26.1 Vectors and Scalars

Time for something completely different now — vectors. You can think of them as straight lines from one point in space to another.

Vectors and Scalars

Learning Objectives — Spec Ref G25:
- Understand and use vector notation.
- Be able to multiply vectors by scalars.

A **vector** has **magnitude** (size or length) and **direction**.

There are various ways to represent a vector — it can be:

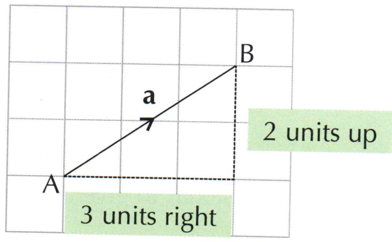
- written as a **column vector** (positive numbers mean right or up, negative numbers mean left or down).

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

x-component (horizontal): 3 units right

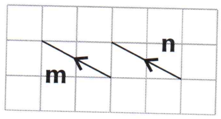
y-component (vertical): 2 units up

- shown on a diagram by an **arrow**.



Vectors can be written using their **end points**, so $\overrightarrow{AB}$ means the vector from **A to B** and $\overrightarrow{BA}$ means the vector from **B to A**. $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are **different** vectors — they have the **same magnitude** but **different directions**. Vectors can also be written using **bold** letters (**a**) or **underlined** letters ($\underline{a}$ or $\underset{\sim}{a}$).

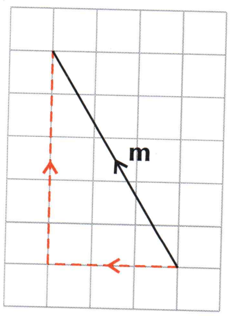
Two vectors are considered to be **equal** if they have the **same magnitude** and **direction**. E.g. vectors **m** and **n** are equal vectors, even though they have different start and end points.



Example 1

On a grid, draw the vector m = $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

1. The positive and negative directions in column vectors are the same as they are for coordinates.
2. So –3 in the x-component means '3 units left', and 5 in the y-component means '5 units up'.
3. Make sure that you label the vector **m**, and add an arrow to show its direction.



Tip: It doesn't matter where the vector is drawn on the grid. You can choose any starting point — as long as the end point is 3 units to the left and 5 units up.

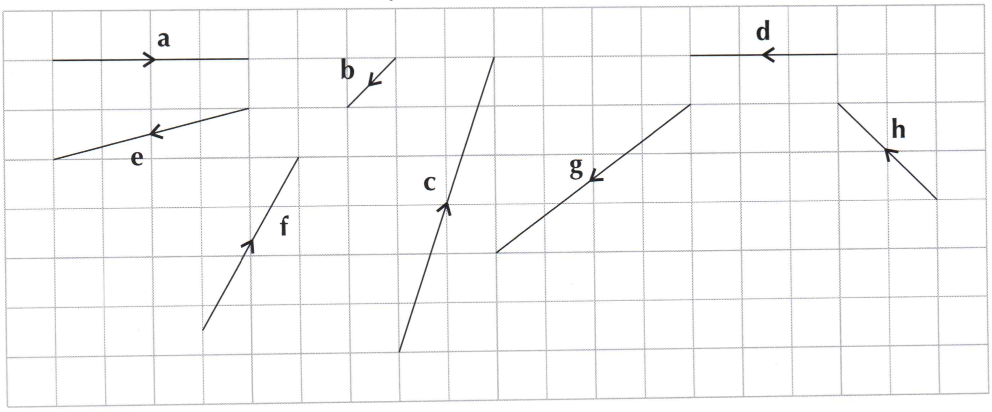
Exercise 1

Q1 On a grid, draw arrows to represent the following vectors.

a) $\begin{pmatrix}1\\4\end{pmatrix}$ b) $\begin{pmatrix}3\\5\end{pmatrix}$ c) $\begin{pmatrix}-2\\4\end{pmatrix}$ d) $\begin{pmatrix}0\\5\end{pmatrix}$

e) $\begin{pmatrix}-3\\-5\end{pmatrix}$ f) $\begin{pmatrix}3\\0\end{pmatrix}$ g) $\begin{pmatrix}-3\\-3\end{pmatrix}$ h) $\begin{pmatrix}0\\-3\end{pmatrix}$

Q2 Write down the column vectors represented by these arrows:



A **scalar** is just a number — scalars have a magnitude (size), but **no** direction. A vector can be **multiplied** by a scalar to give another vector. The resulting vector is **parallel** to the original vector.

E.g. $3 \times \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix}$ so the vectors $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}3\\6\end{pmatrix}$ are **parallel**.

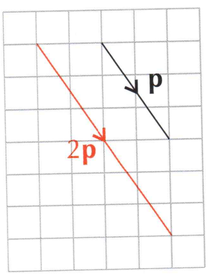
If the scalar is **negative**, the direction of the vector is **reversed**.

Example 2

If vector p = $\begin{pmatrix}2\\-3\end{pmatrix}$, write the following as column vectors: a) 2p b) $\frac{1}{2}$p c) –p

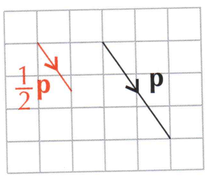
Multiply a vector by a scalar by multiplying the x-component and the y-component separately.

a) $2\mathbf{p} = \begin{pmatrix}2 \times 2\\2 \times -3\end{pmatrix} = \begin{pmatrix}\mathbf{4}\\\mathbf{-6}\end{pmatrix}$



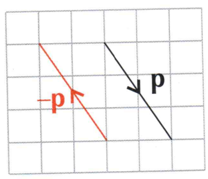
The direction has stayed the same but the magnitude has doubled.

b) $\frac{1}{2}\mathbf{p} = \begin{pmatrix}\frac{1}{2} \times 2\\\frac{1}{2} \times -3\end{pmatrix} = \begin{pmatrix}\mathbf{1}\\\mathbf{-\frac{3}{2}}\end{pmatrix}$



The direction has stayed the same but the magnitude has halved.

c) $-\mathbf{p} = \begin{pmatrix}-1 \times 2\\-1 \times -3\end{pmatrix} = \begin{pmatrix}\mathbf{-2}\\\mathbf{3}\end{pmatrix}$



The direction has reversed but the magnitude has stayed the same.

Exercise 2

Q1 If $\mathbf{q} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, find and draw the following vectors.

a) 3**q** b) 5**q** c) 1.5**q** d) –2**q**

Q2

$\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} 3 \\ 12 \end{pmatrix}$ $\mathbf{d} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$

$\mathbf{e} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $\mathbf{f} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ $\mathbf{g} = \begin{pmatrix} 3 \\ -12 \end{pmatrix}$ $\mathbf{h} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

From the list of vectors above:

a) Which vector is equal to 2**a**?

b) Which vector is equal to –3**b**?

c) Which vector is parallel to **e**?

d) Which two vectors are perpendicular?

Adding and Subtracting Vectors

Learning Objective — Spec Ref G25:
Be able to add and subtract vectors.

To **add** or **subtract** column vectors, you add or subtract the *x*-components and *y*-components separately. The sum of two vectors is called the **resultant vector**.

Vectors can also be added by drawing them in a chain, nose-to-tail. The resultant vector goes in a **straight line** from the **start** to the **end** of the chain of vectors. When you add two vectors, it doesn't matter which comes first, i.e. **a** + **b** = **b** + **a**. Be careful when subtracting though — just like with ordinary numbers **a** – **b** = –**b** + **a**, not **b** – **a**.

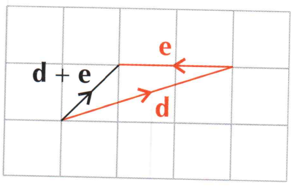
Example 3

If vector d = $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$**, vector e =** $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ **and vector f =** $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$**, write the following as column vectors:**

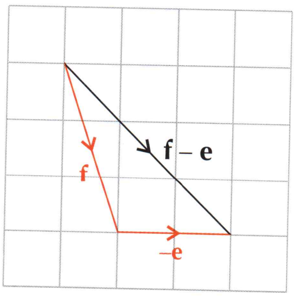
a) d + e **b) f – e**

Add and subtract the *x*-components and the *y*-components separately.

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3+(-2) \\ 1+0 \end{pmatrix} = \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \end{pmatrix}$$



$$\begin{pmatrix} 1 \\ -3 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1-(-2) \\ -3-0 \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{-3} \end{pmatrix}$$



Tip: Subtracting a vector is the same as adding the reverse vector. In part b) the diagram shows that **f** – **e** is the same as **f** + (–**e**).

Example 4

If vector p = $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$, vector q = $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and vector r = $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, write 2q + p – 3r as a column vector.

1. Start by working out the values of the vectors 2**q** and 3**r**.

$$2\mathbf{q} = \begin{pmatrix} 2\times -3 \\ 2\times 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$$3\mathbf{r} = \begin{pmatrix} 3\times 1 \\ 3\times 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

2. Now use vector addition and subtraction to work out the final answer.

$$2\mathbf{q} + \mathbf{p} - 3\mathbf{r} = \begin{pmatrix} -6 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} -6+4-3 \\ 2+2-6 \end{pmatrix} = \begin{pmatrix} \mathbf{-5} \\ \mathbf{-2} \end{pmatrix}$$

Tip: Always remember to multiply, add and subtract the x-components and the y-components of the vectors separately.

Exercise 3

Q1 Write the answers to the following calculations as column vectors.
For each expression, draw arrows to represent the two given vectors and the resultant vector.

a) $\begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ b) $\begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ c) $\begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

d) $\begin{pmatrix} 7 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ e) $\begin{pmatrix} 5 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ f) $\begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

Q2 If $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, work out:

a) **b** + **c** b) **c** – **a** c) 2**c** + **a**

d) **a** + **b** – **c** e) 5**b** + 4**c** f) 4**a** – **b** + 3**c**

Q3 If $\mathbf{p} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, work out:

a) **p** + **r** b) **p** – **q** c) 3**q** + **r**

d) 3**p** + 2**r** e) **p** + **r** – **q** f) 2**p** – 3**q** + **r**

Q4 $\mathbf{u} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

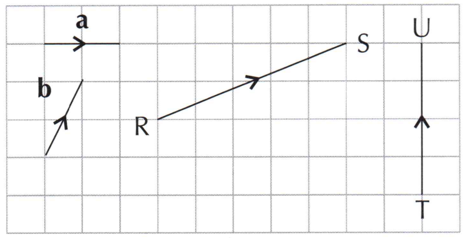
a) Work out **u** + 2**v**.

b) Draw the vectors **u** + 2**v** and **w** on a grid.

c) What do you notice about the directions of the vectors **u** + 2**v** and **w**?

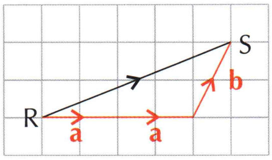
Example 5

The grid on the right shows vectors a, b, $\overrightarrow{RS}$ and $\overrightarrow{TU}$. Describe the following in terms of vectors a and b.



a) $\overrightarrow{RS}$

1. Find a route from R to S using just vectors **a** and **b**.



2. So two lots of vector **a** and then vector **b** takes you from R to S:

$\overrightarrow{RS}$ = **2a + b**

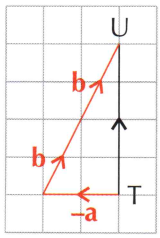
3. Check the answer by doing the vector addition. Work out the component form of the vectors by counting the horizontal and vertical distances.

Check: $2\mathbf{a} + \mathbf{b} = 2\begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}1\\2\end{pmatrix}$

$= \begin{pmatrix}4\\0\end{pmatrix} + \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix} = \overrightarrow{RS}$ ✔

b) $\overrightarrow{TU}$

1. Find a route from T to U using just vectors **a** and **b**.



2. The reverse of vector **a** and two lots of vector **b** take you from T to U:

$\overrightarrow{TU}$ = **–a + 2b**

3. Check the answer by doing the vector addition.

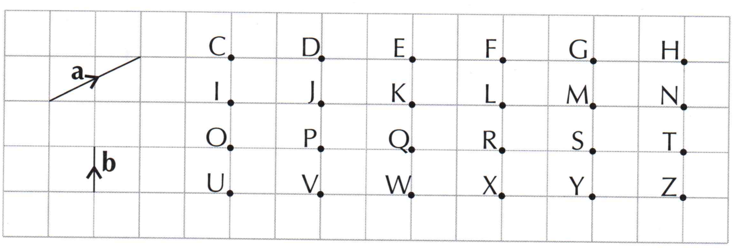
Check: $-\mathbf{a} + 2\mathbf{b} = -\begin{pmatrix}2\\0\end{pmatrix} + 2\begin{pmatrix}1\\2\end{pmatrix}$

$= \begin{pmatrix}-2\\0\end{pmatrix} + \begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix} = \overrightarrow{TU}$ ✔

Tip: If you're struggling to spot a route between the two points using a diagram, try writing out all the column vectors. Then use them to find a combination of **a** and **b** that gives the vector you're looking for.

Exercise 4

Q1 The grid below shows vectors **a**, **b** and points C-Z.



Write the following vectors in terms of **a** and **b**.

a) $\overrightarrow{WH}$ b) $\overrightarrow{ZH}$ c) $\overrightarrow{HR}$ d) $\overrightarrow{FX}$

e) $\overrightarrow{UD}$ f) $\overrightarrow{DU}$ g) $\overrightarrow{EM}$ h) $\overrightarrow{TP}$

i) $\overrightarrow{YJ}$ j) $\overrightarrow{CF}$ k) $\overrightarrow{FZ}$ l) $\overrightarrow{CZ}$

26.2 Vector Geometry

Vectors can be used in geometry to describe the lines that make up shapes. You can use the vectors along with properties of the shape to solve problems and prove other geometrical properties.

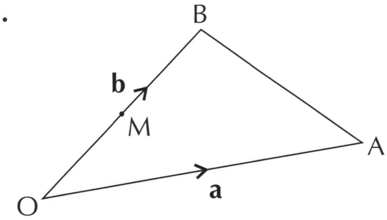
Learning Objective — Spec Ref G25:
Use vectors to solve geometry problems.

Prior Knowledge Check:
Understand and use vector notation (p.339), shape properties (Section 20) and ratios (Section 4).

The first step to any vector geometry problem is working out what information you already **know** and what you need to **find**. Always work with a **diagram** — if you're not given one, **draw your own**. Then it'll be easier to see which vectors you need to add, subtract or multiply to get to the solution.

Example 1

In triangle OAB, $\overrightarrow{OA}$ = a and $\overrightarrow{OB}$ = b. M is the midpoint of OB. Write down $\overrightarrow{AM}$, in terms of vectors a and b:



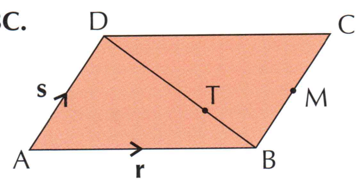
1. To get from A to M, go from A to O, then from O to M. $\overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM}$
2. To get from A to O, you go backwards along the vector **a**. $\overrightarrow{AO} = -\mathbf{a}$
3. To get from O to M, you go halfway along the vector **b**. $\overrightarrow{OM} = \frac{1}{2}\mathbf{b}$
4. Add the vectors together to find $\overrightarrow{AM}$. $\overrightarrow{AM} = \mathbf{-a + \frac{1}{2}b}$

Ratios and **fractions** may be used to describe the position of a point on a line. E.g. if point Z lies on the line XY such that XZ : ZY = 3 : 5, you know that point Z is $\frac{3}{3+5} = \frac{3}{8}$ of the way from X to Y. If you know the vector of the **whole line**, you can use the ratio or fraction to work out the vector for **part of the line**.

To show that three points lie on a **straight line** you need to show that two vectors between the points are **scalar multiples** of each other. E.g. XYZ is a straight line if $\overrightarrow{XY}$ is a scalar multiple of $\overrightarrow{XZ}$ or $\overrightarrow{YZ}$.

Example 2

ABCD is a parallelogram, $\overrightarrow{AB}$ = r and $\overrightarrow{AD}$ = s. M is the midpoint of BC. Point T lies on BD such that BT : BD = 1 : 3.



a) Write down $\overrightarrow{AT}$ in terms of vectors r and s.

1. To get from A to T, go from A to B, then from B to T. $\overrightarrow{AT} = \overrightarrow{AB} + \overrightarrow{BT}$
2. As BT : BD = 1 : 3, BT must be $\frac{1}{3}$ of the length of BD. $\overrightarrow{BT} = \frac{1}{3}\overrightarrow{BD}$
3. To get from B to D, go from B to A, then from A to D. $\overrightarrow{BD} = -\mathbf{r} + \mathbf{s}$
4. Multiply $\overrightarrow{BD}$ by $\frac{1}{3}$ to find $\overrightarrow{BT}$. $\overrightarrow{BT} = -\frac{1}{3}\mathbf{r} + \frac{1}{3}\mathbf{s}$
5. Add $\overrightarrow{AB}$ and $\overrightarrow{BT}$ to find $\overrightarrow{AT}$. $\overrightarrow{AT} = \mathbf{r} - \frac{1}{3}\mathbf{r} + \frac{1}{3}\mathbf{s} = \mathbf{\frac{2}{3}r + \frac{1}{3}s}$

Tip: Be careful when you're using ratios — the ratio given in this question is the ratio of part of the line to the whole line. It's different from the ratio shown in the theory above.

b) Show that ATM is a straight line.

1. First work out $\overrightarrow{AM}$ in terms of $\mathbf{r}$ and $\mathbf{s}$. To get from A to M, go from A to B, then from B to M.

$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM}$

$\overrightarrow{BM} = \frac{1}{2}\mathbf{s}$

$\overrightarrow{AM} = \mathbf{r} + \frac{1}{2}\mathbf{s}$

Tip: Since ABCD is a parallelogram, $\overrightarrow{BC} = \overrightarrow{AD} = \mathbf{s}$.

2. To show that ATM is a straight line you need to show that $\overrightarrow{AT}$ is a scalar multiple of $\overrightarrow{AM}$.

$\frac{2}{3}\overrightarrow{AM} = \frac{2}{3}(\mathbf{r} + \frac{1}{2}\mathbf{s}) = \frac{2}{3}\mathbf{r} + \frac{1}{3}\mathbf{s} = \overrightarrow{AT}$

$\overrightarrow{AT}$ is a scalar multiple of $\overrightarrow{AM}$ so ATM is a straight line.

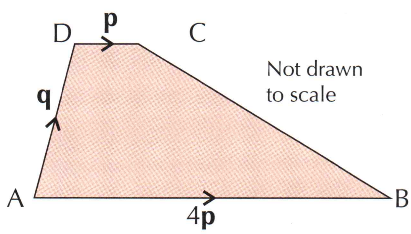
Exercise 1

Q1 ABCD is a trapezium. $\overrightarrow{AB} = 4\mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$ and $\overrightarrow{DC} = \mathbf{p}$.
Write down, in terms of $\mathbf{p}$ and $\mathbf{q}$:

a) $\overrightarrow{CA}$

b) $\overrightarrow{CB}$

c) $\overrightarrow{BD}$

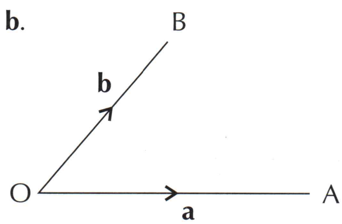


Q2 In the diagram on the right, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
Point C is added such that $\overrightarrow{OC} = \mathbf{a} + \mathbf{b}$.

a) What type of shape is OACB?

b) Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$:

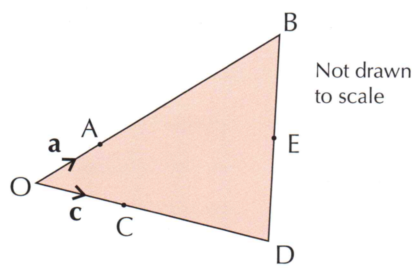
(i) $\overrightarrow{CO}$ (ii) $\overrightarrow{AB}$



Q3 In the triangle OBD, $\overrightarrow{OA} = \mathbf{a}$ is $\frac{1}{4}$ of the length of $\overrightarrow{OB}$,
$\overrightarrow{OC} = \mathbf{c}$ is $\frac{1}{3}$ of the length of $\overrightarrow{OD}$ and E is the midpoint of BD.
Write down, in terms of $\mathbf{a}$ and $\mathbf{c}$:

a) $\overrightarrow{OB}$ b) $\overrightarrow{OD}$ c) $\overrightarrow{AB}$

d) $\overrightarrow{BA}$ e) $\overrightarrow{AC}$ f) $\overrightarrow{OE}$



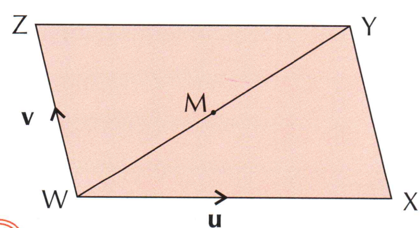
Q4 WXYZ is a parallelogram.
$\overrightarrow{WX} = \mathbf{u}$ and $\overrightarrow{WZ} = \mathbf{v}$. M is the midpoint of WY.

a) Write down, in terms of $\mathbf{u}$ and $\mathbf{v}$:

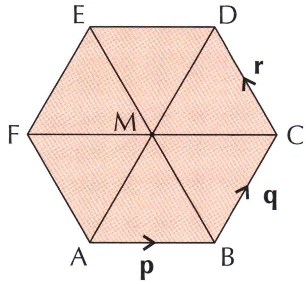
(i) $\overrightarrow{WY}$ (ii) $\overrightarrow{WM}$

(iii) $\overrightarrow{XW}$ (iv) $\overrightarrow{XZ}$

b) Show using vectors that M is the midpoint of XZ. PROBLEM SOLVING



Q5 ABCDEF is a regular hexagon. M is the centre of the hexagon.
$\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{BC} = \mathbf{q}$ and $\overrightarrow{CD} = \mathbf{r}$.



a) Write down, in terms of **p** and **q**:

(i) $\overrightarrow{DE}$ (ii) $\overrightarrow{AC}$

b) Using vectors, show that $\overrightarrow{FD} = \overrightarrow{AC}$.

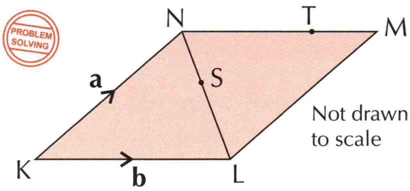
c) Write down, in terms of **q** and **r**:

(i) $\overrightarrow{AM}$ (ii) $\overrightarrow{MB}$

d) Write **p** in terms of **q** and **r**.

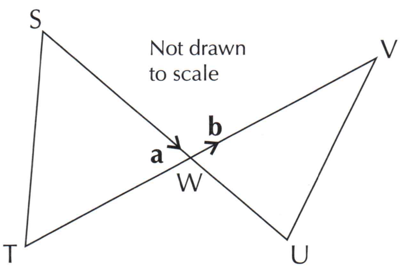
Q6 KLMN is a rhombus. $\overrightarrow{KN} = \mathbf{a}$ and $\overrightarrow{KL} = \mathbf{b}$.
Point S lies on NL such that NS : SL = 2 : 3.
Point T lies on NM such that $\overrightarrow{NT} = \frac{2}{3}\overrightarrow{NM}$.

PROBLEM SOLVING



Show that KST is a straight line.

Q7 In the diagram, $\overrightarrow{SU} = \mathbf{a}$, $\overrightarrow{TV} = \mathbf{b}$, $\overrightarrow{SW} = 2\overrightarrow{WU}$ and $3\overrightarrow{TW} = 2\overrightarrow{WV}$.



a) Write:

(i) $\overrightarrow{WU}$ in terms of $\overrightarrow{SW}$

(ii) $\overrightarrow{SU}$ in terms of $\overrightarrow{SW}$

(iii) $\overrightarrow{SW}$ in terms of **a**

b) Write:

(i) $\overrightarrow{WV}$ in terms of $\overrightarrow{TW}$

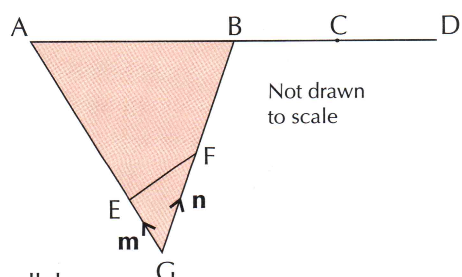
(ii) $\overrightarrow{TV}$ in terms of $\overrightarrow{WV}$

(iii) $\overrightarrow{VW}$ in terms of **b**

c) Write, in terms of **a** and **b**:

(i) $\overrightarrow{ST}$ (ii) $\overrightarrow{UV}$

Q8 $\overrightarrow{GE} = \mathbf{m}$, $\overrightarrow{GF} = \mathbf{n}$, $\overrightarrow{GA} = 5\overrightarrow{GE}$ and $\overrightarrow{GB} = 3\overrightarrow{GF}$.
B is the midpoint of AD, and C is the midpoint of BD.



a) Write down, in terms of **m** and **n**:

(i) $\overrightarrow{GA}$ (ii) $\overrightarrow{GB}$

(iii) $\overrightarrow{AB}$ (iv) $\overrightarrow{BC}$

If E, F and C were on a straight line, $\overrightarrow{EF}$ and $\overrightarrow{FC}$ would be parallel.

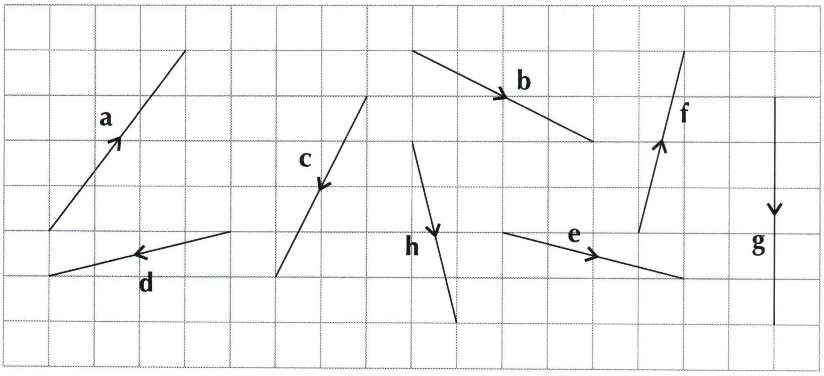
b) Find $\overrightarrow{EF}$ and $\overrightarrow{FC}$ in terms of **m** and **n**, and use your answers to show that E, F and C do not lie on a straight line.

PROBLEM SOLVING

Review Exercise

Q1 Match the following column vectors to the correct vectors in the diagram below.

$\begin{pmatrix}1\\4\end{pmatrix}$ $\begin{pmatrix}-2\\-4\end{pmatrix}$ $\begin{pmatrix}-4\\-1\end{pmatrix}$ $\begin{pmatrix}4\\-2\end{pmatrix}$ $\begin{pmatrix}0\\-5\end{pmatrix}$ $\begin{pmatrix}3\\4\end{pmatrix}$ $\begin{pmatrix}4\\-1\end{pmatrix}$ $\begin{pmatrix}1\\-4\end{pmatrix}$



Q2 If $\mathbf{p} = \begin{pmatrix}4\\-3\end{pmatrix}$, $\mathbf{q} = \begin{pmatrix}0\\2\end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix}-1\\5\end{pmatrix}$, find:

a) $3\mathbf{p}$ b) $2\mathbf{q} + \mathbf{r}$ c) $\mathbf{r} - 2\mathbf{p}$ d) $\mathbf{p} + 5\mathbf{r} - 3\mathbf{q}$

Q3 $\mathbf{a} = \begin{pmatrix}6\\-4\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}-2\\3\end{pmatrix}$ $\mathbf{c} = \begin{pmatrix}1\\2\end{pmatrix}$ $\mathbf{d} = \begin{pmatrix}4\\-2\end{pmatrix}$ $\mathbf{e} = \begin{pmatrix}3\\6\end{pmatrix}$ $\mathbf{f} = \begin{pmatrix}4\\-6\end{pmatrix}$

a) Draw the vector $\mathbf{a} + \mathbf{b}$ and write down the resultant vector as a column vector.

b) Draw the vector $\mathbf{e} - \mathbf{d}$ and write down the resultant vector as a column vector.

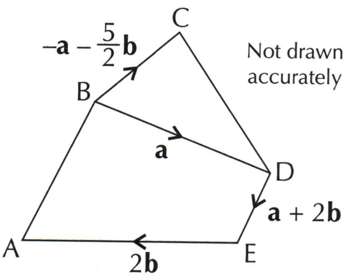
c) Which two vectors above can be added to give the resultant vector $\begin{pmatrix}1\\9\end{pmatrix}$?

d) Which vector above is parallel to **c**?

Q4 The diagram on the right shows pentagon ABCDE.
$\overrightarrow{BD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{a} + 2\mathbf{b}$, $\overrightarrow{EA} = 2\mathbf{b}$ and $\overrightarrow{BC} = -\mathbf{a} - \frac{5}{2}\mathbf{b}$.
Write down, in terms of **a** and **b**:

a) $\overrightarrow{BE}$ b) $\overrightarrow{AB}$

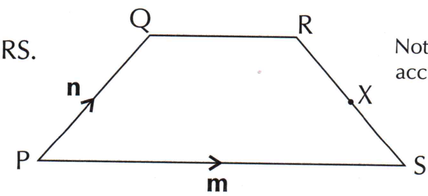
c) $\overrightarrow{CD}$ d) $\overrightarrow{AC}$



Q5 PQRS is a trapezium, where X is the midpoint of RS.
$\overrightarrow{PS} = \mathbf{m}$, $\overrightarrow{PQ} = \mathbf{n}$ and $2\overrightarrow{PS} = 5\overrightarrow{QR}$.

Find $\overrightarrow{PX}$ in terms of **m** and **n**.

PROBLEM SOLVING



Exam-Style Questions

Q1 s and t are column vectors such that $\mathbf{s} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$. Work out the value of:

a) $5\mathbf{s}$

[1 mark]

b) $2\mathbf{s} - \mathbf{t}$

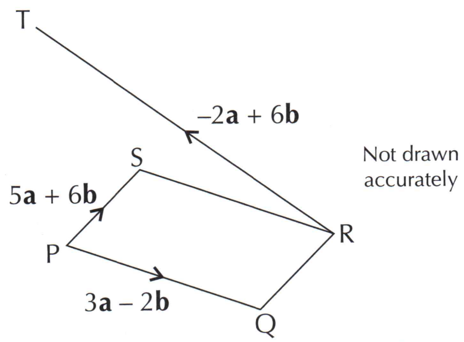
[2 marks]

Q2 **a** and **b** are column vectors such that $\mathbf{a} = \begin{pmatrix} 5p \\ 4q \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -q \\ 0 \end{pmatrix}$ where p and q are integers.

If $2\mathbf{a} - 3\mathbf{b} = \begin{pmatrix} 18 \\ -32 \end{pmatrix}$, find the values of p and q.

[4 marks]

Q3 The diagram shows parallelogram PQRS.

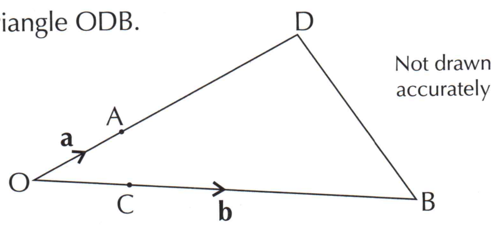


$\overrightarrow{PQ} = 3\mathbf{a} - 2\mathbf{b}$, $\overrightarrow{PS} = 5\mathbf{a} + 6\mathbf{b}$ and T is a point such that $\overrightarrow{RT} = -2\mathbf{a} + 6\mathbf{b}$.

Prove that QST is a straight line.

[4 marks]

Q4 The diagram shows a triangle ODB.



A is a point on OD such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AD} = 3\overrightarrow{OA}$.
$\overrightarrow{OB} = \mathbf{b}$ and C is a point on OB such that OC : OB = 1 : 4.

Prove that ADBC is a trapezium.

[5 marks]