Section 29 Transformations

29.1 Reflections

Transformations can be used to move and resize shapes on a coordinate grid. In this section, you'll meet four different transformations — the first of which is reflection. To reflect a shape, you draw its mirror image.

Learning Objectives — Spec Ref G7:
- Reflect a shape in a given line.
- Describe the reflection that transforms a shape.

To **reflect** a shape, first reflect the **vertices** of the shape in the line of symmetry (also known as the **mirror line**). The **reflected points** (called the **image points**) should be the **same distance** from the line as the original points but on the **other side** of it. Then **join up** the image points to create the reflected shape. The reflected shape will be the **same size** and **shape** as the original — so the shapes are **congruent** (see p.394).

A reflection in the **y-axis** will send a point (x, y) to $\mathbf{(-x, y)}$ and a reflection in the **x-axis** will send a point (x, y) to $\mathbf{(x, -y)}$.

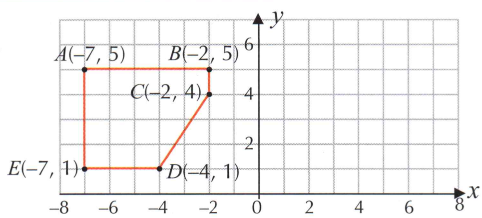
An **invariant point** is one that **doesn't move** under a transformation. For reflections, all the points on the **mirror line** are invariant.

To **describe** a reflection, you just need to find the **equation** of the mirror line.

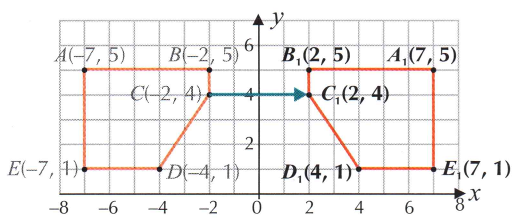
Tip: Mirror lines of the form $x = a$ are vertical and mirror lines of the form $y = a$ are horizontal. See page 177 for more on lines parallel to the axes.

Example 1

Reflect the shape *ABCDE* in the y-axis. Label the image points A_1, B_1, C_1, D_1 and E_1 with their coordinates.

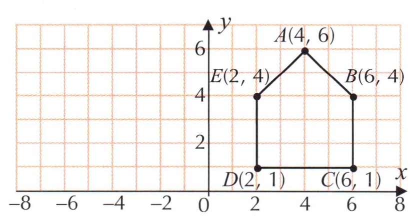


1. Reflect the shape — one vertex at a time.
2. Each image point should be the same distance from the y-axis as the original point. E.g. C is 2 units to the left of the y-axis so its image C_1 should be 2 units to the right of the y-axis.
3. Write down the coordinates of each of the image points. Each point (x, y) becomes $(-x, y)$.



Exercise 1

Q1 Copy the diagram on the right and reflect the shape in the y-axis. Label the image points A_1, B_1, C_1, D_1, E_1 with their coordinates.

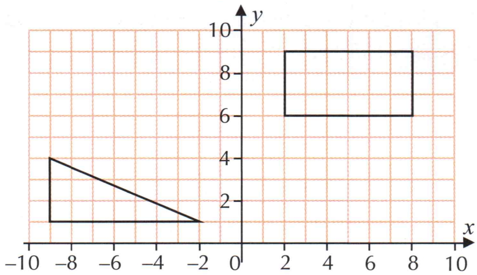


Q2 The following points are reflected in the y-axis. Find the coordinates of the image points.

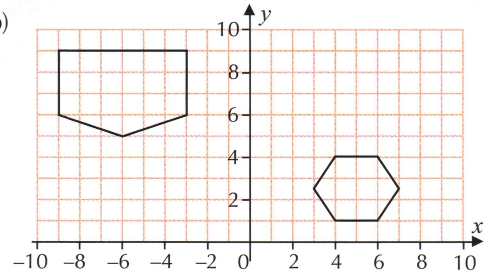
a) (4, 5) b) (7, 2) c) (–1, 3) d) (–3, –1)

Q3 Copy each of the diagrams below and reflect the shapes in the y-axis.

a)



b)

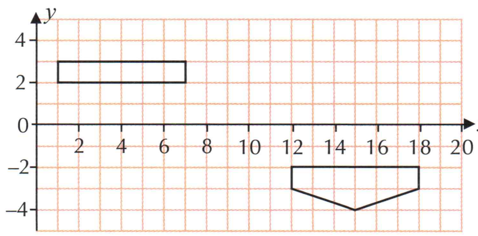


Q4 The following points are reflected in the x-axis. Find the coordinates of the image points.

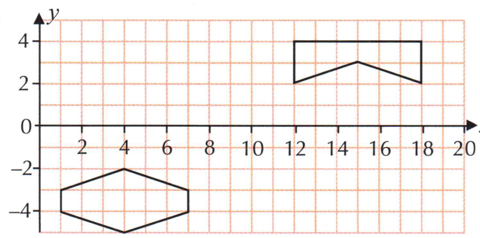
a) (1, 2) b) (3, 0) c) (–2, 4) d) (–1, –3) e) (–2, –2)

Q5 Copy each of the diagrams below and reflect the shapes in the x-axis.

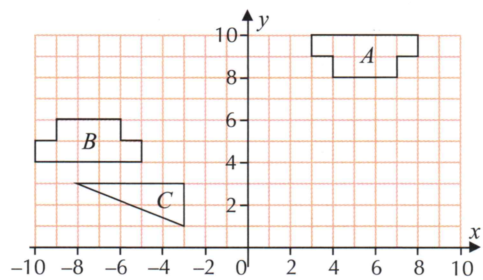
a)



b)



Q6 Copy the diagram shown on the right.



a) Reflect shape A in the line $x = 2$. Label the image A_1.

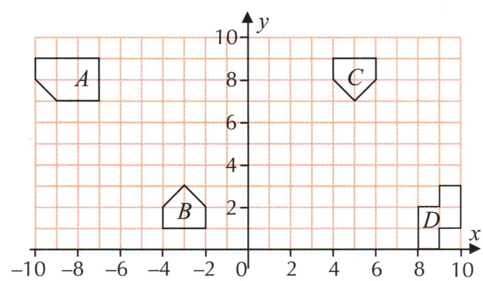
b) Reflect shape B in the line $x = –1$. Label the image B_1.

c) Reflect shape C in the line $x = 1$. Label the image C_1.

d) Shape B_1 is the reflection of shape A in which mirror line?

e) Give the coordinates of the two invariant points on Shape B when it is reflected in the line $x = –7$.

Q7 Copy the diagram shown on the right.



a) Reflect shape A in the line $y = 6$. Label the image A_1.

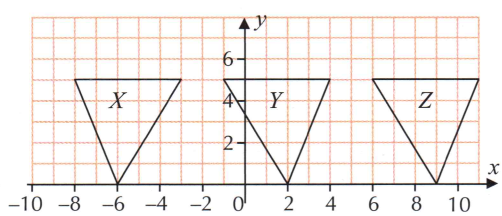
b) Reflect shape B in the line $y = 4$. Label the image B_1.

c) Reflect shape C in the line $y = 5$. Label the image C_1.

d) Reflect shape D in the line $y = 3$. Label the image D_1.

e) Shape C_1 is the reflection of shape B in which mirror line?

Q8



a) Describe the reflection that would take triangle X to the triangle Y.

b) The image of triangle Z under a single reflection is either triangle X or triangle Y. Determine which and describe the reflection. PROBLEM SOLVING

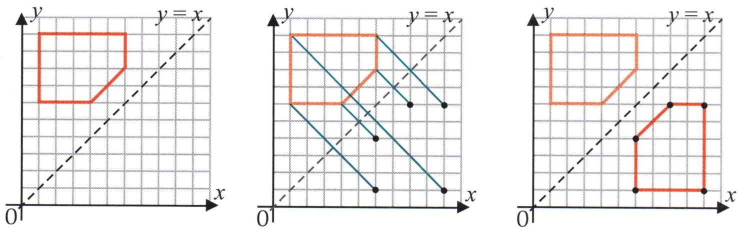
c) Which points of triangle X are invariant if it is reflected in the line $y = 5$?

The mirror line isn't always one of the axes or a line parallel to an axis — it could be a **diagonal** line such as $y = x$ or $y = -x$. A reflection in $\boldsymbol{y = x}$ sends (x, y) to $\boldsymbol{(y, x)}$ and a reflection in $\boldsymbol{y = -x}$ sends (x, y) to $\boldsymbol{(-y, -x)}$. Follow the same method of **reflecting each vertex** and then joining up the **image points**.
Make sure the image points are the **same distance** from the mirror line as the original ones — you measure this distance **perpendicular** to the mirror line.

Example 2

Reflect the shape in the line $y = x$ on the grid below.

1. Reflect the shape one vertex at a time.
2. Join the image points. They should be the same perpendicular distance from the line $y = x$ as the original points.



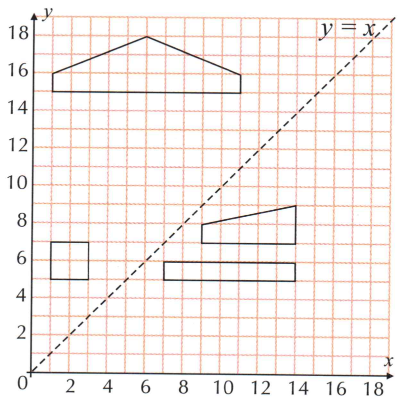
Exercise 2

Q1 The points below are reflected in the line $y = x$. Find the coordinates of their reflections.

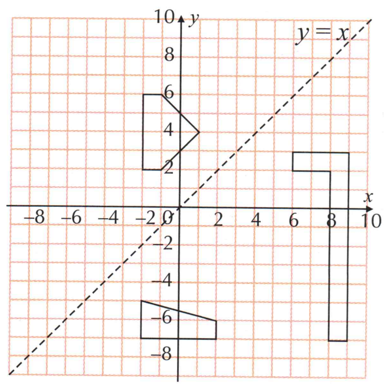
a) (1, 2) b) (3, 0) c) (–2, 4) d) (–1, –3) e) (–2, –2)

Q2 Copy each of the diagrams below and reflect the shapes in the line $y = x$.

a)



b)



Questions 3 and 4 refer to the diagram on the right.

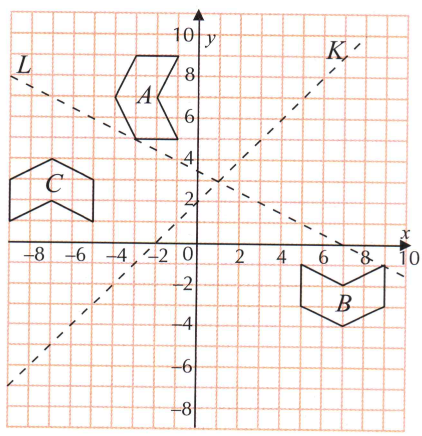
Q3 Describe the reflection that would map:

a) Shape A onto shape B.

b) Shape C onto shape A.

Q4 a) Copy the diagram and reflect each of the shapes A, B and C in line K. Label them A_1, B_1 and C_1 respectively.

b) Give the points of invariance on shapes A, B or C when they are reflected in line L.



29.2 Rotations

The second transformation to learn about is rotation. Rotations spin everything around a fixed point.

Learning Objectives — Spec Ref G7:
- Rotate a shape around a given point.
- Describe the rotation that transforms a shape.

When an object is **rotated** about a point, its size and shape stay the same — so the new shape is **congruent** to the original shape. Also, the **distance** of each vertex from the centre of rotation doesn't change. To describe a rotation, you need to give **three** pieces of information:

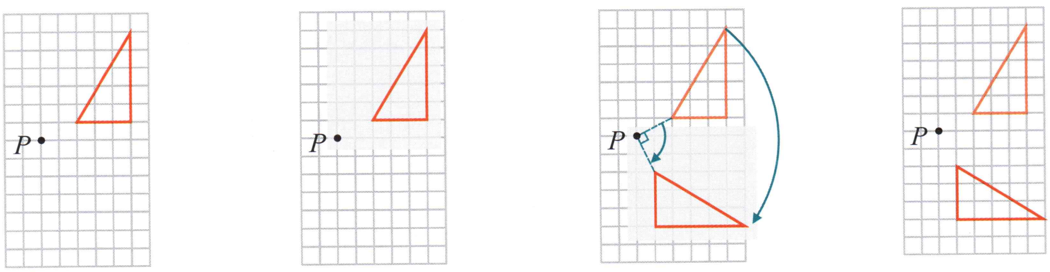
(i) the **centre** of rotation (ii) the **direction** of rotation (iii) the **angle** of rotation

The **centre** of rotation can be **any point** — e.g. the origin (0, 0) or (5, 1). The **direction** of rotation will be either **clockwise** or **anticlockwise** and the **angle** might be given in **degrees** or as a **fraction of a turn** (e.g. 90° or a quarter-turn). A rotation of **180°** is the same in both directions so you don't need a direction.

If a point on the shape lies on the **centre of rotation**, it will be an **invariant point** under any rotation.

Example 1

Rotate the shape below 90° clockwise about point *P*.

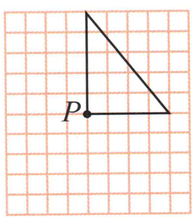


1. Draw the shape on a piece of tracing paper. (Or imagine a drawing of it.)
2. Rotate the tracing paper 90° clockwise about P. ('About P' means P doesn't move.)
3. Draw the image in its new position.

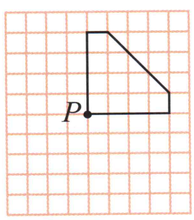
Exercise 1

Q1 a) Copy the diagrams below, then rotate the shapes 180° about P.

(i)

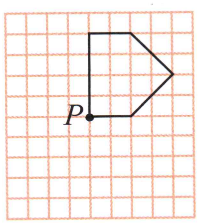


(ii)

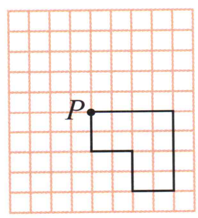


b) Copy the diagrams below, then rotate the shapes 90° clockwise about P.

(i)



(ii)



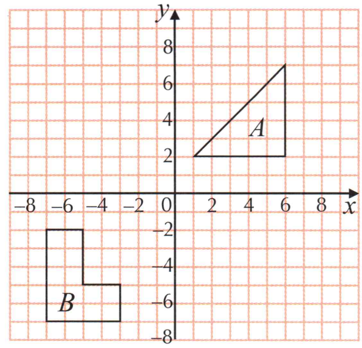
Q2 Find the new position of the points with the coordinates below if they are rotated anticlockwise about the origin by: (i) 90° (ii) 180° (iii) 270°

a) (1, 0) b) (0, 2) c) (3, 1)

Q3 Copy the diagram on the right, then:

a) Rotate A 90° clockwise about the origin.

b) Rotate B 270° anticlockwise about the origin.



Q4 The triangle ABC has vertices $A(-2, 1)$, $B(-2, 6)$ and $C(4, 1)$.

a) Draw the triangle on a pair of axes with x-values from –4 to 8 and y-values from –6 to 6.

b) Rotate the triangle a quarter-turn clockwise about (5, 4). Label the image $A_1B_1C_1$.

c) Write down the coordinates of A_1, B_1 and C_1.

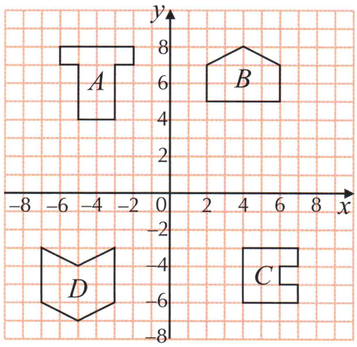
Q5 a) Copy the diagram below, then:

(i) Rotate A 90° clockwise about (–8, 5).

(ii) Rotate B 90° clockwise about (1, 4).

(iii) Rotate C 90° clockwise about (8, –4).

(iv) Rotate D 180° about (–2, –5).



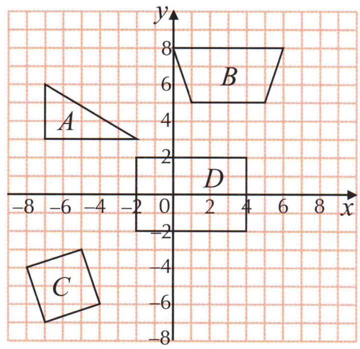
b) Copy the diagram below, then:

(i) Rotate A 180° about (–5, 6).

(ii) Rotate B 90° anticlockwise about (5, 9).

(iii) Rotate C 180° about (–3, –5).

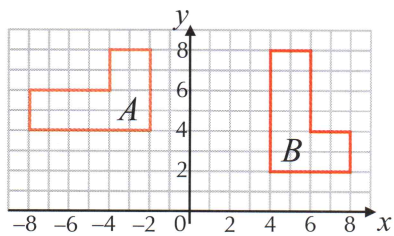
(iv) Rotate D 180° about (3, –1).



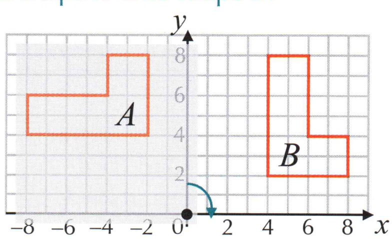
Q6 The triangle DEF has coordinates $D(-2, -2)$, $E(-2, 5)$ and $F(3, 5)$. DEF is rotated a half-turn about (2, 0) to create the image $D_1E_1F_1$. Find the coordinates of D_1, E_1 and F_1

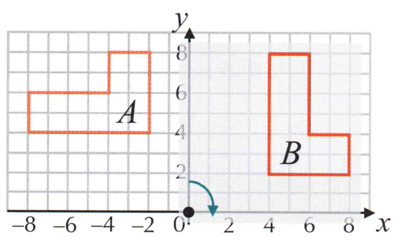
Example 2

Describe fully the rotation that transforms shape A to shape B.



1. The shape looks like it has been rotated clockwise by 90°.
2. Trace shape A using tracing paper. Put your pencil on different centres of rotation and turn the tracing paper 90° clockwise until you find a centre that takes shape A onto shape B.
3. To fully describe the rotation, you need to write down the centre, direction and angle of rotation.

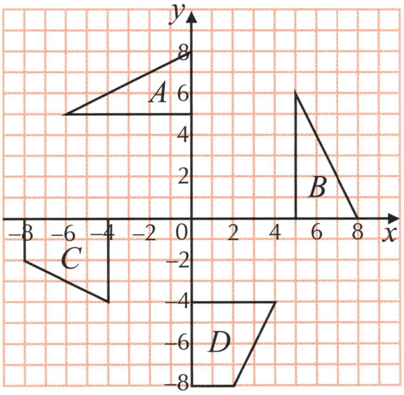




So A is transformed to B by a rotation of **90° clockwise** (or **270° anticlockwise**) about **the origin (0, 0)**.

Exercise 2

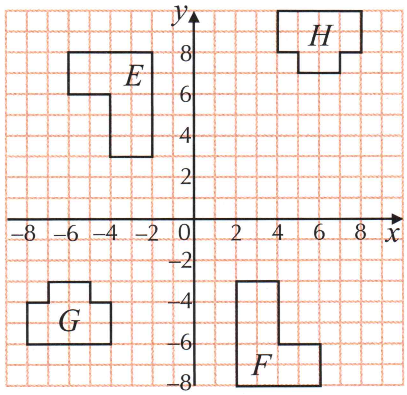
Q1 a)



(i) Describe fully the transformation that maps shape *A* to shape *B*.

(ii) Describe fully the transformation that maps shape *C* to shape *D*.

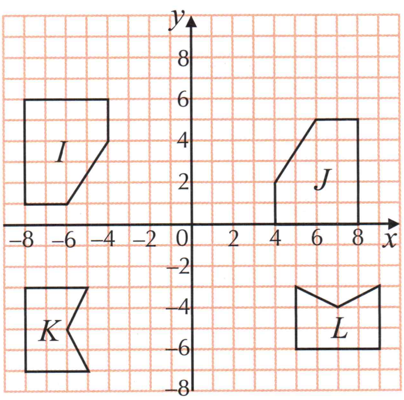
b)



(i) Describe fully the transformation that maps shape *E* to shape *F*.

(ii) Describe fully the transformation that maps shape *G* to shape *H*.

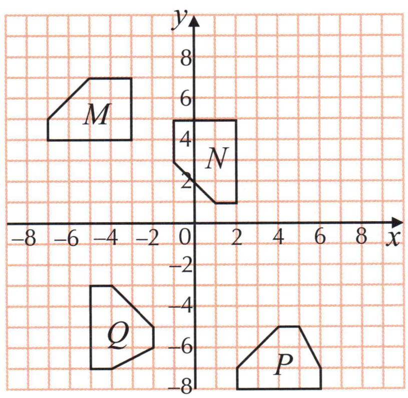
c)



(i) Describe fully the transformation that maps shape *I* to shape *J*.

(ii) Describe fully the transformation that maps shape *K* to shape *L*.

d)



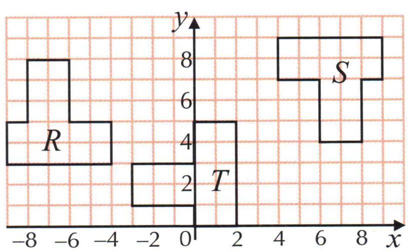
(i) Describe fully the transformation that maps shape *M* to shape *N*.

(ii) Describe fully the transformation that maps shape *P* to shape *Q*.

Q2 a) Describe fully the transformation that maps shape *R* to shape *S*.

b) Describe fully the transformation that maps shape *R* to shape *T*.

c) Describe fully the transformation that maps shape *S* to shape *T*.



Q3 The triangle *UVW* has vertices *U*(1, 1), *V*(3, 5) and *W*(–1, 3).
The triangle *XYZ* has vertices *X*(–2, 4), *Y*(–6, 6) and *Z*(–4, 2).

a) Draw the two triangles on a pair of axes.

b) Describe fully the transformation that maps *UVW* to *XYZ*.

29.3 Translations

Translations are simple — they just slide a shape up/down and left/right.

Learning Objectives — Spec Ref G7/G24:
- Translate a shape on a coordinate grid.
- Find the vector that describes a translation.

Prior Knowledge Check:
Be familiar with column vectors (see p.339).

To **translate** an object, you need to know the **distance** that it moves and in which **direction** — these are broken down into **horizontal** and **vertical** movements.

- For horizontal movements, **positive** numbers move the shape **right** and **negative** numbers move it **left**.
- For vertical movements, **positive** numbers move the shape **up** and **negative** numbers move it **down**.

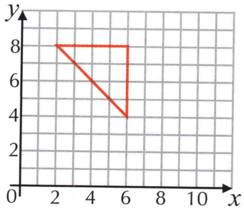
You can use **column vectors** to represent this information. For example:

$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
- 1: the object moves 1 unit to the right (**positive x-direction**)
- −2: the object moves 2 units down (**negative y-direction**)

An object and its image after a translation are **congruent** — so it doesn't change shape or size. Since a translation moves every point on the shape, there are **no invariant points**.

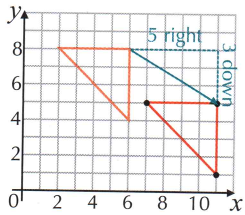
Example 1

Translate the shape on the axes below by the vector $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$.



$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$ is a translation of: (i) 5 units to the right,
(ii) 3 units down.

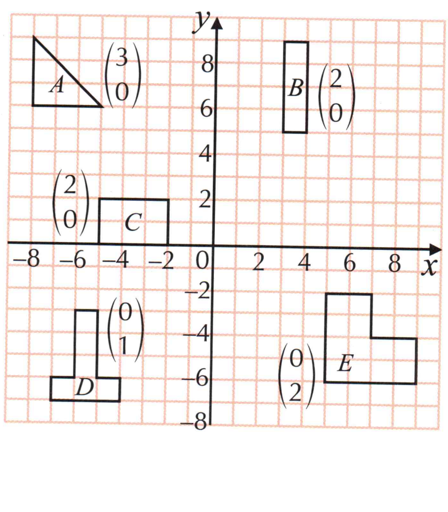
Move each vertex 5 units right and 3 units down and then join them up to create the translated shape.



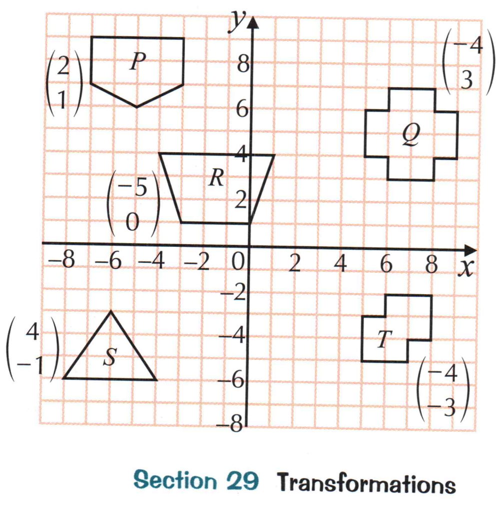
Exercise 1

Q1 Copy the diagrams below, then translate each shape by the vector written next to it.

a)



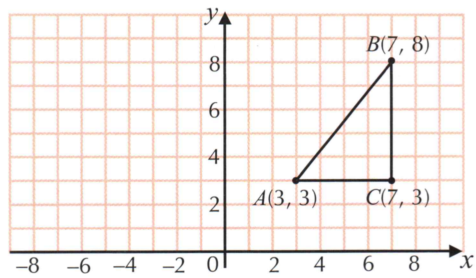
b)



Q2 a) Copy the diagram on the right, then translate the triangle ABC by the vector $\begin{pmatrix} -10 \\ -1 \end{pmatrix}$. Label the image $A_1B_1C_1$.

b) Label A_1, B_1 and C_1 with their coordinates.

c) Describe a rule connecting the coordinates of A, B and C and the coordinates of A_1, B_1 and C_1.



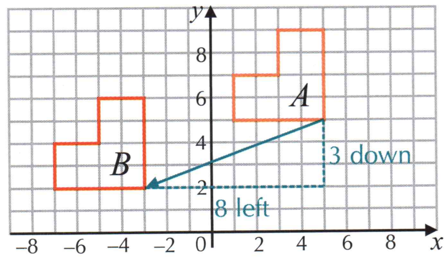
Q3 The triangle DEF has vertices $D(1, 1)$, $E(3, –2)$ and $F(4, 0)$. After the translation $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$, the image of DEF is $D_1E_1F_1$. Find the coordinates of D_1, E_1 and F_1.

PROBLEM SOLVING

Example 2

Describes the transformation that maps shape *A* onto shape *B*.

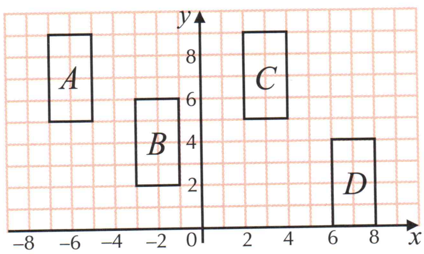
1. The shape hasn't been rotated or reflected and hasn't changed size, so this must be a translation.
2. Choose a pair of corresponding vertices on the two shapes and count how many units horizontally and vertically the point on *A* has moved to become the point on *B*.
3. Write the translation as a vector.



A has moved 8 units to the left and 3 units down so the transformation is a **translation described by the vector** $\begin{pmatrix} \mathbf{-8} \\ \mathbf{-3} \end{pmatrix}$.

Exercise 2

Q1 a)



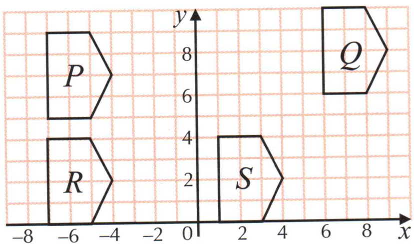
Give the vector that describes each of the following translations.

(i) *A* onto *B* (ii) *A* onto *C*

(iii) *C* onto *B* (iv) *C* onto *D*

(v) *D* onto *A* (vi) *D* onto *B*

b)



Give the vector that describes each of the following translations.

(i) *P* onto *R* (ii) *R* onto *S*

(iii) *P* onto *Q* (iv) *S* onto *R*

(v) *Q* onto *R* (vi) *S* onto *P*

Q2 The triangle DEF has vertices $D(–3, –2)$, $E(1, –1)$ and $F(0, 2)$. The triangle GHI has vertices $G(0, 2)$, $H(4, 3)$ and $I(3, 6)$. Give the vector that describes the translation that maps DEF onto GHI.

Q3 Shape W is the image of shape Z after the translation $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$.
Write the translation that maps shape W onto shape Z as a vector.

PROBLEM SOLVING

29.4 Enlargements

Enlargements can be trickier than the other transformations. They depend on a scale factor (which can be positive or negative, and sometimes a fraction) and a centre of enlargement.

Learning Objectives — Spec Ref G7:

- Enlarge a shape on a coordinate grid with a given centre of enlargement.
- Enlarge a shape using positive, fractional and negative scale factors.
- Describe an enlargement.

When an object is **enlarged**, its shape stays the same, but its **size changes** — so the image of a shape after an enlargement is **similar** to the original shape (see page 396 for more on similar shapes).

The **scale factor** of an enlargement tells you **how many times longer** the sides of the new shape are **compared** to the old shape. It also tells you **how much further** the points on the new shape are from the **centre of enlargement** than the points on the old shape. For example, enlarging by a **scale factor of 2** makes each side **twice as long** and each point **twice as far** from the centre of enlargement.

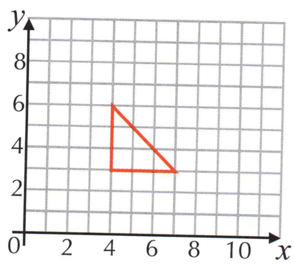
To enlarge a shape by a **positive** scale factor, follow this method:

- **Draw lines** from the centre of enlargement to **each vertex** of the shape.
- **Extend** each line depending on the scale factor (e.g. if the scale factor is 3 the line needs to be 3 times as long). Mark the vertices of the new shape at the ends of these extended lines.
- **Join up** the new vertices to create the enlarged shape.

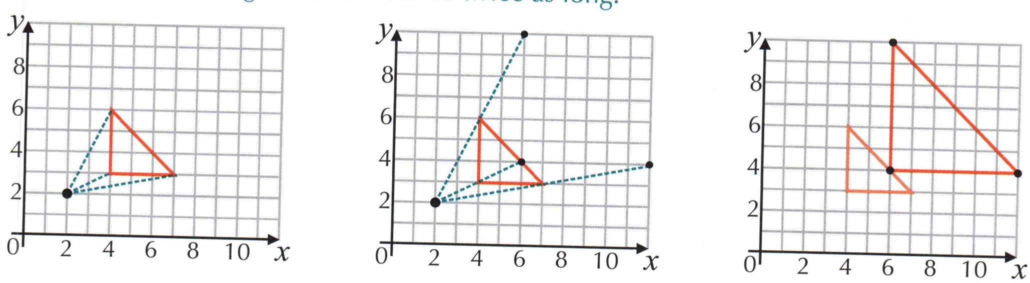
If a point on the shape lies on the **centre of enlargement**, it will be the only **invariant point** under an enlargement.

Example 1

Enlarge the triangle on the axes on the right by scale factor 2 with centre of enlargement (2, 2).

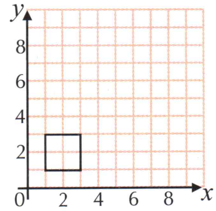


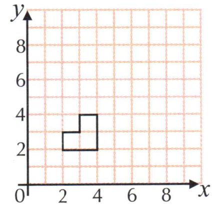
1. Draw a line from (2, 2) through each vertex of the shape.
2. The scale factor is 2, so extend the lines so they are twice as long and mark the new vertices at the ends.
3. Join up the points to create the enlarged triangle. Each side of the triangle should now be twice as long.

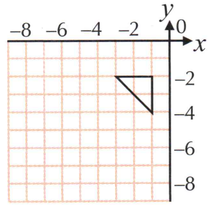


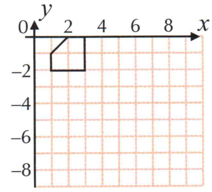
Exercise 1

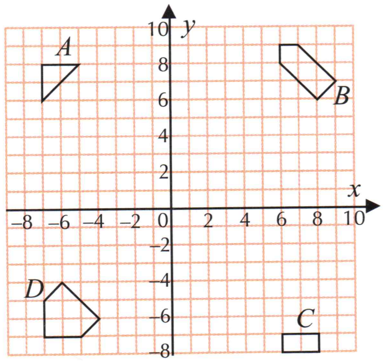
Q1 Copy the diagrams below and enlarge each shape by scale factor 2 with centre of enlargement (0, 0).

a) 

b) 

c) 

d) 



Q2 Copy the diagram on the right.

a) Enlarge *A* by scale factor 2 with centre of enlargement (–8, 9).

b) Enlarge *B* by scale factor 2 with centre of enlargement (9, 9).

c) Enlarge *C* by scale factor 3 with centre of enlargement (9, –8).

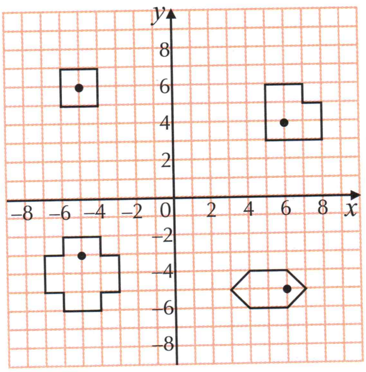
d) Enlarge *D* by scale factor 4 with centre of enlargement (–8, –8).

Q3 The triangle *PQR* has corners at *P*(1, 1), *Q*(1, 4) and *R*(4, 2).

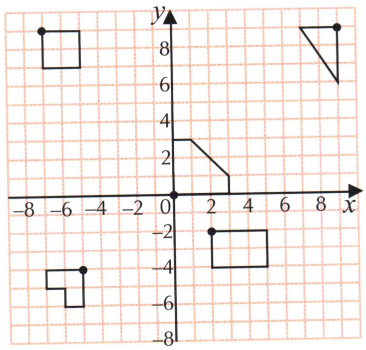
a) Draw *PQR* on a pair of axes with *x*- and *y*-values from –1 to 9.

b) Enlarge *PQR* by scale factor 2 with centre of enlargement (–1, 1).

Q4 a) Copy the diagram below, then enlarge each shape by scale factor 2, using the centre of enlargement marked inside the shape.



b) Copy the diagram below, then enlarge each shape by scale factor 2, using the centre of enlargement marked on the shape's vertex.



Q5 The shape *WXYZ* has corners at *W*(0, 0), *X*(1, 3), *Y*(3, 3) and *Z*(4, 0).

a) Draw *WXYZ* on a pair of axes with *x*-values from –5 to 9 and *y*-values from –5 to 6.

b) Enlarge *WXYZ* by scale factor 3 with centre of enlargement (2, 2).

Q6 The shape *KLMN* has corners at *K*(3, 4), *L*(3, 6), *M*(5, 6) and *N*(5, 5).

a) Draw *KLMN* on a pair of axes with *x*-values from 0 to 10 and *y*-values from 0 to 7.

b) Enlarge *KLMN* by scale factor 3 with centre of enlargement (3, 6). Label the shape $K_1L_1M_1N_1$.

c) Enlarge *KLMN* by scale factor 2 with centre of enlargement (5, 6). Label the shape $K_2L_2M_2N_2$.

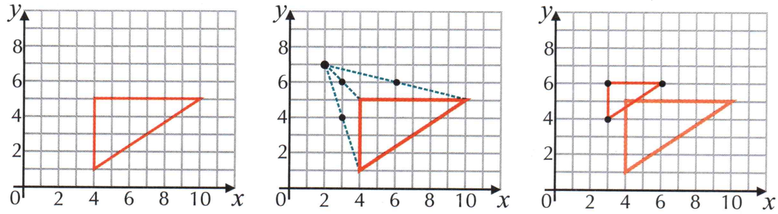
If the scale factor is a **fraction** then the enlargement will actually **shrink** the shape.
E.g. a scale factor of $\frac{1}{2}$ makes all the sides of the new shape **half as long**, and each point on the new shape will be **half as far** from the centre of enlargement.

Just like before, **draw lines** from the **centre of enlargement** to **each vertex**. Then mark the vertices of the **new shape** a **fraction** of the way along these lines (depending on the **scale factor**) and join them up.
E.g. a scale factor of $\frac{1}{4}$ means the vertices of the new shape are a quarter of the way along the lines.

Example 2

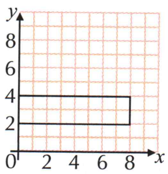
Enlarge the shape on the axes below by scale factor $\frac{1}{2}$ with centre of enlargement (2, 7).

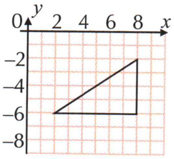
1. Draw lines from (2, 7) to each vertex.
2. Mark the vertices of the new shape half as far from the centre of enlargement as the original vertices. Join them up to create the enlarged shape.

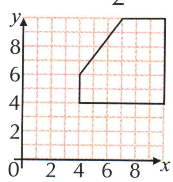


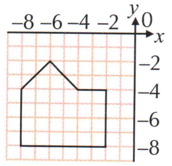
Exercise 2

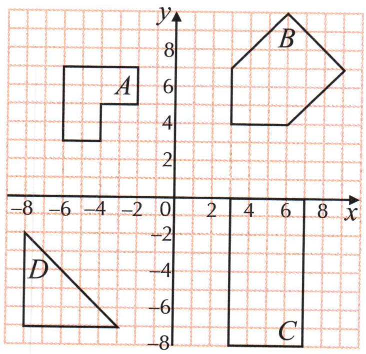
Q1 Copy the diagrams below. Enlarge each shape by scale factor $\frac{1}{2}$ with centre of enlargement (0, 0).

a) 

b) 

c) 

d) 

Q2 

Copy the diagram on the left.

a) Enlarge A by scale factor $\frac{1}{2}$ with centre of enlargement (–8, 9).

b) Enlarge B by scale factor $\frac{1}{3}$ with centre of enlargement (0, 1).

c) Enlarge C by scale factor $\frac{1}{4}$ with centre of enlargement (–1, –8).

d) Enlarge D by scale factor $\frac{1}{5}$ with centre of enlargement (2, 3).

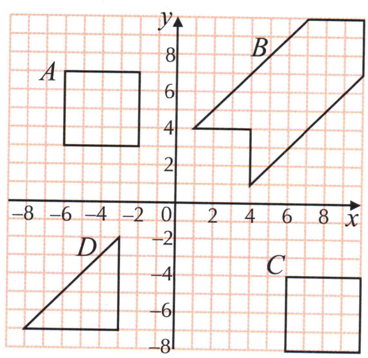
Q3 Copy the diagram on the right.

a) Enlarge A by scale factor $\frac{1}{2}$ with centre of enlargement (–4, 5).

b) Enlarge B by scale factor $\frac{1}{3}$ with centre of enlargement (7, 7).

c) Enlarge C by scale factor $\frac{1}{4}$ with centre of enlargement (6, –4).

d) Enlarge D by scale factor $\frac{1}{5}$ with centre of enlargement (–3, –2).



If you have a **negative** scale factor (i.e. $-k$), the sides of the enlarged shape are **k times longer**. Each point moves **k times further away** from the centre of enlargement but in the **opposite direction**.

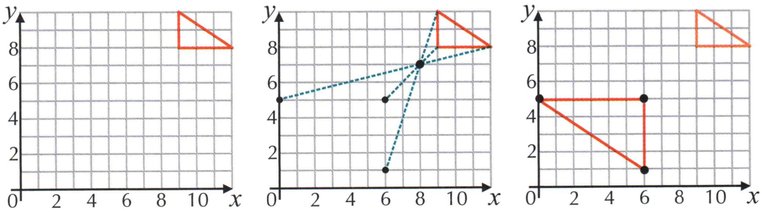
To enlarge a shape by scale factor $-k$, **draw lines** from **each vertex** of the shape to the centre of enlargement. **Extend** each line **k times further** out of the **other side** of the centre of enlargement. The vertices of the new shape are at the ends of these lines — draw them in and **join them up** to create the new shape.

Tip: Enlarging by a scale factor of $-k$ is the same as enlarging by a scale factor of k and then rotating 180° about the centre of enlargement.

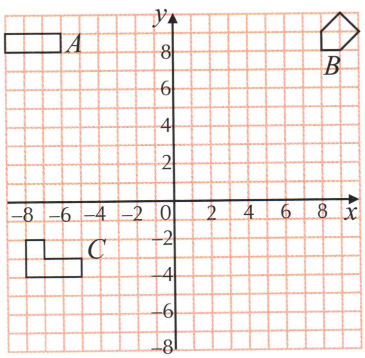
Example 3

Enlarge the shape on the axes below by scale factor –2 with centre of enlargement (8, 7).

1. Draw lines from each vertex of the shape to (8, 7). The scale factor is –2, so extend the lines twice as far out of the other side of the centre of enlargement.
2. Mark the new vertices and join them up to create the enlarged shape — it's twice as big but on the opposite side of the centre of enlargement.



Exercise 3

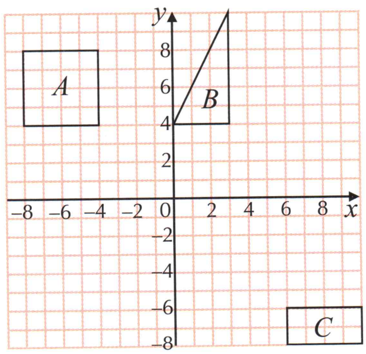


Q1 Copy the diagram on the left.

a) Enlarge A by scale factor –2 with centre of enlargement (–6, 7).

b) Enlarge B by scale factor –3 with centre of enlargement (7, 7).

c) Enlarge C by scale factor –2 with centre of enlargement (–2, –4).



Q2 Copy the diagram on the right.

a) Enlarge A by scale factor $-\frac{1}{2}$ with centre of enlargement (–4, 4).

b) Enlarge B by scale factor $-\frac{1}{3}$ with centre of enlargement (6, 4).

c) Enlarge C by scale factor $-\frac{1}{2}$ with centre of enlargement (4, –4).

Q3 The shape $TUVW$ has corners at (1, 0), (4, 6), (2, 5) and (0, 2).

a) Draw $TUVW$ on a pair of axes with x-values from –4 to 4 and y-values from –6 to 6.

b) Enlarge $TUVW$ by scale factor –1, centred at the origin, and label it $T_1U_1V_1W_1$.

c) What other single transformation would map $TUVW$ to $T_1U_1V_1W_1$? PROBLEM SOLVING

To describe an enlargement, you need to give the **scale factor** and the **centre of enlargement**.

To find the **scale factor**, take the length of **any side** on the new shape and the length of its corresponding side on the old shape and use the **formula**:

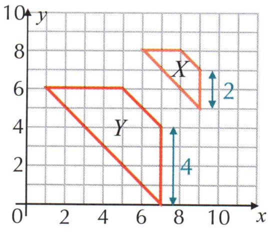
$$\textbf{scale factor} = \frac{\textbf{new length}}{\textbf{old length}}$$

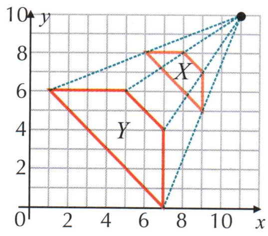
For the **centre of enlargement**, **draw** and **extend lines** that go through corresponding vertices of both shapes and see where they all **intersect**. If you're dealing with a **negative scale factor**, the centre of enlargement will lie **between** the two shapes.

Example 4

Describe the enlargement that maps shape *X* onto shape *Y*.

1. Pick any side on shape *Y* and its corresponding side on shape *X* (it's best to pick a horizontal or vertical side). Measure their lengths and put them into the formula to work out the scale factor.
2. Draw and extend lines from each vertex on shape *Y* through the corresponding vertex on shape *X*. The point where these lines meet is the centre of enlargement.

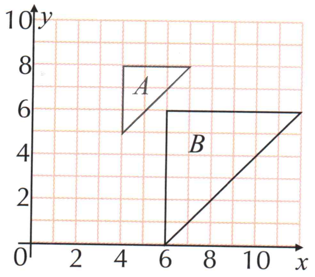


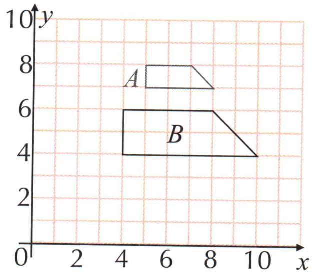


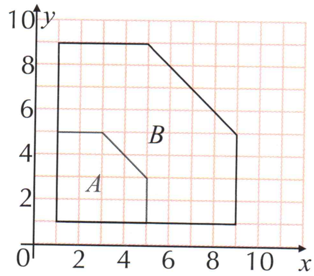
An enlargement by scale factor $\frac{4}{2}$ = **2**, centre **(11, 10)**.

Exercise 4

Q1 For each of the following, describe fully the transformation that maps shape *A* onto shape *B*.

a) 

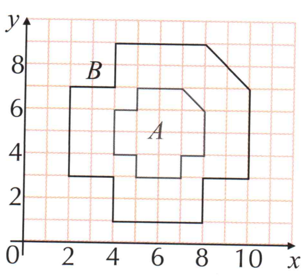
b) 

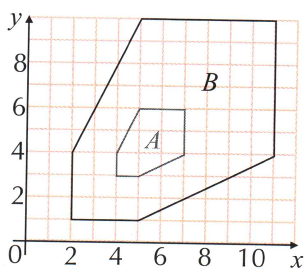
c) 

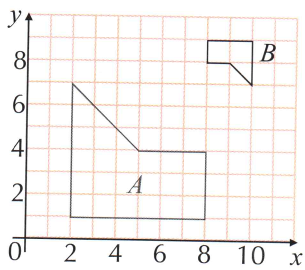
Q2 For each of the following, describe fully the transformation that maps:

(i) shape *A* onto shape *B*,

(ii) shape *B* onto shape *A*.

a) 

b) 

c) 

29.5 Combinations of Transformations

Once you've got your head around the transformations covered in this section, the next step is doing them one after another — and figuring out how you could do them as a single transformation.

Learning Objectives — Spec Ref G7/G8:

- Apply combinations of transformations to shapes.
- Describe a combination of transformations as a single transformation.
- Recognise points of invariance under a combination of transformations.

Prior Knowledge Check: Be familiar with rotation, reflection and translation (see p.377-384).

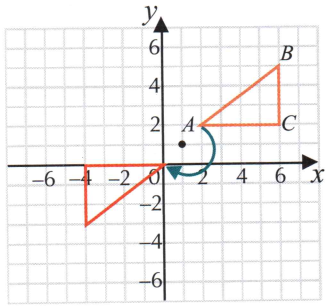
Doing a **combination** of transformations is as simple as doing **one after the other**, and you can often describe a combination using a **single** transformation. Just make sure you **specify** all the **details** when describing the single transformation — for a **reflection** you need the **equation of the mirror line**, for a **rotation** you need the **centre**, **angle** and **direction of rotation** and for a **translation** you need a **vector**.

Example 1

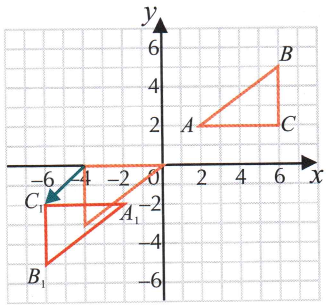
a) Rotate triangle ABC on the axes below 180° about the point (1, 1), then translate the image by $\begin{pmatrix}-2\\-2\end{pmatrix}$. Label the final image $A_1B_1C_1$.

b) Describe the single rotation that transforms triangle ABC onto the image $A_1B_1C_1$.

a)

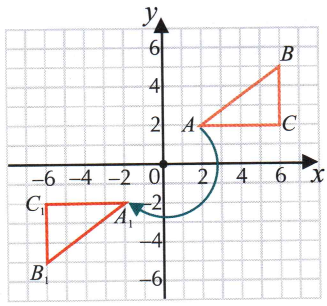


Rotate ABC 180° about (1, 1)...



... then translate by $\begin{pmatrix}-2\\-2\end{pmatrix}$.

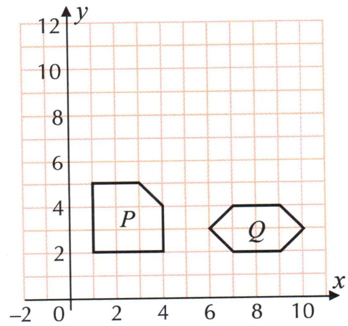
b)



A **rotation** of **180°** about the **origin** maps ABC onto $A_1B_1C_1$.

Exercise 1

Q1 Copy the diagram on the right.



a) (i) Rotate shape P 180° about (4, 5).

(ii) Translate the image by $\begin{pmatrix}2\\2\end{pmatrix}$. Label the final image P_1.

b) Rotate shape P 180° about (5, 6). Label the final image P_2.

c) (i) Rotate shape Q 180° about (4, 5).

(ii) Translate the image by $\begin{pmatrix}2\\2\end{pmatrix}$. Label the final image Q_1.

d) Rotate shape Q 180° about (5, 6). Label the final image Q_2.

e) What do you notice about the images P_1 and P_2 and about the images Q_1 and Q_2?

Q2 Triangle ABC has its corners at $A(2, 1)$, $B(6, 4)$ and $C(6, 1)$.

a) Draw triangle ABC on a pair of axes, where both the x- and y-axes are labelled from –6 to 6.

b) Reflect ABC in the y-axis. Label the image $A_1B_1C_1$.

c) Reflect the image $A_1B_1C_1$ in the x-axis. Label the image $A_2B_2C_2$.

d) Find a single rotation that transforms triangle ABC onto the image $A_2B_2C_2$.

Q3 Draw triangle PQR with corners at $P(2, 3)$, $Q(4, 3)$ and $R(4, 4)$ on a pair of axes with x- and y-values from –6 to 6.

a) (i) Rotate PQR 90° clockwise about the point (1, 3). Label the image $P_1Q_1R_1$.

(ii) Write down any points of PQR that are invariant under this transformation.

b) (i) Translate $P_1Q_1R_1$ by $\begin{pmatrix}1\\1\end{pmatrix}$. Label the new image $P_2Q_2R_2$.

(ii) Write down any points of $P_1Q_1R_1$ that are invariant under this transformation.

c) (i) Describe a single transformation that maps triangle PQR onto the image $P_2Q_2R_2$.

(ii) Write down any points of PQR that are invariant under this transformation.

Q4 Triangle WXY has its corners at $W(-5, -5)$, $X(-4, -2)$ and $Y(-2, -4)$.

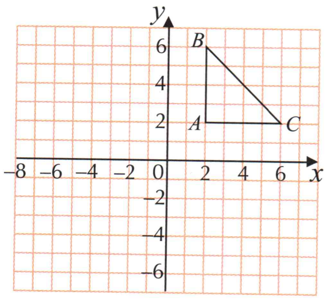
a) Draw triangle WXY on a pair of axes, where both the x- and y-axes are labelled from –6 to 6.

b) Reflect WXY in the line $y = x$. Label the image $W_1X_1Y_1$.

c) Reflect the image $W_1X_1Y_1$ in the y-axis. Label the image $W_2X_2Y_2$.

d) Find a single transformation that maps WXY onto the image $W_2X_2Y_2$.

Q5 Copy the diagram on the right.



a) Rotate triangle ABC 180° about (0, 0). Label the image $A_1B_1C_1$.

b) $A_1B_1C_1$ is translated such that the point B on the triangle ABC is invariant under the combination of the rotation followed by the translation. Describe this translation. PROBLEM SOLVING

Q6 Shape $WXYZ$ has its corners at $W(-6, 2)$, $X(-3, 3)$, $Y(-2, 6)$ and $Z(-2, 2)$.

a) Draw $WXYZ$ on a pair of axes, where both the x- and y-axes are labelled from –6 to 6.

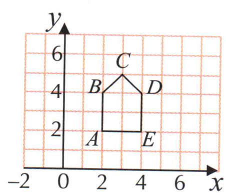
b) Reflect $WXYZ$ in the y-axis. Label the image $W_1X_1Y_1Z_1$.

c) Rotate the image $W_1X_1Y_1Z_1$ 90° clockwise about (0, 0). Label the image $W_2X_2Y_2Z_2$.

d) Find a single transformation that maps $WXYZ$ onto $W_2X_2Y_2Z_2$.

Q7 Copy the diagram on the right.

a) Translate shape $ABCDE$ by $\begin{pmatrix}-2\\-2\end{pmatrix}$. Label the image $A_1B_1C_1D_1E_1$.



b) The shape $A_1B_1C_1D_1E_1$ is rotated to become $A_2B_2C_2D_2E_2$. Under the combination of the translation from a) and this rotation, point C is invariant and point D maps to point B_2. Describe the rotation. PROBLEM SOLVING

Review Exercise

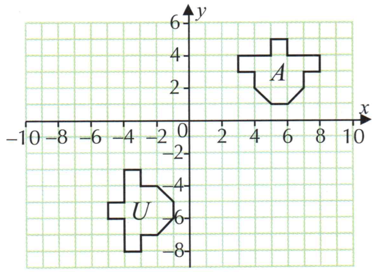
Q1 a) Copy the grid and shape A on the right and then:

(i) Reflect shape A in the x-axis and label it A_1.

(ii) Reflect shape A in the line $x = 3$ and label it A_2.

b) For each reflection in part a), write down the coordinates of any points of A that are invariant.

c) Describe the reflection that maps shape A to shape U.



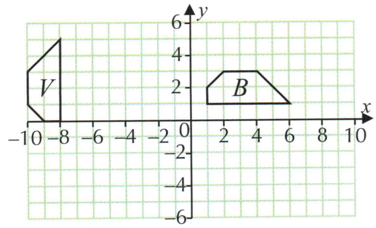
Q2 a) Copy the grid and shape B on the right and then:

(i) Rotate shape B 180° about the origin and label it B_1.

(ii) Rotate shape B 90° clockwise about (6, 1) and label it B_2.

b) For each rotation in part a), write down the coordinates of any points of B that are invariant.

c) Describe the rotation that maps shape B to shape V.

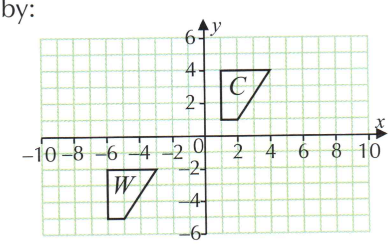


Q3 a) Copy the grid and shape C on the right. Translate shape C by:

(i) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and label it C_1, (ii) $\begin{pmatrix} -10 \\ 0 \end{pmatrix}$ and label it C_2.

b) For each translation in part a), write down the coordinates of any points of C that are invariant.

c) Describe the translation that maps shape C to shape W.



Q4 a) Copy the grid and shape D on the right and then:

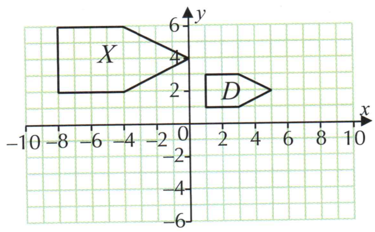
(i) Enlarge shape D by scale factor 2, with centre of enlargement the origin, and label it D_1.

(ii) Enlarge shape D by scale factor $\frac{1}{2}$, with centre of enlargement (–9, 3), and label it D_2.

(iii) Enlarge shape D by scale factor –3, with centre of enlargement (3, 1), and label it D_3.

b) For each enlargement in part a), write down the coordinates of any points of D that are invariant.

c) Describe the enlargement that maps shape D to shape X.



Q5 By considering triangle XYZ with vertices $X(2, 2)$, $Y(4, 4)$ and $Z(6, 2)$, find the single transformation equivalent to a reflection in the line $y = 2$, followed by a reflection in the line $x = 2$.

Q6 By considering shape $ABCD$ with corners at $A(2, -2)$, $B(4, -2)$, $C(5, -5)$ and $D(2, -4)$, find the single transformation equivalent to a rotation of 90° anticlockwise about (2, –2), followed by a reflection in the y-axis, followed by a translation by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$.

PROBLEM SOLVING

Exam-Style Questions

Q1 a) Describe fully the single transformation equivalent to

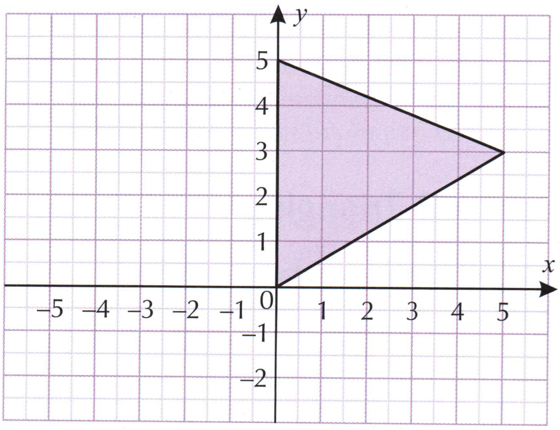
- a reflection in the line $x = 0$, followed by
- a reflection in the line $y = -x$.

Use the grid to help you.

[3 marks]

b) State the coordinates of any points of invariance under this transformation.

[1 mark]



Q2 A triangle with vertices $A(0, 0)$, $B(3, 0)$ and $C(3, 6)$ is reflected in the line $y = k$ and then translated 10 units in the positive y-direction. Point C is invariant under this combination of transformations.

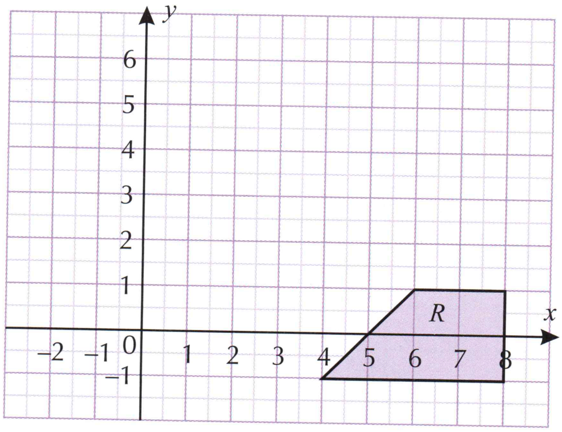
PROBLEM SOLVING

Find the value of k.

[2 marks]

Q3 A shape P is transformed by a translation of $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$ to produce an image Q.

Shape Q is then transformed by an enlargement of scale factor 2 with centre of enlargement $(-2, 3)$ to give shape R. Shape R is shown on the diagram below.



Copy the diagram and draw shape P. Show all your working.

PROBLEM SOLVING

[3 marks]