

27.1 Triangles and Quadrilaterals

You should already be familiar with area and perimeter, but for GCSE you'll need to work them out for all sorts of shapes, including composite shapes. We'll start with the basics — squares, rectangles and triangles.

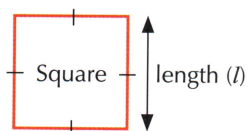
Triangles, Squares and Rectangles

Learning Objective — Spec Ref G16/G17:

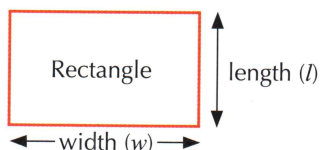
Find the area and perimeter of rectangles, triangles and composite shapes.

Prior Knowledge Check:
Be familiar with properties of triangles and quadrilaterals.
See Section 20.

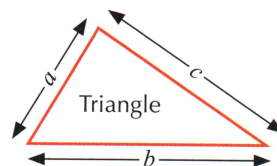
Perimeter (P) is the distance around the outside of a shape.



$$P = 4l$$

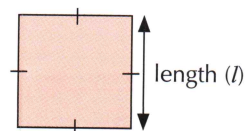


$$P = 2l + 2w$$



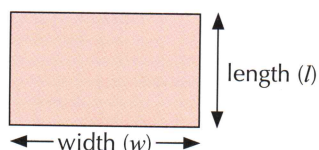
$$P = a + b + c$$

Area (A) is the amount of space inside a shape.



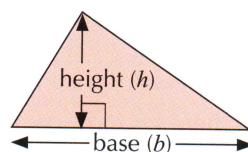
$$\text{Area} = (\text{side length})^2$$

$$A = l^2$$



$$\text{Area} = \text{length} \times \text{width}$$

$$A = lw$$

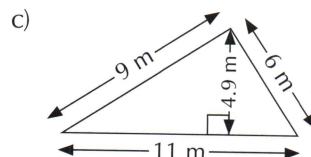
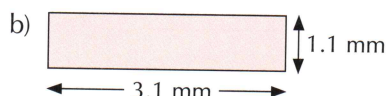
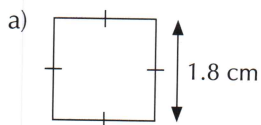


$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

$$A = \frac{1}{2}bh$$

Exercise 1

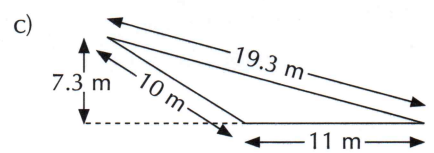
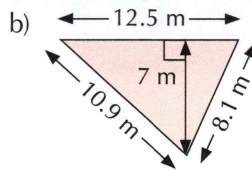
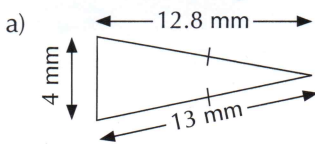
Q1 For each shape below, find: (i) its perimeter, (ii) its area.



Q2 For each shape described below, find: (i) its perimeter, (ii) its area.

- | | |
|---|--|
| a) a square with sides of length 4 cm | b) a rectangle of width 6 m and length 8 m |
| c) a rectangle 23 mm long and 15 mm wide | d) a square with 17 m sides |
| e) a rectangle 22.2 m long and 4.3 m wide | f) a rectangle of length 9 mm and width 2.4 mm |

Q3 For each of the triangles below, find: (i) its perimeter, (ii) its area.

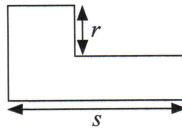


You might also have to find the area or perimeter of a **composite shape** (a shape that can be split up into two or more basic shapes). To find the **perimeter**, use the lengths you're given to find any **missing** ones, then **add** all the side lengths up. For the **area**, **split the shape up** into smaller pieces, then work out the area of each piece separately and add them together.

Example 1

For the shape on the right, find: a) its perimeter, b) its area.

- a) 1. Label the missing sides, and find their lengths.



$$r = 10 - 4 = 6 \text{ cm}, \quad s = 7 + 11 = 18 \text{ cm}$$

2. Add up the lengths of all the sides. Perimeter = $7 + 6 + 11 + 4 + 18 + 10 = 56 \text{ cm}$

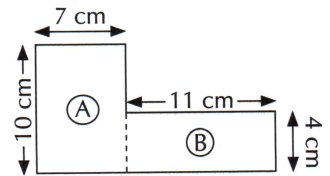
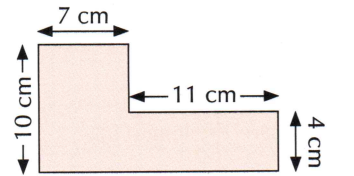
- b) 1. Split the shape into rectangles A and B, and find their areas.

$$\text{Area of rectangle A} = 10 \times 7 = 70 \text{ cm}^2$$

$$\text{Area of rectangle B} = 11 \times 4 = 44 \text{ cm}^2$$

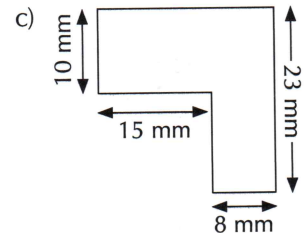
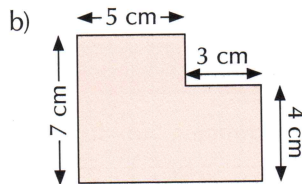
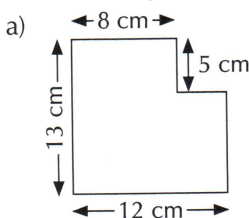
2. Add these to find the total area.

$$\text{Total area of shape} = 70 + 44 = 114 \text{ cm}^2$$

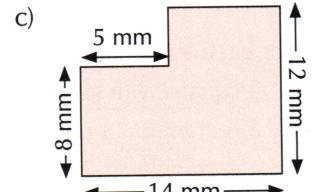
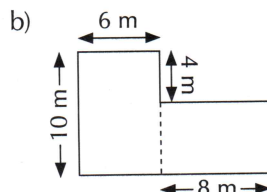
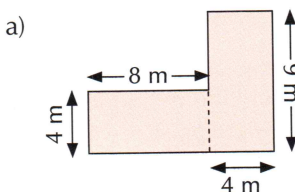


Exercise 2

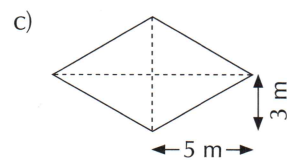
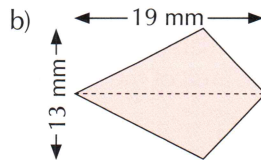
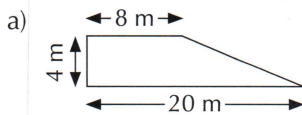
Q1 For each shape below, find: (i) its perimeter, (ii) its area.



Q2 Find the area of each shape below.



Q3 Find the areas of the shapes below. The dashed lines show lines of symmetry.



Parallelograms and Trapeziums

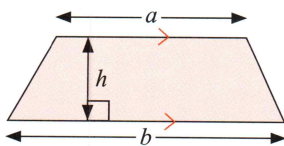
Learning Objective — Spec Ref G16/G17:

Find the area and perimeter of parallelograms, trapeziums and composite shapes.

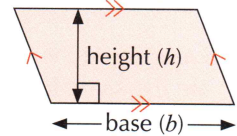
You also need to know the area formulas for **parallelograms** and **trapeziums** — remember that trapeziums always have **one pair** of parallel sides, and parallelograms have **two pairs** of parallel sides (see p.255-256).

The **area** of a **parallelogram** is given by the formula: $A = bh$

Here, h is the **perpendicular height** — it's measured at **right angles** to the base.



The **area** of a **trapezium** is given by the formula: $A = \frac{1}{2}(a + b) \times h$



Example 2

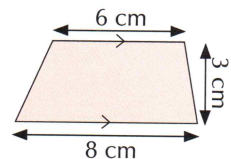
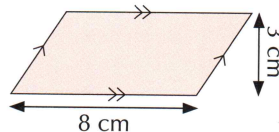
Find the area of:

a) the parallelogram, b) the trapezium.

Just put the numbers into the formulas.

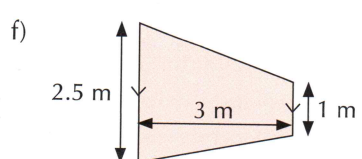
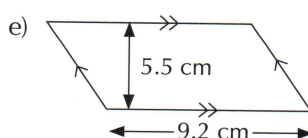
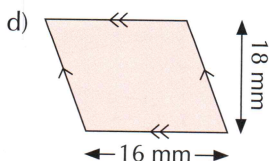
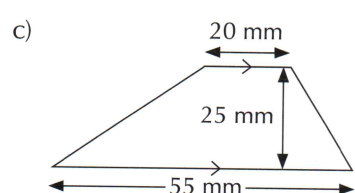
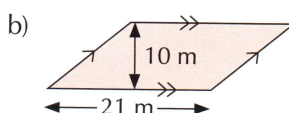
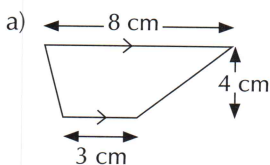
a) $A = bh = 8 \times 3 = 24 \text{ cm}^2$

b) $A = \frac{1}{2}(a + b) \times h = \frac{1}{2}(6 + 8) \times 3 = \frac{1}{2} \times 14 \times 3 = 21 \text{ cm}^2$



Exercise 3

Q1 Find the area of each shape below.



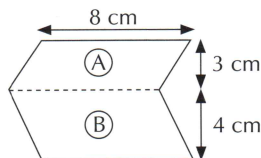
Composite shapes made up of parallelograms and trapeziums work exactly the same as ones made of rectangles and triangles. When finding perimeters of these shapes, remember that **opposite sides** on a parallelogram are the **same length** — this can be helpful for finding missing lengths. If a big shape has a smaller shape cut out of it, **subtract** the area of the **smaller** shape from the area of the bigger one.

Example 3

For the composite shape on the right, find:

a) its area, b) its perimeter.

- a) 1. Split the shape up into two parallelograms — call them A and B.
Find the area of each one.



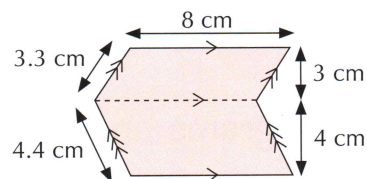
$$\text{Area of A} = bh = 8 \times 3 = 24 \text{ cm}^2$$

$$\text{Area of B} = bh = 8 \times 4 = 32 \text{ cm}^2$$

2. Add these areas to find the total area.

$$\text{Total area} = 24 + 32 = 56 \text{ cm}^2$$

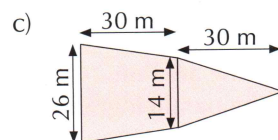
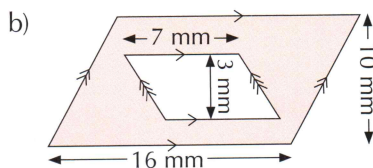
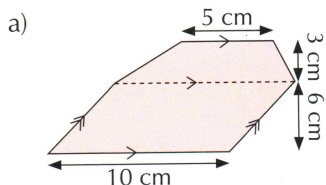
- b) Opposite sides of a parallelogram are the same length. $P = 8 + 3.3 + 4.4 + 8 + 4.4 + 3.3 = 31.4 \text{ cm}$



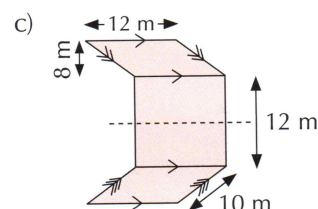
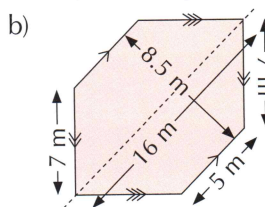
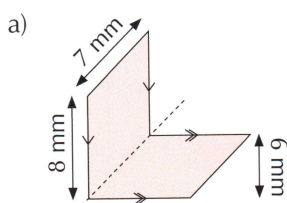
Tip: Make sure you don't accidentally add the dotted line — it's not part of the perimeter.

Exercise 4

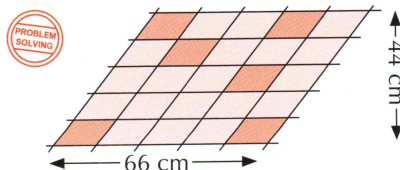
Q1 Find each shaded area below.



Q2 For each shape below, find: (i) the area, (ii) the perimeter. The dotted lines show lines of symmetry.

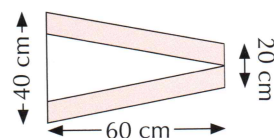


Q3 The picture shows part of a tiled wall. All the tiles are identical parallelograms. Find the area of one tile.



Q4 The flag shown is in the shape of a trapezium. The coloured strips along the top and bottom edges are identical parallelograms.

- a) Find the total area of the flag.
b) Find the total area of the coloured strips.



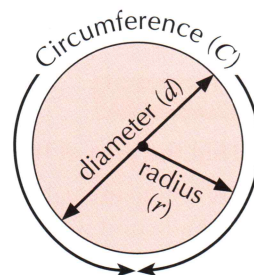
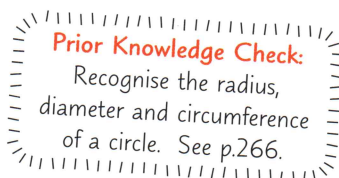
27.2 Circles and Sectors

There are more formulas you need to know on the next few pages, this time featuring your good friend π . Make sure you know where the π button lives on your calculator — you'll need it a lot for circle questions.

Circumference of a Circle

Learning Objective — Spec Ref G17:

Find the circumference of a circle.



You met the **radius**, **diameter** and **circumference** of a circle on p.266. You can find the circumference (C) of a circle from its diameter (d) or its radius (r) using the formula:

$$C = \pi d = 2\pi r$$

(since $d = 2r$)

Example 1

A circle has a radius of 13 m. Find its circumference to 1 d.p.

You're given r , so use the

' $C = 2\pi r$ ' version of the formula.

$$C = 2\pi r = 2 \times \pi \times 13$$

$$= 81.681... = \mathbf{81.7 \text{ m}} \text{ (1 d.p.)}$$

Tip: If you're asked for an **exact** answer, leave it in terms of π : $C = 26\pi$.

Example 2

The shape on the right consists of a semicircle on top of a rectangle. Find the perimeter of the shape.

1. Find the curved length.

This is half the circumference of a circle with diameter 6 cm.

$$\text{Curved length} = \pi d \div 2 = \pi \times 6 \div 2$$

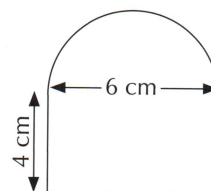
$$= 9.424... \text{ cm}$$

$$\text{Total of straight sides} = 4 + 6 + 4$$

$$= 14 \text{ cm}$$

2. Find the total length of the straight sides and add the two parts.

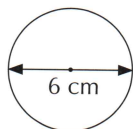
$$\text{Total length} = 9.424... + 14 = 23.424... = \mathbf{23.4 \text{ cm}} \text{ (1 d.p.)}$$



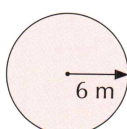
Exercise 1

Q1 Find the circumference of each circle. Give your answers to 1 decimal place.

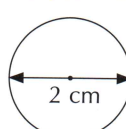
a)



b)



c)



d)



Q2 Find, to 1 d.p., the circumference of the circles with the diameter (d) or radius (r) given below.

a) $d = 4 \text{ cm}$

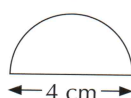
b) $r = 11 \text{ m}$

c) $r = 0.1 \text{ km}$

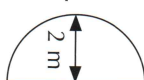
d) $d = 6.3 \text{ mm}$

Q3 Find the perimeter of each shape below. Give your answers to 1 decimal place.

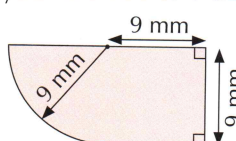
a)



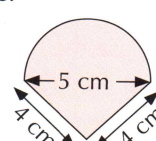
b)



c)



d)



Area of a Circle

Learning Objective — Spec Ref G17:

Find the area of a circle.

The **area** (A) of a circle with radius r is given by the formula: $A = \pi r^2$

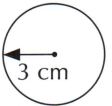
If you're given the **diameter** of the circle, you need to **divide it by 2** before you can use the formula.

Example 3

Find the area of the circle shown. Give your answer to 1 decimal place.

Substitute the values into the formula and work out the answer.

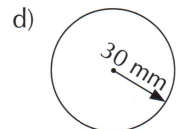
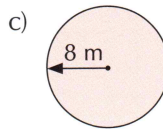
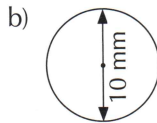
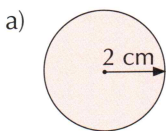
$$A = \pi r^2 = \pi \times 3^2 = 28.274... \\ = 28.3 \text{ cm}^2 \text{ (1 d.p.)}$$



Exercise 2

In the following questions, give your answers to 1 decimal place.

Q1 Find the area of each circle.



Q2 Find, to 1 decimal place, the areas of the circles with the diameter (d) or radius (r) given below.

a) $r = 6 \text{ mm}$

b) $r = 5 \text{ cm}$

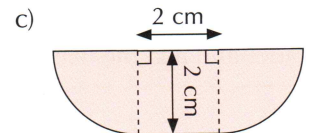
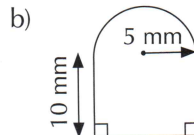
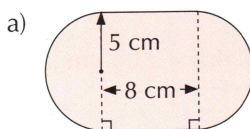
c) $r = 4 \text{ m}$

d) $d = 8.5 \text{ mm}$

e) $d = 3.5 \text{ m}$

f) $d = 1.2 \text{ mm}$

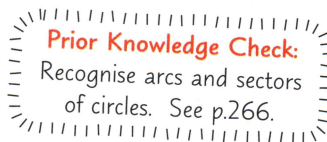
Q3 Find the area of each shape below to 1 decimal place.



Arcs and Sectors of Circles

Learning Objectives — Spec Ref G17:

- Find the area of a sector of a circle.
- Find the length of an arc of a circle.

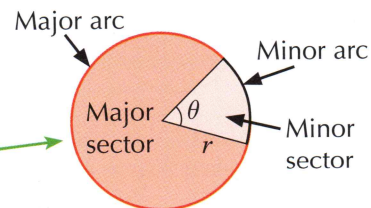


You saw on p.266 that a **sector** is a 'slice' of a circle and an **arc** is a part of the circumference. If the angle in a sector is **less than 180°** then it's a **minor** sector, and if the angle is **more than 180°** then it's a **major** sector.

You can use the following formulas to find an arc length and a sector area:

$$\text{Length of arc} = \frac{\theta}{360^\circ} \times \text{circumference of circle} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \text{area of circle} = \frac{\theta}{360^\circ} \times \pi r^2$$



Example 4

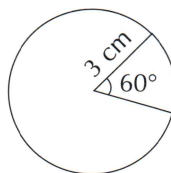
For the circle shown, calculate the exact area of the minor sector and the exact length of the minor arc.

Substitute the values into the formulas and work out the answers.

$$\text{Area of sector} = \frac{60^\circ}{360^\circ} \times \pi \times 3^2$$

$$= \frac{1}{6} \times 9\pi = \frac{3}{2}\pi \text{ cm}^2$$

$$\text{Length of arc} = \frac{60^\circ}{360^\circ} \times (2 \times \pi \times 3) = \frac{1}{6} \times 6\pi = \pi \text{ cm}$$

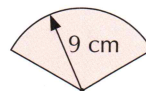


Tip: The question asks for exact solutions, so leave them in terms of π .

Example 5

The shape shown is a sector of a circle with radius 9 cm.

The area of the shape is $27\pi \text{ cm}^2$. Find the exact perimeter of the shape.



1. You're given the sector area, so use the formula to find the sector angle.

$$27\pi = \frac{\theta}{360^\circ} \times \pi \times 9^2 \Rightarrow \theta = \frac{360^\circ \times 27\pi}{81\pi} = 120^\circ$$

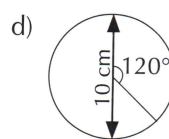
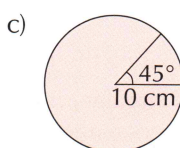
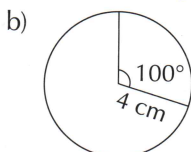
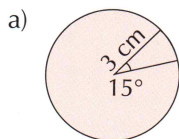
2. Find the length of the arc and add the two straight sides to find the perimeter.

$$\text{Arc length} = \frac{120^\circ}{360^\circ} \times (2 \times \pi \times 9) = 6\pi \text{ cm}$$

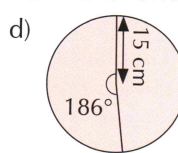
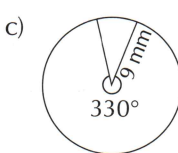
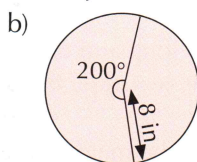
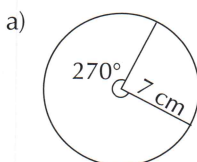
$$\text{Perimeter} = 9 + 9 + 6\pi = (18 + 6\pi) \text{ cm}$$

Exercise 3

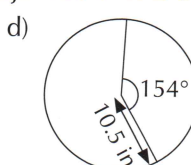
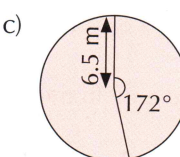
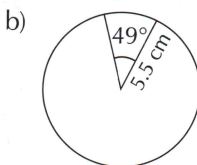
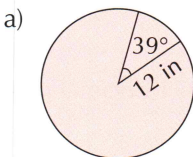
- Q1 For each circle below, find the exact length of the minor arc and exact area of the minor sector.



- Q2 Find the exact length of the major arc and the exact area of the major sector for the following circles.



- Q3 For the circles below, find the length of the major arc and the area of the major sector to 2 d.p.



- Q4 A sector of a circle has an arc length of $15\pi \text{ cm}$. The radius of the circle is 12 cm. Find the angle of the minor sector.

- Q5 A circular park has an area of $400\pi \text{ m}^2$. A children's playground is a sector of the circle and has an area of $80\pi \text{ m}^2$. Calculate:



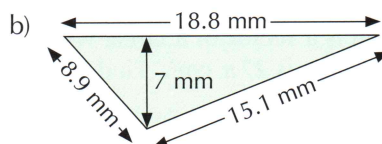
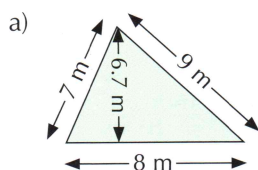
- a) the radius of the park, b) the sector angle of the children's playground,
c) the perimeter of the children's playground, to 1 decimal place.

Review Exercise

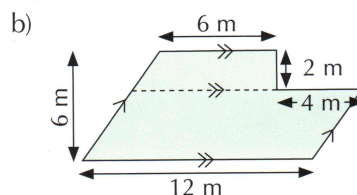
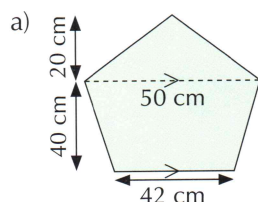
Q1 Barbie has a rectangular lawn that is 23.5 m long by 17.3 m wide. She is going to mow the lawn and then put a fence around the outside.

- What area will Barbie have to mow (to the nearest m^2)?
- How long will the fence need to be?

Q2 Find the perimeter and area of the triangles below:



Q3 Calculate the area of the following composite shapes:



Q4 Homer and Ned are comparing the flower beds in their gardens.

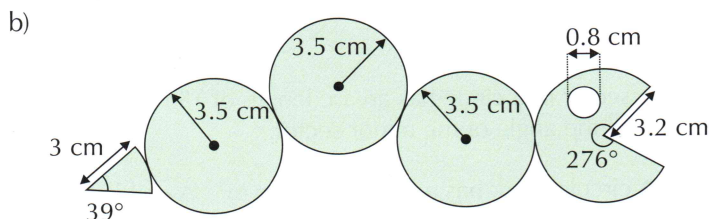
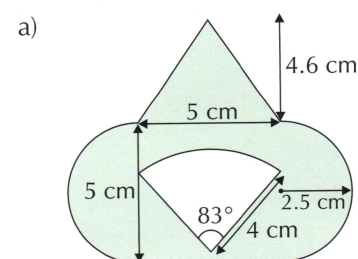
- Homer's flower bed is circular with a diameter of 2.7 m. Calculate the circumference and area of Homer's flower bed to 1 d.p.
- Ned's flower bed is in the shape of a semicircle. Its area is 19.6 m^2 . What is the length of its straight edge? Give your answer to 2 d.p.



Q5 Timi has a rectangular sheet of metal that is 50 cm long and 80 cm wide. She cuts a quarter circle of radius 15 cm from each corner. Calculate, to 1 d.p., the remaining area of metal.

- Q6**
- Find the length of the major arc and the area of the major sector for a circle with radius 13 cm and minor sector angle 14° . Give your answers to 2 d.p.
 - Find the exact length of the minor arc and the exact area of the minor sector for a circle with diameter 10 m and major sector angle 320° .

Q7 Calculate the total shaded area in the shapes below, giving your answers to 1 decimal place.



Exam-Style Questions

- Q1** A rectangular floor measures 9 m by 7.5 m.
It is to be tiled using square tiles with sides of length 0.5 m.
- a) What is the area of: (i) the floor, (ii) one of the tiles?



[2 marks]

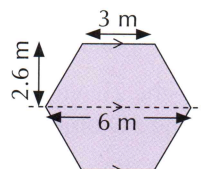
- b) How many tiles are needed to cover the floor?

[1 mark]

- Q2** Alec has built a semicircular sheep pen. A wall forms the straight side of the semicircle and Alec used wooden fencing to construct the rest of the pen. The radius of the semicircle is 16 m. How many metres of fencing has Alec used?

[2 marks]

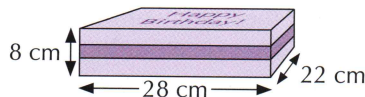
- Q3** The shape shown in the diagram has a line of symmetry, marked by the dotted line. Find the area of the shape.



[2 marks]

- Q4** Ali bakes a rectangular cake, as shown.

- a) What length of ribbon is needed to wrap once around the outside of the cake, as shown in the diagram to the right?

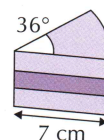


[2 marks]

- b) If the top and the four sides are to be iced, what area of icing will be needed?

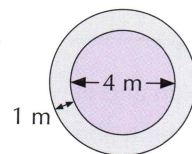
[2 marks]

- c) Tyler bakes a circular cake, and cuts it into 10 segments, one of which is shown. What length of ribbon is needed to wrap once around the segment? Give your answer to 1 decimal place.



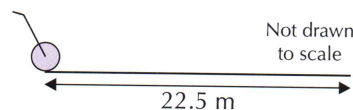
[3 marks]

- Q5** A circular pond of diameter 4 m is surrounded by a path 1 m wide, as shown in the diagram. What is the area of the path, to 1 d.p.?



[3 marks]

- Q6** A groundsman uses a roller to help dry a flat 22.5 m long cricket pitch. He pushes the roller in a straight line. If it takes exactly 17 revolutions of the roller to cover one length of the pitch, work out the radius of the roller in cm. Give your answer to an appropriate degree of accuracy.



[3 marks]