35.1 The AND Rule for Independent Events

One of the most useful probability rules is the AND rule — it lets you work out the probability that one event AND another will both happen. However, it only works when the events don't affect each other.

Learning Objectives — Spec Ref P8:
- Know what it means for events to be independent.
- Use the AND rule to find probabilities.

Prior Knowledge Check:
Be able to use set notation.
See Section 19.

Events are **independent** if one of them happening has **no effect** on the probability that the others happen.

For example, imagine drawing **two cards** at random from a standard deck of playing cards. If you **put the card back** and shuffle the deck, then the first card drawn **doesn't affect** the second card drawn, so the events are **independent**. However, if you **don't** put the first card back, then the probabilities for the second draw **depend** on the result of the first draw — e.g. the probability of selecting a heart on the second draw is $\frac{12}{51}$ (if the first card **was** a heart) or $\frac{13}{51}$ (if the first card **wasn't** a heart). So the events are **not** independent.

If A and B are **independent** events, then the **AND rule** says that the probability of **both A and B** happening is equal to the probability of A happening **multiplied** by the probability of B happening.

P(A and B) = P(A) × P(B) or **$P(A \cap B) = P(A) \times P(B)$** in **set notation**.

Example 1

A biased dice has a probability of 0.2 of landing on 6. The dice is rolled twice.
What is the probability that: a) both rolls are a 6? b) neither roll is a 6?

a) The first roll has no effect on the second roll, so the events '6 on first roll' and '6 on second roll' are independent.

P(both rolls are 6) = P(6 on first roll) × P(6 on second roll)
= 0.2 × 0.2 = **0.04**

b) 1. On any roll, either a 6 is rolled or a 6 isn't rolled, so P(6) + P(not 6) = 1.

P(not 6) = 1 – P(6) = 1 – 0.2 = 0.8

2. The events 'not 6 on first roll' and 'not 6 on second roll' are independent.

P(neither roll is 6)
= P(not 6 on first roll) × P(not 6 on second roll)
= 0.8 × 0.8 = **0.64**

The AND rule works for **more than 2 independent events** — e.g. P(A and B and C) = P(A) × P(B) × P(C).

Example 2

Jeff, Britta and Annie go to the same library. On any day, the probability that Jeff goes to the library is 0.6, the probability that Britta goes is 0.4 and the probability that Annie goes is 0.75.

a) In context, describe what assumption needs to be made in order to use the AND rule to find the probability of more than one of Jeff, Britta and Annie visiting library on any given day.

You need to assume that the probability that each person goes to the library on any day is **independent** of whether or not the others go — i.e. they are not more (or less) likely to go if one of the others goes.

b) Find the probability that Annie and Britta both go to the library on any one day.

Use the AND rule: P(Annie and Britta go to the library) = P(Annie goes) × P(Britta goes)
= 0.75 × 0.4 = **0.3**

c) Find the probability that all three go to the library on any one day.

The AND rule works for 3 independent events too.

P(All three go to the library)
= P(Jeff goes) × P(Britta goes) × P(Annie goes) = 0.6 × 0.4 × 0.75 = **0.18**

Exercise 1

Q1 Say whether each of these pairs of events are independent or not.

a) Tossing a coin and getting heads, then tossing the coin again and getting tails.

b) Selecting a coffee-flavoured chocolate at random from a box of assorted chocolates. Then, after eating the first chocolate, randomly selecting another coffee-flavoured chocolate from the same box.

c) Rolling a 6 on a dice and randomly selecting a king from a pack of cards.

Q2 A fair coin is tossed and a fair six-sided dice, numbered 1 to 6, is rolled. Find:

a) P(heads ∩ 6)
b) P(heads ∩ odd number)
c) P(tails ∩ square number)
d) P(tails ∩ prime number)
e) P(heads ∩ multiple of 3)
f) P(tails ∩ factor of 5)

Q3 Prince is doing a survey in his school. He picks pupils at random to do the survey. The probability of picking a left-handed pupil is 10% and the probability of picking a pupil who wears glasses is 15%. Assuming that the hand they write with and glasses wearing are independent, find the probability that a pupil picked at random:

a) is right-handed
b) doesn't wear glasses
c) wears glasses and is left-handed
d) doesn't wear glasses and is right-handed

Q4 A bag contains ten coloured balls. Five of them are red and three of them are blue. A ball is taken from the bag at random, then replaced. A second ball is then selected at random. Find the probability that:

a) both balls are red
b) neither ball is red
c) the first ball is red and the second ball is blue

Q5 The probability of Selvi passing some exams is shown in the table. Assuming that her passing any subject is independent of her passing any of the other subjects, find the probability that she:

Subject	Maths	English	Geography	Science
Probability	0.8	0.6	0.3	0.4

a) passes English and Maths
b) passes Maths and Geography
c) passes Maths and fails Science

Q6 The probability of randomly selecting an ace from a pack of cards is $\frac{4}{52}$. Len claims that the probability of randomly selecting an ace, then randomly selecting another ace (without replacing the first ace) is $\frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$. Say whether you agree with Len and explain why.

35.2 The OR Rule

The OR rule helps you to work out the probability that either one event OR another will happen. How it works depends on whether or not it's possible for both events to happen at the same time.

The OR Rule for Mutually Exclusive Events

Learning Objectives — Spec Ref P4/P8:
- Know what it means for events to be mutually exclusive.
- Use the OR rule to find probabilities for mutually exclusive events.

Two events are **mutually exclusive** if they **can't both happen** at the same time (see p.452) — for example, rolling a 2 and a 3 on the same dice roll, or scoring over 70% and less than 40% on the same test.

The **OR rule** (or **addition rule**) says that if events are **mutually exclusive**, then the probability that **at least** one event happens is the **sum** of the probabilities of each event happening.

P(A or B) = P(A) + P(B) or **P(A ∪ B) = P(A) + P(B)** in **set notation**.

Example 1

A bag contains red, yellow and blue counters. The probabilities of randomly selecting each colour are shown in the table opposite.

Red	Yellow	Blue
0.3	0.5	0.2

Find the probability that a randomly selected counter is red or blue.

The counter can't be both red and blue, so the events 'counter is red' and 'counter is blue' are mutually exclusive.

P(red or blue) = P(red) + P(blue)
= 0.3 + 0.2 = **0.5**

Exercise 1

Q1 A fair spinner has eight sections labelled 1 to 8.
Say whether these pairs of events are mutually exclusive or not.
a) The spinner landing on 6 and the spinner landing on 3.
b) The spinner landing on 2 and the spinner landing on a factor of 6.
c) The spinner landing on a number less than 4 and the spinner landing on a number greater than 3.

Q2 A bag contains some coloured balls. The probability of randomly selecting a pink ball is 0.5, a red ball is 0.4 and an orange ball is 0.1. A ball is picked out at random. Find:
a) P(pink ∪ orange) b) P(pink ∪ red) c) P(red ∪ orange)

Q3 Chocolates in a box are wrapped in four different colours of foil — gold, silver, red and blue. The table shows the probabilities of randomly picking a chocolate wrapped in each colour.
Find the probability of picking a chocolate wrapped in:

Colour	Gold	Silver	Red	Blue
Probability	0.4	0.26	0.14	0.2

a) red or gold foil b) silver or red foil c) gold or blue foil d) silver or gold foil

Q4 On sports day, pupils are split into three equal teams — the Eagles, the Falcons and the Ospreys. What is the probability that a pupil picked at random belongs to:
a) the Eagles? b) the Eagles or the Falcons? c) the Falcons or the Ospreys?

Q5 Jane is told that in a class of 30 pupils, 4 wear glasses and 10 have blonde hair. She says that the probability of a pupil picked at random from the class having blonde hair or wearing glasses is $\frac{10}{30} + \frac{4}{30} = \frac{14}{30}$. PROBLEM SOLVING
Say whether you agree with Jane and explain why.

Q6 The owner of a cafe records the sandwich fillings chosen by customers who buy a sandwich one lunchtime. The table shows the probabilities that a randomly chosen sandwich buyer chose each of five fillings.

Cheese	Tuna	Salad	Pickle	Mayo
0.54	0.5	0.26	0.22	0.28

Customers can choose any combination of fillings, apart from pickle and mayonnaise together.

a) Find the probability that a randomly selected customer chose pickle or mayonnaise.

b) Explain why the probabilities add up to more than 1.

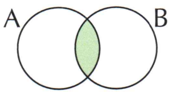
The General OR Rule

Learning Objective — Spec Ref P8:
Use the OR rule for events that aren't mutually exclusive.

When two events are **not mutually exclusive** (i.e. they **can both happen** at the same time), you need a different version of the OR rule. The **general OR rule** says that for any two events, the probability that **at least one event happens** is equal to the **sum** of the probabilities of **each** event happening **minus** the probability that **both** events happen.

P(A or B) = P(A) + P(B) – P(A and B) or **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$** in **set notation**.

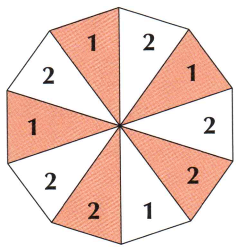
To help you visualise this, think about **sets A and B** on a Venn diagram. If there is an **overlap** between sets A and B (i.e. things can be in both set A and set B), then **n(A) + n(B)** will add things in the overlap **twice**, so you need to **subtract n(A and B)**.



The OR rule for **mutually exclusive** events (see previous page) is actually a **special case** of this rule — if A and B can't happen at the same time, then P(A and B) = 0, so the OR rule becomes:
P(A or B) = P(A) + P(B) – 0 = P(A) + P(B).

Example 2

The fair spinner shown on the right is spun once.
What is the probability that it lands on a 2 or a shaded sector?



1. The spinner can land on both 2 and a shaded sector at the same time, so the events are not mutually exclusive. Find the probability of the events you're interested in.

   Six sectors on the spinner are numbered 2, so $P(2) = \frac{6}{10}$

   There are five shaded sectors, so $P(\text{shaded}) = \frac{5}{10}$

   There are two shaded sectors numbered 2, so $P(2 \text{ and shaded}) = \frac{2}{10}$

2. Now use the OR rule.

   P(2 or shaded) = P(2) + P(shaded) – P(2 and shaded)

   $= \frac{6}{10} + \frac{5}{10} - \frac{2}{10} = \mathbf{\frac{9}{10}}$

Tip: In this example it would be easy to just count the outcomes, but if you only knew the probabilities you'd have to use the OR rule.

Example 3

The two-way table on the right shows the probabilities for various events. Find $P(X \cup A)$.

	A	A′	Totals
X	0.15	0.15	
Y			0.45
Z	0.1		
Totals		0.55	1

1. Use the OR rule to rewrite $P(X \cup A)$ in terms of $P(X)$, $P(A)$ and $P(X \cap A)$. — $P(X \cup A) = P(X) + P(A) - P(X \cap A)$
   $P(X \cap A) = 0.15$ from the table.
2. $P(X)$ is the total of the X row of the table. — $P(X) = 0.15 + 0.15 = 0.3$
3. $P(A) + P(A')$ should be 1. — $P(A) = 1 - 0.55 = 0.45$
4. Use the formula to find $P(X \cup A)$. — $P(X \cup A) = 0.3 + 0.45 - 0.15 = \mathbf{0.6}$

Exercise 2

Q1 A fair 20-sided dice is rolled. What is the probability that it lands on:

a) a multiple of 3 or an odd number?

b) a factor of 20 or a multiple of 4?

Q2 A card is randomly chosen from a standard pack of 52 playing cards.

a) What's the probability that it's a red suit or a queen?

b) What's the probability that it's a club or a picture card?

Q3 The table shows the number of boys and girls in each year group at a school. What is the probability that a randomly chosen pupil is:

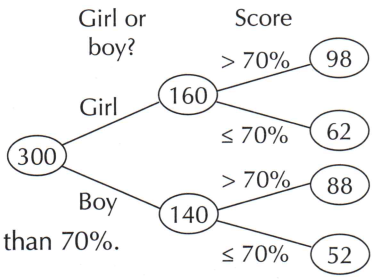
a) in Year 11 or a girl?

b) a boy or not in Year 9?

	Boys	Girls
Year 9	240	310
Year 10	305	287
Year 11	212	146

Q4 Sumi organises some elements into two sets, A and B.
$n(A) = 28$, $n(B) = 34$, $n(A \cap B) = 12$, $n(\xi) = 100$.
Calculate the probability that a randomly chosen element is in set A or set B.

Q5 The frequency tree shows some information about scores achieved by girls and boys on a maths test. Use the frequency tree to find the probability that a randomly chosen student is:

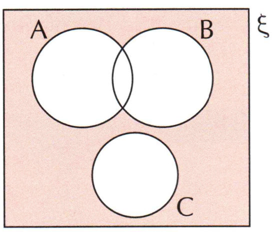


a) a girl or scored no more than 70% on the maths test.

b) a boy or scored more than 70% on the maths test.

c) a boy who scored no more than 70% or a girl who scored more than 70%.

Q6 The Venn diagram on the right shows three events A, B and C.
A and B are independent.
A and C are mutually exclusive.
B and C are mutually exclusive.



Use the Venn diagram to derive an expression for $P(A \cup B \cup C)$ in terms of $P(A)$, $P(B)$ and $P(C)$.

35.3 Using the AND/OR Rules

A lot of tricky probability questions require you to use both the AND rule and the OR rule together. Don't forget that you need to know that the events are independent before you can use the AND rule.

Learning Objective — Spec Ref P8:
Use the AND and OR rules to solve probability problems.

You can work out more complicated probability questions by **breaking them down** into smaller chunks.

You should always start by figuring out what **events** are happening and whether or not they are **independent** or **mutually exclusive**. Remember that independent events **don't affect each other** and mutually exclusive events **can't both happen** at the same time.

Once you know what your events are, write down what **probabilities** you're looking for, then use the **AND** and **OR rules** as necessary to work them out.

Example 1

A biased dice lands on each number with the probabilities shown in the table opposite. The dice is rolled twice. Find the probability of rolling exactly one 2 and one 3.

1	2	3	4	5	6
0.1	0.15	0.2	0.2	0.3	0.05

1. There are two options for rolling one 2 and one 3.

   (2 and then 3) or (3 and then 2)

2. Find the probability of getting 2 on the first roll and 3 on the second roll. These events are independent, so you can multiply the probabilities.

   P(2 and then 3) = P(2) × P(3)
   = 0.15 × 0.2 = 0.03

3. Find the probability of getting 3 on the first roll and 2 on the second roll.

   P(3 and then 2) = P(3) × P(2)
   = 0.2 × 0.15 = 0.03

4. Find the probability of getting the first option or the second option. These events are mutually exclusive, so you can add the probabilities.

   P((2 and then 3) or (3 and then 2))
   = 0.03 + 0.03 = **0.06**

Example 2

In an online game, you can purchase armour boxes containing one piece of armour. Armour is given a material and a type at random, with the following independent probabilities:

Material: P(Leather) = 0.5 P(Steel) = 0.35 P(Gold) = 0.15

Type: P(Chest) = 0.6 P(Leg) = 0.4

a) Wizards can only use leather or gold armour. Find the probability that an armour box contains a piece of leg armour that a Wizard can use.

1. There are two possibilities — leather leg armour or gold leg armour. The probabilities for each are independent, so use the AND rule.

   P(Leather and legs) = P(Leather) × P(Leg)
   = 0.5 × 0.4 = 0.2

   P(Gold and legs) = P(Gold) × P(Leg)
   = 0.15 × 0.4 = 0.06

2. Now you can use the OR rule.

   P(Leather legs or gold legs) = 0.2 + 0.06 = **0.26**

b) Fliss wants to get some gold chest armour for her Wizard character, so she buys two boxes. What is the probability that she gets gold chest armour from exactly one box?

1. First, write down what you're trying to find — use the OR rule to add the results that give exactly 1 gold chest.

   P(one gold chest) = P(gold chest and not gold chest) + P(Not gold chest and gold chest)

2. Find the probability that a box contains gold chest armour using the AND rule.

   P(gold chest) = P(gold) × P(chest) = 0.15 × 0.6 = 0.09

3. Use this to work out the probability that a box doesn't give gold chest armour.

   P(not gold chest) = 1 – P(gold chest) = 1 – 0.09 = 0.91

4. Use the AND rule to find the probabilities that the gold chest comes from each box.

   P(gold chest and not gold chest) = 0.09 × 0.91 = 0.0819
   P(not gold chest and gold chest) = 0.91 × 0.09 = 0.0819

5. Finally add the probabilities of each result.

   P(one gold chest) = 0.0819 + 0.0819 = **0.1638**

Exercise 1

Q1 A fair coin is tossed twice. Work out the probability of getting:

a) 2 heads b) 2 tails c) both tosses the same

Q2 All of Justin's shirts are either white or black and all his trousers are either black or grey.
The probability that he chooses a white shirt on any day is 0.8.
The probability that he chooses black trousers on any day is 0.55.
His choice of shirt colour is independent of his choice of trousers colour.

On any given day, find the probability that Justin chooses:

a) a white shirt and black trousers
b) a black shirt and black trousers
c) a black shirt and grey trousers
d) either a black shirt or black trousers, but not both

Q3 A spinner has four sections labelled A, B, C and D.
The probabilities of landing on each section are shown in the table below.

A	B	C	D
0.5	0.15	0.05	0.3

If the spinner is spun twice, find the probability of spinning:

a) A both times b) B both times c) not C on either spin
d) C, then not C e) not C, then C f) C on exactly one spin

Q4 A class of 30 pupils were asked how they normally travel to school.
The table below shows their responses.

	Car	Bus	Walk	Cycle
Girls	5	6	3	1
Boys	2	7	2	4

One boy and one girl from the class are picked at random. Find the probability that:

a) both travel by bus
b) the girl walks and the boy travels by car
c) exactly one of them cycles
d) at least one of them cycles

35.4 Tree Diagrams

Tree diagrams can be used to show all the possible results from an experiment — they're really useful for working out probabilities of combinations of events.

Learning Objective — Spec Ref P6/P8:
Use tree diagrams to represent events and to find probabilities.

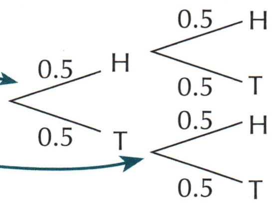
Tree diagrams are similar to **frequency trees** (see p.459), except they show the **probabilities** of the events rather than the frequency from an experiment. The really useful thing about tree diagrams is that you can find the probability of specific **results** (e.g. P(A happens and B doesn't happen)) by **multiplying along the branches** that you follow to get to that result. Since branches from the **same point** show all the outcomes of a single activity, their probabilities should **add up to 1**.

Example 1

Two fair coins are tossed.

a) Draw a tree diagram showing all the possible results for the two tosses.

1. Draw a set of branches for the first toss. You need 1 branch for each of the 2 results.
2. Draw a set of branches for the second toss. Again, there are two possible results.
3. Write on the probability for each branch.



b) Find the probability of getting 2 heads.

Multiply along the branches for heads then heads. P(H then H) = 0.5 × 0.5 = **0.25**

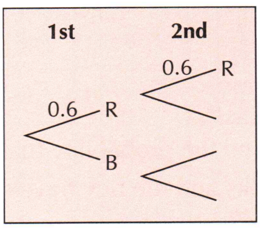
c) Find the probability of getting heads and tails in any order.

1. Multiply along the branches for heads then tails. P(H then T) = 0.5 × 0.5 = 0.25
2. Multiply along the branches for tails then heads. P(T then H) = 0.5 × 0.5 = 0.25
3. Both these results give H and T, so add the probabilities. P(1H and 1T) = 0.25 + 0.25 = **0.5**

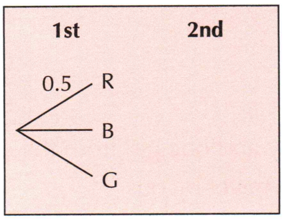
Exercise 1

Q1 Copy and complete the following tree diagrams.

a) A fair spinner has five equal sections — three are red and two are blue. The spinner is spun twice.



b) A bag contains ten coloured balls — five red, three blue and two green. A ball is selected at random and replaced, then a second ball is selected.



Q2 For each of these questions, draw a tree diagram showing all the possible results. Write the probability on each branch.

a) A biased coin lands on heads with a probability of 0.65. The coin is tossed twice.

b) A fair six-sided spinner has two sections labelled 1, two labelled 2 and two labelled 3. The spinner is spun twice.

c) The probability a football team wins is 0.7, draws is 0.1 and loses is 0.2. The team plays three matches.

Q3 The probability that Freddie beats James at snooker is 0.8. They play two games of snooker.

a) Draw a tree diagram to show all the possible results for the two games.

b) Find the probability that Freddie wins: (i) both games (ii) exactly one of the games

Q4 On any Saturday, the probability that Alex goes to the cinema is 0.7. Use a tree diagram to work out the probability that Alex either goes to the cinema on all three of the next three Saturdays, or doesn't go to the cinema on any of the next three Saturdays.

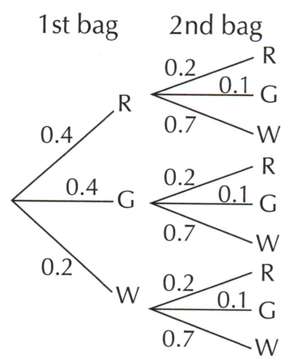
Q5 Layla rolls a fair six-sided dice three times. Use a tree diagram to find the probability that she rolls:

a) 'less than 3' on all three rolls b) 'less than 3' once and '3 or more' twice

If working out the probability of an event happening involves adding up probabilities for **lots of results**, it might be quicker to find the probability of the event **not happening** and then **subtracting** it from 1.

Example 2

Two bags each contain red, green and white balls. This tree diagram shows all the possible results when one ball is picked at random from each bag. Find the probability that the two balls picked are different colours.



1. The quickest way to do this is to work out the probability that they're the same colour, then subtract this from 1.
2. There are 3 combinations which give the same colour.

   P(same) = P(R, R) + P(G, G) + P(W, W)

   = (0.4 × 0.2) + (0.4 × 0.1) + (0.2 × 0.7) = 0.26
3. P(different colours) = 1 – P(same colour)

   P(different colours) = 1 – 0.26 = **0.74**

Exercise 2

Q1 Sally owns 12 DVDs, four of which are comedies, and Jesse owns 20 DVDs, eight of which are comedies. They each select one of their DVDs at random to watch over the weekend.

a) Draw a tree diagram showing the probabilities of each choice being 'comedy' or 'not a comedy'.

b) Find the probability that: (i) neither chooses a comedy (ii) at least one chooses a comedy

Q2 A fair spinner has six equal sections — three red, two blue and one yellow. It is spun twice. Use a tree diagram to find the probability of spinning:

a) blue then red b) red once and blue once c) the same colour twice

d) at least two different colours e) not yellow then not yellow f) yellow at least once

35.5 Conditional Probability

Earlier, you saw the AND rule for events that were independent. When you're looking at events that are dependent, you need to figure out how they affect each other by working out the conditional probability.

The AND Rule for Dependent Events

Learning Objective — Spec Ref P8/P9:
Understand conditional probability and use it to find probabilities of dependent events.

Conditional probabilities tell you the probability of an event, **given that** another has happened. They're used when dealing with **dependent events**, where the probabilities **change** depending on what else happens. For example, the probability that it rains tomorrow might depend on whether or not it has rained today. In this case, you might find that P(rain tomorrow, **given that** it rained today) is **higher** than P(rain tomorrow) — **knowing** that it rained today **increases** how likely you think it is to rain tomorrow.

If **A** and **B** are **dependent** events: **P(A and B) = P(A) × P(B given A)**

The AND rule for **independent** events from p.463 is a **special case** of this — if A and B are independent, then P(B given A) will be **the same** as P(B), since A happening doesn't affect the probability of B happening. Then the AND rule becomes: P(A and B) = P(A) × P(B given A) = **P(A) × P(B)**.

Conditional probabilities often come up in situations where objects are selected **without replacement**, where the probabilities for the second selection **depend** on what the first selection was.

Example 1

A box of chocolates contains 10 white chocolates and 10 milk chocolates.

a) Jane picks a chocolate at random. Find the probability that it's a white chocolate.

Use the formula for equally likely outcomes.

20 chocolates in the box, 10 are white chocolates

P(Jane picks white) = $\frac{10}{20} = \mathbf{\frac{1}{2}}$

b) Given that Jane has picked a white chocolate, what's the probability that Geoff randomly picks a white chocolate?

1. Consider the situation where Jane has already picked a white chocolate.
2. Now use the formula for equally likely outcomes again.

19 chocolates in the box, 9 are white chocolates

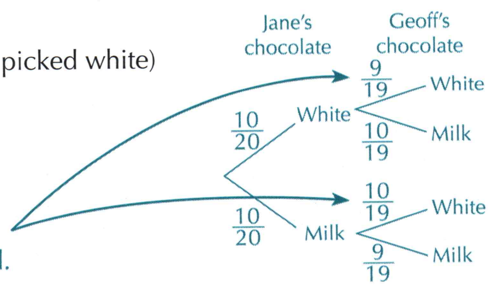
P(Geoff picks white given Jane picked white) = $\mathbf{\frac{9}{19}}$

c) Find the probability that both Jane and Geoff pick a white chocolate.

The events are dependent so use the formula given above and your answers to a) and b).

P(Jane and Geoff both pick white)
= P(Jane picks white) × P(Geoff picks white given Jane picked white)
$= \frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = \mathbf{\frac{9}{38}}$

The tree diagram for every possible result is shown on the right — notice that the probability of Geoff picking a white chocolate changes depending on what Jane picked.



Exercise 1

Q1 A bag contains ten balls numbered 1 to 10. Ball 8 is selected at random and not replaced.
Find the probability that the next ball selected at random is ball 7.

Q2 In a Year 11 class there are 16 boys and 14 girls. Two names are picked at random.
Given that the first student picked is a girl, what is the probability that the second student picked is:

a) a girl? b) a boy?

Q3 The members of a drama group are choosing their characters for a pantomime. There are nine left to choose — four wizards, three elves and two toadstools. Nobody wants to play the toadstools, so the names of the nine characters are put into a bag so they can be selected at random.
John is going to pick first, followed by Kerry.

a) If John picks an elf, what is the probability that Kerry picks:

(i) an elf? (ii) a wizard? (iii) a toadstool?

b) If John picks a toadstool, what is the probability that Kerry also picks a toadstool?

Q4 A club is drawn at random from a standard pack of cards and not replaced.
Find the probability that the next card drawn at random:

a) is a club b) is not a club

c) is a diamond d) is a black card

e) is a red card f) has the same value as the first card

Q5 The probability of rolling a 4 on an unfair dice is 0.1.
The probability that in two rolls you score a total of 10, given that a 4 is rolled first, is 0.4.
What is the probability of rolling a 4 on your first roll and having a total score of 10 after two rolls?

Q6 The probability that Shaun has a stressful day at work is 0.6. If Shaun doesn't have a stressful day at work, then the probability that he eats ice cream in the evening is 0.2.
What is the probability that Shaun doesn't have a stressful day and eats ice cream?

Q7 A teacher wants to randomly select two students from his class to be on the school quiz team.
There are 12 boys and 8 girls in the class.

a) If the first student chosen is a girl, find the probability that the second one chosen is a girl.

b) If the first student chosen is a boy, find the probability that the second one chosen is a girl.

c) Draw a tree diagram showing all the possible results for the two choices.

d) Find the probability that: (i) two girls are chosen. (ii) two boys are chosen.

Q8 Three cards are selected at random from ten cards labelled 1 to 10. Find the probability that:

a) all three cards are even numbers b) one card is odd and the other two are even

Q9 When Tom goes to his favourite Italian restaurant he always orders pizza or pasta.
The probability that he orders pizza is 0.5 if he ate pizza last time, but 0.9 if he ate pasta last time.
Given that he ate pizza last time, find the probability that:

a) He orders pizza on each of the next three times he eats there.

b) He doesn't order pizza on any of the next three times he eats there.

c) He orders pizza on two of the next three times he eats there.

Conditional Probability Using Venn Diagrams

Learning Objective — Spec Ref P4/P6/P9:
Use Venn diagrams to find conditional probabilities.

Prior Knowledge Check:
Be able to interpret and draw Venn diagrams. See p.238-244.

Venn diagrams are really helpful for working out probabilities, particularly **conditional probabilities**. If you've got a **complete** Venn diagram for your events, then it's really easy to find the probability of one event given another:

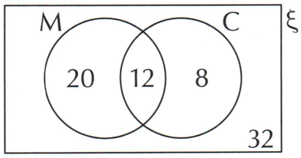
$$\text{P(A given B)} = \frac{\text{Number of outcomes in both A and B}}{\text{Number of outcomes in B}} = \frac{n(A \cap B)}{n(B)}$$

You can think of this as the **probability** of getting something from **A** when randomly picking from all the things in **B** (**ignoring** all the things that **aren't in B**).

Example 2

The Venn diagram shows the number of males (M) and the number of people who are drinking coffee (C) at a cafe.

What is the probability that a randomly selected person at the cafe is male, given that they are drinking coffee?



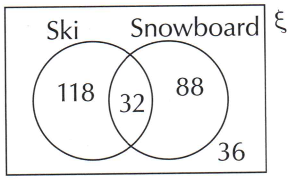
Use the formula — divide the number of people that are both males and drinking coffee by the total number of people drinking coffee.

$$\text{P(M given C)} = \frac{n(\text{M and C})}{n(\text{C})} = \frac{12}{12+8} = \frac{12}{20} = \mathbf{\frac{3}{5}}$$

Tip: Remember, n(M and C) or $n(M \cap C)$ is the number in the bit where M and C overlap.

Exercise 2

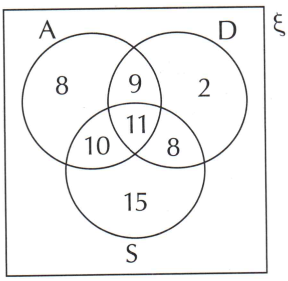
Q1 A survey was taken at a hotel in a ski resort which asked 274 people if they could ski and if they could snowboard. The results are shown in the Venn diagram on the right.



What is the probability that a randomly chosen hotel guest:

a) can ski?
b) can snowboard?
c) can ski and can snowboard?
d) can ski given that they can snowboard?
e) can't snowboard given that they can't ski?

Q2 The Venn diagram below shows the number of members in a drama group who can act (A), dance (D) and sing (S). Each member of the group can do at least one of these.



What is the probability that a randomly chosen member:

a) can act given that they can dance?
b) can sing given that they can't dance?
c) can't dance given that they can act?
d) can't act given that they can't sing?
e) can act and dance given that they can sing?
f) can sing and act given that they can't dance?
g) can act or dance given that they can't sing?

Review Exercise

Q1 The probability that Colin wins any game of chess is 0.8. Find the probability that he wins:

a) both of his next two games

b) neither of his next two games

c) his next game but loses the one after

d) one of his next two games

Q2 A board game has four categories of question, each split into two difficulty levels. The table shows the probability of randomly selecting a question card of each type.

	Easy	Challenge
TV	0.20	0.10
Music	0.15	0.07
Sport	0.12	0.08
Literature	0.15	x

a) Find the missing probability, x, from the table.

b) Which of the four categories has the most easy questions?

c) Mark randomly selects one card from all the question cards. What is the probability that he selects:

(i) a music question?

(ii) a sport question?

(iii) a music or a sport question?

(iv) a music question or a challenging question?

(v) an easy question or a sport question?

d) Dahlia plays two games of the board game. Each game starts with the first question card being randomly selected from all the cards. Find the probability that both starting cards are:

(i) challenge music questions

(ii) easy literature questions

Q3 Eight friends have to pick three from the group to represent them at a meeting. Five of the friends are in Year 10 and three are in Year 11. If they pick the three representatives at random, find the probability that:

a) all three are in Year 10

b) all three are in Year 11

c) two are in Year 10 and one is in Year 11

d) two are in Year 11 and one is in Year 10

Q4 A box contains ten coloured marbles — five blue, four white and one red. Two marbles are picked at random.

a) Draw a tree diagram showing all the possible results.

b) Work out the probability that:

(i) both are blue

(ii) neither is blue

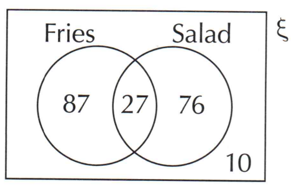
(iii) exactly one is blue

(iv) at least one is blue

(v) one is red and one is white

(vi) they are different colours

Q5 At a fast food restaurant there are two options for side dishes — fries or salad. The side dish choices of 200 customers are recorded in this Venn diagram. What is the probability that a randomly chosen customer:



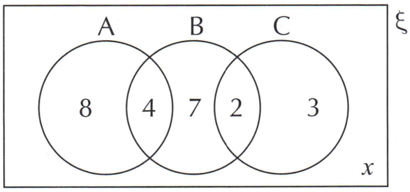
a) had fries given that they had salad?

b) didn't have salad given that they had fries?

c) didn't have a side given that they didn't have salad?

Exam-Style Questions

Q1 30 Sixth Form students were asked if they are studying any of the subjects Art (A), Biology (B) or Chemistry (C) for A-level. The results of this survey are shown in the Venn diagram, where each entry represents the number of students.



a) Work out the value of x.

[1 mark]

b) Explain clearly why the Venn diagram shows that studying Art and studying Chemistry are "mutually exclusive" events for these 30 students.

[2 marks]

c) Given that a student is studying Biology, work out the probability that they also study Art or Chemistry.

[2 marks]

Q2 Tariq and Udai are researching driving test statistics. The probability of passing a driving test when it is taken for the first time is 0.5. When taken a second time, the probability of passing increases to 0.6. When taken a third time, the probability of passing is 0.8.

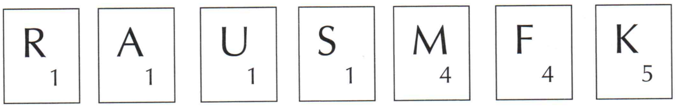
a) Work out the probability that Tariq will pass his driving test on his third attempt.

[2 marks]

b) Work out the probability that Tariq and Udai will both pass their driving tests on the third attempt. You may assume that the events are independent of each other.

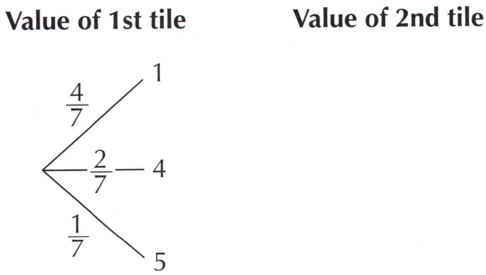
[1 mark]

Q3 Naasir is playing a board game. He picks two letter tiles at random from a bag. The bag contains the following tiles:



The numbers on each tile represent how many points that tile is worth.

a) Copy and complete the tree diagram below. The first set of branches is drawn for you.



[3 marks]

b) Work out the probability that the two tiles that Naasir picks will be worth less than 6 points in total.

[3 marks]