Ratio and Proportion

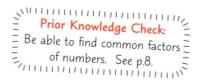
4.1 Ratios

Along with fractions and percentages, ratios are a way of showing proportion. They tell you how the size of one thing relates to the size of another thing, or sometimes several other things. They crop up in lots of practical problems, in real life as well as in maths exams — such as when using map scales and recipes.

Simplifying Ratios

Learning Objectives — Spec Ref R4:

- Write ratios in their simplest form.
- Write ratios in the form 1:n.



Ratios are used to compare quantities. The ratio of quantity a compared to quantity b is written a:b. You can also compare more than two quantities in a ratio by writing them in the form a:b:c, or even a:b:c:d. You **simplify** ratios by dividing the numbers by a **common factor**, just like you do with fractions. E.g. 2:4 simplifies to 1:2 by dividing both sides by 2. A ratio is in its **simplest form** when all of the parts are **whole numbers** but they have **no common factors** greater than 1.

Example 1

There are 15 fiction books and 10 non-fiction books on a shelf.

Write down the ratio of fiction books to non-fiction books in its simplest form.

- 1. Write down the ratio with the quantities in the order that you're asked to give them.
- 2. Divide both sides by the same number— a common factor of the two sides.
- 3. Stop when you can't divide any further and leave whole numbers on each side.

fiction : non-fiction 15 : 10

5 is a common factor:

$$\div 5 \binom{15:10}{3:2} \div 5$$

The simplest form is 3:2

Tip: Dividing by the highest common factor (p.13) gets you to the simplest form in one go. Here, 5 is the HCF of 15 and 10.

- Q1 Write down each of the following ratios in its simplest form.
 - a) 2:6
- b) 40:10
- c) 24:6
- d) 7:28

- e) 16:12
- f) 48:36
- g) 80:32
- h) 121:33
- Q2 Write down each of the following ratios in its simplest form.
 - a) 6:2:4
- b) 15:12:3
- c) 16:24:80
- d) 21:49:42
- Q3 There are 18 boys in a class of 33 pupils. Find the ratio of boys to girls in its simplest form.
- Q4 Isabel and Sophia share a bag of 42 sweets. Isabel has 16 sweets, and Sophia has the rest. Find the ratio of Sophia's sweets to Isabel's sweets, giving your answer in its simplest form.

When you have a ratio involving **different units**, **convert** all quantities to the **same unit** before simplifying.

Example 2

Write the ratio 1 m: 40 cm in its simplest form.

- 1. Rewrite the ratio so that the units are the same.
- 2. Remove the units altogether.
- 3. Simplify as usual by dividing each side by the highest common factor, 20.

1 m = 100 cm, so 1 m: 40 cm is the same as 100 cm: 40 cm

$$\div 20 \begin{pmatrix} 100:40 \\ 5:2 \end{pmatrix} \div 20$$

So the simplest form is **5:2**

Tip: You'll need to know your unit conversions — see Section 22.

Exercise 2

- Q1 Write these ratios in their simplest form.
 - a) 10p:£1
- b) 20 mm:4 cm
- c) 2 weeks:7 days
- d) 18 mins: 1 hour

- e) 30 cm: 2 m
- f) 1 m: 150 mm
- g) 6 months: 5 years
- h) 2.5 hours: 20 mins

- i) 8 cm: 1.1 m
- j) 9g:0.3 kg
- k) 65 m: 1.56 km
- l) 1.2 kg:480 g
- Q2 Emma's mass is 54 kg. Her award-winning pumpkin has a mass of 6000 g. Find the ratio of the pumpkin's mass to Emma's mass. Give your answer in its simplest form.
- Q3 The icing for some cupcakes is made by mixing 1.6 kg of icing sugar with 640 g of butter. Find the ratio of butter to icing sugar. Give your answer in its simplest form.

Ratios in the form 1:n

Practical problems can often involve finding 'how much of b is needed for **every one** of a'. Instead of simplifying the ratio a:b by dividing by a common factor, you need to divide both sides by a, to leave a ratio in the form $1:\frac{b}{a}=1:n$, where n could be a **decimal** or a **fraction**. You can also give the ratio a:b in the form n:1 by dividing both sides by b.

Example 3

Nigel makes a smoothie by mixing half a litre of blueberry juice with 100 millilitres of plain yoghurt. How much yoghurt does he use for every millilitre of juice?

- 1. Write down the ratio of juice to yoghurt and remove the units.
- 2. You want the juice side (the left-hand side) to equal 1, so divide both sides by 500.

1 litre = 1000 ml, so 0.5 litres:100 ml is the same as 500 ml:100 ml

$$\div 500 \left(\begin{array}{c} 500:100 \\ 1:0.2 \end{array} \right) \div 500$$

3. Use this simplified ratio to answer the question.

So he uses 0.2 ml of yoghurt for 1 ml of juice.

Exercise 3

Q1 Write down the following ratios in the form 1:n.

a) 2:6

b) 30:120

c) 8:26

d) 6:21

e) 2:1

f) 10:3

g) 8:5

h) 5:9

i) 10 mm: 5 cm

i) 30 mins : 2 hours

k) 90 m: 7.2 km

l) 50p:£6.25

Q2 Two towns 4.8 km apart are shown on a map 12 cm apart. Find the ratio of the map distance to the true distance in the form 1:n.

Q3 A recipe uses 125 ml of chocolate syrup and $2\frac{1}{2}$ litres of milk. How much milk does it use for every millilitre of chocolate syrup?

Ratios and Fractions

Learning Objectives — Spec Ref N11/R8:

- Use ratios to find fractions.
- Use fractions to find ratios.

Prior Knowledge Check:

Be able to simplify, add and subtract fractions, and write
one quantity as a fraction of another. See p.28, 31 and 32.

If you have a ratio of quantities a:b (or a:b:c), you can find the **fraction** of one quantity out of the total:

- Add up all the quantities in the ratio to get the total number of parts, i.e. a + b, or a + b + c.
- Put the number of parts for the quantity you're interested in as the fraction's numerator, and the total number of parts as the fraction's denominator.
- **Simplify** the fraction if necessary.

Example 4

A box of doughnuts contains jam doughnuts and chocolate doughnuts in the ratio 3:5. What fraction of the doughnuts are chocolate flavoured?

1. Find the total number of 'parts'.

There are 3 + 5 = 8 parts altogether.

2. Write the number of parts that are chocolate as the numerator, and the total number of parts as the denominator. The fraction is in its simplest form.

5 of the parts are chocolate, so

 $\frac{5}{\Omega}$ of the doughnuts are chocolate flavoured.

Exercise 4

Q1 A recipe uses white flour and brown flour in the ratio 2:1. What fraction of the flour is brown?

Q2 A tiled floor has blue and white tiles in the ratio 9:4. What fraction of the tiles are blue?

Q3 In a tournament the numbers of home wins, away wins and draws were in the ratio 7:2:5. What fraction of the games were home wins?

- Q4 Amy's sock drawer has spotty, stripy and plain socks in the ratio 5:1:4. What fraction are stripy?
- Q5 At a birthday party there are purple, red and blue balloons in the ratio 3:8:11.
 - a) What fraction of the balloons are blue?
 - b) What fraction of the balloons are purple?
 - c) What fraction of the balloons aren't red?

You can also express quantities given as fractions in the form of a ratio.

- Work out the fractions of each quantity you're interested in and put them all over a **common denominator**.
- The **numerators** of each quantity are the numbers to go in the ratio.

Tip: The denominator is the total number of parts for the ratio.

Example 5

Of Hannah's DVDs, $\frac{2}{7}$ are horror films and the rest are comedies. Find the ratio of comedies to horror films.

- 1. Find the fraction of films that are comedies by subtracting the fraction of horror films from 1. $\frac{2}{7}$ are horror, so $1 \frac{2}{7} = \frac{5}{7}$ are comedies.
- 2. Write the numerators of the fractions in ratio form, making sure they're in the order you're asked for. So ratio of **comedies to horror** is **5:2**.

- Q1 In a bag of red sweets and green sweets, $\frac{1}{3}$ are red. Find the ratio of red to green sweets.
- Q2 All watched $\frac{3}{10}$ of the episodes in a series. Find the ratio of episodes he watched to ones he didn't.
- Q3 $\frac{1}{8}$ of some counters are red, $\frac{5}{8}$ are blue and the rest are green. Find the ratio of red to blue to green.
- Q4 Last season, a rugby team won half of their matches by more than 10 points and a quarter of their matches by less than 10 points. They lost the rest. Find the team's ratio of wins to losses.
- Q5 A cycling challenge has three routes A, B and C. $\frac{7}{10}$ of the competitors choose route A, $\frac{1}{5}$ choose route B and the rest choose route C.

 What is the ratio of competitors choosing route A to those choosing route C?
- Q6 In a bowl of salad there are $\frac{5}{2}$ as many tomatoes as there are lettuce leaves.
 - a) Write this as a ratio of tomatoes to lettuce leaves. Give your ratio in its simplest form.
 - b) There are 45 tomatoes in the salad bowl. How many lettuce leaves are there?
- One third of the counters in a box are red, one fifth are white and the rest are beige. Find the ratio of red to beige counters.

4.2 Using Ratios

You should be feeling more confident with ratios by now, so it's time to see how they can be used to solve different types of problems. They're not too tricky as long as you understand what a given ratio tells you — so if you're not sure, have a look back at the earlier pages in this section.

Part: Whole Ratios

Learning Objective — Spec Ref R5:

Be able to change between part: part and part: whole ratios.

A **part:part** ratio a:b contains a lot more information than it appears to at first. For example, you can use it to write **fractions** of each quantity out of the total (a + b), as you saw on page 44. Similarly, you can form **part:whole** ratios of each quantity to the total — a:a+b and b:a+b.

To turn a **part:whole** ratio back into a **part:part** ratio, you **subtract** the known 'part' from the 'whole' to find the **unknown part**. E.g. if the part:whole ratio is 3:9 then the unknown part is 9-3=6, and the part ratio is 3:6.

Example 1

A shop only sells green and brown garden sheds. The ratio of green sheds sold to the total number of sheds sold is 5:7. What is the ratio of green sheds sold to brown sheds sold?

1. Write down the ratio you know, and the one you need to know.

The total (green + brown) is 7, so subtract to find the number of brown sheds for every 5 green.

3. Write this in ratio form, and simplify if necessary.

green:total green:brown 5:7 5:?

7 - 5 = 2 brown for every 5 green

green: brown = 5:2

- Q1 A reptile house only contains snakes and lizards. The ratio of snakes to the total number of reptiles is 3:8.
 - a) What fraction of the reptiles are snakes?
- b) What is the ratio of snakes to lizards?
- Q2 A biscuit tin only contains digestives and bourbons. The ratio of digestives to the total number of biscuits is 4:7. What is the ratio of digestives to bourbons?
- Q3 When Dan makes a cup of tea, the ratio of water to milk is 15:2. What is the ratio of water to the total amount of liquid in the cup?
- Q4 A rock album is sold as a digital download and on a CD. The ratio of digital downloads to CDs sold is 53:46. What is the ratio of CDs sold to the total number of albums sold?
- Q5 Ashley and Cameron have a combined age of 20. Cameron is 10 years older than Ashley.



- a) Find the ratio of Ashley's age to Cameron's age in its simplest form.
- b) Find the ratio of Cameron's age to their combined age in its simplest form.

Scaling Up Ratios

Learning Objectives — Spec Ref R5/R8:

- Be able to scale up ratios to find unknown amounts.
- Be able to connect ratios to linear graphs and use them for scaling.



You've seen how all the numbers in a ratio can be divided by the same number to get a **simplified** ratio. You can also **multiply** all the numbers in a ratio by the **same number** to get a scaled up **equivalent ratio**, which can then be used to solve problems. This is similar to finding equivalent fractions — see page 27.

Example 2

The ratio of men to women in an office is 3:4. If there are 9 men in the office, how many women are there?

- 1. Write down the ratio as it is, and what you need it to be. Here you need the 'men' (left-hand) side to be 9.
- 2. Work out what you have to multiply the left-hand side by to get 9.
- 3. Multiply the right-hand side by the same number.
- 4. Use your ratio to answer the question.

Ratio of men: women is 3:4 But you need 9:?

$$9 \div 3 = 3$$

So you need to multiply by 3.
$$\times 3 \left(\begin{array}{c} 3:4 \\ 9:12 \end{array} \right) \times 3$$

So there are 12 women.

If two variables are in a constant ratio, they have a linear relationship. If you plot a graph of one variable against the other, you'll get a straight line that goes through the origin. To represent a ratio on a graph, follow these steps:

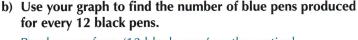
- If the ratio between two variables is **a**:**b**, plot the **coordinate** (**a**, **b**), where the first variable is along the horizontal axis, and the second variable is up the vertical axis.
- Draw a **straight line** through the **origin** and **(a, b)**.

For any value of one variable, you can now **read off the graph** to get the value of the other variable. Every **coordinate** on the graph can be written as a **scaled ratio** equivalent to a:b.

Example 3

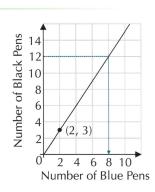
A factory produces blue pens and black pens in the ratio 2:3.

- a) Draw a graph to show this relationship.
 - 1. Draw a set of axes with 'number of blue pens' on the horizontal axis, and 'number of black pens' on the vertical axis.
 - 2. For every 2 blue pens, there are 3 black pens, so draw a straight line through the origin and the point (2, 3).



Read across from '12 black pens' on the vertical axis until you hit the graph, then read down to the horizontal axis to find the number of blue pens.

So there are **8 blue pens** for every 12 black pens (the ratio of blue pens: black pens = 8:12).



Tip: The two variables

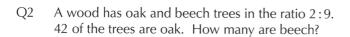
are in direct proportion

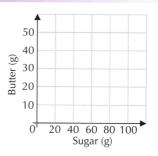
- there's more about

this on page 53.

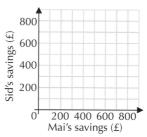
Exercise 2

- Q1 A recipe uses sugar and butter in the ratio 2:1.
 - a) How much butter is needed for 100 g of sugar?
 - b) Show the ratio as a graph on the axes on the right.
 - c) Use the graph to find the amount of sugar needed for 40 g of butter.





- Q3 The ages of a father and son are in the ratio 8:3. If the father is 48, how old is the son?
- Q4 A cardboard cut-out of a footballer is 166 cm tall. The height of the cut-out and the footballer's actual height are in the ratio 5:6. How tall is the footballer?
- Q5 Mai and Sid split their savings in the ratio 5:4.
 - a) Show this ratio on the axes on the right.
 - b) Use the graph to find how much Sid gets when Mai gets £400.
 - c) Sid gets £60. How much do they get in total?



- Q6 A fruit punch is made by mixing pineapple juice, orange juice, and lemonade in the ratio 1:3:6. If 500 ml of pineapple juice is used, how much orange juice is needed?
- Q7 Jem orders a pizza with olives, slices of courgette and slices of goat's cheese in the ratio 8:3:4. How many slices of courgette and how many slices of goat's cheese would she get with 24 olives?
- Q8 Mo, Liz and Dee's heights are in the ratio 32:33:37. Mo is 144 cm tall. What is the combined height of the three people?
- Q9 Max, Molly and Maisie are at a bus stop. The number of minutes they have waited can be represented by the ratio 3:7:2. Molly has been waiting for 1 hour and 10 minutes. How long have Max and Maisie been waiting?
- Q10 The ratio of children to adults in a swimming pool must be 5:1 or less.
 - a) Draw a graph to show this ratio.
 - b) If there are 18 children, use your graph to work out how many adults there must be.
- A TV-show producer is selecting a studio audience.

 He wants the ratio of under-30s to over-30s to be at least 8:1.

 If 100 under-30s are selected, find the maximum number of over-30s.

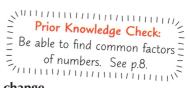


Olga is allowed no more than 5 minutes of reality TV for every minute of news programmes she watches. She watches 45 minutes of reality TV. What's the least amount of news she should watch?

Changing Between Ratios

Learning Objective — Spec Ref R5:

Be able to change ratios when one or more quantities change.



If two or more quantities are in a given ratio and some of the quantities **change** then the **ratio** will change too.

Example 4

Ashraf collects coins. He has 45 gold coins, and the ratio of gold coins to silver coins is 9:13. If he sells 15 silver coins, what will be the new ratio of gold coins to silver coins? Give your ratio in its simplest form.



- 1. First find the original number of silver coins. Write down the ratio as it is, and what it needs to be. Here you need the 'gold' (left-hand) side to be 45.
- 2. Work out what you have to multiply by to get 45.
- 3. Multiply the right-hand side by the same number to get the original number of silver coins.
- 4. Work out the remaining number of silver coins by subtracting the number sold, 15.
- 5. Write out the new amounts in ratio form.
- 6. Simplify your answer by dividing by the HCF, 5.

Ratio of gold:silver is 9:13 But you need 45:?

$$45 \div 9 = 5$$
.

So you need to multiply by 5.

$$\times 5$$
 $\left(\begin{array}{c}9:13\\45:65\end{array}\right) \times 5$ So there were 65 silver coins.

65 - 15 = 50 silver coins remaining

So new ratio gold: silver is 45:50

This simplifies to:



Exercise 3

For the following questions, give each ratio in its simplest form.

- Q1 In a bakery, the ratio of fruit scones to cheese scones is 3:4.
 - a) Given that there are 24 cheese scones, how many fruit scones are there?
 - b) If 6 fruit scones were sold, what would be the new ratio of fruit scones to cheese scones?
- Q2 An aquarium contains fish and crabs. The ratio of fish to crabs is 4:1. There are 16 fish in the aquarium. If 2 more crabs were added to the aquarium, what would be the new ratio of fish to crabs?
- Q3 A village has detached and terraced houses in the ratio 5:7. There are 56 terraced houses. If 12 new detached houses were built, what would be the new ratio of detached houses to terraced houses?
- Q4 At the beginning of the week, a bookshop which sells school books has 48 English books in stock and the ratio of English to Maths books is 2:5. At the end of the week, the ratio of English to Maths books is 3:4. Given that the shop sold 12 English books during the week, how many Maths books did it sell?
- A bag contains black and white tokens in a ratio of 3:2. 8 black tokens and 2 white tokens are added to the bag. The ratio of black to white tokens is now 2:1. How many white tokens were in the bag to begin with?

4.3 Dividing in a Given Ratio

Ratios can be used to share things out unequally. When dividing an amount in a given ratio, one side of the ratio gets a greater share than the other(s). You can use part:part and part:whole ratios for sharing, but you might have to come up with the ratio yourself from information about how things are related.

Learning Objectives — Spec Ref R5/R6:

- Divide quantities in a given part:part or part:whole ratio.
- Be able to write a ratio given a multiplicative relationship between two quantities.

Part: part ratios can be used to divide an amount into two or more **shares**. The numbers in the ratio show how many parts of the whole each share gets. To share an amount in a given ratio:

- Add up all the quantities in the ratio to get the total number of parts, i.e. a + b, or a + b + c.
- Divide the amount to be shared by the total number of parts to get the amount for one part.
- Multiply the amount for one part by the number of parts for each share.

Example 1

Divide £54 in the ratio 4:5.

1. Find the total number of parts.

2. Work out the amount for one part.

3. Multiply by the number of parts for

each share, i.e. the numbers in the ratio.

4. Check that the shares add up

to the initial amount.

4 + 5 = 9 parts altogether

9 parts = £54.

So 1 part = £54 \div 9 = £6

The ratio is 4:5.

 $£6 \times 4 = £24$ $£6 \times 5 = £30$

The shares are £24 and £30.

 $(£24 + £30 = £54 \checkmark)$

Tip: Using fractions, the shares are $\frac{4}{9}$ and $\frac{5}{9}$. So $\frac{4}{9} \times £54 = £24$, and $\frac{5}{9} \times £54 = £30$.

Exercise 1

- Q1 Divide £48 in the following ratios.
 - a) 2:1
- b) 1:3

c) 5:1

d) 7:5

- Q2 Divide 72 cm in the following ratios.
 - a) 2:3:1
- b) 2:2:5
- c) 5:3:4
- d) 7:6:5

- Q3 Share £150 in these ratios.
 - a) 1:4:5
- b) 15:5:30
- c) 6:7:2
- d) 13:11:6
- Q4 Find the smallest share when each amount below is divided in the given ratio.
 - a) £22 in the ratio 5:6

b) 450 g in the ratio 22:28

c) 45 kg in the ratio 2:3:4

- d) 1800 ml in the ratio 5:6:7
- Q5 Find the largest share when each amount below is divided in the given ratio.
 - a) £30 in the ratio 1:3

b) 36 g in the ratio 3:2

c) 150 kg in the ratio 10:7:3

d) 24 000 ml in the ratio 5:7:20

'Dividing into ratios' questions are often set in a **real-life context**, but you can still use the usual method if you're given a **part:part** ratio. If you're given a **part:whole** ratio, you can just **divide** the amount by the total number of parts (the whole) and **multiply** it by the number of parts in the share you're trying to find.

Example 2

A box of 45 chocolates contains dark and milk chocolates in the ratio 2:7. How many of the chocolates are milk chocolate?

- 1. To find the number of chocolates that are milk chocolates, you need to divide 45 into the ratio 2:7.
- 2. First find the total number of parts...
- 2 + 7 = 9 parts altogether
- 3. ... then divide to work out the amount for one part.
- 9 parts = 45 chocolates. 1 part = $45 \div 9 = 5$ chocolates
- 4. Milk chocolates make up 7 parts, so multiply the amount for one part by 7.
- 5 chocolates × 7 = 35 milk chocolates

Tip: Using fractions, $\frac{7}{9}$ of the chocolates are milk chocolates, so find $\frac{7}{9}$ of 45 = 35.

- Q1 Share 32 sandwiches in the ratio 3:5.
- Q2 Kat and Lindsay share 30 cupcakes in the ratio 3:2. How many do they each get?
- Q3 There are 112 dogs at a show. For every 14 dogs at the show, 9 are male. How many male and how many female dogs are at the show?
- Q4 Lauren is 16 and Cara is 14. Their grandad gives them £1200 to share in the ratio of their ages. How much money do they each get?
- Q5 Tangerine paint is made from yellow and red paint. In a 42 litre batch of tangerine paint, the ratio of red paint to tangerine paint is 3:7. How many litres of yellow paint were used?
- Q6 A quiz team share their £150 prize money in the ratio 2:4:6. What is the least amount that any team member receives?
- Q7 a) The angles in a triangle are in the ratio 2:1:3. Find the sizes of the three angles.
 - b) The four angles in a quadrilateral are in the ratio 1:5:2:4. Calculate the sizes of all four angles.
- Q8 A rope is cut into three pieces in the ratio 2:1:4. The third piece is 22 cm longer than the first piece. What is the length of the second piece?



- The length and width of a rectangle are in the ratio 5:1. If the perimeter of the rectangle is 72 cm, calculate the length and the width of the rectangle.
- Andrew, Pierre and Susanne are making tomato sauce.
 They share out tomatoes in the ratio 2:3:5. Susanne gets 16 more tomatoes than Pierre. How many tomatoes do they have in total?



Ali and Max earn £200 one weekend. They share it according to the hours they worked. Ali worked 10 hours and Max worked 6 hours. Ali had help from her brother Tim, so she splits her share with him in the ratio 4:1. How much do Ali, Max and Tim each get?



Dan, Stan and Jan are aged 8, 12 and 16. They get pocket money each week in the ratio of their ages. Dan normally gets £6, but has been naughty, so this week all the money is split between Stan and Jan, in the ratio of their ages, to the nearest penny. How much do they each receive this week?



You might be given the **multiplicative relationship** between two or more variables instead of a ratio. To divide an amount into shares using this relationship, you need to write it as a **ratio** first. E.g. if there are **five times** as many sparrows as pigeons then the ratio of sparrows to pigeons is 5:1.

Example 3

Mika is twice as old as Nate, and Nate's age is one third of Orla's age. They have a combined age of 72. Find each person's age.

- 1. You need to divide 72 in the ratio of their ages, so first find this ratio from what you're told.
- 2. For 1 year of Nate's age, Mika has 2. For 1 year of Orla's age, Nate has $\frac{1}{3}$.
- 3. Multiply the second ratio by 3 so the parts for Nate are the same in both ratios (1).
- 4. Now you can combine them into a single ratio.
- 5. As usual, find the total number of parts then divide to work out the amount for one part.
- 6. Finally, multiply by the number of parts for each share.
- 7. Check the answer works with the given information.

Mika: Nate = 2:1
Nate: Orla =
$$\frac{1}{3}$$
:1
×3

Mika: Nate: Orla = 2:1:3

$$2 + 1 + 3 = 6$$
 parts altogether 6 parts = 72. So 1 part = $72 \div 6 = 12$

Mika is $2 \times 12 = 24$, Nate is $1 \times 12 = 12$, and Orla is $3 \times 12 = 36$.

(24 is twice 12, 12 is one third of 36 **✓**)



- Q1 In a school of 600 pupils, there are seven times as many right-handed pupils as left-handed pupils. How many pupils are right-handed?
- Q2 Daniel puts one-and-a-half times as much money into a business as his partner Elsie. They share the profits in that same ratio. How much of a £5700 profit does Daniel get?
- Q3 There are 28 passengers on a bus. There are two-and-a-half times as many passengers on the phone as not on the phone. How many passengers are on the phone?
- Q4 A fruit salad has twice the mass of raspberries as redcurrants, and three times the mass of strawberries as redcurrants. How much of each fruit is needed to make 450 g of fruit salad?
- Q5 Nicky, Jacinta and Charlie share a bag of 35 sweets so that Nicky gets half as much as Jacinta and Charlie gets half as much as Nicky. How many sweets do each of them get?

4.4 Proportion

Proportion tells you how one thing changes in relation to another — either in direct or inverse proportion.

Direct Proportion

Learning Objective — Spec Ref R7/R10:

Understand and use direct proportion.

If the ratio between two things (variables) is always the same, then they're in direct proportion. For example, the **diameter** and **circumference** of a circle are always in the ratio 1: π , so circumference is **directly proportional** to diameter. If two variables are in direct proportion then increasing one of the variables increases the other by the same scale factor, e.g. doubling one thing doubles the other. You can use the **unitary method** to solve simple problems where two things are in direct proportion:

- **Divide** to find the amount of one variable for every **one** of the other (e.g. the price per item).
- Then **multiply** to find the scaled amount.

Example 1

If 8 chocolate bars cost £6, calculate the cost of 10 chocolate bars.

Divide by 8 (the number of bars) to find the cost of one bar.

 $£6 \div 8 = £0.75$

Multiply by 10 (the new number of bars) to find the cost of 10 bars.

 $£0.75 \times 10 = £7.50$

Example 2

A bus travels 50 km in 40 minutes. Assuming the bus travels at the same average speed:

a) Calculate the time it would take to travel 65 km.

1. Divide by 50 to find the time taken for 1 km.

1 km takes $40 \div 50 = 0.8$ minutes

2. Multiply by 65 to find the time taken for 65 km. 65 km takes $0.8 \times 65 = 52$ minutes

b) Calculate the distance the bus would travel in 16 minutes.

1. This time, divide by 40 to find the distance travelled in 1 minute.

In 1 minute it travels $50 \div 40 = 1.25 \text{ km}$

2. Multiply by 16 to find the distance travelled in 16 minutes.

In 16 minutes it travels $1.25 \times 16 = 20 \text{ km}$

Tip: In a) you found the number of minutes per km, but in b) you need the km per minute, which is a measure of the average speed.

- Q1 The cost of 8 identical books is £36. What is the cost of 12 of these books?
- If it takes 1.8 kg of flour to make 3 loaves of bread, how much flour is needed to make 5 loaves? Q2
- Q3 5 litres of water make 4 jugs of squash. How much water is needed to make 3 jugs of squash?
- Q4 Ryan earns £192 for cleaning 12 cars. How much more will he earn if he cleans another 5 cars?
- Q5 If 7 of the same DVDs cost £84, how many DVDs can be bought for £48?

- Q6 A car uses 35 litres of petrol to travel 250 km. Assuming its fuel consumption stays constant:
 - a) Calculate how far, to the nearest km, the car can travel on 50 litres of petrol.
 - b) Calculate how many litres of petrol the car would use to travel 400 km.
- Q7 A 58 g bar of chocolate contains 11 g of fat.
 - a) To the nearest gram, how much fat is this per 100 g?
 - b) If my diet only allows me to eat 5 g of fat, how many grams (to 1 d.p.) of the bar can I eat?

Exchange rates and **conversions** are also direct proportion problems, so you can use a similar method.

Example 3

Oliver has 30 euros (€) left over from a holiday in France. Assuming the exchange rate is £1 = €1.14, how many pounds can be exchange his euros for?

- 1. Divide £1 by 1.14 to find the number of pounds per €1. $€1 = £1 \div 1.14 = £0.877...$
- 2. Multiply by the number of euros, 30, and round to 2 d.p. $\in 30 = 30 \times £0.877... = £26.32$ (2 d.p.)

Exercise 2

- Q1 If £1 is worth €1.14, convert the following into pounds: a) €10 b) €100 c) €250
- Q2 1 kg \approx 2.2 pounds (lbs). Calculate the approximate mass of 8.25 lbs of sugar in kg.
- Q3 Use the exchange rate £100 = 1055 Chinese yuan to convert 65 Chinese yuan into pounds.
- Q4 Philip changed £50 into Swiss francs. The exchange rate was £1 = 1.47 Swiss francs.
 - a) How many Swiss francs did he get?

The next week, he changed 30 Swiss francs into pounds with exchange rate £1 = 1.50 Swiss francs.

b) How many pounds did he get back?

When **two variables** are changing, split the calculation up into **stages** and change one variable at a time. Then use the **unitary method** on **each stage**.

Example 4

Two people can build four identical walls in three days. At this rate, how many of these walls could ten people build in 3.75 days?

- 1. First change the number of people. Find how many walls 1 person, then 10 people, can build in 3 days, by dividing by 2 and multiplying by 10.
- 2. Then change the number of days. Find how many walls 10 people can build in 1 day, then 3.75 days, by dividing by 3 then multiplying by 3.75.

In 3 days, 2 people can build 4 walls, 1 person can build $4 \div 2 = 2$ walls, and 10 people can build $2 \times 10 = 20$ walls.

10 people can build 20 walls in 3 days. They can build $20 \div 3 = 6.66...$ walls in 1 day, and $6.66... \times 3.75 = 25$ walls in 3.75 days.

Exercise 3

- Q1 It takes three people five minutes to eat six hotdogs.
 At this rate, how many hotdogs could five people eat in ten minutes?
- Q2 Ten people can count 850 coins in two minutes. At this rate, how many coins could 22 people count in five minutes?
- Q3 It takes two people 90 minutes to plant 66 flower bulbs. At this rate, how many bulbs could three people plant in three hours?
- Q4 Two toy makers can make eight toy soldiers in two weeks. How many soldiers could seven toy makers make in three weeks if they work at the same rate?

Inverse Proportion

Learning Objective — Spec Ref R7/R10: Understand and use inverse proportion.

If one variable decreases as another increases, then they're **inversely proportional**. E.g. **increasing** your speed **decreases** the time it will take to finish your journey, so **speed** and **time** are inversely proportional.

With inverse proportion problems, think about whether the variable should be **increasing** (so you need to **multiply** by something) or **decreasing** (so you need to **divide**). Then check that your answer **makes sense**.

Example 5

Four people take five and a quarter hours to dig a hole. How long would it take seven people to dig the same sized hole at the same rate?

- 1. Work out how long 1 person will take. The fewer people there are, the longer it will take, so you need to multiply the time by 4.
- 2. 7 people will dig the hole in a shorter time than 1 person, so divide the time by 7.

4 people take 5.25 hours 1 person will take: $5.25 \times 4 = 21$ hours

7 people will take: $21 \div 7 = 3$ hours

Tip: Check it makes sense: 7 people should take less time than 4, and 3 hours < 5.25 hours ✔

- Q1 It takes three people two hours to paint a wall. How long would it take five people to paint the same wall at the same rate?
- Q2 It takes four teachers two and a half hours to mark a set of test papers. How long would it take six teachers to mark the same set of papers at the same rate?
- Q3 Four waiters can set all the tables in a restaurant in twenty minutes. They hire an extra waiter. How long will it take all five of them to set all of the tables, assuming they all work at the same rate?

- Q4 A journey takes two and a quarter hours when travelling at an average speed of 30 mph. How long would the same journey take when travelling at an average speed of 45 mph?
- Q5 It will take five builders sixty-two days to complete a particular project.
 - a) At this rate, how long would the project take if there were only two builders?
 - b) If the project needed completing in under forty days, what is the minimum number of builders that would be required?



As on page 54, when **two variables change**, split the calculation up into **stages** and change one variable at a time. Decide if the amount should go **up** or **down** at each stage of the calculation.

Example 6

It takes 5 examiners 4 hours to mark 125 exam papers. How long would it take 8 examiners to mark 200 papers at the same rate?

- 1. First change the number of examiners. Find how long it will take 1 examiner, then 8 examiners, to mark 125 papers, by multiplying by 5 and dividing by 8.
- 2. Then change the number of papers. Find how long it will take 8 examiners to mark 1 paper, then 200 papers, by dividing by 125 and multiplying by 200.
- 5 examiners take 4 hours to mark 125 papers. So 1 examiner will take $4 \times 5 = 20$ hours, and 8 examiners will take $20 \div 8 = 2.5$ hours
- 8 examiners take 2.5 hours to mark 125 papers. So they take $2.5 \div 125 = 0.02$ hours to mark 1 paper, and $0.02 \times 200 = 4$ hours to mark 200 papers.

- Q1 It takes 45 minutes for two people to clean six identical rooms. How long would it take five people to clean 20 of the same rooms at the same rate?
- Q2 It takes two electricians nine days to rewire two identical houses. How long would it take three electricians to rewire three of the same houses at the same rate?
- Q3 It takes four bakers 144 minutes to ice 72 cakes. How long would it six bakers to ice 96 of these cakes?
- Q4 Three people take twenty minutes to shovel snow from two identical driveways.

 At this rate, how long would it take two people to shovel snow from five of the same driveways?
- Q5 Fourteen people can paint 35 identical plates in two hours. At this rate, how many people would be needed to paint 60 of the same plates in two and a half hours?
- It will take twelve workers three weeks to complete four stages of a ten-stage project. Each stage of the project takes the same number of hours of work to complete.
 - a) At this rate, how long would it take fifteen workers to complete the whole project?
 - b) The project needs completing in under four weeks.

 What is the minimum number of workers that would be required, given that they all work at the same rate as the original workers?



Review Exercise

- Q1 A builder mixes 10 kg of cement with 25 kg of sand. Give the ratio of cement to sand:
 - a) in its simplest form
- b) in the form 1:*n*
- c) in the form n:1
- d) Find the fraction of the mix that is cement, in its simplest form.
- Q2 A picture has a width of 120 mm. An enlargement of the picture has a width of 35 cm. Write the ratio of the original width to the enlarged width in its simplest form.
- Q3 Alf has $\frac{2}{3}$ as many nails as screws.
 - a) Write the ratio of nails to screws in its simplest form.
 - b) Write the ratio of the number of screws to the total number of nails and screws.
- **Q4** Evie is making green cards and purple cards in the ratio 7:3. She has made 12 green cards so far and has 72 more green cards to make. How many purple cards will she need to make?
- Q5 A farm has sheep, cows and pigs in the ratio 8:1:3. If there are 16 sheep on the farm, how many animals are there altogether?
- On a normal day, 20 planes depart from an airport to go to Spain and Finland in the ratio 4:1. One day, 2 additional flights went to Spain, and the ratio of Spanish to Finnish flights was 3:1. How many additional flights departed for Finland on this day?
- Q7 Luiz, Seth and Fran share £1440 in the ratio 3:5:7. Fran splits her share with Ali in the ratio 4:1.
 - a) How much does each person end up with?
 - b) Find the ratio of Fran's amount to Seth's amount in its simplest form.
- Q8 The only ingredients of a particular cereal are raisins, nuts and oats. The mass of raisins is three times the mass of nuts, and the mass of oats is half the mass of raisins. How many grams of nuts, to the nearest gram, are needed to make 450 g of the cereal?
- **Q9** It costs £20 to put 12.5 litres of petrol in my car.
 - a) How much will it cost me for a full tank of petrol if the tank holds 60 litres?
 - b) How much petrol can I put in my car for £30?
- **Q10** Emma changed £500 into rand before going on holiday, at an exchange rate of £1 = 20.8 rand.
 - a) How many rand did she get for her £500?

Emma spent 8000 rand on holiday. When she got home, she changed her leftover rand into pounds at the new exchange rate of £1 = 19.6 rand.

- b) How much money did she get back in pounds?
- Q11 Eight lumberjacks can chop down three identical trees in an hour and a half.
 - a) At this rate, how many trees could eight lumberjacks chop down in six hours?
 - b) How long would it take eight lumberjacks to chop down eight trees at the same rate?
 - c) At this same rate, how many trees could six lumberjacks chop down in an eight-hour day?

Exam-Style Questions

Q1 Write the ratio 0.75:1.2 in the form 1:n where n is a decimal.



[2 marks]

Q2 The ratio of female to male teachers at a school is 7:2. There are 40 more female teachers than male teachers.

Work out how many teachers there are in total at the school.

[2 marks]

Q3 At a wildlife park there are badgers, foxes and hares.
There are three times as many badgers as there are foxes.
For every two badgers there are five hares.



Work out the ratio of the number of badgers to the number of foxes to the number of hares. Give your ratio in its simplest form, using integers.

12 marks1

- **Q4** On the right is part of a recipe for cupcakes.
 - a) How much butter is needed to make 10 cupcakes?

[2 marks]

b) How much flour is needed to make 35 cupcakes?

Makes 25 Cupcakes:

200 g butter 250 g caster sugar 280 g flour 4 eggs

[2 marks]

Q5 A team of eight people can deliver the morning newspapers in 20 minutes. One morning, one of the team is off sick.

How long will it take the rest of them to deliver all the newspapers at their usual rate? Assume each person delivers newspapers at the same rate. Give your answer to the nearest second.

[3 marks]

Q6 A brand of apple juice has a sugar content of 12 grams per 100 millilitres. A brand of lemonade has a sugar content of 2.5 grams per 100 millilitres.

Work out the ratio in which this apple juice and lemonade should be mixed to produce a drink with a sugar content of 6 grams per 100 millilitres.

[4 marks]