# **Probability**

# 34.1 Calculating Probabilities

Probability is all about measuring how likely it is for things to happen. Those things could be simple, like rolling a 6 on a dice roll, or they could be more complicated, like rolling a total of 20 over 5 dice rolls.

## **Calculating Probabilities**

## Learning Objectives — Spec Ref P2/P3:

- Find the probability of a given event.
- Know that probabilities must be between 0 and 1.

In probability, an **outcome** is the result of an activity (e.g. 'getting tails on a coin flip' or 'rolling 3 on a dice roll') and an **event** is a set of one or more outcomes to which you assign a **probability**. The **probability** of an event is a number that represents how **likely** it is for that event to happen. All probabilities are between **0** and **1**. An **impossible** event has a probability of **0** and an event that's **certain** to happen has a probability of **1**. You can show probabilities on a **scale** (i.e. a **number line** going from 0 to 1).

The probability of an event can be written as **P(event)**, and can be given as a **fraction**, **decimal** or **percentage**. For example, if you flip a fair coin, P(heads) =  $\frac{1}{2}$  = 0.5 = 50%.

When **all** the possible outcomes are **equally likely**, you can work out probabilities of events using this formula:

$$P(\text{event}) = \frac{\text{number of ways the event can happen}}{\text{total number of possible outcomes}}$$

## Example 1

A box contains 20 counters, numbered 1 to 20. If one counter is selected at random, work out the probability that the number is less than 12.

- work out the probability that the number is less than 12.
  1. All the possible outcomes are equally likely, so use the formula.
  The total number of possible outcomes is the number of counters.
  Total possible
- 2. Count the number of outcomes that are less than 12.
- 3. Put the numbers into the formula.

Total possible outcomes = 20

11 ways of getting less than 12

P(less than 12) =  $\frac{11}{20}$ 

If you're asked for the probability of one event **or** another, you can just **add up** the number of outcomes for each event — as long as both events **can't** happen at the **same time** (see next page).

## Example 2

Aidan has 6 apples, 3 bananas, 4 pears and 2 oranges in his fruit bowl. He picks out one piece of fruit at random to eat. What is the probability that it will be an apple or a pear?

- 1. Work out the total number of possible outcomes (how many pieces of fruit are in the bowl).
- 2. Count the number of fruit that are either apples or pears.
- 3. Use the formula and simplify the fraction.

Total fruit = 
$$6 + 3 + 4 + 2 = 15$$

Apples or pears 
$$= 6 + 4 = 10$$

P(apple or pear) = 
$$\frac{10}{15} = \frac{2}{3}$$

#### Exercise 1

- O1 Calculate the probability of rolling a fair, six-sided dice and getting each of the following.
  - a) 6

b) 7

- c) 4 or 5
- d) a factor of 6
- O2 A standard pack of 52 playing cards is shuffled and one card is selected at random. Find the probability of selecting each of the following.
  - a) a club

b) an ace

c) a red card

- d) the two of hearts
- e) not a spade

- f) a 4 or a 5
- A bag contains 16 coloured balls 2 black, 4 blue, 2 green, 3 red, 2 yellow, 1 orange, 1 brown O3 and 1 purple. If one ball is selected at random, find the probability of getting the following colours.
  - a) green
- b) blue or green
- c) not purple
- d) red, green or brown
- For each of these questions, draw a copy of the spinner on the **O**4 right and number the sections in a way that fits the given rule.
  - a) The probability of getting 2 is  $\frac{3}{8}$ . b) The probability of getting 3 is  $\frac{1}{2}$ .
  - c) The probability of getting 5 and the probability of getting 6 are both  $\frac{1}{4}$ .



- Diane has 20 pairs of socks and each pair is different. She picks one sock at random. If she then picks Q5 another sock at random from the remaining socks, what is the probability that the 2 socks make a pair?
- A box of identically wrapped chocolates contains 8 caramels, 6 truffles and 4 pralines. Half of each Q6 type of chocolate are coated in milk chocolate and half are coated in white chocolate. Chelsea selects a chocolate at random. She doesn't like pralines or white chocolate, but she likes all the others.
  - a) What is the probability that she gets a white chocolate-coated praline?
  - b) What is the probability that she gets a chocolate that she likes?
- This table shows information about the books Harry owns. Q7 If one of Harry's books is selected at random, what is the probability that it's a:
  - a) paperback fiction book?
- b) non-fiction book?

	Paperback	Hardback
Fiction	14	6
Non-fiction	2	8

# Probabilities that Add Up to 1

## Learning Objective — Spec Ref P4:

Know that the probabilities of mutually exclusive events sum to 1.

Events that can't happen at the same time are called mutually exclusive (there's more on mutually exclusive events on p.465). For example, rolling a 1 and rolling a 3 in one dice roll are mutually exclusive events.

The probabilities of mutually exclusive events that cover all possible outcomes always add up to 1.

For any event, there are only two possibilities — it either happens or it doesn't happen. So:

The probability that an event doesn't happen is equal to 1 minus the probability that it does happen.

## Example 3

The probability that Kamui's train is late is 0.05. Work out the probability that his train is not late.

The train is either late or not late, so P(train is late) + P(train is not late) = 1.

P(train is not late) = 1 - P(train is late)= 1 - 0.05 = 0.95

You can also use the fact that the probabilities for every possible outcome add up to 1 to find **missing probabilities**.

## Example 4

A bag contains red, green, blue and white counters. This table shows the probabilities of randomly selecting a red, green or white counter from the bag.

Work out the probability of selecting a blue counter.

The probabilities of the 4 colours add up to 1, so the probability of blue is 1 minus the other 3 probabilities.

Colour	Red	Green	Blue	White
Probability	0.2	0.1		0.5

$$P(blue) = 1 - (0.2 + 0.1 + 0.5)$$
  
= 1 - 0.8 = **0.2**

## Exercise 2

- Q1 The probability that it will snow in a particular Canadian town on a particular day is  $\frac{5}{8}$ . What is the probability that it won't snow there on that day?
- Q2 The probability that Clara wins a raffle prize is 25%. Find the probability that she doesn't win a raffle prize.
- Q3 If the probability that Jed doesn't finish a crossword is 0.74, what's the probability he does finish it?
- Q4 The probability that Gary wins a tennis match is twice the probability that he loses it. Work out the probability that he wins a tennis match.



Q5 Everyone taking part in a certain lucky dip wins a prize.

The table on the right shows the probabilities of winning the four possible prizes.

Prize	Lollipop	Pen	Cuddly toy	Gift voucher
Probability	0.4	0.1		0.2

- a) Find the missing probability.
- b) What's the probability of winning a prize that's not a pen?
- Q6 When two football teams play each other the probability that Team A wins is 0.4 and the probability that Team B wins is 0.15. What is the probability that the match is a draw?
- One counter is selected at random from a box containing blue, green and red counters. The probability that it's a blue counter is 0.5 and the probability that it's a green counter is 0.4. If there are four red counters in the box, how many counters are there altogether?
- A spinner has three sections, coloured pink, blue and green. The probability it lands on pink is 0.1. If the probability it lands on blue is half the probability it lands on green, find the probability that it lands on green.



# 34.2 Listing Outcomes

One of the trickiest bits of probability is figuring out what all the possible outcomes are, especially if you've got more than one activity. You'll need to find a way to list or count all the different possible outcomes.

## **Listing Outcomes**

**Learning Objective** — Spec Ref P6/P7: List outcomes of two or more events.

When **two things** are happening at once, e.g. a coin toss and a dice roll, it's much easier to work out probabilities if you **list all the possible outcomes** in a systematic way, so that you don't miss any outcomes. A coin toss has two outcomes (heads 'H' and tails 'T'), and a dice roll has 6 ('1', '2', '3', '4', '5' and '6'), so if both happen together, there will be 12 possible outcomes: 'H and 1', 'T and 1', 'H and 2', 'T and 2', 'H and 3', 'T and 3', etc.

It's often a good idea to use a **sample space diagram** (also called a **possibility diagram**) to write the outcomes in an **ordered** and **logical** way. It can be in the form of a **list**, a **grid** or a **two-way table**.

#### Example 1

Anne has three tickets for a theme park. She chooses two friends at random to go with her. She chooses one girl from Belinda, Claire and Dee, and one boy from Fred and Greg. What is the probability that Anne chooses Claire and Fred to go with her?

- 1. Use a simple two-column table to list the outcomes. Write each girl in turn and fill in all the possibilities for the boys. Each row of the table is a possible outcome.
- Count the number of rows that Claire and Fred both appear in. Then divide by the total number of rows.

There's 1 outcome that includes both Claire and Fred, and 6 outcomes in total. So P(Claire and Fred) =  $\frac{1}{6}$ .

Girls	Boys
Belinda	Fred
Belinda	Greg
Claire	Fred
Claire	Greg
Dee	Fred
Dee	Greg

#### Exercise 1

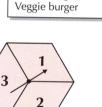
- Q1 A burger bar offers the meal deal shown on the right. Jana picks one combination of burger and drink at random.
  - a) What is the probability that she chooses a veggie burger and cola?
  - b) What is the probability that she chooses at least one of cheeseburger or coffee?
- Q2 The fair spinner shown on the right is spun twice.
  - a) What is the probability of spinning 1 on both spins?
  - b) What is the probability of getting less than 3 on both spins?

Q3 A fair coin is tossed three times. Work out the probability of getting:

a) three tails

b) no tails

c) one head and two tails



Burgers

Hamburger

Cheeseburger

Choose 1 burger and 1 drink

Drinks

Lemonade

Cola

Coffee

## Example 2

Two fair, four-sided dice, one white and one blue, are rolled and their scores are added together. Both are numbered 1-4.

- a) List all the possible total scores.
  - 1. Draw a two-way table with the outcomes for one dice across the top and those for the other dice down the side.
  - 2. Fill in each square with the score for the row plus the score for the column.

		White dice				
		1 2 3 4				
е	1	2	3	4	5	
dice	2	3	4	5	6	
ine (	3	4	5	6	7	
B	4	5	6	7	8	

b) What is the probability of scoring a total of 4?

Count how many times a total of 4 appears in the table, then divide by the total number of outcomes.

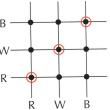
3 of the outcomes are 4, and there are 16 outcomes in total. So P(total of 4) =  $\frac{3}{16}$ .

## Example 3

A spinner with three equal sections, coloured red, white and blue, is spun twice. Find the probability of spinning the same colour both times.

- 1. Draw a two-way table or grid to show all the possible outcomes.
- 2. Highlight and count the number of ways of spinning the same colour both times, then divide by the total number of outcomes.

There are 3 ways of spinning the same colour, and there are 9 outcomes in total. So P(same colour) =  $\frac{3}{9} = \frac{1}{3}$ .



- Q1 Two fair six-sided dice are rolled.
  - a) Copy and complete the table on the right to show all the possible outcomes when the scores are added together.
  - b) Find the probability of each of the following total scores.
    - (i) 6
- (ii) 12
- (iii) less than 8
- (iv) more than 8



- Oz Craig likes to eat curries, but he always finds it difficult to choose what type of rice to have. He is equally happy to eat boiled, lemon, pilau or vegetable rice. He decides that from now on, he's going to select his rice at random from those four options.
  - a) Draw a diagram to show all the possible combinations of rice he could eat with his next two curries. Assume he only has one type of rice per curry.
  - b) Find the probability that he eats the following types of rice with his next two curries:
    - (i) pilau rice both times
- (ii) the same type both times
- (iii) lemon rice at least once
- Hayley and Asha are playing a game. In each round they both spin a spinner with five equal sections labelled 1 to 5, and whoever gets the higher score wins the round. If the winner spins a 5, she scores 2 points, and if the winner spins less than 5, she scores 1 point. If both players spin the same number, no one wins a point. What is the probability that Hayley wins:
  - a) exactly 1 point in the first round?
- b) 2 points in the first round?

## **Product Rule for Counting Outcomes**

#### Learning Objective — Spec Ref N5:

Use the product rule to find the number of outcomes.

When you have **lots** of events happening, listing **all** of the outcomes might be **too difficult** or take **too long**. In situations like this, you can use the **product rule** to count the total number of outcomes:

The number of **possible outcomes** for multiple events is equal to the number of possible outcomes of **each** event **multiplied** together.

**Tip:** The product rule only works when the outcome of one event doesn't affect the number of outcomes of another event

#### Example 4

Gary flips a coin, rolls a standard six-sided dice and draws a card from a standard 52-card deck. How many different possible outcomes are there?

- 1. Work out what the different events are, and how many possible outcomes each has.
- 2. Use the product rule to find the number of possible combinations.

Flipping the coin = 2 possible outcomes Rolling the dice = 6 possible outcomes Drawing the card = 52 possible outcomes

Total possible outcomes =  $2 \times 6 \times 52 = 624$ 

#### Example 5

How many different six-digit numbers are there that only include the digits 4, 8 and 9?

1. Treat each digit of the number as its own event.

There are 6 'events', each with 3 possible outcomes: 4, 8 or 9.

2. Use the product rule.

**Tip:** It would take too long to list all 729 outcomes individually.

- Q1 On a menu in a restaurant there are 4 starters, 8 main courses and 3 desserts to choose from.
  - a) How many different three-course meals could you choose from?
  - b) In a two-course meal one of the meals has to be a main course. How many different two-course meals could you choose from?
- Q2 A combination lock has five rotating wheels which can each be set to one of the digits 0-6.
  - a) How many different combinations could you set for the lock?
  - b) How many different combinations could you set that only include odd digits?
  - c) If you randomly choose a combination, what is the probability that all the wheels will be set to odd digits?
- Q3 A vending machine contains 6 different flavours of fruit juice. Trish buys 4 cans of fruit juice from the vending machine. She says, "There are 1296 different combinations of fruit juice I could get." Is Trish correct? Explain your answer.



# 34.3 Probability from Experiments

All the probability so far in this section has been theoretical. In reality, it's difficult to know for sure that the outcomes are equally likely — for example, how can you be certain that a dice isn't more likely to roll a 6 than a 1? The best way to find out is to do experiments and see if the results agree with what you expect.

## Relative Frequency

Learning Objective — Spec Ref PI/P5:

Find the relative frequency of an event.

You can **estimate** probabilities using the results of an experiment or what you know has already happened. Your estimate is the **relative frequency** (also called the **experimental probability**), which you work out using this formula:

Relative frequency =  $\frac{\text{Number of times the result happens}}{\text{Number of times the experiment is done}}$ 

The more times you do the experiment, the more accurate the estimate should be.

## Example 1

A biased dice is rolled 100 times. The results are on the right. Estimate the probability of rolling a 1 with this dice.

Score	1	2	3	4	5	6
Frequency	11	14	27	15	17	16

Use the relative frequency as an estimate of the probability — divide the number of times 1 was rolled by the total number of rolls.

1 was rolled 11 times, so  $P(1) = \frac{11}{100}$ 

## Example 2

Dan caught 64 butterflies in the butterfly house at the zoo. 16 were red, 15 were blue, 17 were green and the rest were purple. After catching each butterfly, he let it go. Estimate the probability that the next butterfly he catches is either blue or green.

1. Find the total number of blue or green butterflies that Dan caught.

- Blue or Green = 15 + 17 = 32
- 2. Use the relative frequency to estimate the probability.
- P(Blue or Green) =  $\frac{32}{64} = \frac{1}{2}$

- Q1 A spinner with four sections is spun 100 times. The results are shown in the table below.
  - a) Estimate the probability of spinning each colour.
  - b) How could the estimates be made more accurate?
- ColourRedGreenYellowBlueFrequency493489
- Q2 Stacy rolls a six-sided dice 50 times and 2 comes up 13 times. Jason rolls the same dice 100 times and 2 comes up 18 times.
  - a) Use Stacy's results to estimate the probability of rolling a 2 on this dice.
  - b) Use Jason's results to estimate the probability of rolling a 2 on this dice.
  - c) Explain whose estimate should be more accurate.

Q3 Jamal records the colours of the cars passing his school. Estimate the probability, as a decimal, that the next car passing Jamal's school will be:

Colour	Silver	Black	Red	Blue	Other
Frequency	452	124	237	98	89

a) silver

b) red

c) not silver, black, red or blue

- Q4 Jack and his dad have played tennis against each other 15 times. Jack has won 8 times.
  - a) Estimate the probability that Jack will win the next time they play.
  - b) Estimate the probability that Jack's dad will win the next time they play.
- Q5 Describe how Lilia could estimate the probability that the football team she supports will win a match.

## **Expected Frequency**

Learning Objective — Spec Ref P2/P3:

Find the expected frequency of an event.

If you know (or have an estimate of) the **probability** of an event, you can work out how many times you would **expect** the event to happen in a given number of experiments. This isn't always going to happen in reality — it's only an **estimate** of roughly how many times you think it will occur.

Expected frequency = number of times the experiment is done × probability of the event happening

## Example 3

Ezra rolls a dice 60 times. Estimate the number of times he would expect to roll a 1, if the dice he is rolling is:

a) biased with P(1) = 0.2

Multiply the number of rolls by the probability of rolling a 1.

Expected frequency =  $60 \times 0.2 = 12$  times

b) fair, 20-sided and numbered 1-20

2. Find the expected frequency.

1. Work out the probability of rolling a 1.

 $P(1) = \frac{1}{20}$ Expected frequency =  $60 \times \frac{1}{20} = 3$  times

- Q1 The probability that a biased dice lands on 4 is 0.75. How many times would you expect to roll 4 in:
  - a) 20 rolls?
- b) 60 rolls?
- c) 100 rolls?
- d) 1000 rolls?

- Q2 The spinner on the right has three equal sections. How many times would you expect to spin 'penguin' in:
  - a) 60 spins?
- b) 300 spins?
- c) 480 spins?



- Q3 A fair, six-sided dice is rolled 120 times. How many times would you expect to roll:
  - a) a 5?
- b) a 6?

- c) an even number?
- d) higher than 1?

## Frequency Trees

#### Learning Objective — Spec Ref PI:

Use frequency trees to display outcomes and to find probabilities.

When you have data relating to **multiple events**, you can use a **frequency tree** to record it. The **branches** of a frequency tree show the **different possible outcomes** of each event, and the **numbers** at the end of the branches show **how many times** that event or **combination** of events happened.

## Example 4

James asks 200 people of various ages to roll a dice. There were 120 people under the age of 20 and 16 of them rolled a six. Of the people aged 20 and over, 18 rolled a six.

Draw a frequency tree to show this information.

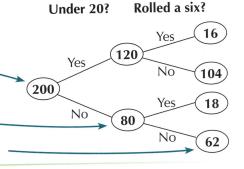
. Draw branches to show the different possibilities.

- Fill in the bits of the tree given in the question first
   — the total number of people goes at the start.
- 3. The numbers at the end of a set of branches should always add up to the number at the start of the branches.

200 - 120 = 80 people are aged 20 and over -

120 - 16 = 104 people under 20 didn't roll a six

80 - 18 = 62 people aged 20 and over didn't roll a six



Rainy?

30

Windy?

Yes

No

Yes

No

11

## Example 5

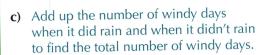
The frequency tree on the right shows weather data for Statston collected on 30 random days of the year. Use the frequency tree to estimate the probability of any day in Statston being:

- a) rainy
- b) rainy and windy
- c) windy

There were 12 rainy days So P(rainy) =  $\frac{12}{30} = \frac{2}{5}$ 

**b)** Follow the top branches to find the number of days that were both rainy and windy.

8 days were both rainy and windy So P(rainy and windy) =  $\frac{8}{30} = \frac{4}{15}$ 



Read the number of rainy days from

the frequency tree and use the relative

frequency to estimate the probability.

Number of windy days = 8 + 7 = 15So P(windy) =  $\frac{15}{30} = \frac{1}{2}$ 

## Exercise 3

Q1 Information on 800 UK residents' hair and eye colour was collected. The data was recorded in the two-way table shown on the right.

Has brown hair?

Yes No

Has brown eyes?

No 335 140

- a) Show this information on a frequency tree.
- b) Find the relative frequency of someone having brown hair and brown eyes.
- c) If you collected the same data on another group of 2000 UK residents, how many would you estimate won't have brown eyes or brown hair?

720 truck drivers were asked to do an eye test. 640 said their vision was fine. 180 of the drivers Q2 who said their vision was fine failed the eye test. 30 of the drivers who said their vision wasn't fine passed the eye test. Draw and complete a frequency tree to represent this information.

## Fair or Biased?

#### Learning Objective — Spec Ref P2:

Use experimental data to decide if outcomes are fair or biased.

Things like dice and spinners are fair if they have the same chance of landing on each side or section. If they're more likely to give some outcomes than others, then they're called biased. To decide whether something is fair or biased, you need to do an experiment. You can then compare the experimental results with what you would expect in theory.

For example, if you flip a coin, you expect it to land on heads about half of the time so if you flipped it 100 times, you'd expect about 50 heads. If you actually got 80 heads, you'd be pretty convinced that the coin was biased.

#### Example 6

The table shows the results of 60 rolls of a 6-sided dice.

a) Calculate the relative frequency of each outcome, giving your answers as decimals.

Score	1	2	3	4	5	6
Frequency	12	3	9	15	9	12

score 
$$1 = \frac{12}{60} = 0.2$$

score 
$$2 = \frac{3}{60} = 0.05$$

score 
$$3 = \frac{9}{60} = 0.15$$

score 
$$4 = \frac{15}{60} = 0.2$$

score 
$$5 = \frac{9}{60} = 0.1$$

score 
$$6 = \frac{12}{60} = 0.2$$

b) Do you think the dice is fair or biased? Explain your answer.

Compare the relative frequencies to the theoretical probability of  $\frac{1}{6} = 0.166...$  The relative frequency of a score of 2 is v different from the theoretical probability,

The relative frequency of a score of 2 is very so the experiment suggests that the dice is biased.

- A spinner has four sections: blue, green, white and pink. The table shows the results of 100 spins. Q1
  - a) Work out the relative frequencies of the four colours.
  - b) Explain whether you think the spinner is fair or biased.

Colour	Blue	Green	White	Pink
Frequency	22	21	18	39

- A six-sided dice is rolled 120 times and 4 comes up 32 times. Q2
  - a) How many times would you expect 4 to come up on a fair dice in 120 rolls?
  - b) Use your answer to part a) to explain whether you think the dice is fair or biased.
- Three friends each toss a coin and record the number of heads they get. Q3 The table shows their results.
  - a) Copy and complete the table.
  - b) Explain whose results are the most reliable.
  - c) Explain whether you think the coin is fair or biased.

		Hal
20	60	100
12	33	49

# Review Exercise

Q1 280 pupils have to choose whether to study history or geography. The numbers choosing each subject are shown in the table.

A pupil is chosen at random. Find the probability that the pupil is:

	History	Geography
Girls	77	56
Boys	63	84

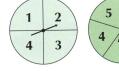
- a) a girl studying geography
- b) a boy studying geography
- c) a girl

d) a boy

- e) studying history
- f) not studying history
- Amy's CD collection is organised into four categories groups, male vocal, female vocal and compilations. The table shows the probability that she randomly picks a CD of each category.

Category	Groups	Male vocal	Female vocal	Compilations
Probability		0.15	0.1	

- a) Find the missing probability from the table.
- b) Amy picks one CD at random. What is the probability that it isn't a male vocal CD?
- c) Given that Amy has 80 CDs in total, how many female vocal CDs does she have?
- How many different 4 digit even numbers are there where the first digit is odd and the second digit is a multiple of 3?
- Q4 The probability that a particular component produced in a factory is faulty is 0.002.
  - a) What is the probability that the component is not faulty?
  - b) 60 000 of these components are produced in a week. How many of the components would you expect to be: (i) faulty? (ii) not faulty?
- Q5 The two fair spinners shown on the right are both spun once.
  - a) Show all the possible outcomes using a sample space diagram.



- b) Find the probabilities of spinning the following:
  - (i) double 4

(ii) a 2 and a 3

(iii) both numbers the same

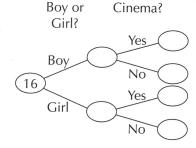
- (iv) two different numbers
- (v) at least one 5
- (vi) no fives
- c) If the five-sided spinner is spun 100 times, how many times would you expect to spin 3?
- Q6 The results of rolling a four-sided dice 200 times are shown in the table on the right.

Score	1	2	3	4
Frequency	56	34	54	56

- a) Work out the relative frequencies of the dice scores.
- b) Explain whether these results suggest that the dice is fair or biased.
- c) Estimate the probability or rolling a 1 or a 2 with this dice.

# **Exam-Style Questions**

- Q1 lan asked 16 students in one of the Year 11 classes at his school whether they went to the cinema last week. 5 boys said they did. The other 2 boys said they did not. Two thirds of the girls said they did whilst the other girls said they did not.
  - a) Copy and complete the frequency tree below to show lan's data.



[3 marks]

b) If one of the 16 students is picked at random, work out the probability that they went to the cinema last week.

[2 marks]

- Q2 Ellie is playing a game where she has to roll a 6-sided dice, with sides labelled A, B, C, D, E and F. She rolls the dice 4 times in a row to make a 4-letter sequence (the first result is the first letter, the second result is the second letter, etc.).
  - a) How many different 4-letter sequences can she make?

[2 marks]

b) What is the probability that she gets the sequence 'BEAD'?

[1 mark]

Ellie thinks the dice might not be fair. She writes down the next 10 sequences she makes: CEAA, ABAA, ACFD, AECE, AFAC, DAEA, DFAE, ADED, AABF, CCAC

c) Copy and complete this table showing the frequency and relative frequency of each letter.

Letter	Α	В	С	D	Е	F
Frequency						
Relative Frequency						

[3 marks]

d) Do you think the dice is fair or biased? Explain your answer.

[2 marks]

e) If the dice was rolled another 200 times, estimate how many more times it would land on A than B.

[2 marks]