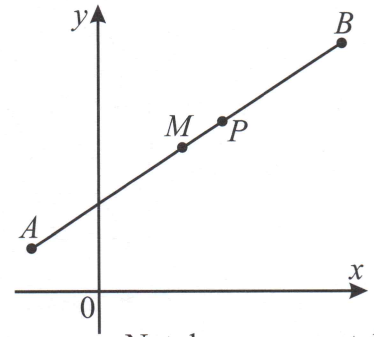
Coordinates and Ratio

1 The line segment AB is shown below. M is the midpoint of AB and has coordinates (4, 5).

The coordinates of point A are (–2, 2).
a) Find the coordinates of point B.



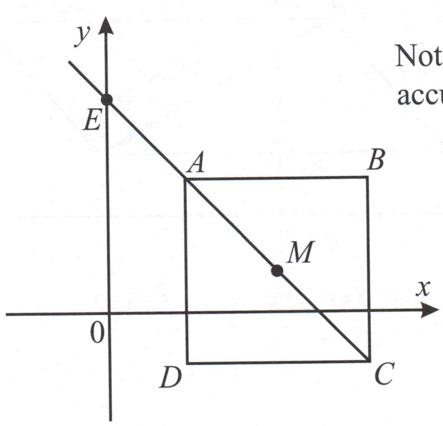
Not drawn accurately

(..................... , ...)
[2]

Point P lies on the line segment AB such that $MP:PB = 1:2$.
b) Find the ratio $AP:AB$.

....
[3]
[Total 5 marks]

2 $ABCD$ is a square with side length 4 units. The coordinates of point D are (2, –1). M is the centre of the square and point E has coordinates (0, 5).



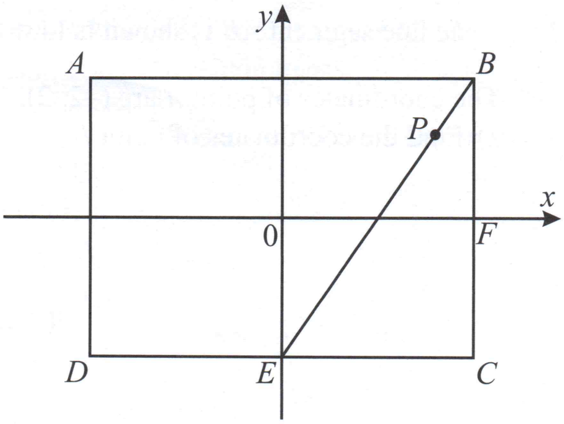
Not drawn accurately

Find the ratio $EM:MC$. Give your answer in its simplest form.

....
[Total 3 marks]

3 The diagram shows rectangle $ABCD$. Point E has coordinates (0, –4), point F has coordinates (6, 0) and point B has coordinates (6, 4).

Not drawn accurately



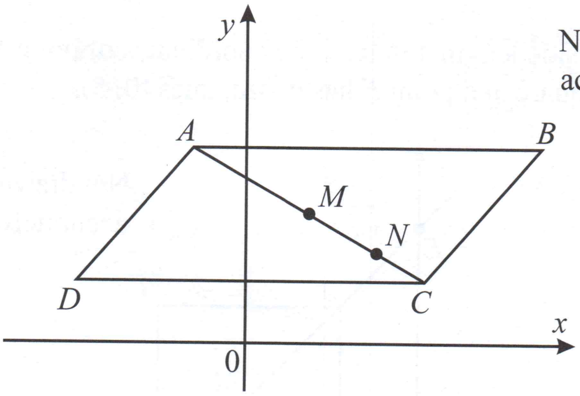
P is the point on the line EB such that $EP:PB = 3:1$.
Calculate the length of the line segment PF.

....

[Total 4 marks]

4 $ABCD$ is a parallelogram. The coordinates of point A are (–2, 7) and the coordinates of point D are (–5, 2). M is the midpoint of line AC and has coordinates (3, 4.5).

Not drawn accurately



Point N lies on the line AC such that $AM:MN:NC = 5:3:2$.
Find the exact length NB.

....

[Total 6 marks]

Score: $\frac{\quad}{18}$

Perpendicular Lines

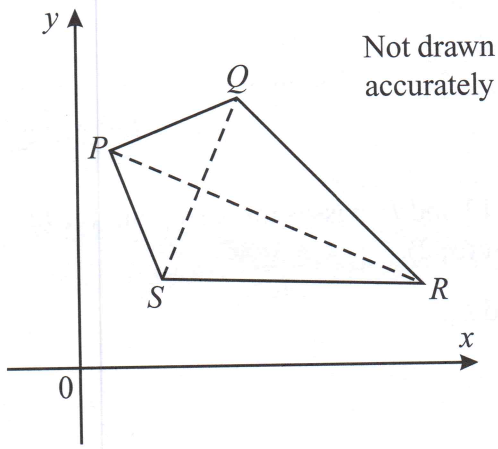
1 Line L_1 passes through the points (4, 6) and (11, 20).
Line L_2 is perpendicular to L_1 and intersects the x-axis at (28, 0).

Find the equation of line L_2.

....

[Total 3 marks]

2 The line SQ is a diagonal of the kite $PQRS$ and has equation $y = 4x - 3$.
The coordinates of point R are (8, 15). Find the equation of the other diagonal PR.



....

[Total 3 marks]

3 Lines L_1 and L_2 are perpendicular and intersect at point M.
L_1 has equation $x + 5y = 100$ and L_2 passes through point (2, 4).

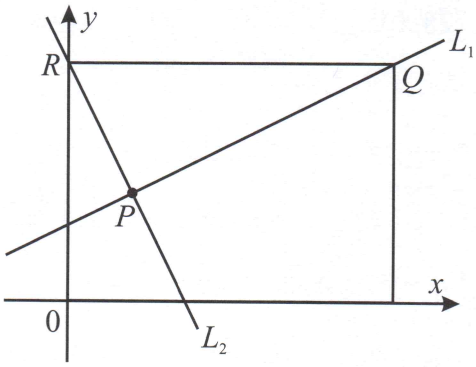
Find the coordinates of point M.

(.................... , ...)

[Total 5 marks]

2y − x = 4 Edex

4 The line L_1 has equation $2y - x = 14$ and passes through the points $P(6, 10)$ and Q. L_2 is the line that is perpendicular to L_1 and passes through point P. L_2 intercepts the y-axis at R. RQ is horizontal.



Not drawn accurately

Find the coordinates of Q.

(.................... , ...)

[Total 5 marks]

5 Lines L_1 and L_2 are parallel. L_1 has equation $2x + 3y = 12$ and L_2 passes through point (6, 13). Line L_3 is perpendicular to L_1 and L_2 and intersects L_1 at (3, 2).

Find the coordinates of the point of intersection of L_2 and L_3.

(.................... , ...)

[Total 6 marks]

Exam Practice Tip

Remember — the gradients of two perpendicular lines multiply to give –1. Once you know that, use whatever information you're given to find the equation of the line. You sometimes have to do quite a bit of work to find the equation, so if you're asked to find a point, don't forget to do the final step and find the coordinates.

Score

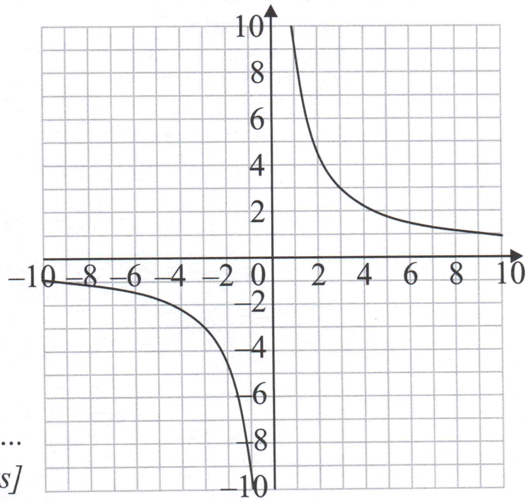
22

Harder Graphs

1 The graph shows the curve $y = \frac{9}{x}$.

Find the smallest possible distance between the two sections of the graph. Give your answer as a simplified surd.

The closest points will lie on the line $y = x$.



....

[Total 3 marks]

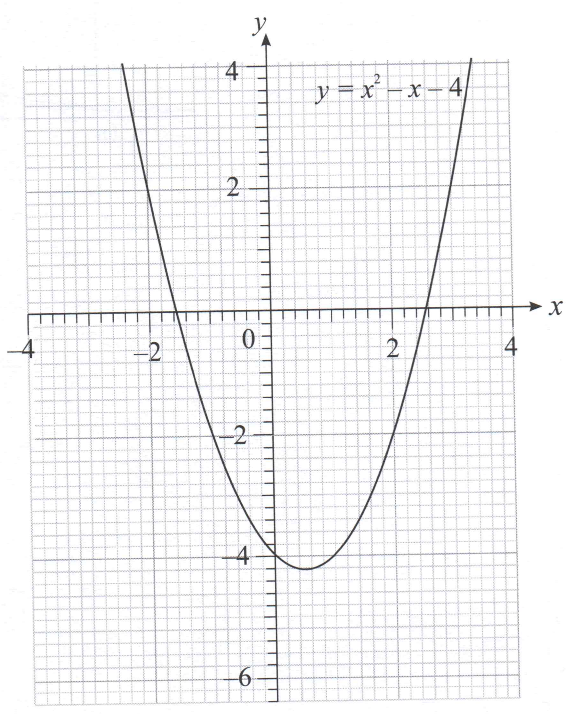
2 The point (7, 24) lies on a circle with centre (0, 0). Find the radius and equation of the circle.

Radius = ...

Equation: ....

[Total 2 marks]

3 The graph of the curve $y = x^2 - x - 4$ is shown. Use the graph to estimate the solutions to $x^2 + x = 1$.



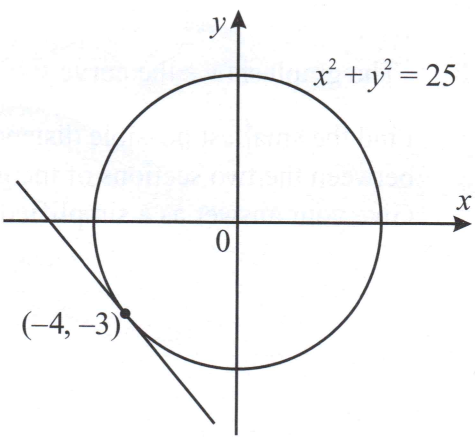
x = ...

x = ...

[Total 4 marks]

4 Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(-4, -3)$.

Start by finding the equation of the radius that goes from (0, 0) to (–4, –3).

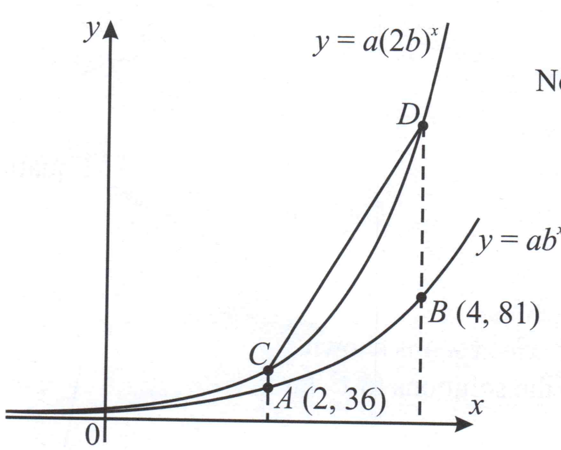


....

[Total 3 marks]

5 The diagram shows the curves $y = ab^x$ and $y = a(2b)^x$, where a and b are constants. The curve $y = ab^x$ passes through the points $A(2, 36)$ and $B(4, 81)$. The curve $y = a(2b)^x$ passes through the points C and D. C and D lie vertically above A and B.

Calculate the gradient of the line segment CD.



Not drawn accurately

....

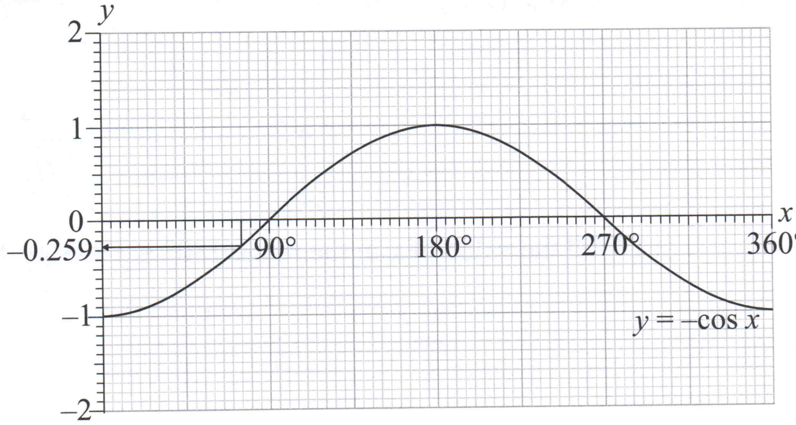
[Total 5 marks]

Score: /17

Trig Graphs

1 The graph of $y = -\cos x$ is shown below for $0° \leq x \leq 360°$.

As shown on the graph,
$-\cos 75° = -0.259$.

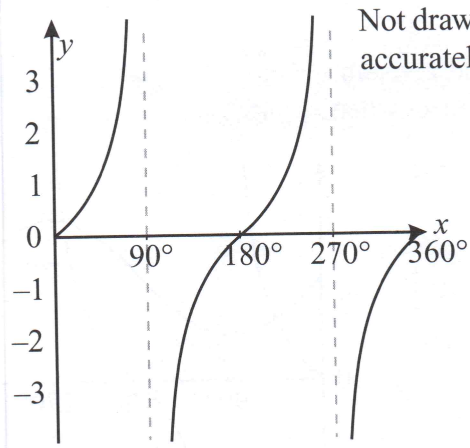


Give another value of x, found on this graph, where $-\cos x = -0.259$.

$x =$ .... °

[Total 1 mark]

2 The diagram shows a sketch of $y = \tan x$ for $0° \leq x \leq 360°$.



Not drawn accurately

$\tan 105° = -3.732$.

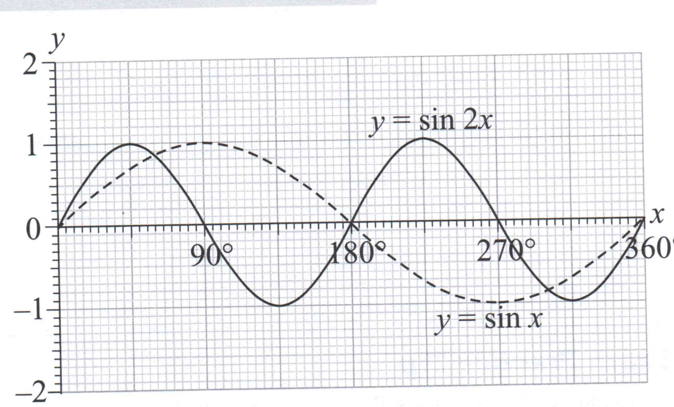
Write down the two solutions to the equation $\tan x = 3.732$ for $0° \leq x \leq 360°$.

$x =$ .... ° and $x =$ .... °

[Total 2 marks]

3 The graphs of $y = \sin x$ and $y = \sin 2x$ for $0° \leq x \leq 360°$ are shown below.

For the graph of $y = \sin 2x + 3$, find the exact y-value when $x = 22.5°$

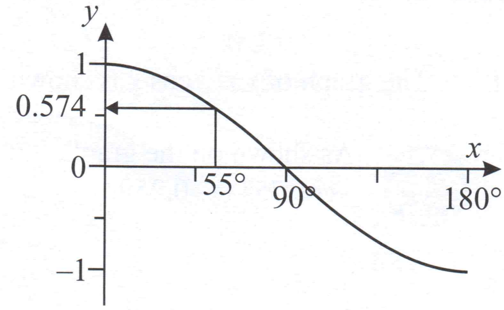


$y =$ ....

[Total 2 marks]

4 The diagram shows a sketch of $y = \cos x$ for $0° \leq x \leq 180°$.

As shown on the graph, $\cos 55° = 0.574$.



a) Find the value of x in the range $0° \leq x \leq 180°$ for which $\cos x = -0.574$.

$x =$ .... °

[1]

b) Find the value of x in the range $180° \leq x \leq 360°$ for which $\cos x = -0.574$.

$x =$ .... °

[1]

c) Find the value of x in the range $-180° \leq x \leq 0°$ for which $\cos x = 0.574$.

$x =$ .... °

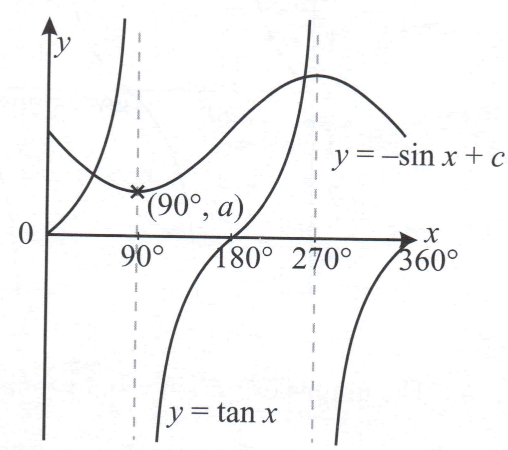
[1]

[Total 3 marks]

5 The sketch below shows the graphs of $y = \tan x$ and $y = -\sin x + c$, where c is a positive number. The two graphs intersect when $x = 45°$.

The point $(90°, a)$ lies on the curve $y = -\sin x + c$. Work out the exact value of a.

Remember the common trig values — they'll come in handy for this question.



Not drawn accurately

$a =$ ....

[Total 4 marks]

Exam Practice Tip

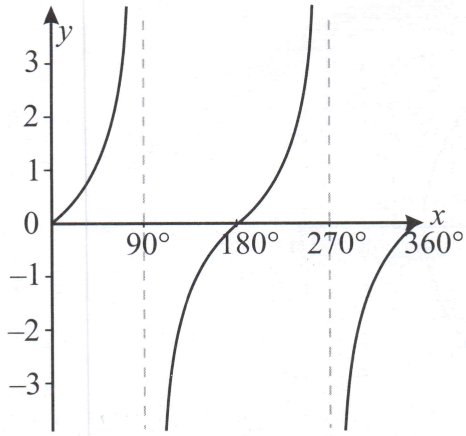
It's really, really important that you know all the properties of the sin, cos and tan graphs — their shapes, where they cross the x- and y-axes, any symmetry they have, where the pattern repeats etc. If you're not given the graph over a big enough range to solve the question, you can always draw a quick sketch to help you.

Score

/12

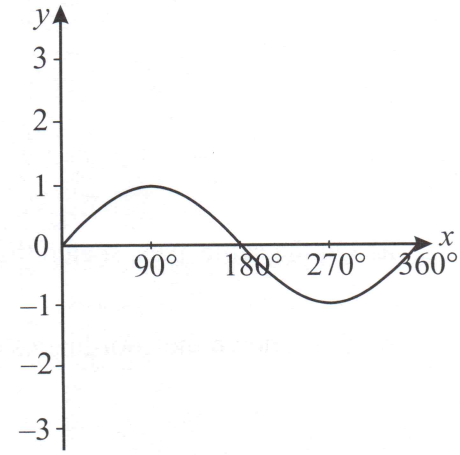
Graph Transformations

1 For parts a) and b) below, draw the transformed graphs on the same axes as the original graphs.



a) The graph on the left shows a sketch of $y = \tan x$ for $0° \leq x \leq 360°$. Sketch the graph of $y = -\tan x$.

[1]

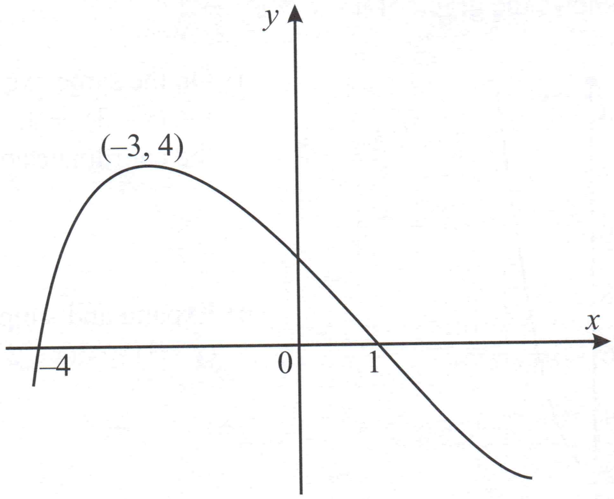


b) The graph on the right shows a sketch of $y = \sin x$ for $0° \leq x \leq 360°$. Sketch the graph of $y = \sin x + 2$.

[1]

[Total 2 marks]

2 The diagram shows a sketch of $y = f(x)$, which crosses the x-axis at –4 and 1, and has a turning point at (–3, 4).



a) On the same axes, sketch the graph of $y = f(-x)$, labelling the turning point and where it crosses the x-axis.

[3]

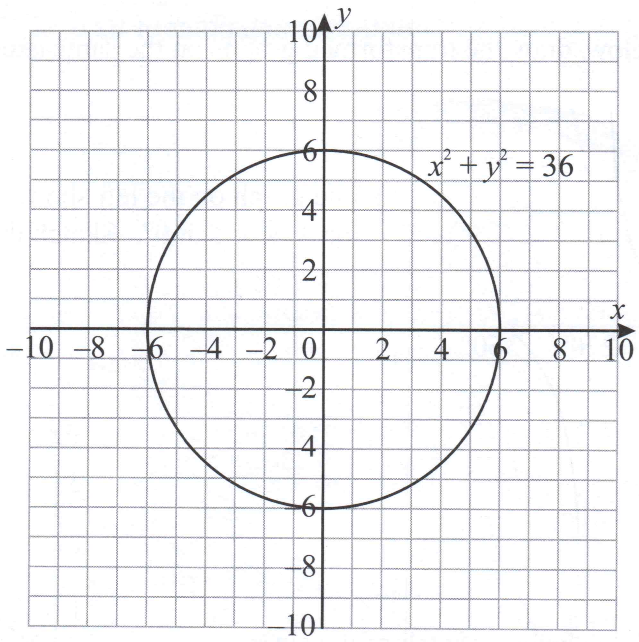
b) Write down the coordinates of the turning point of $y = f(x + 3) + 2$.

(..................... , ...)

[2]

[Total 5 marks]

3 The circle $x^2 + y^2 = 36$ is shown below.



Don't be put off by the fact that the equation isn't in the form $y = f(x)$ — you can use the transformation rules in the same way.

a) On the same axes, sketch the graph of $(x - 2)^2 + y^2 = 36$.

[2]

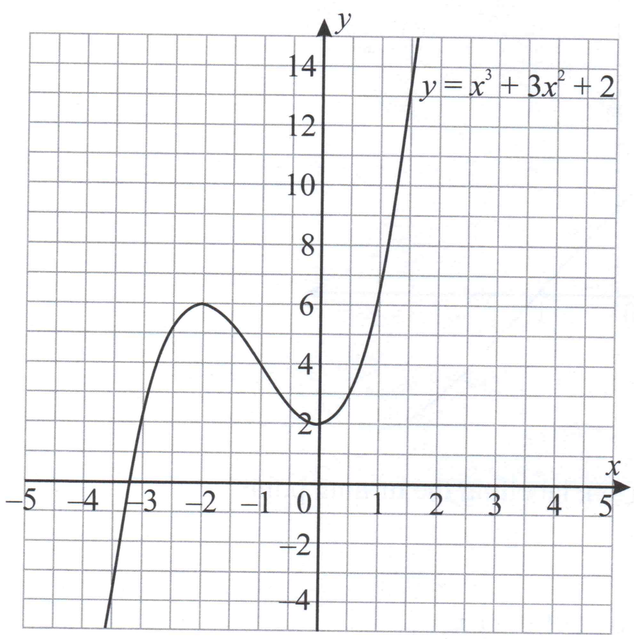
b) Write down the coordinates of the centre of the graph $(x + 6)^2 + y^2 = 36$.

(.................... , ...)

[1]

[Total 3 marks]

4 The diagram below shows the graph of $y = x^3 + 3x^2 + 2$.



a) On the same axes, draw the graph of $y = (x - 3)^3 + 3(x - 3)^2 + 1$, showing clearly the coordinates of any turning points.

[3]

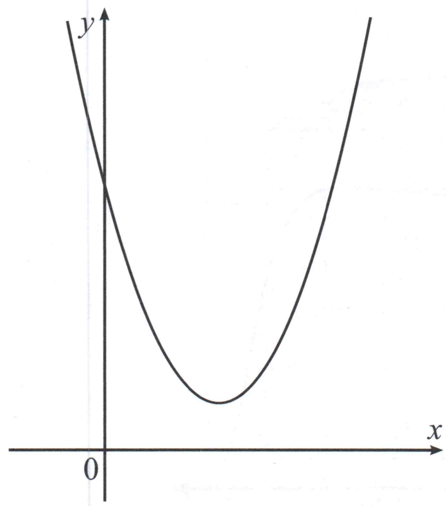
b) Expand and simplify $(x - 3)^3 + 3(x - 3)^2 + 1$.

....

[4]

[Total 7 marks]

5 The diagram shows a sketch of $y = f(x)$, where $f(x) = x^2 - 5x + 7$.



a) By completing the square, find the coordinates of the turning point of $f(x)$.

(..................... , ...)

[3]

b) Hence find the coordinates of the turning point of $y = f(x + 3) - 2$.

(..................... , ...)

[2]

c) Find the x-values of the points where the graph of $y = f(x + 3) - 2$ intersects the x-axis.

$x =$ .... and $x =$ ....

[3]

[Total 8 marks]

6 The graph $y = \frac{6}{x}$ is transformed into the graph of $y = \frac{3x}{x-2}$.

a) Show that $\frac{ab}{x-a} + b \equiv \frac{bx}{x-a}$.

[2]

b) Describe the transformation that maps the graph of $y = \frac{6}{x}$ to the graph of $y = \frac{3x}{x-2}$.

....

....

Start by using the identity in part a) to rewrite the second equation in part b).

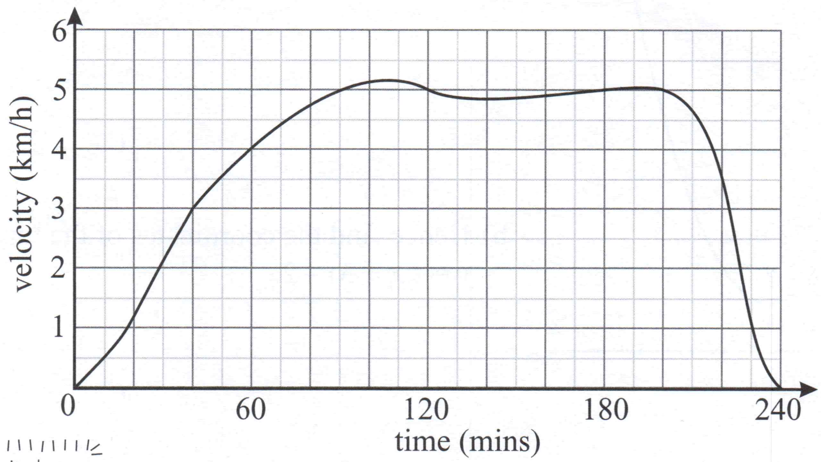
[3]

[Total 5 marks]

Score: ☐ / **30**

Velocity-Time Graphs

1 Estimate the distance covered by the tractor whose journey is shown on the velocity-time graph below.



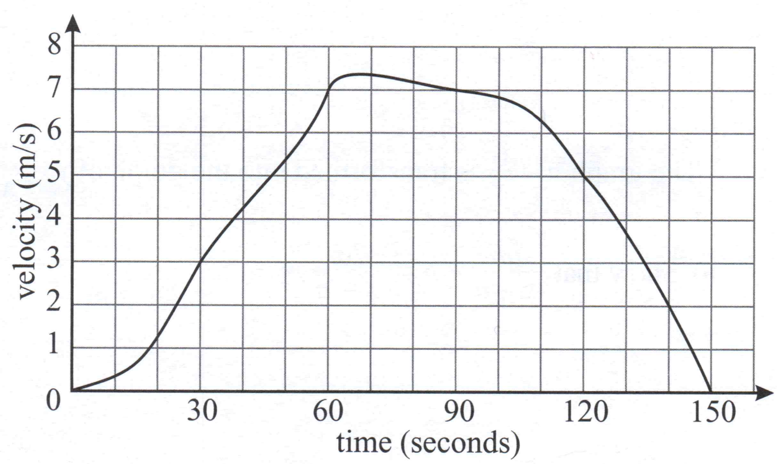
Be careful with the units here.

.... km

[Total 3 marks]

2 This velocity-time graph models the first 150 seconds of a journey.

a) Calculate an estimate for the distance travelled during these 150 seconds.



.... m

[3]

b) Is your answer to part a) an underestimate or an overestimate? Give a reason for your answer.

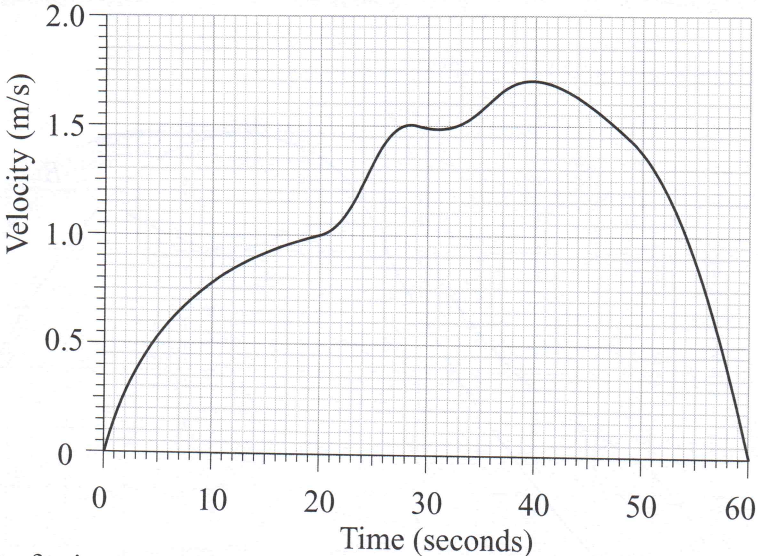
....

....

[2]

[Total 5 marks]

3 A go-kart's velocity over one lap is plotted on the velocity-time graph below.



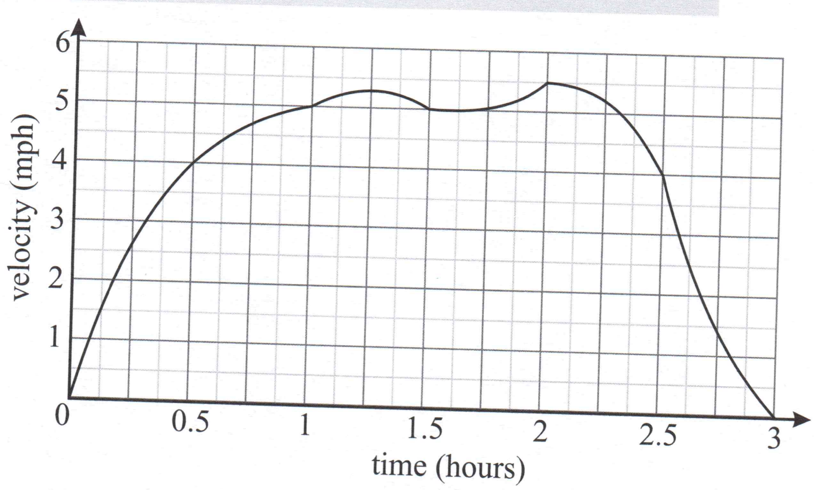
Calculate an estimate for its average speed.

You'll need to find the total distance the go-kart travelled first.

.... m/s

[Total 4 marks]

4 The velocity-time graph below shows the journey of a hot-air balloon.

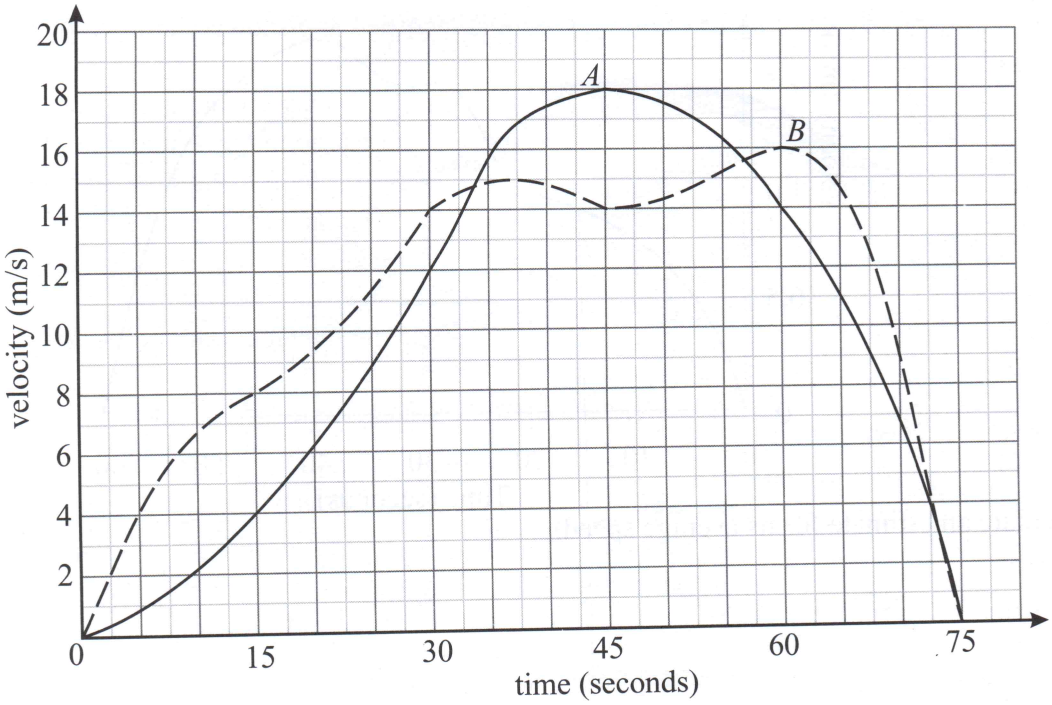


Find the percentage decrease between the estimates for the total distance travelled when the area is split into six strips and into three strips of equal width. Give your answer to 1 decimal place.

.... %

[Total 6 marks]

5 The velocity-time graph below shows the journeys of two different objects, A and B. Object A's journey is shown by the solid curve and object B's journey is shown by the dashed curve.



By dividing each area into 5 strips of equal width, calculate the ratio of the estimated average speed of object A to the estimated average speed of object B. Give your answer in its simplest form.

....

[Total 6 marks]

Exam Practice Tip

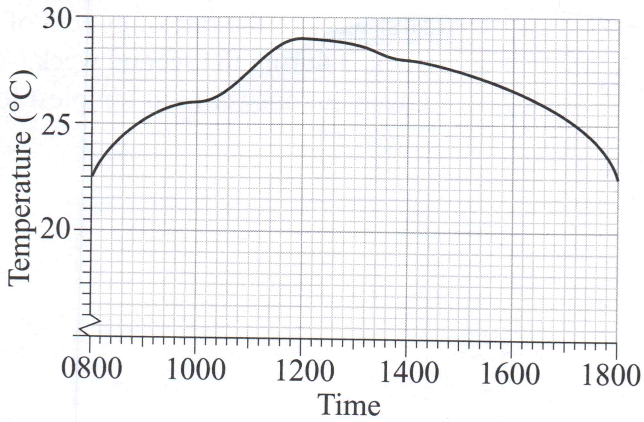
Finding the area under velocity-time graphs is much harder if the graphs are curved, not straight. You can't find the exact area, only an estimate by dividing it up into triangles, rectangles and trapeziums. Remember, the gradient of the line shows the acceleration — and a negative gradient means the object is slowing down.

Score

24

Gradients

1 The temperature of an indoor swimming pool is recorded and shown on the graph below.



The manager claims that the average rate of change of the temperature between 8 am and 12 noon is the same as the average rate of change of the temperature between 4 pm and 6 pm.

Is the manager correct? Explain your answer.

....

....

....

....

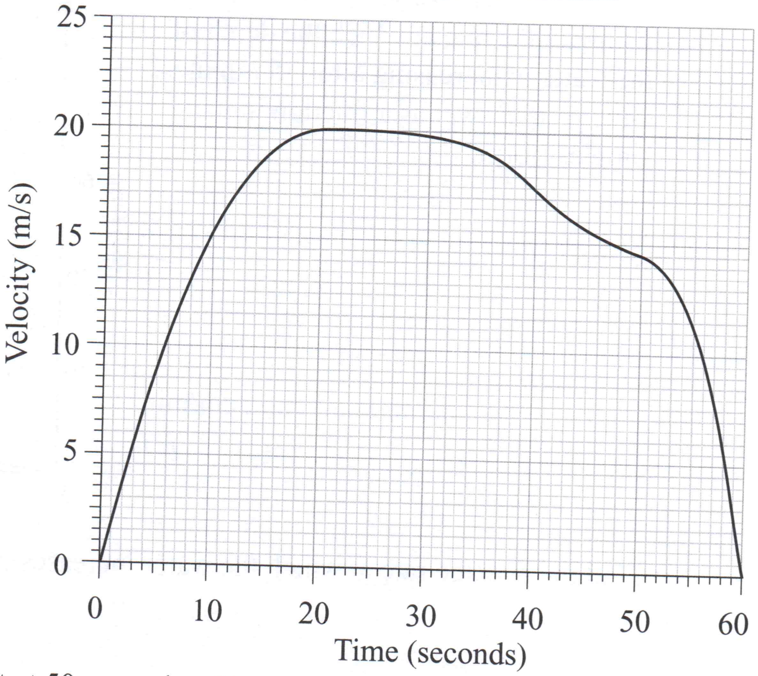
[Total 3 marks]

2 The first 60 seconds of a cyclist's journey are shown on the velocity-time graph below.

a) Find the average acceleration of the cyclist between 10 and 20 seconds.

.... m/s^2

[2]



b) Estimate the acceleration of the cyclist at 50 seconds. Give your answer to 3 s.f.

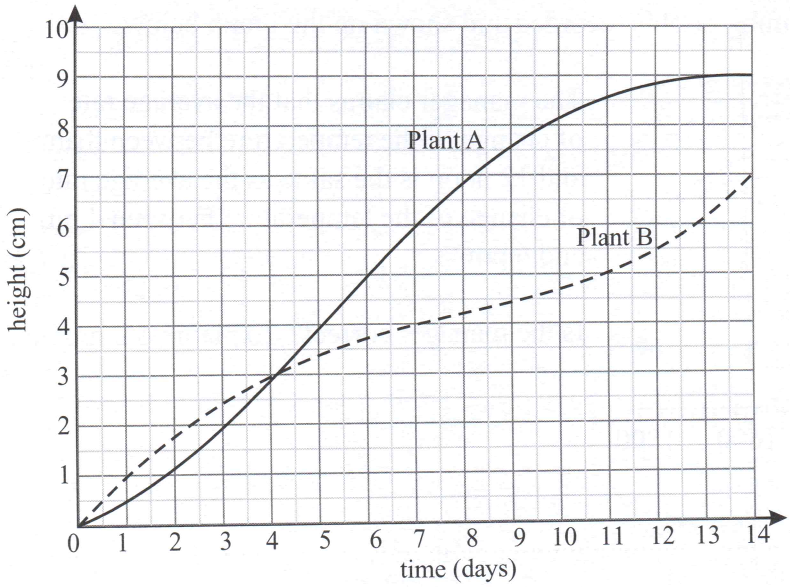
Draw a tangent to the curve at 50 seconds.

.... m/s^2

[2]

[Total 4 marks]

3 A scientist measures the growth of two different plants over two weeks. Her results are shown in the graph below.



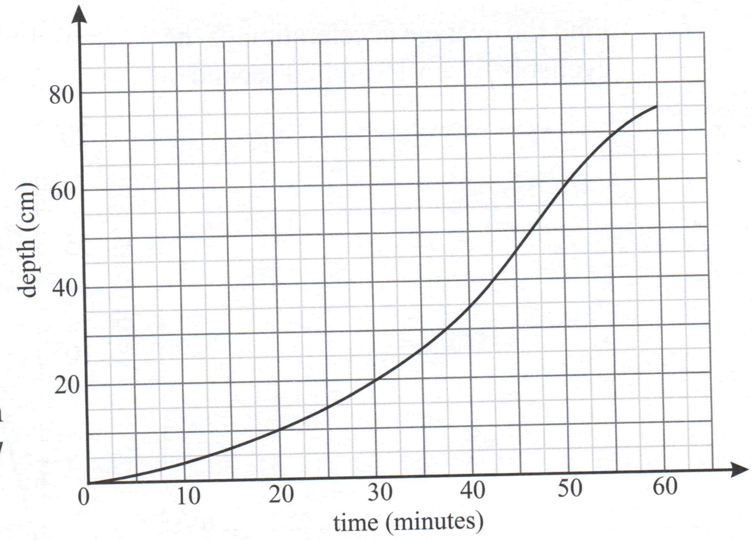
Find an estimate of the ratio of the rate of growth of plant A during the first week to the rate of growth of plant B during the second week. Give your answer in its simplest form.

....

[Total 3 marks]

4 The graph shows the depth of water in a container.

a) Estimate the rate at which the depth of the water is increasing after 35 minutes. Give your answer as a fraction in its simplest form.



.... cm/min

[2]

b) Find the average rate at which the water increases over the 60 minute period.

.... cm/min

[2]

[Total 4 marks]

Score: ☐ / **14**