Section 24 Constructions

24.1 Scale Drawings

It's important for things like maps to show distances accurately, but scaled down to a manageable size. Drawing diagrams to the correct scale is all about ratios and converting between different units of length.

Learning Objectives — Spec Ref R2/G15:
- Interpret and construct scale drawings.
- Work out real-life distances from maps and vice versa.

Prior Knowledge Check: Use ratio notation and apply ratios to problems with real contexts (Section 4). Convert between metric units of length (p.276).

The **scale** on a map or plan tells you the **relationship** between the distances on the **map** and in **real life**.

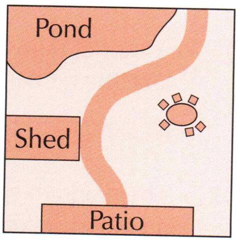
For example, a map scale of 1 cm : 100 m means that 1 cm on the **map** represents an **actual distance** of 100 m. A scale **without units** (e.g. 1 : 100) means you can use **any units** as long as they're the **same** — e.g. 1 cm : 100 cm, 1 mm : 100 mm, etc.

To work out what a map distance **represents in real life**, **multiply** the map distance by the real-life distance **represented by 1 cm** on the map — so for a map with scale 1 cm : 10 km, a map distance of 2.5 cm would be 2.5 × 10 = 25 km in real life. To convert **real-life distances** to **map distances**, you **divide** by this value — so 50 km in real life would be represented by 50 ÷ 10 = 5 cm on a map with scale 1 cm : 10 km.

Example 1

The diagram shows a plan of a square garden, drawn to a scale of 1 cm : 5 m.

a) With the help of a ruler, find the actual perimeter of the garden.



1. Measure a side of the garden with your ruler. Both the length and width should be the same, since the diagram is a square. — Length = width = 3 cm
2. Use the scale to work out the actual length of one side. — 1 cm represents 5 m, so 3 cm represents 3 × 5 = 15 m
3. Each length is 15 m, so use this to work out the perimeter. — Perimeter = 4 × 15 m = **60 m**

b) Two of the trees in the garden are 2.5 m apart. How far apart should they be drawn on the plan?

Use the scale to work out the length on the plan that represents an actual distance of 2.5 m. — 5 m is shown as 1 cm, so 2.5 m is shown as 2.5 ÷ 5 = **0.5 cm**

Example 2

A plan of the grounds of a palace has a scale of 1 : 4000.

a) Represent this scale in the form 1 cm : *n* m.

1. Write down the scale using centimetres. — 1 cm : 4000 cm
2. Convert the right-hand side to metres by dividing by 100. — 1 cm : (4000 ÷ 100) m
   1 cm : 40 m

Tip: You could also do:
1 m : 4000 m
100 cm : 4000 m
1 cm : 40 m

b) On the plan, one of the lakes is 11.5 cm wide. Calculate the actual width of the lake.

Use the scale to work out what 11.5 cm represents in real life.

1 cm represents 40 m, so
11.5 cm represents 11.5 × 40 = **460 m**

Exercise 1

Q1 The scale on a map of Europe is 1 cm : 50 km. Find the distance used on the map to represent:

a) 150 km b) 600 km c) 1000 km

d) 25 km e) 10 km f) 15 km

Q2 The floor plan of a house is drawn to a scale of 1 cm : 2 m.
Find the actual dimensions of the rooms if they are shown on the plan as:

a) 2.7 cm by 1.5 cm b) 3.2 cm by 2.2 cm c) 1.85 cm by 1.4 cm d) 0.9 cm by 1.35 cm

Q3 A bridge of length 0.8 km is to be drawn on a map with scale 1 cm : 0.5 km.
What length will the bridge appear on the map?

Q4 A set of toy furniture is made to a scale of 1 : 40. Find the dimensions of the actual furniture when the toys have the following measurements.

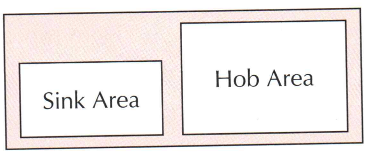
a) Width of bed: 3.5 cm b) Length of table: 3.2 cm c) Height of chair: 2.4 cm

Q5 A road of length 6.7 km is to be drawn on a map. The scale of the map is 1 : 250 000.
How long will the road be on the map in centimetres?

Q6 A model railway uses a scale of 1 : 500.
Use the actual measurements given below to find measurements for the model in centimetres.

a) Length of footbridge: 100 m b) Height of signal box: 6 m

Q7 The diagram on the right shows a plan for a kitchen surface.
The scale is 1 mm : 3 cm.



Use the plan to find the actual dimensions of: a) the sink area, b) the hob area.

Q8 A path of length 4.5 km is shown on a map as a line of length 3 cm.
Express the scale in the form 1 cm : n km.

Q9 The plan for a school has a scale of 1 : 1500.

a) Express this scale in the form 1 cm : n m.

b) The school playground is 60 m in length. Find the corresponding length on the plan.

Q10 On the plans for a house, 30 cm represents the length of a garden with actual length 18 m.

a) Find the map scale in the form 1 : n.

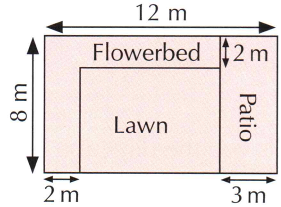
b) On the plan, the width of the garden is 12 cm. What is its actual width in metres?

c) The lounge has a width of 4.5 m. What is the corresponding length on the plan?

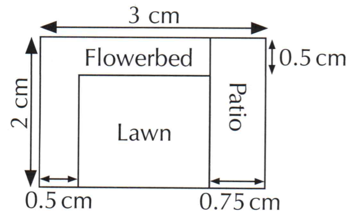
If you know the **measurements** of something, you can draw an **accurate plan** using a given scale by first **converting** all the distances to lengths on the plan, then using your **ruler** to draw the lines to the required lengths.

Example 3

The diagram shows a rough sketch of a garden.
Use the scale 1 : 400 to draw an accurate plan of the garden.

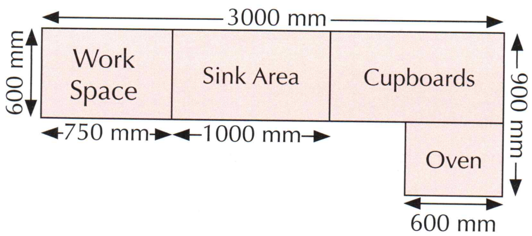


1. Write down the scale in cm. — 1 cm : 400 cm
2. Change the right-hand side to metres by dividing by 100. — 1 cm : 4 m
3. Use the scale to work out the lengths on the plan, then use these lengths to draw your plan. — 4 m is shown as 1 cm, so:
   12 m is shown as 12 ÷ 4 = 3 cm
   8 m is shown as 8 ÷ 4 = 2 cm
   2 m is shown as 2 ÷ 4 = 0.5 cm
   3 m is shown as 3 ÷ 4 = 0.75 cm

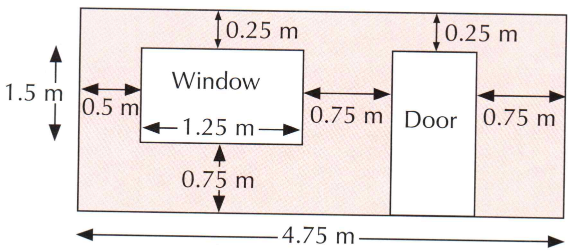


Exercise 2

Q1 Using a scale of 1 : 20, draw an accurate plan of the kitchen shown below.



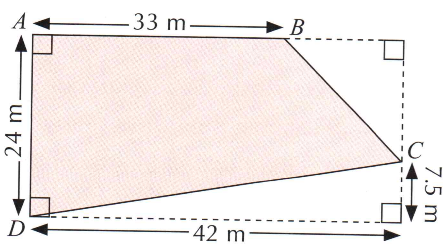
Q2 Use the scale 1 : 25 to draw a scale drawing for the house extension shown in the rough sketch below.



Q3 The diagram on the right shows a sketch of a park lake.

a) Draw an accurate plan of the lake using the scale 1 cm : 3 m.

b) There is a duck house at the intersection of AC and BD. Find the actual distance between the duck house and point B.



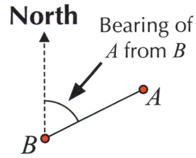
24.2 Bearings

Bearings are often used in navigation to describe which direction something is in, relative to a north line.

Learning Objective — Spec Ref G15:
Understand and use bearings.

Prior Knowledge Check: Use properties of angles and lines. See p.248-251.

A **bearing** tells you the direction of one point from another. Bearings are given as **three-figure angles**, measured **clockwise** from the **north line**. Draw the north line at the point you're finding the angle '**from**', then go clockwise to draw the angle you want. To work out a bearing, use the information you're given along with **properties of angles** (angles **around a point**, on a **straight line**, and on **parallel lines** — see p.248-251).

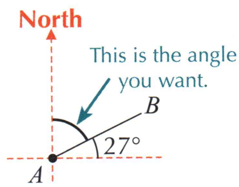


Example 1

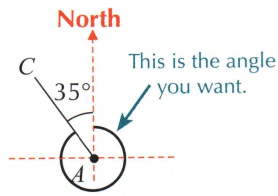
a) **Find the bearing of *B* from *A*.**

b) **Find the bearing of *C* from *A*.**

1. Find the clockwise angle from the north line.
2. Give the bearing as a three-figure number.



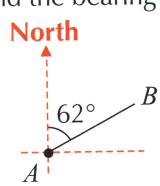
a) 90° – 27° = 63°, so the bearing of *B* from *A* is **063°**.

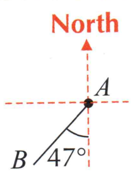


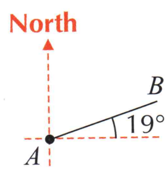
b) 360° – 35° = 325°, so the bearing of *C* from *A* is **325°**.

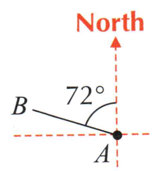
Exercise 1

Q1 Find the bearing of *B* from *A* in the following diagrams.

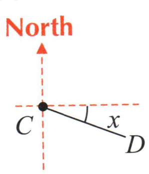
a) 

b) 

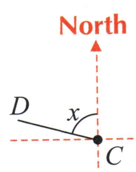
c) 

d) 

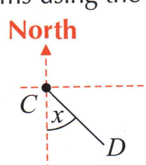
Q2 Find the size of angle *x* in each of the following diagrams using the information given.

a) 

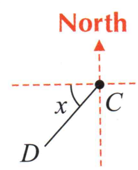
Bearing of *D* from *C* is 111°

b) 

Bearing of *D* from *C* is 285°

c) 

Bearing of *D* from *C* is 135°

d) 

Bearing of *D* from *C* is 222°

Q3 Liverchester is 100 km due south of Manpool. King's Hill is 100 km due east of Liverchester.

a) Sketch the layout of the three locations.

b) Find the bearing of King's Hill from Liverchester.

c) Find the bearings from King's Hill of: (i) Liverchester (ii) Manpool

Q4 Mark a point *O* and draw in a north line. Use a protractor to help you draw the points a) to d) with the following bearings from *O*.

a) 040° b) 321° c) 163° d) 283°

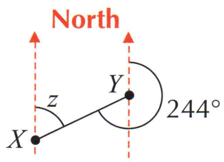
Since **north lines** are always **parallel**, you can use the properties of **alternate**, **allied** and **corresponding** angles (see p.250-251) to work out the bearing of A from B, **given** the bearing of B from A.
If you're not given a **diagram**, it's always a good idea to **sketch** one to help you to see what to do.

You might notice that the bearing of A from B is always either **180° more** or **180° less** than the bearing of B from A. If the given bearing is **less than 180°**, then you can just **add** 180° to get the other bearing, and if the given bearing is **more than 180°**, then **subtract** 180° to find the one you want.

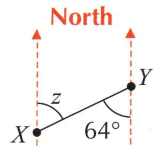
Example 2

The bearing of X from Y is 244°. Find the bearing of Y from X.

1. Draw a diagram showing what you know. Label the angle you're trying to find.

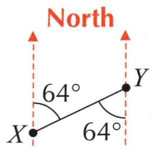


2. Find the alternate angle to the one you're looking for.
244° – 180° = 64°



Tip: There are other methods you could use — for example, doing 360° – 244° to find the allied angle to the one you're looking for.

3. Alternate angles are equal, so these two are the same. Make sure you give your answer as a three-figure bearing.

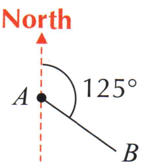


So the bearing of Y from X is **064°**.

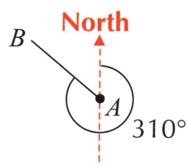
Exercise 2

Q1 Find the bearing of A from B in the following diagrams.

a)



b)

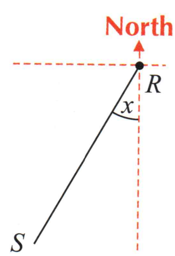


Q2 The bearing of H from G is 023°. Find the bearing of G from H.

Q3 The bearing of K from J is 101°. Find the bearing of J from K.

Q4 Find the bearing of N from M, given that the bearing of M from N is:
a) 200° b) 310° c) 080°
d) 117° e) 015° f) 099°

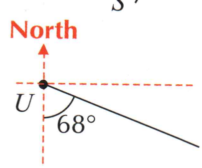
Q5 a) Measure the angle x in the diagram on the right.
b) Write down the bearing of R from S.



Q6 The point Q lies due west of point P. Find the bearing of:
a) Q from P b) P from Q.

Q7 The point Z lies southeast of the point Y. Find the bearing of:
a) Z from Y b) Y from Z

Q8 In the diagram on the right, find the bearing of:
a) V from U b) U from V

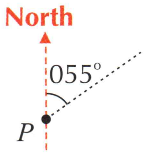


Bearings can also be used to draw **scale diagrams** (see method on p.301).
Use your **protractor** to draw any bearings and make sure any **north lines** are vertical.

Example 3

The points P and Q are 75 km apart. Q lies on a bearing of 055° from P.
Use the scale 1 cm : 25 km to draw an accurate scale diagram of P and Q.

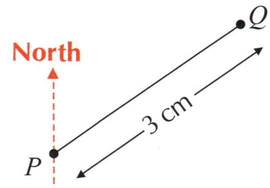
1. Draw P and a north line. Use your protractor to measure the required angle clockwise from north.



2. Use the scale to work out the distance between the two points.

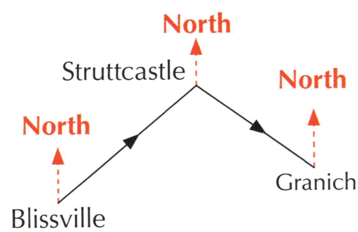
25 km is shown by 1 cm,
so 75 km is shown by
$75 \div 25 = 3$ cm

3. Draw Q the correct distance and direction from P.



Exercise 3

Q1 A pilot flies from Blissville to Struttcastle, then on to Granich. This journey is shown by the scale diagram on the right, which is drawn using a scale of 1 cm : 100 km.



a) Find the distance and bearings from:

(i) Blissville to Struttcastle (ii) Struttcastle to Granich

b) The pilot returns directly from Granich to Blissville. Find the actual distance travelled in this stage.

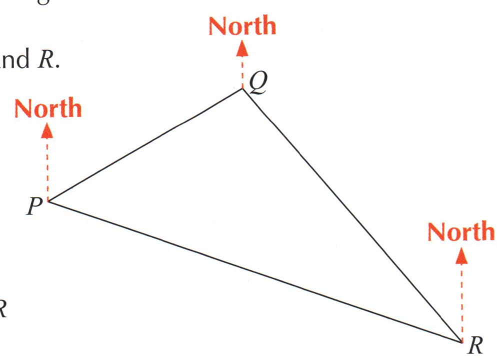
Q2 Paul's house is 150 km from Tirana on a bearing of 048°.
Draw an accurate scale diagram of Paul's house and Tirana using the scale 1 cm : 30 km.

Q3 Paradise City lies 540 km from Pretty Grimville on a bearing of 125°.
Draw an accurate scale diagram of the two locations using the scale 1 cm : 90 km.

Q4 A pilot flies 2000 km from Budarid to Madpest on a bearing of 242°.
Draw an accurate scale diagram of the journey using the scale 1 : 100 000 000.

Q5 Use the scale 1 : 22 000 000 to draw an accurate scale diagram of an 880 km journey from Budarid to Blissville on a bearing of 263°.

Q6 The scale drawing on the right shows three cities, P, Q and R.
The scale of the diagram is 1 : 10 000 000.



a) Use the diagram to find the following actual distances in km.

(i) PQ (ii) QR (iii) PR

b) Use a protractor to find the following bearings.

(i) Q from P (ii) R from Q (iii) P from R

24.3 Constructions

Constructing a shape means drawing it using just a pencil, a ruler, a pair of compasses and a protractor. You can also use your ruler and compasses to draw perpendicular lines, split angles in half, and much more.

Constructing Triangles

Learning Objective — Spec Ref G1/G2:
Construct triangles using a ruler, a pair of compasses and a protractor.

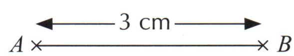
To construct a triangle, you need to know **three** pieces of information about it — these could be **lengths** of the sides or **angles** between them. If you're given a **side length**, use your **ruler** to measure the length and draw an arc with your **compasses** if necessary. Measure any **angles** that you're given with your **protractor**.

When constructing shapes and lines, you should **always** leave **compass arcs** and other **construction lines** on your finished drawing to show that you've used the right method.

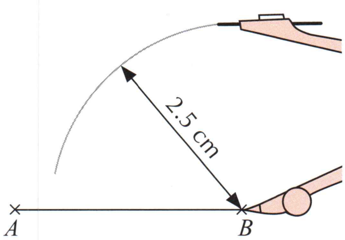
Example 1

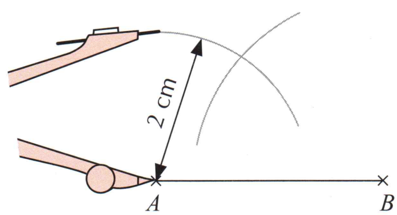
Draw triangle *ABC*, where *AB* is 3 cm, *BC* is 2.5 cm and *AC* is 2 cm.

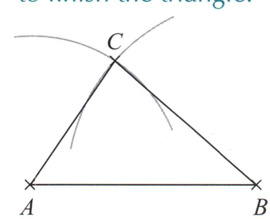
1. Draw and label side *AB*.



2. Set your compasses to 2.5 cm. Draw an arc 2.5 cm from *B*.
3. Now set your compasses to 2 cm. Draw an arc 2 cm from *A*.
4. *C* is where your arcs cross — so draw lines to finish the triangle.

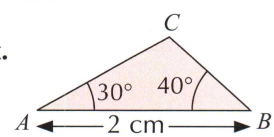




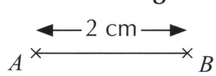


Example 2

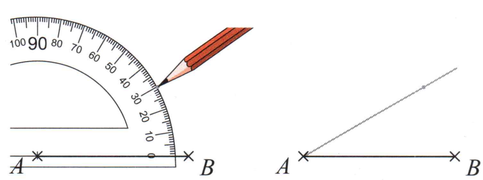
Accurately construct the triangle *ABC* shown in the rough sketch on the right.



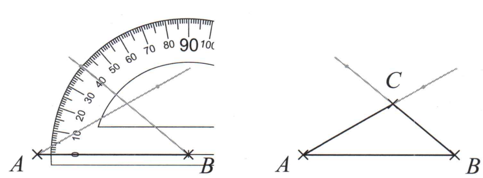
1. Start by drawing side *AB*.



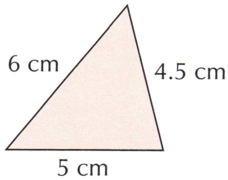
2. Measure an angle of 30° at *A* with a protractor. Mark the angle with a dot.
3. Draw a faint line from *A* through the dot.



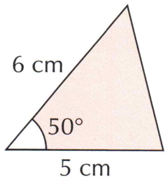
4. Do the same for the 40° angle at *B*.
5. *C* is where these lines cross, so draw darker lines to complete the triangle.



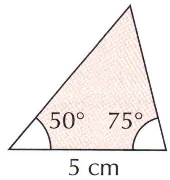
When you're given any of the following **three pieces of information,** there's **only one triangle** you can draw (ignoring reflections and rotations).



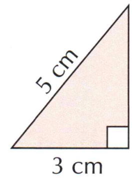
SSS — 3 sides.



SAS — 2 sides and the angle between them.



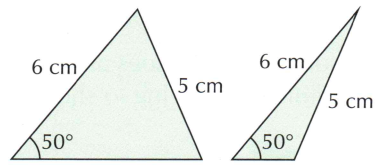
ASA — 2 angles and the side between them.



RHS — a right angle, the hypotenuse and another side.

If you're given 2 angles and a **side** that **isn't between them** (AAS), you can use the fact that angles in a triangle add up to 180° (see p.252) to work out the **third angle** and turn it into an **ASA** triangle.

However, if you're given two sides and an **angle** that **isn't between them** (SSA), then there are often **two possible triangles** you could draw, as shown in the diagram on the right.

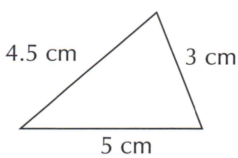


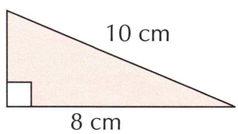
Both triangles have a 6 cm side and a 5 cm side with a 50° angle opposite it.

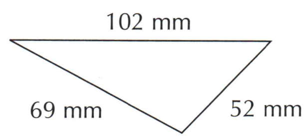
Three angles isn't enough information to draw a triangle because you only know its **shape** — you don't know how big it is.

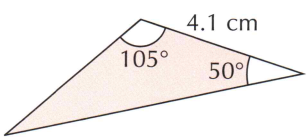
Exercise 1

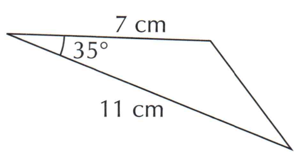
Q1 Draw the following triangles accurately.

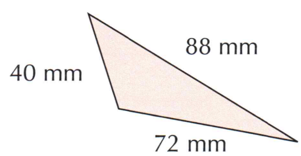
a) 

b) 

c) 

d) 

e) 

f) 

Q2 Draw each of the triangles ABC described below.

a) AB is 4 cm, AC is 9 cm, angle BAC is 50°

b) Angle BAC is 40°, angle ABC is 25°, AB is 7 cm

c) AB is 9 cm, BC is 9 cm, AC is 5 cm

d) Angle BAC is 90°, AB is 35 mm, BC is 61 mm

e) AB is 4.6 cm, BC is 5.4 cm, AC is 8.4 cm

f) AB is 4.4 cm, angle ACB is 32°, angle ABC is 16°

Q3 Draw an isosceles triangle with:

a) one side of length 7 cm and two sides of length 5 cm.

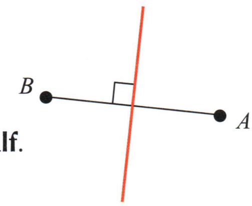
b) two angles of 75° and a side of length 45 mm between them.

c) two sides of length 7.1 cm and an angle of 22° between them.

Q4 In triangle ABC, AB is 10 cm, AC is 6 cm and angle ABC is 30°. Use an accurate drawing to find the two possible lengths of BC. PROBLEM SOLVING

Constructing a Perpendicular Bisector

Learning Objective — Spec Ref G2:
Be able to construct a perpendicular bisector.



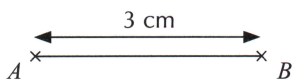
The **perpendicular bisector** of a line *AB* is at **right angles** to the line, and cuts it in **half**.

All points on the perpendicular bisector are **equally far** from both *A* and *B* (this is important when drawing loci — see page 313). You can use this fact to draw perpendicular bisectors **without measuring** the distances — just use your compasses to find two points that are the **same distance** from *A* and *B*, then the perpendicular bisector will **pass through both** of these points.

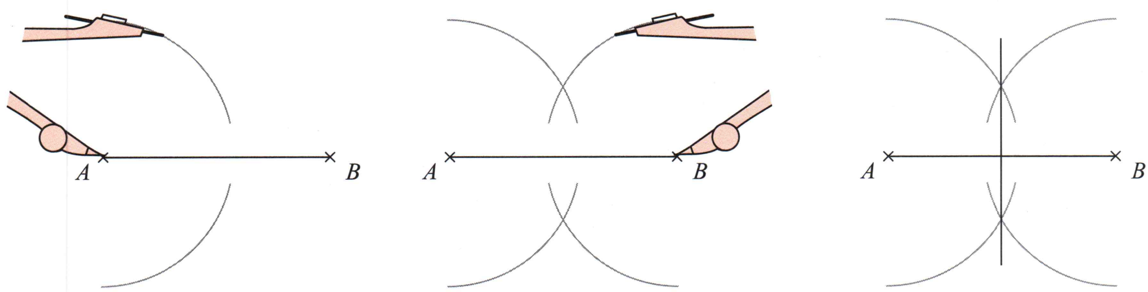
Example 3

Draw a line *AB* which is 3 cm long and construct its perpendicular bisector.

1. Draw *AB*.



2. Place the compass point at *A*, with the radius set at more than half of the length *AB*. Draw two arcs as shown.
3. Keep the radius the same and put the compass point at B. Draw two more arcs.
4. Use a ruler to draw a straight line through the points where the arcs meet. This is the perpendicular bisector.



Exercise 2

Q1 Draw the following lines and construct their perpendicular bisectors using only a ruler and a pair of compasses.

a) A horizontal line *PQ* 5 cm long. b) A vertical line *XY* 9 cm long. c) A line *AB* 7 cm long.

Q2 a) Draw a line *AB* 6 cm long. Construct the perpendicular bisector of *AB*.

b) Draw the rhombus *ACBD* with diagonals 6 cm and 8 cm.

Q3 a) Draw a circle with radius 5 cm and draw any two chords. Label your chords *AB* and *CD*.

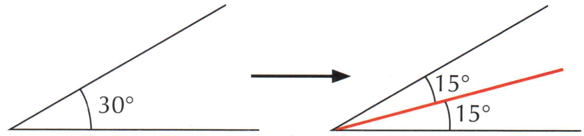
b) Construct the perpendicular bisector of chord *AB*.

c) Construct the perpendicular bisector of chord *CD*.

d) Where do the two perpendicular bisectors meet?

Constructing an Angle Bisector

Learning Objective — Spec Ref G2:
Be able to construct an angle bisector.

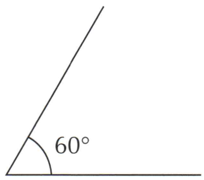


An **angle bisector** is the line that cuts an angle in half. All points on the angle bisector are the **same distance** from each of the two lines that enclose the angle (this is also useful for drawing loci — see page 313). Just like the perpendicular bisector, you can use your ruler and compasses to construct an angle bisector **without measuring** the angles with a protractor.

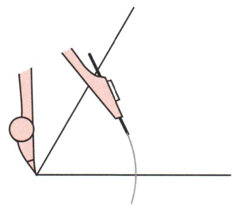
Example 4

Draw an angle of 60° using a protractor, then construct the angle bisector using only a ruler and compasses.

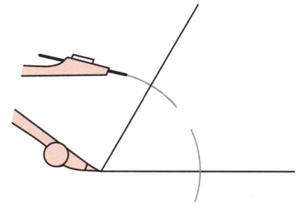
1. Place the point of the compasses on the angle...



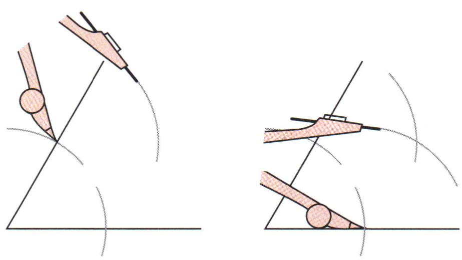
...and draw arcs crossing both lines...



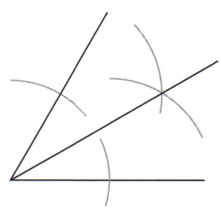
...using the same radius.



2. Now place the point of the compasses where your arcs cross the lines and, from each point, draw a new arc (using the same radius).



3. Draw the angle bisector through the point where the arcs cross.



Tip: You can check your answer by measuring the angles with a protractor — they should both be 30°.

Exercise 3

Q1 Draw each angle using a protractor, then construct its angle bisector using a ruler and compasses.

a) 100° b) 140° c) 96° d) 44°
e) 50° f) 70° g) 20° h) 65°

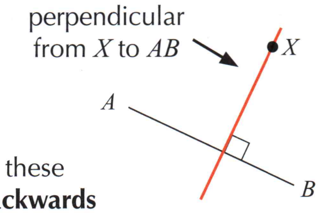
Q2 Draw a triangle and construct the bisectors of each of the angles.
What do you notice about these bisectors?

Q3 a) Draw an angle *ABC* of 110°, with *AB* = *BC* = 5 cm. Construct the bisector of angle *ABC*.

b) Mark point *D* on your drawing, where *D* is the point on the angle bisector with *BD* = 8 cm.
What kind of quadrilateral is *ABCD*?

Constructing a Perpendicular from a Point to a Line

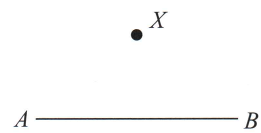
Learning Objective — Spec Ref G2:
Be able to construct a perpendicular from a point to a line.



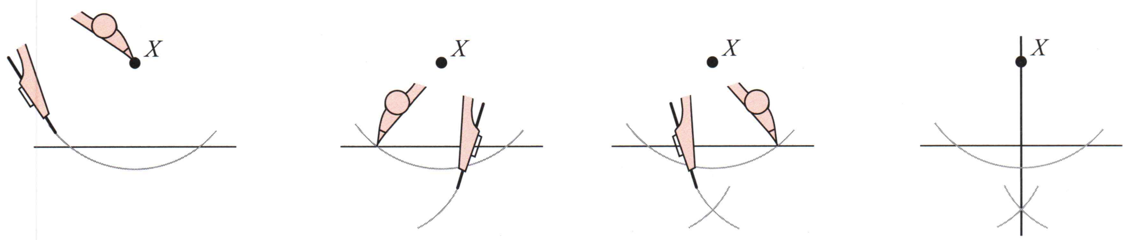
The **perpendicular** from a point to a line is the **shortest path** between them. It should **pass through** the point, and meet the line at **90°**. Constructing one of these is similar to constructing a **perpendicular bisector**, except you need to work **backwards** — first, you find two points **on the line**, then you use them to find the perpendicular.

Example 5

Construct the perpendicular from the point X to the line AB using only a ruler and a pair of compasses.



1. Draw an arc centred on X cutting the line twice (you may need to extend the line).
2. Draw an arc centred on one of the points where your arc meets the line.
3. Do the same for the other point, keeping the radius the same.
4. Draw the perpendicular to where the arcs cross.



Exercise 4

Q1 Use a ruler to draw any triangle. Label the corners of the triangle X, Y and Z. Construct the perpendicular from X to the line YZ.

Q2 Draw three points that do not lie on a straight line. Label your points P, Q and R.
Draw a straight line passing through points P and Q.
Construct the shortest possible line from R to the line PQ.

Q3 a) On squared paper, draw axes with x-values from 0 to 10 and y-values from 0 to 10.

b) Plot the points $A(1, 2)$, $B(9, 1)$ and $C(6, 8)$, and draw the line AB.

c) Construct the perpendicular from point C to the line AB.

Q4 Draw any triangle. Construct a perpendicular from each of the triangle's corners to the opposite side. What do you notice about these lines?

Q5 a) Construct triangle DEF, where $DE = 5$ cm, $DF = 6$ cm and angle $FDE = 55°$.

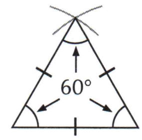
b) Construct the perpendicular from F to DE.
Label the point where the perpendicular meets DE as point G.

c) Measure the length FG. Use your result to work out the area of the triangle.

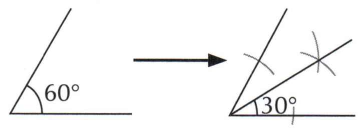
Constructing 60° and 30° Angles

Learning Objective — Spec Ref G2:
Be able to construct 60° and 30° angles.

To construct a **60° angle** you can take advantage of the fact that all the interior angles of an **equilateral triangle** are 60°. Start by drawing a line, then set your compasses to match its **length** and draw an arc from **each end** of the line. The point where the arcs cross will form an **equilateral triangle** with the **end points** of the line.



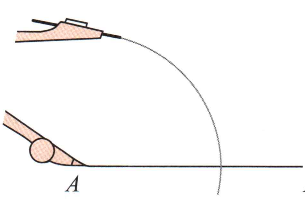
To construct a **30° angle**, first construct a 60° angle. You can then construct the **angle bisector** (see page 308) to get **two** angles of 30°.



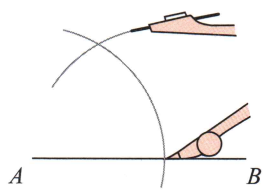
Example 6

Draw a line *AB* and construct an angle of 30° at *A*.

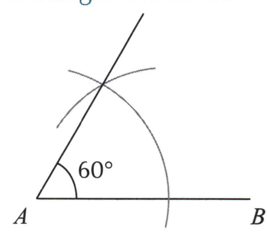
1. First construct an angle of 60°. Place the compass point on *A* and draw a long arc that crosses the line *AB*.



2. Place the compass point where the arc meets the line and draw another arc of the same radius.

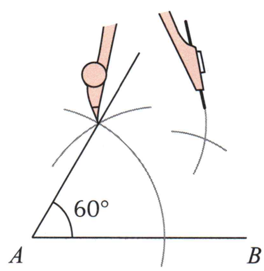


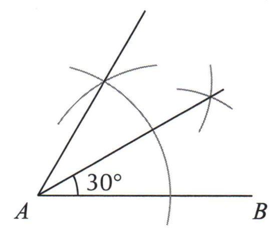
3. Draw a straight line through *A* and the point where your arcs cross. The angle will be 60°.



4. Now bisect this angle. You can use the arc from step 1 — so place the compass point at the points where it crosses each line and draw an arc from each of the same radius.

5. Finally, draw a straight line through *A* and the point where your arcs cross to get a 30° angle.





Exercise 5

Q1 Draw a line *AB* measuring 5 cm. Construct an angle of 60° at *A*.

Q2 Draw an equilateral triangle with sides measuring 6 cm.

Q3 Draw a line *AB* measuring 6 cm. Construct an angle of 30° at *A*.

Q4 Construct the triangle *ABC* where *AB* = 7 cm, angle *CAB* = 60° and angle *CBA* = 30° using only a ruler and compasses.

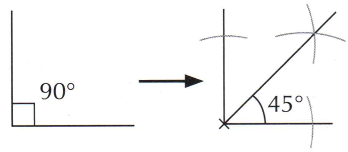
Q5 Construct an isosceles triangle *PQR* where *PQ* = 8 cm and the angles *RPQ* and *RQP* are both 30°, using only a ruler and compasses.

Constructing 90° and 45° Angles

Learning Objective — Spec Ref G2:
Be able to construct 90° and 45° angles.

Constructing a **90° angle** is a bit like constructing a **perpendicular from a point to a line** (see p.309) except now the point is **on the line**.

If you need to construct an angle of 45°, just construct the **angle bisector** (p.308) of the 90° angle.

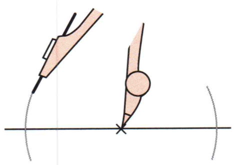


Example 7

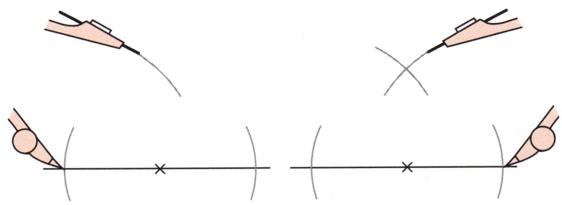
Construct an angle of 45° using only compasses and a ruler.

1. Draw a straight line and mark the point where you want to form the angle.

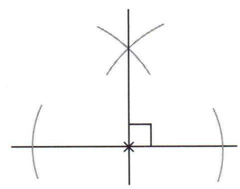
2. Draw arcs of the same radius either side of the point.



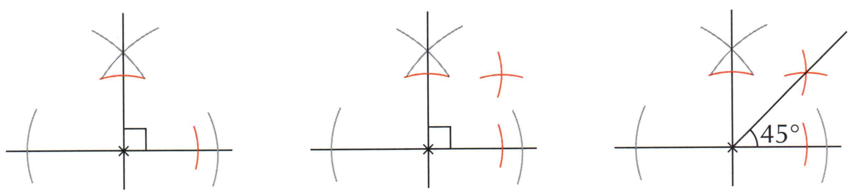
3. Increase the radius of your compasses, and draw two arcs of the same radius — one arc centred on each of the intersections.



4. Draw a straight line to complete the right angle.



5. Now bisect the right angle to get an angle of 45°.



Exercise 6

Q1 Draw a straight line and mark a point roughly halfway along it. Label the point X. Construct a right angle at X using only a ruler and compasses.

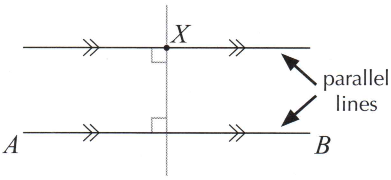
Q2 Using ruler and compasses only, construct a rectangle with sides of length 5 cm and 7 cm.

Q3 Draw a straight line, and mark a point roughly halfway along it. Label the point X. Construct an angle of 45° at X using only a ruler and compasses.

Q4 Construct an isosceles triangle ABC where AB = 8 cm and the angles CAB and CBA are both 45°, using only a ruler and compasses.

Constructing Parallel Lines

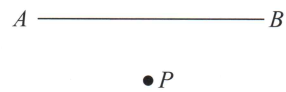
Learning Objective — Spec Ref G2:
Be able to construct parallel lines.



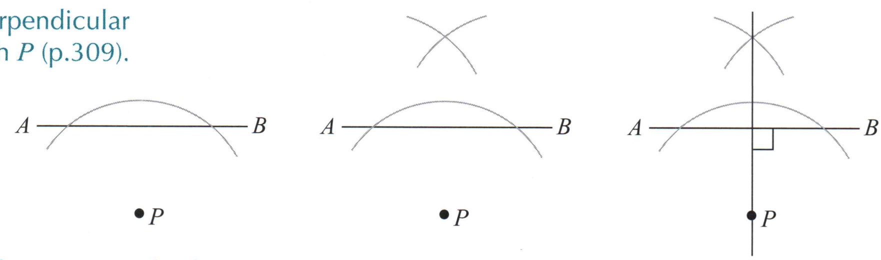
To construct a line that is **parallel** to another, passing **through a given point**, the first step is to construct the **perpendicular from the point to the line** (see p.309). Once you've done that, you just need to construct a **right angle** at the point using the method on the previous page.

Example 8

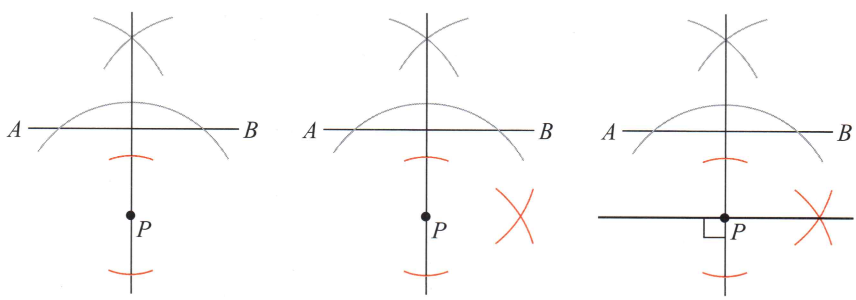
Construct a line parallel to *AB* through the point *P*.



1. Construct the line perpendicular to *AB* passing through *P* (p.309).



2. Construct a right angle (p.311) to this line at point *P*. This will be parallel to *AB*.

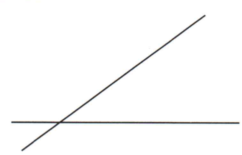


Tip: You could also construct the parallel line through *P* by constructing a right angle anywhere on line *AB*, then constructing the perpendicular from *P* to this line.

Exercise 7

Q1 Draw a horizontal line *AB* and mark a point *P* approximately 4 cm from your line. Construct a line parallel to *AB* through the point *P*.

Q2 Copy the two straight lines shown on the right. By adding two lines, one parallel to each of these, construct a parallelogram.



Q3 a) Use only a ruler and compasses to construct the quadrilateral *ABCD* such that:
- *AB* = 10 cm
- angle *ABC* = 60° and *BC* = 3 cm
- angle *BAD* = 30°
- *AB* and *CD* are parallel

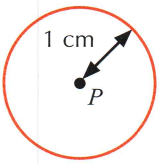
b) What type of quadrilateral is *ABCD*?

24.4 Loci

One of the most common uses for constructions is to find all the points that are a given distance from a shape or the same distance from two shapes. These sets of points are called loci (pronounced low-kai).

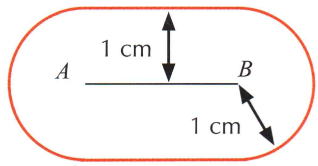
Learning Objectives — Spec Ref G2:
- Construct the locus of points that are a given distance from a point or a line.
- Construct the locus of points that are equidistant from two points or two lines.
- Solve problems involving loci.

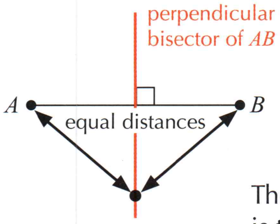
A **locus** (plural **loci**) is a **set of points** which satisfy a particular condition. The types of loci you need to know are the sets of points that are a **fixed distance away** from a point or a line (or another kind of shape), and the sets of points that are **equidistant** (i.e. the **same distance**) from two points or two lines.



The locus of points that are a fixed distance, e.g. 1 cm, from a **point** P is a **circle** with radius 1 cm centred on P. To construct this, set your **compasses** to the given distance and draw a circle around the point.

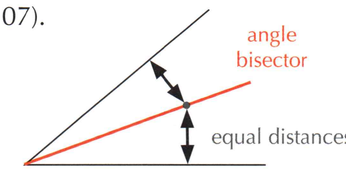
The locus of points that are a fixed distance from a **line** AB is a 'sausage shape'. To construct this, use your compasses to draw the ends, which are **semicircles**, then join them up with your ruler.





The locus of points equidistant from **two points** A and B is the **perpendicular bisector** of AB (see page 307).

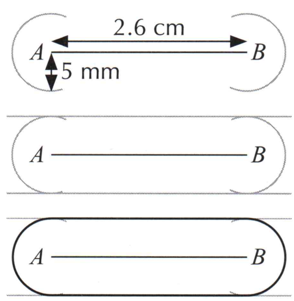
The locus of points equidistant from **two lines** is their **angle bisector** (see page 308).



Example 1

The line AB is 2.6 cm long. Construct the locus of points that are 5 mm from AB.

1. Draw AB, then set your compasses to 5 mm and draw arcs around each end of the line. Make sure each arc is slightly more than a semicircle.
2. Using your ruler, join the tops and bottoms of the arcs with straight lines.
3. Mark the locus, leaving your construction lines on the diagram.



Exercise 1

Q1 Draw a line, AB, that is 7 cm long. Construct the locus of all the points 2 cm from the line.

Q2 a) Mark a point X on your page. Draw the locus of all points which are 3 cm from X.
b) Shade the locus of all points on the page which are less than 3 cm from X.

Q3 Mark two points A and B that are 6 cm apart.
Construct the locus of all points which are equidistant from A and B.

Q4 Draw two lines that meet at an angle of 50°.
Construct the locus of all points which are equidistant from the two lines.

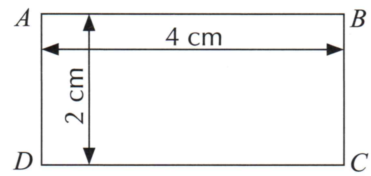
Q5 Draw a line AB 6 cm long. Draw the locus of all points which are 3 cm from AB.

Q6 a) Draw axes on squared paper with x- and y-values from 0 to 10.
Plot the points $P(2, 7)$ and $Q(10, 3)$.
b) Construct the locus of points which are equidistant from P and Q.

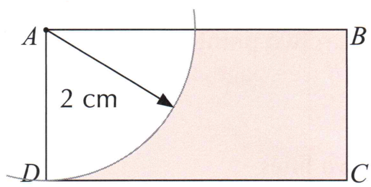
You might have to draw **more than one locus** to find the region that satisfies **multiple conditions**.

Example 2

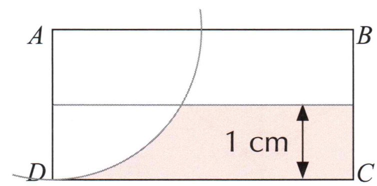
Shade the locus of points inside rectangle *ABCD* that are:
- **more than 2 cm from point *A*,**
- **less than 1 cm from side *CD*.**



1. Use your compasses to construct an arc of radius 2 cm around A. The first condition means that points within this arc are excluded.



2. Use your ruler to draw a line 1 cm from CD. The second condition means that points on or above this line are also excluded.



Exercise 2

Q1 a) Mark points P and Q that are 5 cm apart.
b) Draw the locus of points which are 3 cm from P.
c) Draw the locus of points which are 4 cm from Q.
d) Show clearly which points are both 3 cm from P and 4 cm from Q.

Q2 Draw a line RS which is 5 cm long. Construct the locus of points that are equidistant from R and S and less than 4 cm from R.

Q3 a) Construct a triangle with sides 4 cm, 5 cm and 6 cm.
b) Draw the locus of all points which are exactly 1 cm from any of the triangle's sides.

Q4 a) Construct an isosceles triangle DEF with $DE = EF = 5$ cm and $DF = 3$ cm.
b) Draw the locus of points which are equidistant from D and F and less than 2 cm from E.

Q5 Lines AB and CD are both 6 cm long. AB and CD are perpendicular to each other and cross at the midpoint of each line, M.
a) Construct lines AB and CD.
b) Draw the locus of points which are less than 2 cm from AB and CD.

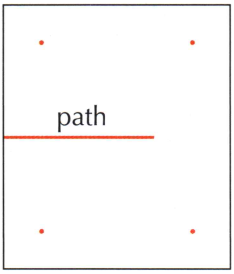
Loci can also be used to solve **real-life problems**, particularly on **scale diagrams** (see p.299-301).

Example 3

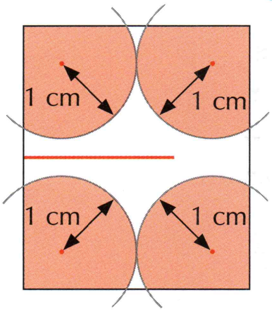
The diagram shows a plan of a greenhouse, drawn at a scale of 1 : 400. The greenhouse has a path through the middle modelled by a straight line, and four sprinklers, shown as dots on the plan.

The sprinklers can water plants within a 4 m radius. The gardener can water anything up to 6 m away from the path using a hosepipe.

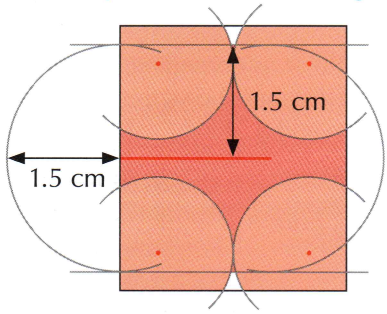
Shade the area on the diagram that can be watered.



1. At a scale of 1 : 400, 1 cm : 400 cm = 4 m. So draw arcs of radius 1 cm around each sprinkler, and shade the region inside.



2. 6 m in real life is 6 ÷ 4 = 1.5 cm on the diagram. Construct the locus of points that are 1.5 cm away from the path and shade the region inside.



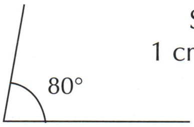
Exercise 3

Q1 A ship sails so that it is always the same distance from a port P and a lighthouse L. The lighthouse and the port are 3 km apart.

a) Draw a scale diagram showing the port and lighthouse. Use a scale of 1 cm : 1 km.

b) Show the path of the ship on your diagram.

Q2 Two walls of a field meet at an angle of 80°. A bonfire has to be the same distance from each wall and 3 m from the corner. Copy the diagram on the right, then use a ruler and pair of compasses to show the position of the fire.



Scale
1 cm : 0.5 m

Q3 Two camels set off at the same time from towns A and B, located 50 miles apart in the desert.

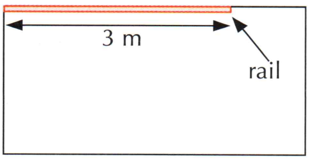
a) Draw a scale diagram showing towns A and B. Use a scale of 1 cm : 10 miles.

b) If a camel can walk up to 40 miles in a day, show on your diagram the region where the camels could possibly meet each other after walking for one day.

Q4 A walled rectangular yard has length 4 m and width 2 m. A dog is secured by a lead of length 1 m to a post in a corner of the yard.

a) Show on an accurate scale drawing the area in which the dog can move. Use the scale 1 cm : 1 m.

b) The post is replaced with a 3 m rail mounted horizontally along one of the long walls, with one end in the corner as shown. If the end of the lead attached to the rail is free to slide, show the area in which the dog can move now.



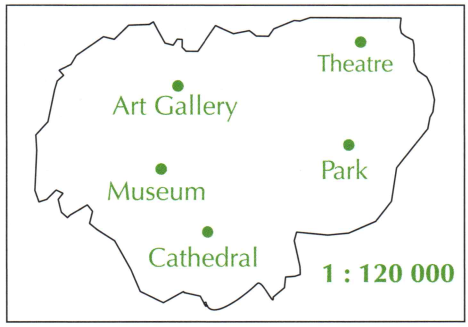
Review Exercise

Q1 The map shows tourist attractions in a city.
Find the actual distances between:

a) the museum and the cathedral,

b) the art gallery and the theatre,

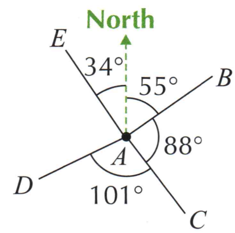
c) the cathedral and the theatre.



Q2 A ship, A, is communicating with four other nearby ships, B, C, D and E. The diagram shows the positions of each ship and some angles between them and A. Find the bearing of:

a) B from A b) C from A c) D from A d) E from A

e) A from B f) A from C g) A from D h) A from E



Q3 Using a scale of 1 cm : 10 miles, draw a map showing the relative positions of the three places described on the right.
(All distances are 'as the crow flies'.)

- High Cross is 54 miles due west of Low Cross.
- Very Cross is 48 miles from Low Cross and 27 miles from High Cross
- Very Cross lies to the south of High Cross and Low Cross.

Q4 Draw a line AB that is 8 cm long.

a) Construct an angle of 60° at A.

b) Complete a construction of a rhombus $ABCD$ with sides of length 8 cm.

Q5 a) Construct an equilateral triangle DEF with sides of length 5.8 cm.

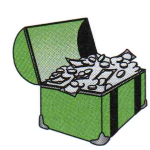
b) Construct a line that is parallel to side DE and passes through point F.

Q6 Construct the triangle ABC with AB = 7.4 cm, angle CAB = 60° and angle ABC = 45°, using only a ruler and a pair of compasses.

Q7 Construct an angle of 15° using a ruler and a pair of compasses only. PROBLEM SOLVING

Q8 Some students are doing a treasure hunt. They know the treasure is:

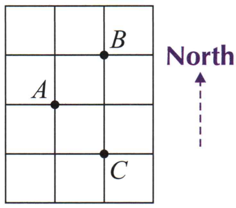
- located in a square region $ABCD$, which measures 10 m × 10 m
- the same distance from AB as from AD
- 7 m from corner C.



Draw a scale diagram to show the location of the treasure. Use a scale of 1 cm : 1 m.

Exam-Style Questions

Q1 Many American cities have a road design of square grids. The diagram shows part of such a design which is a rectangle made up of 12 congruent squares. 3 locations in this city are labelled as the points *A*, *B* and *C*.



a) Write down the three figure bearing of *B* from *A*.

[1 mark]

b) Work out the three figure bearing of *A* from *C*.

[1 mark]

Q2 On a 1 : 25 000 scale map, the length of a reservoir is 3.8 cm.
Work out the actual length of the reservoir, giving your answer in metres.

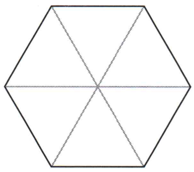
[2 marks]

Q3 Accurately construct a triangle with sides of length 37 mm, 52 mm, and 60 mm.

[3 marks]

Q4 A regular hexagon can be made up of 6 identical equilateral triangles, as shown in the diagram.

Construct a regular hexagon with sides of length 3 cm using a ruler and compasses.

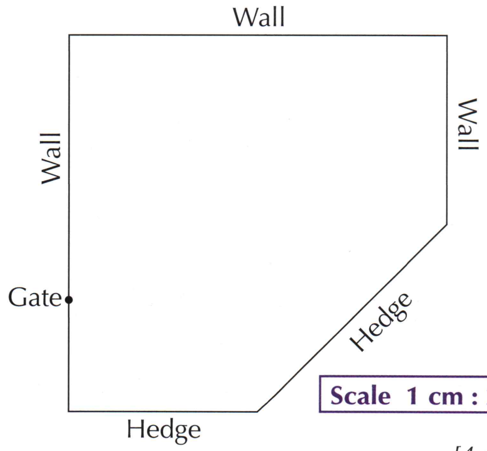


[3 marks]

Q5 A farmer decides to put a scarecrow in a field to protect her crop of wheat. The field is shown in the diagram on the right.

She positions the scarecrow so that it is an equal from each hedge and 40 m from the gate.

Copy the diagram and use a ruler and a pair of compasses to accurately show the two possible positions of the scarecrow.



[4 marks]