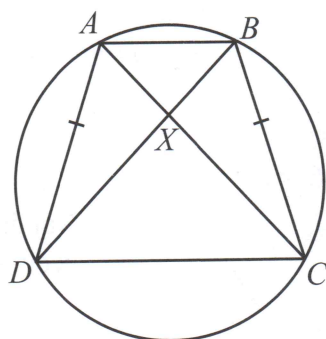
- 1 The diagram below shows parallelogram $ABCF$. AFE is a straight line.

Prove that triangles BCD and ABE are similar.



[Total 3 marks]

- 2 Points A , B , C and D are points on the circumference of the circle below, and $ABCD$ is an isosceles trapezium.

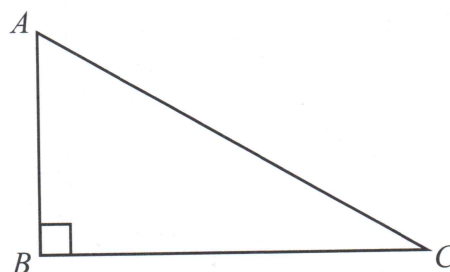


Prove that triangles AXD and BXC are congruent.

[Total 3 marks]

- 3 ABC is a right-angled triangle.

- a) Construct a line that is perpendicular to AC and passes through point B . Label the point where the line crosses side AC point D .



[2]

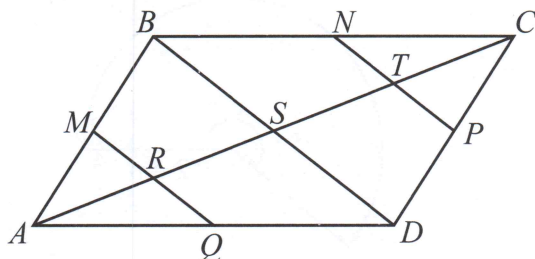
- b) Show that triangles ABD and BCD are similar.

[3]

[Total 5 marks]

- 4 $ABCD$ is a parallelogram. M, N, P and Q are the midpoints of AB, BC, CD and DA respectively. R is the midpoint of MQ, S is the midpoint of BD and T is the midpoint of NP .

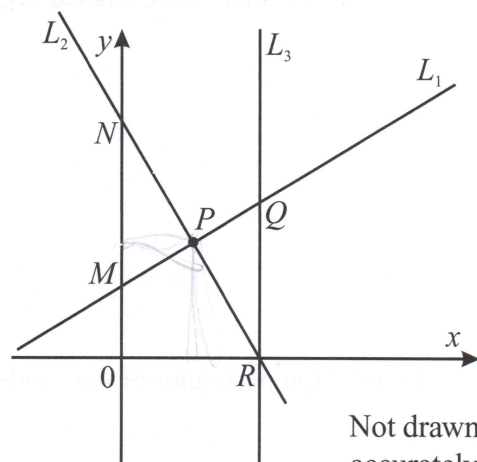
Prove that triangles AMQ and NCP are congruent.



[Total 4 marks]

- 5 Lines L_1 and L_2 are perpendicular, and line L_3 is a vertical line. Lines L_1 and L_2 intersect at point P , which has coordinates $(2, 2.5)$. Lines L_2 and L_3 intersect at point R , which lies on the x -axis. The equation of line L_2 is $5x + 4y = 20$.

Prove that triangles MNP and QRP are congruent.

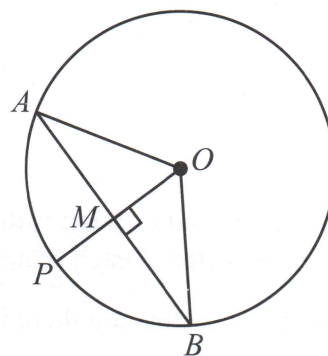


[Total 6 marks]

Not drawn accurately

- 6 The diagram shows a circle with centre O . AB is a chord, and the radius OP is perpendicular to AB .

Prove that OP bisects the chord AB .



[Total 4 marks]

Exam Practice Tip

Remember the four conditions for congruence — SSS, AAS, SAS and RHS. Then all you have to do is find any side lengths and angles that you can, and see which condition they fit. Don't forget to state the condition at the end of your answer — e.g. 'the condition SSS holds, so the triangles are congruent'.

Score

25

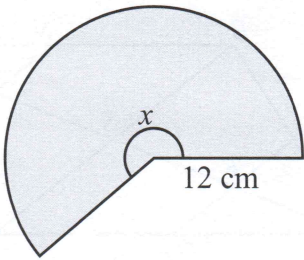


Arcs, Sectors and Segments

1 The major sector on the right has an area of $88\pi\text{ cm}^2$.

Find the size of angle x .

Not drawn accurately

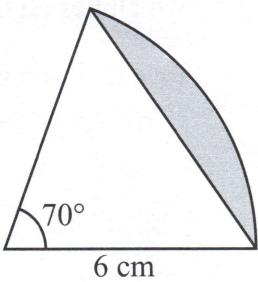


$x = \dots\dots\dots^\circ$
[Total 3 marks]

2 The diagram on the right shows a sector of a circle.

a) Find the area of the shaded segment. Give your answer to 3 s.f.

Not drawn accurately



$\dots\dots\dots\text{ cm}^2$
[3]

b) Find the perimeter of the shaded segment. Give your answer to 3 s.f.

You need to use the cosine rule for part b).

$\dots\dots\dots\text{ cm}$
[4]
[Total 7 marks]

3 A circle of radius 8 cm is divided up into equal sectors. The perimeter of each sector is 21.6 cm to 3 s.f.

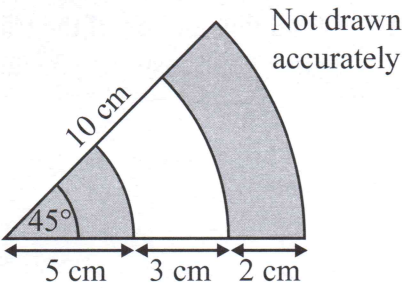
How many sectors are there in total?

$\dots\dots\dots$
[Total 3 marks]

4 Hannah cuts a slice of cake. The cake has a radius of 10 cm, and Hannah's slice is a sector with angle 45° . The cake is covered in two different colours of icing, as shown below.

- a) Find the area of white icing on Hannah's slice of cake.
Give your answer to 3 s.f.

..... cm^2
[3]



- b) The height of the cake is 8 cm. Find the volume of Hannah's slice.
Give your answer to 3 s.f.

..... cm^3
[2]

[Total 5 marks]

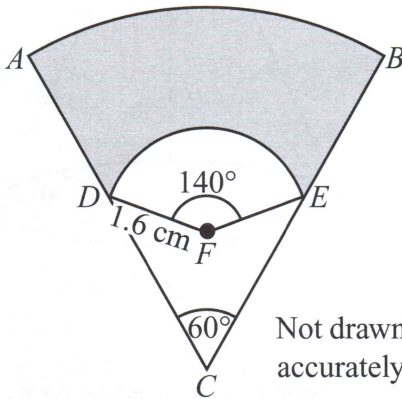
5 The diagram shows sector ABC of a circle with centre C and sector DEF of a circle with centre F . The diagram has a vertical line of symmetry, and D is the midpoint of AC .

$DF = 1.6 \text{ cm}$, angle $DFE = 140^\circ$ and angle $DCE = 60^\circ$.

- a) By considering triangle DFC , calculate the length of DC .
Give your answer to 3 s.f.

You'll have to use
the sine rule here.

..... cm
[3]



- b) Calculate the perimeter of the shaded region $ADEB$. Give your answer to 3 s.f.

..... cm
[5]

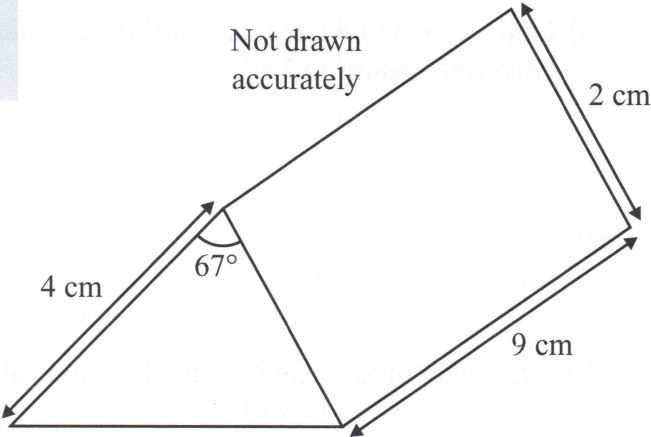
[Total 8 marks]

Score:

26

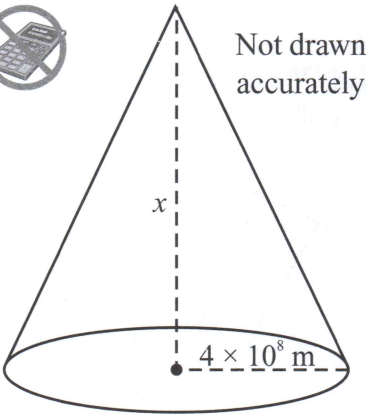
3D Shapes — Surface Area and Volume

- 1 Find the volume of the triangular prism below. Give your answer to 3 significant figures.



..... cm³
[Total 3 marks]

- 2 The cone below has a volume of $(3.2 \times 10^{26})\pi \text{ m}^3$ and a radius of $4 \times 10^8 \text{ m}$. Find x , the vertical height of the cone. Give your answer in standard form.



The formula for the volume of a cone is $V = \frac{1}{3} \times \pi r^2 \times h_v$

..... m
[Total 3 marks]

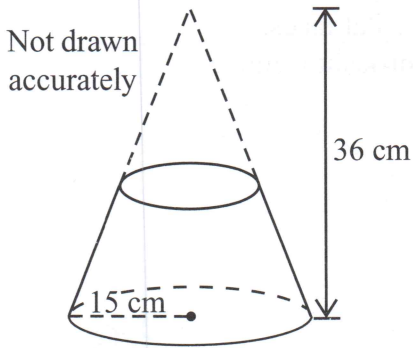
- 3 Sphere A has radius 6 cm. The volume of sphere B is 60% greater than the volume of sphere A. What is the radius of sphere B?

Give your answer to 3 significant figures.

The formula for the volume of a sphere is $V = \frac{4}{3} \pi r^3$.

..... cm
[Total 4 marks]

- 4 A cone has radius 15 cm and height 36 cm. The top of the cone is removed to create a frustum two-thirds of the height of the original cone.



a) Find the exact volume of the frustum.

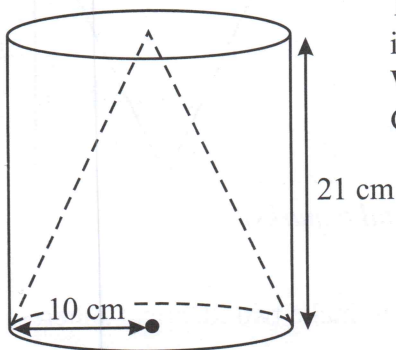
..... cm^3
[3]

b) Find the exact surface area of the frustum.

..... cm^2
[3]

[Total 6 marks]

- 5 A piece of apparatus for an experiment is made up of a cone within a cylinder. The cone and cylinder have the same radius, and the vertical height of the cone is the same as the vertical height of the cylinder (as shown in the diagram).



The space within the cylinder not taken up by the cone is filled with a gas. The gas has a density of 0.52 kg/m^3 .

What is the mass of the gas in the cylinder?

Give your answer in grams to 3 significant figures.

Be careful with the units here.

Not drawn accurately

..... g
[Total 6 marks]

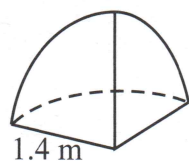
- 6 A cone and a cylinder have the same radius, r . The slant height of the cone is 7 cm and the height of the cylinder is 6 cm. The combined surface area of both shapes is $110\pi \text{ cm}^2$.

Find the radius, r , of the cone and the cylinder.

$r =$ cm
[Total 6 marks]

- 7 A sphere with radius 1.4 m is cut into 8 identical pieces, as shown below. The weight of the whole sphere is 5000 N.

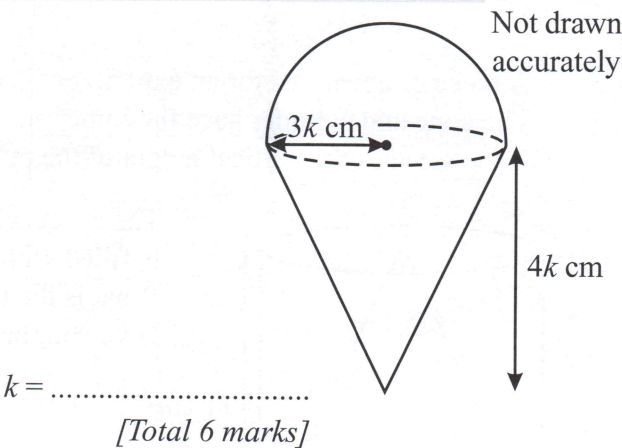
One piece of the sphere is resting on horizontal ground on one of the flat faces. Find the pressure exerted on the ground. Give your answer to 3 significant figures.



..... N/m²
[Total 4 marks]

- 8 The diagram shows a solid object made up of a hemisphere of radius $3k$ cm and a cone with vertical height $4k$ cm. The total surface area of the object is 3993π cm².

Work out the value of k .



- 9 Josie melts down 1200 cm³ of steel. She uses 30% of the steel to make two identical spheres. She uses $\frac{1}{3}$ of the steel to make four identical cones with the same radius as the spheres.

Work out whether Josie has enough steel left to make one cube with side length equal to the height of the cone.

[Total 6 marks]

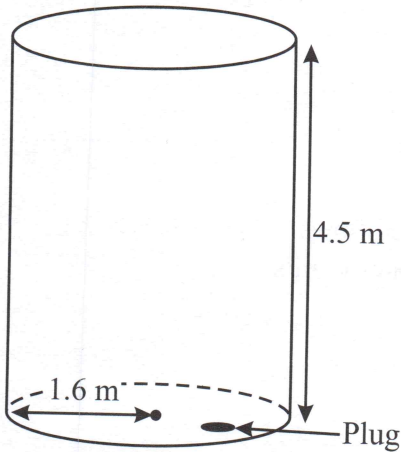
Score:

44

Rates of Flow

- 1 The cylindrical tank below is full of water. The plug is removed, and the tank empties. It takes 25 minutes before the tank is completely empty.

Find the rate of flow of the water. Give your answer in litres per minute to 3 significant figures.



Not drawn accurately

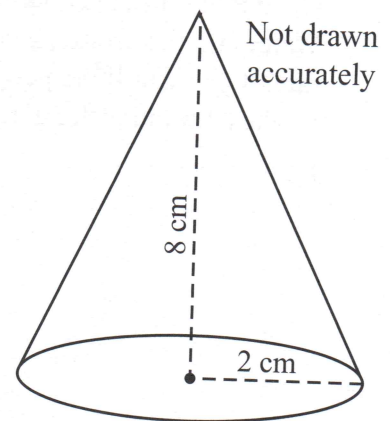
..... litres/min

[Total 4 marks]

- 2 The bottom half of an egg-timer is the shape of the cone below. The circular base is resting on a flat horizontal surface. Liquid flows from the top half of the timer into the bottom half at a rate of $0.5 \text{ cm}^3/\text{s}$.

When the timer is started, the cone is empty.

- a) How long does it take for the liquid in the bottom half of the timer to reach a depth of 2 cm? Give your answer in seconds to 3 s.f.



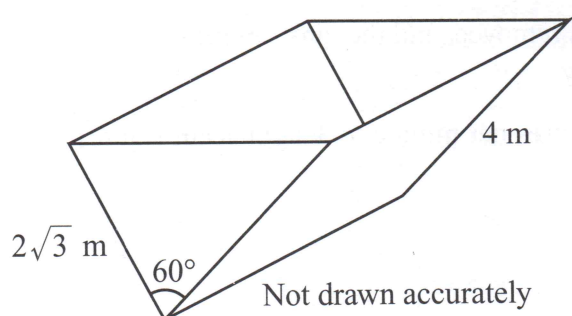
..... s
[3]

- b) When the timer has finished, liquid has filled 89.5% (by volume) of the bottom half of the egg-timer to 3 s.f. How long does the timer last for?

..... s
[2]

[Total 5 marks]

- 3 A flood channel is built to divert excess water from a river. The flood channel is the shape of a triangular prism.



- a) Find the exact cross-sectional area of the flood channel.

..... m^2
[2]

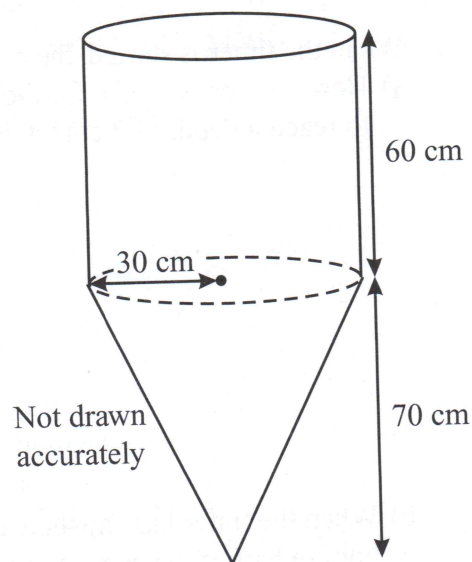
- b) At the peak of a flood, the flood channel is full of water, which flows at a rate of 90 000 litres per minute. Find the speed of the water in m/s.

..... m/s
[4]

[Total 6 marks]

- 4 A container is in the shape of a cylinder placed on top of a cone. The radius of each shape is 30 cm, the height of the cylinder is 60 cm and the vertical height of the cone is 70 cm.

The container is filled at a rate of $x\pi$ litres per second. Once it is full, the container is then emptied. Water empties from the container at a rate of $(x - 2)\pi$ litres per second. It takes 2 minutes longer to empty than it takes to fill. Find the value of x .



$x =$

[Total 7 marks]

Score:

22



Enlargement

1 Three cylinders are mathematically similar. Their radii are in the ratio 1 : 3 : 5. The smallest cylinder has a vertical height of 6 cm.

a) Write down the ratio of the surface areas of the three cylinders.

.....
[1]

b) The volume of the largest cylinder is $6750\pi \text{ cm}^3$. Find the radius of the middle cylinder.

..... cm
[3]
[Total 4 marks]

2 An Egyptian pyramid has a volume of $2.5 \times 10^6 \text{ m}^3$. A museum is building a scale model of the pyramid.

a) The model has a volume of 160 m^3 . Find the scale factor used for the model. Give your answer as a fraction in its simplest form.

||||| The model is smaller than the original pyramid, so the scale factor will be less than 1. |||||

.....
[2]

b) The surface area of the model is 98 m^2 . What is the surface area of the original pyramid? Give your answer in standard form.

..... m^2
[2]
[Total 4 marks]

3 Two cones are mathematically similar. Cone A has a volume of $648\pi\text{ cm}^3$ and cone B has a volume of $1536\pi\text{ cm}^3$. The surface area of cone A is $198\pi\text{ cm}^2$.

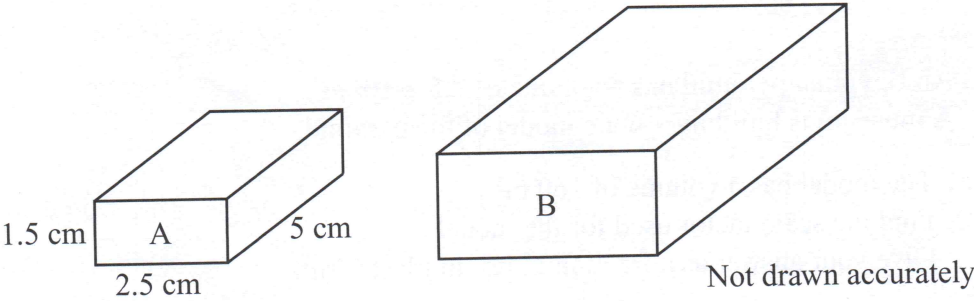
a) Find the exact surface area of cone B.

..... cm^2
[2]

b) Write down the ratio of the radius of cone A to the radius of cone B.
Give your answer in its simplest form.

.....
[1]
[Total 3 marks]

4 Cuboids A and B below are similar, and cuboid A has dimensions as shown.
The scale factor of enlargement from A to B is $\frac{8}{5}$.



a) Find the volume of cuboid B.

..... cm^3
[2]

b) The cuboids are made out of different types of metal and both have a mass of 0.06 kg.
Find the percentage decrease in density from cuboid A to cuboid B.
Give your answer to 3 significant figures.

..... %
[3]
[Total 5 marks]

- 5 Will buys a set of three vases. The vases are mathematically similar and have bases with areas of 90 cm^2 , 160 cm^2 and 1440 cm^2 . The volume of the largest vase is 0.016 m^3 and the height of the medium vase is 20 cm .

Find the height and volume of the smallest vase.



height = cm

volume = cm^3

[Total 4 marks]

- 6 Anna makes necklaces using spherical beads. She has two different sizes of beads. Small beads have a volume of 2.4 cm^3 and large beads have a volume of 8.1 cm^3 .

The time taken to decorate a bead is proportional to the surface area of the bead.
It takes 8 minutes to decorate a small bead.

She uses 5 small beads and 4 large beads to make a necklace.

Can she decorate all the beads needed for a necklace in $1\frac{3}{4}$ hours?
Show how you worked out your answer.

[Total 4 marks]

Exam Practice Tip

Just remember — for a scale factor of n , side lengths are n times bigger, areas (or surface areas) are n^2 times bigger and volumes are n^3 times bigger. If you have a fractional scale factor, don't forget to square and cube both the numerator and denominator of the fraction to find the area and volume of the enlarged shape.

Score

24