Mixed Exam-Style Questions

PROBLEM SOLVING

Q1 A brand of breakfast cereal is sold in two different sizes. The 'family' size box contains 750 g. The 'standard' box normally contains 500 g, but as a special offer, these boxes currently contain an extra 25% of cereal. The 'family' box costs £2.25. The special offer 'standard' boxes cost £2.

Work out which size of box is better value for money.
You must show your working.

[4 marks]

Q2 The ratio of boys to girls in a primary school class of 28 children is 3 : 4. Half of the boys and 75% of the girls eat school dinners. The rest eat packed lunches.

Show this information as a frequency tree.

[4 marks]

Q3 Jemima is looking to buy a new tablet computer on the internet. A UK seller offers the model she wants with free delivery. She also sees the same model available from an American seller priced at 432 US dollars ($). Delivery from America will cost $30 and there will also be a 20% import tax on both the cost of the computer and the delivery charge.
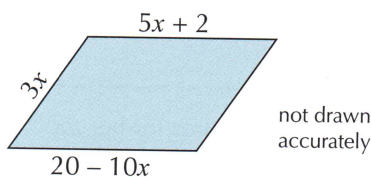
The exchange rate is $1 = £0.75.

The price of the computer from the UK seller is £415.
Which seller should Jemima buy the computer from?

[3 marks]

Q4 The diagram below shows a parallelogram with the lengths of three of its sides given in centimetres in terms of x:

$5x + 2$

$3x$

not drawn accurately

$20 - 10x$

Show that the perimeter of the parallelogram is 23.2 cm.

[4 marks]

Q5 The flag of the Asian country Laos is a rectangle with a width to height ratio of 3 : 2. It consists of three horizontal stripes. The top and bottom stripes are the same height and are half the height of the middle stripe.

In the centre is a circle which has a diameter of $\frac{4}{5}$ of the height of the middle stripe.
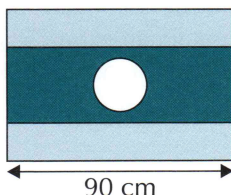
not drawn to scale

90 cm

Work out the area of this circle for a flag of width 90 cm.
Give your answer in cm² as a multiple of π.

[4 marks]

Q6 Andrea has a small city car and Zac has a large sports car.
The efficiency of any car can be measured using the formula:

$$\text{efficiency} = \frac{\text{distance travelled (km)}}{\text{volume of fuel used (litres)}}$$

On a particular trip, Andrea's car travels 81.9 km and uses 2.6 litres of fuel.
The efficiency of Zac's car is two thirds of the efficiency of Andrea's car.

Zac plans to drive a distance of 64.7 km on his next trip. Work out how much fuel he should expect to use on this trip, giving your answer in litres to 2 significant figures.

[3 marks]

Q7 Erica is deciding between having a mobile phone contract or choosing 'pay as you go'.
She sees two offers from the companies FF and H2:

FF — contract

- £51.99 per month
- 4GB of data free per month then £4.99 per GB
- 5 hours of free calls per month then 7p per minute
- Unlimited texts

H2 — Pay As You Go

Great price on data

Calls cost 8p per minute

Texts costs 4p each

Erica makes the decision based on the figures for her mobile phone use last month when she used 5 gigabytes (GB) of data, made 7 hours of calls and sent 60 texts.

Work out the maximum price that H2 could charge per GB of data that would make H2 the better deal. You must show your working.

[5 marks]

Q8 Three interior angles of a pentagon are right angles.
The other two angles are in the ratio $4:11$.

Work out the size of the largest interior angle in the pentagon.

[4 marks]

Q9 A metal shipping container is in the shape of a cuboid with six flat faces.
It has a length of 6 m, a width of 2.6 m and a height of 2.4 m.
The face *ABCD* is on the ground, as shown.

6 m

not drawn to scale

D   *C*

2.4 m

A   *B*
2.6 m
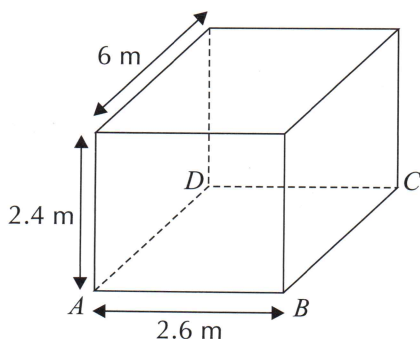
A worker is asked to paint the other five faces to protect the container from rust.
She will apply 0.1 litres of paint per square metre.
She is going to paint the four vertical faces first and the top face last.

If she only has 5 litres of paint, work out, to the nearest whole number,
the percentage of the top face she will be able to paint.

[4 marks]

Q10 A teacher randomly chooses three pupils from her class
to take part in a school debating competition.

The teacher chooses one boy and two girls.
There are n boys in the class.
The number of girls in the class is two fewer than the number of boys.

Show that the number of different teams the teacher can
choose is given by the expression $n^3 - 5n^2 + 6n$.

[3 marks]

Q11 The line segment *AB* is such that *A* and *B* have coordinates $(-1, 6)$ and $(3, 4)$ respectively.
Find the equation of the line perpendicular to *AB* which goes through the midpoint of *AB*.

[4 marks]

Q12 Weedkiller is applied to a rectangular lawn at a rate of 20 ml/m². The length of the lawn is four times its width.

If 8 litres of weedkiller is used in total, calculate the length of the lawn, in metres.

[4 marks]

Q13 When Roger plays tennis against Stan on a grass court, the ratio of Roger's chances of winning a set to Stan's is 5:2. When they play on a clay court, this ratio is 4:5. They play one set on a grass court and one set on a clay court. The events of Roger or Stan winning or losing either set are assumed to be independent.
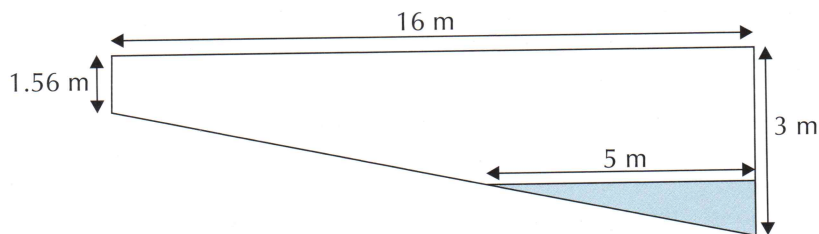
a) Explain what is meant by 'independent' events in probability.

[1 mark]

b) Find the probability that Roger and Stan win one set each.

[4 marks]

Q14 The diagram shows the cross-section of a swimming pool which is in the shape of a trapezium.

not drawn to scale

The shallow end has a depth of 1.56 m and the deep end has a depth of 3 m. The pool is 16 m long. The pool is being filled and the length of the surface of the water is currently 5 m.

Work out how far the surface of the water is from the top of the pool.

[4 marks]

Q15 Solve the equation: $9^{4x} = 3^{3x+4}$

[3 marks]

Q16 A bag contains 40 sweets. 18 are red and the rest are yellow.

Danesh only likes the red sweets. He eats 9 of the red sweets from the bag and then is given 5 more yellow sweets from a friend. He puts these in the bag.

By how much has the percentage of yellow sweets in the bag increased?

[3 marks]

Q17 For the sequence of triangular numbers, the *n*th term is given by:

$$u_n = \frac{n(n+1)}{2}, \qquad n > 0.$$

Use this to prove that the sum of two consecutive triangular numbers is a square number.
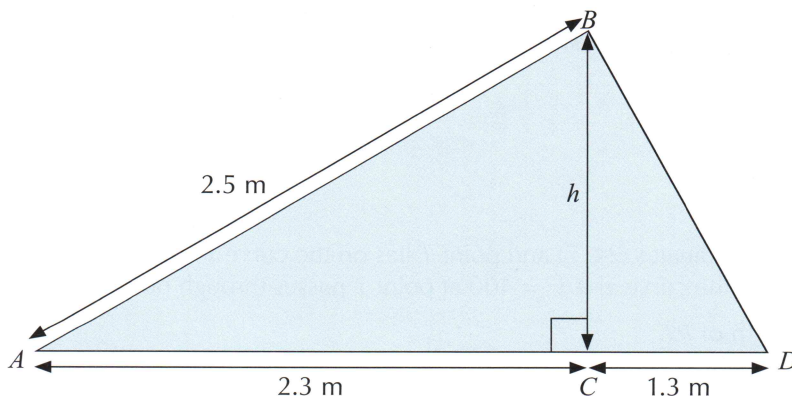
[3 marks]

Q18 The diagram below shows the cross-section of a garage roof, where *ABD* is a triangle.
The lengths *AB* = 2.5 m, *AC* = 2.3 m, *CD* = 1.3 m are marked on the diagram.
All lengths are accurate to 1 decimal place.

a) Show that the upper bound for the perpendicular height *h* of the roof is 1.2 m.

[3 marks]

b) Hence show that the upper bound for the angle *BDC* is 43.8° to 1 decimal place.
You may use the fact that tan *x* increases as *x* increases.
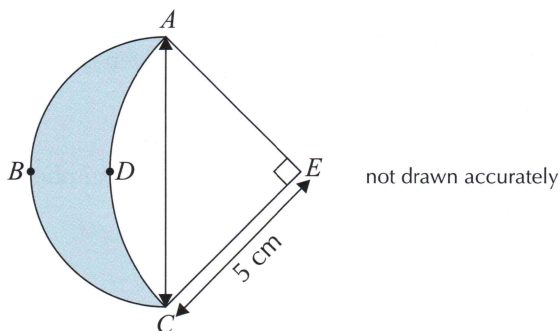
[3 marks]

Q19 Eight teams are in a draw for the quarter-finals of a football competition and x of the teams are British. The first two teams are randomly selected. The probability that neither team is British is $\frac{5}{14}$.

Show that $x^2 - 15x + 36 = 0$.

[4 marks]

Q20 A costume jeweller is designing a pendant, as shown in the diagram.

not drawn accurately

ABCA is a semicircle. *ADCEA* is a sector of a circle of radius 5 cm with centre E and angle $AEC = 90°$. The pendant is the area between the arcs *ABC* and *ADC*.

Find the area of the pendant, giving your answer as an improper fraction in its simplest form.

[5 marks]

Q21 Two variables m and h are related by the formula:

$$m^{\frac{1}{4}}h^{-2} = k, \text{ where } k \text{ is a constant.}$$

When $m = 4096$, $h = 2$.

Show that an expression for m in terms of h is $m = ah^8$, where a is a constant to be found.

[4 marks]

Q22 Point P has coordinates $(24, 7)$ and point T lies on the curve $x^2 + y^2 = 400$. The tangent to the curve $x^2 + y^2 = 400$ at point T passes through point P.

Find the length of *PT*.

[5 marks]