

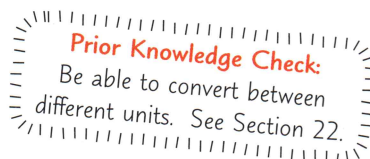
## 23.1 Compound Measures

Compound measures are made up of two or more other measurements. For example, speed is a compound measure because it's a combination of the two measurements distance and time.

### Speed, Distance and Time

#### Learning Objective — Spec Ref NI3/RI/RII:

Know and use the formula linking speed, distance and time.



The **speed** of an object is the **total distance** it travels divided by the **time** it takes to travel this distance. The units for speed are **distance per unit time**, e.g. **km per hour** or **metres per second**. The **formula** to calculate the (average) speed of an object given the distance and time is:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The formula gives the **average speed** of the journey — the actual speed is likely to change throughout.

#### Example 1

A car travels 81 km in 45 minutes. What is the average speed of the car in km/h?

1. Convert the time to hours.  $45 \text{ minutes} = 45 \div 60 = 0.75 \text{ hours}$

2. Substitute the distance and time into the formula. The units of speed are a combination of the units of distance (km) and time (hours).

$$\begin{aligned} \text{Speed} &= \frac{\text{Distance}}{\text{Time}} \\ &= \frac{81 \text{ km}}{0.75 \text{ hours}} = 108 \text{ km/h} \end{aligned}$$

**Tip:** Convert the distance and time into the relevant units before doing the calculation.

### Exercise 1

- Q1 Find the average speed of the following:
- a plane flying 1800 miles in 4.5 hours
  - a lift travelling 100 m in 80 seconds
  - a cyclist travelling 34 km in 1.6 hours
  - an escalator step moving 15 m in 24 seconds
- Q2 Find the average speed of the following in km/h:
- a train travelling 300 000 m in 2.5 hours
  - a river flowing 5.25 km in 45 minutes
  - a fish swimming 0.5 km in 12 minutes
  - a balloon rising 700 m in 3 minutes
- Q3 A spacecraft travels 332 900 miles to the moon in 13.7 hours. Calculate the average speed of the spacecraft.
- Q4 A bobsleigh covers 1.4 km in 65 seconds. Find its average speed in metres per second. Give your answer to 2 significant figures.
- Q5 A tortoise walks 98 cm in 8 minutes. Find the tortoise's average speed in m/s to 1 significant figure.

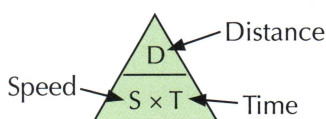
You can **rearrange** the speed formula to give formulas for **distance** and **time**:

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

You can use the **formula triangle** given below to help you remember all three of the formulas. To use the formula triangle, **cover up** the measurement that you want to find and **write down** the two measurements that are left.

- To find **speed**, cover up **S** to leave  $\frac{D}{T}$
- To find **distance**, cover up **D** to leave  $S \times T$
- To find **time**, cover up **T** to leave  $\frac{D}{S}$



Remember to always **check the units** before doing a calculation. If you have **speed** in **mph** and **time** in **minutes**, then doing speed  $\times$  time won't give you the correct answer.

### Example 2

**A man sprints for 25 seconds with an average speed of 8 m/s. How far does he sprint?**

- |   |   |
|---|---|
| 1. Write down the formula for distance.   | Distance = Speed $\times$ Time                      |
| 2. Check units — you have speed in m/s and time in seconds, so your answer will be in metres. | Speed = 8 m/s<br>Time = 25 seconds                  |
| 3. Substitute the numbers into the formula.   | Distance = 8 m/s $\times$ 25 seconds = <b>200 m</b> |

### Example 3

**A car travels 85 miles at an average speed of 34 mph. How many minutes does the journey take?**

- |  |  |
|--|--|
| 1. Write down the formula for time.  | Time = $\frac{\text{Distance}}{\text{Speed}}$                        |
| 2. Check units — you have speed in mph and distance in miles, so your answer will be in hours. | Distance = 85 miles<br>Speed = 34 mph                                |
| 3. Substitute the numbers into the formula.  | Time = $\frac{85 \text{ miles}}{34 \text{ mph}} = 2.5 \text{ hours}$ |
| 4. Finally, convert hours to minutes.  | 2.5 hours = 2.5 $\times$ 60 = <b>150 minutes</b>                     |

## Exercise 2

- Q1 For each of the following, use the speed and time given to calculate the distance travelled.
- |                                      |                                       |
|--------------------------------------|---------------------------------------|
| a) speed = 98 km/h, time = 3.5 hours | b) speed = 25 mph, time = 2.7 hours   |
| c) speed = 15 m/s, time = 9 minutes  | d) speed = 72 mph, time = 171 minutes |
- Q2 For each of the following, use the speed and distance given to calculate the time taken.
- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| a) speed = 2.5 km/h, distance = 4 km | b) speed = 5 m/s, distance = 9.3 m   |
| c) speed = 9 m/s, distance = 61.2 km | d) speed = 8 cm/s, distance = 1.96 m |
- Q3 A dart is thrown with speed 15 m/s and hits a dartboard 2.4 m away. For how long is the dart in the air?
- Q4 A flight to Spain takes 2 hours and 15 minutes. If the plane travels at an average speed of 480 mph, how far does the plane travel?
- Q5 A girl skates at an average speed of 7.5 mph. How far does she skate in 75 minutes?
- Q6 A train travels at 78 km/h for 5.6 km. How long does the journey take to the nearest minute?



# Density, Mass and Volume

## Learning Objective — Spec Ref N13/R11:

Know and use the formula linking density, mass and volume.

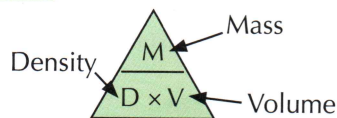
**Density** is another compound measure — it's the **mass per unit volume** of a substance and is usually measured in **kg/m<sup>3</sup>** or **g/cm<sup>3</sup>**. Different substances have **different densities** — for example, gold has a **higher density** than ice. The **formulas** that connect density, mass and volume are:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Mass} = \text{Density} \times \text{Volume}$$

The **formula triangle** on the right gives a summary of all the formulas. Remember to **check the units** of the measurements that you're putting into a formula — that way you'll know the units of the measurement that comes out.



### Example 4

**A 1840 kg concrete block has a volume of 0.8 m<sup>3</sup>. Calculate the density of the concrete block.**

1. Write down the formula for density.
2. Check units — the mass is in kg and the volume is in m<sup>3</sup>, so your answer will be in kg/m<sup>3</sup>.
3. Substitute the numbers into the formula.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Mass} = 1840 \text{ kg, Volume} = 0.8 \text{ m}^3$$

$$\text{Density} = \frac{1840 \text{ kg}}{0.8 \text{ m}^3} = \mathbf{2300 \text{ kg/m}^3}$$

### Example 5

**The mass of a bathtub filled with water is 225 kg. Water has a density of 1000 kg/m<sup>3</sup>. If the empty bathtub has a mass of 45 kg, what is the volume of water in the bathtub?**



1. Calculate the mass of the water on its own.
2. Write down the formula for volume.
3. Check units — the mass is in kg and the density is in kg/m<sup>3</sup>, so your answer will be in m<sup>3</sup>.
4. Substitute the numbers into the formula.

$$225 \text{ kg} - 45 \text{ kg} = 180 \text{ kg}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Mass} = 180 \text{ kg, Density} = 1000 \text{ kg/m}^3$$

$$\text{Volume} = \frac{180 \text{ kg}}{1000 \text{ kg/m}^3} = \mathbf{0.18 \text{ m}^3}$$

## Exercise 3

- Q1** For each of the following, use the mass and volume given to calculate the density.
- |  |   |
|--|---|
| a) mass = 200 kg, volume = 540 m <sup>3</sup>  | b) mass = 23 kg, volume = 0.5 m <sup>3</sup>    |
| c) mass = 1088 kg, volume = 1.6 m <sup>3</sup> | d) mass = 2498 g, volume = 0.25 cm <sup>3</sup> |
- Q2** For each of the following, use the density and mass given to calculate the volume.
- |  |  |
|--|--|
| a) density = 8 kg/m <sup>3</sup> , mass = 1 kg   | b) density = 1510 kg/m <sup>3</sup> , mass = 3926 kg |
| c) density = 90 g/cm <sup>3</sup> , mass = 540 g | d) density = 240 kg/m <sup>3</sup> , mass = 14.4 kg  |
- Q3** A limestone statue has a volume of 0.4 m<sup>3</sup>. Limestone has a density of 2610 kg/m<sup>3</sup>. Calculate the mass of the statue.



# Pressure, Force and Area

## Learning Objective — Spec Ref NI3/R11:

Know and use the formula linking pressure, force and area.

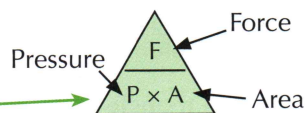
**Prior Knowledge Check:**  
Be familiar with areas of shapes. See Section 27.

Like speed and density, **pressure** is a compound measure — it's the **force of an object per unit area**. Pressure is usually measured in  $\text{N/m}^2$  (also known as pascals, **Pa**), or  $\text{N/cm}^2$ , where **N** is **Newtons**, the unit of force. The **formulas** that connect pressure, force and area are:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Area} = \frac{\text{Force}}{\text{Pressure}}$$

$$\text{Force} = \text{Pressure} \times \text{Area}$$



The **formula triangle** on the right gives a summary of these formulas.

### Example 6

An object is resting with its base on horizontal ground. The area of the object's base is  $20 \text{ cm}^2$  and the object weighs  $60 \text{ N}$ . What pressure is the object exerting on the ground?

1. Write down the formula for pressure.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

2. Check units — you have force in  $\text{N}$  and area in  $\text{cm}^2$ , so your answer will be in  $\text{N/cm}^2$ .

$$\begin{aligned}\text{Force} &= 60 \text{ N} \\ \text{Area} &= 20 \text{ cm}^2\end{aligned}$$

3. Substitute the numbers into the formula.

$$\text{Pressure} = \frac{60 \text{ N}}{20 \text{ cm}^2} = 3 \text{ N/cm}^2$$

**Tip:** Weight is the force of the object on the ground due to gravity.

### Example 7

A laptop with a base of  $0.07 \text{ m}^2$  is resting on a desk and exerting a pressure of  $330 \text{ Pa}$ . How much does the laptop weigh?

1. Write down the formula for force.

$$\text{Force} = \text{Pressure} \times \text{Area}$$

2. Check units — you have area in  $\text{m}^2$  and pressure in  $\text{Pa}$  (which is  $\text{N/m}^2$ ), so your answer will be in  $\text{N}$ .

$$\begin{aligned}\text{Pressure} &= 330 \text{ Pa} \\ \text{Area} &= 0.07 \text{ m}^2\end{aligned}$$

3. Substitute the numbers into the formula.

$$\text{Force} = 330 \text{ Pa} \times 0.07 \text{ m}^2 = 23.1 \text{ N}$$

## Exercise 4

Q1 For each of the following, calculate the missing measure.

- Pressure = ? Pa, Area =  $4 \text{ m}^2$ , Force =  $4800 \text{ N}$
- Pressure = ?  $\text{N/cm}^2$ , Area =  $80 \text{ cm}^2$ , Force =  $640 \text{ N}$
- Pressure =  $180 \text{ N/m}^2$ , Area = ?  $\text{m}^2$ , Force =  $540 \text{ N}$
- Pressure =  $36 \text{ N/cm}^2$ , Area =  $30 \text{ cm}^2$ , Force = ?  $\text{N}$

Q2 A cube of metal with a volume of  $512 \text{ cm}^3$  is resting with one of its faces on horizontal ground. The cube has a weight of  $1792 \text{ N}$ . What pressure is the cube exerting on the ground?

Q3 A cylinder with a height of  $80 \text{ cm}$  is resting with one of its circular faces on horizontal ground. The weight of the cylinder is  $560 \text{ N}$  and it exerts a pressure of  $70\,000 \text{ N/m}^2$  on the ground. What is the volume of the cylinder in  $\text{cm}^3$ ?

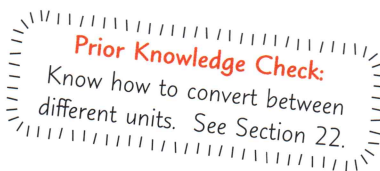




# Converting Compound Measures

## Learning Objective — Spec Ref R1/R11:

Convert units for compound measures.



To **convert** the units of a compound measure, you can convert the **individual units** that make up compound measure separately. For example, to convert **km/h to m/s** you would need to do **two separate conversions** — first convert **km** to **metres** (to give m per hour) and then convert **hours** to **seconds** (to give m/s). It doesn't matter which unit you convert first, but **always check your conversions** to make sure your answers are sensible.

If you were converting from, say, **km/h to mph** you would just need to do the conversion from **km** to **miles** because the unit of time (hours) is the **same**.

**Tip:** Other common compound measures include rates of pay (e.g. £ per hour) and prices per unit mass/volume (e.g. £ per kg or pence per ml).

### Example 8

Using the conversion 1 mile  $\approx$  1.6 km, what is 22 m/s in mph?

1. Use the conversion factor for metres to miles. There'll be fewer miles per second than m/s, so divide.

$$1 \text{ mile} \approx 1.6 \text{ km} = 1600 \text{ metres}$$

$$22 \text{ m/s} = \frac{22}{1600} = 0.01375 \text{ miles per second}$$

2. Work out the conversion factor for seconds to hours. There'll be more mph than miles per second, so multiply.

$$1 \text{ hour} = 60 \text{ mins} = 3600 \text{ seconds}$$

$$0.01375 \text{ miles/s} = 0.01375 \times 3600 = \mathbf{49.5 \text{ mph}}$$

### Example 9

What is 20 kg/m<sup>3</sup> in g/cm<sup>3</sup>?

1. Work out the conversion factor for kg to grams. There'll be more g/m<sup>3</sup> than kg/m<sup>3</sup>, so multiply.
2. Work out the conversion factor for m<sup>3</sup> to cm<sup>3</sup>. There'll be fewer g/cm<sup>3</sup> than g/m<sup>3</sup>, so divide.

$$1 \text{ kg} = 1000 \text{ g}$$

$$20 \text{ kg/m}^3 = 20 \times 1000 = 20\,000 \text{ g/m}^3$$

$$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3 (= 100^3)$$

$$20\,000 \text{ g/m}^3 = \frac{20\,000}{1\,000\,000} = \mathbf{0.02 \text{ g/cm}^3}$$

## Exercise 5

Q1 For each of the following, convert the compound units:

- |   |   |                              |
|---|---|------------------------------|
| a) £5 per gram to £ per kg                    | b) £36/hour to £ per minute                   | c) 62 m/s to metres per hour |
| d) 54 km/h to m/s                             | e) 3000 N/cm <sup>2</sup> to N/m <sup>2</sup> | f) 156 m per hour to cm/s    |
| g) 830 kg/m <sup>3</sup> to g/cm <sup>3</sup> | h) £12/kg to pence per gram                   | i) 29.25 mph to m/s          |

Q2 An object has a density of 97 g/cm<sup>3</sup>. What is the density of the object in kg/m<sup>3</sup>?

Q3 Barney is paid 12p for every minute he works. What is his rate of pay in pounds per hour?

Q4 A train travels from Edinburgh to Newcastle at 40 m/s in 1.5 hours and from Newcastle to London at 46 m/s in 3 hours. What is the average speed in mph?

Q5 A car travels at  $x$  km/h for the first half of a journey and then at  $y$  km/h for the second half. Write an expression for the average speed of the whole journey in m/s.



## 23.2 Distance-Time Graphs

Distance-time graphs are used to show the journey of an object over a given period of time.

### Learning Objective — Spec Ref A15:

Draw and interpret distance-time graphs.

### Prior Knowledge Check:

Be able to read off graphs.  
See p.214-215.

A **distance-time** graph shows how far an object has travelled in a particular period of time.

- When the graph is **going up**, the object is **moving away**.  
When the graph is **going down**, the object is **coming back**.
- The **gradient** of a distance-time graph shows the **speed** of the object (see p.285). The **steeper** the graph, the **faster** the object is moving.
- A **straight** line shows the object is moving at a **constant speed**.  
A **horizontal** line means the object is **stationary**.
- A **curved** graph shows that the **speed** of the object is **changing**. If the curve is getting **steeper**, the object is **accelerating**. If the curve is getting **flatter**, the object is **decelerating**.

**Tip:** Be careful when reading distance-time graphs — they show distance from a point, not always total distance travelled.

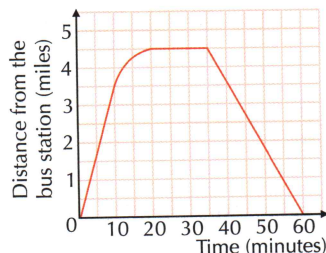
### Example 1

The graph on the right represents a bus journey.

a) How far does the bus travel before it stops?

The bus stops when the graph is a horizontal line. Find the first point where the line becomes horizontal and read the distance off the vertical axis.

4.5 miles



b) Describe the motion of the bus during the journey.

- At first the graph is a straight line upwards, so the bus is moving at a constant speed away from its starting point.
- As the graph curves, the gradient decreases — the bus is slowing down.
- When the graph is flat, the gradient is 0 so the bus has no speed.
- From 35 minutes, the graph is a straight line again, this time downwards — so the bus must be coming back to its starting point.

Between 0 and 10 minutes the bus was travelling at a **constant speed**.

Between 10 and 20 minutes the bus was **decelerating** (slowing down).

Between 20 and 35 minutes the bus was **stationary**.

Between 35 and 60 minutes the bus travels at a **constant speed** in the opposite direction to before.

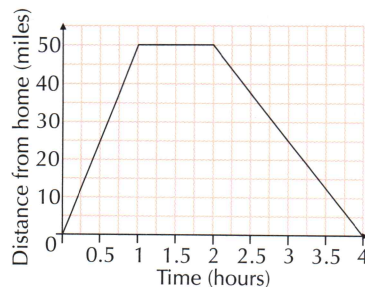
### Exercise 1

- Q1 Harry walks 3 km in 50 minutes at a constant speed to his friend's house. He stays there for 1 hour. He then walks at a constant speed back towards home for 30 minutes until he gets to the shop, 1 km from home. He stays at the shop for 10 minutes before walking home at a constant speed, which takes a further 15 minutes.

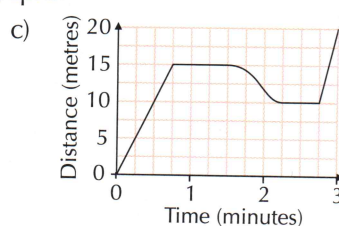
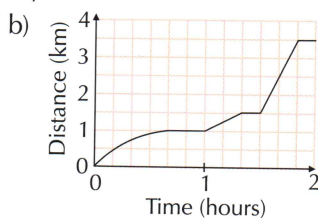
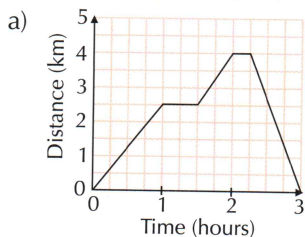
Draw a graph to represent Harry's journey.



- Q2 The graph on the right shows a family's car journey. The family left home at 8:00 am.
- (i) How long did the family travel for before stopping?  
(ii) How far had they travelled when they stopped?
  - How long did the family stay at their destination before setting off home?
  - (i) What time did they start the journey back home?  
(ii) How long did the journey home take?
  - Without doing any calculations, state whether the family travelled at a greater speed on the way to their destination or on the way back. Explain your answer.



- Q3 Describe the journey represented by each of these distance-time graphs.



## Finding Speed from a Distance-Time Graph

### Learning Objective — Spec Ref A15/R14:

Find speed from a distance-time graph.

### Prior Knowledge Check:

Be able to find the gradient of a line (see p.180) and estimate the gradient of a curve (see p.223).

The **gradient** of a distance-time graph shows the **speed** of the object. You can work out the speed at any stage of the journey by dividing the **distance travelled** by the **time taken**. To calculate the **average speed** across the whole journey, divide the **total distance travelled** by the **total time taken**.

If the graph is **curved**, you can **estimate** the speed at any point by drawing a **tangent** to the curve at that point and measuring its **gradient** (see p.223).

**Tip:** If the graph goes up and down, you'll have to add up the distance travelled at each stage to find the total distance travelled.

### Example 2

The graph on the right represents a train journey from Clumpton Station to Hillybrook Station.

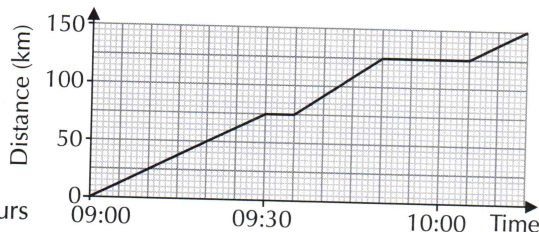
- a) Find the speed of the train (in km/h) at 9:15.

The speed is constant from 9:00 until 9:30, so use the graph to find the distance travelled in this time.

Distance = 75 km

Time = 30 mins = 0.5 hours

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{75 \text{ km}}{0.5 \text{ hours}} = 150 \text{ km/h}$$



- b) Find the average speed of the train (in km/h) for this journey.

1. First work out the total distance travelled and the total time taken.

Distance =  $150 - 0 = 150 \text{ km}$

Time = 09:00 to 10:15 = 1 hour 15 mins = 1.25 hours

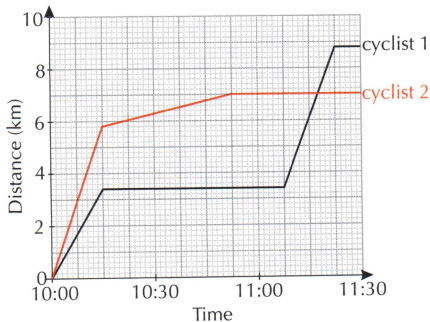
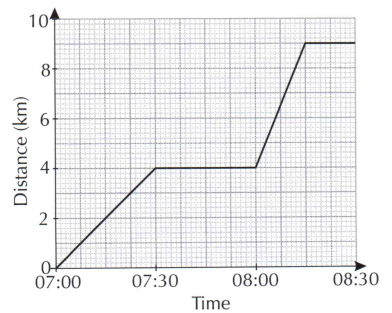
2. Then use the formula for speed.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{150 \text{ km}}{1.25 \text{ hours}} = 120 \text{ km/h}$$

## Exercise 2

Q1 The graph on the right shows a commuter's journey to work. His journey consists of two stages of travelling, separated by a break of 30 minutes.

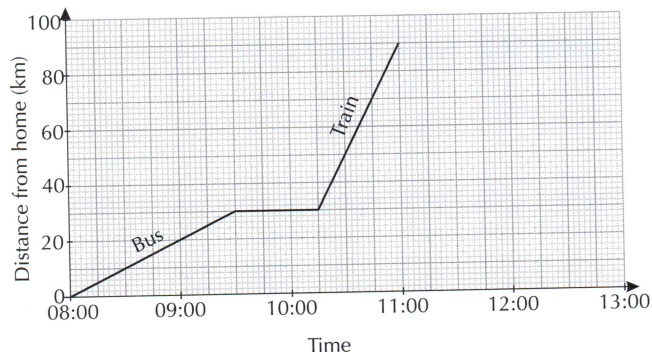
- What was his speed (in km/h) during the first stage of his journey?
- What was his speed (in km/h) during the second stage of his journey?



- Q2 The graph on the left shows two cyclists' journeys.
- Which cyclist had the highest maximum speed?
  - Find the difference in the speeds of the cyclists during the first 15 minutes of their journeys.

Q3 Chay travels to Clapham by taking a bus and a train. His journey is shown in the graph below.

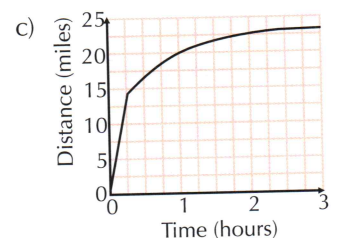
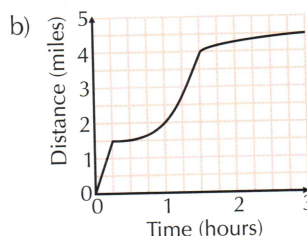
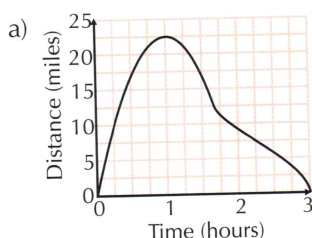
- Find the average speed Chay travelled:
  - by bus
  - by train
- Chay spends 30 minutes in Clapham before he returns home by taxi at an average speed of 60 km/h.
  - Find the time it took Chay to travel home by taxi.
  - Copy the graph and extend the line to show Chay's time in Clapham and the taxi ride home.



Q4 Draw a distance-time graph to show each of the following journeys:

- Ash catches a train at 10:00, then travels for 280 miles at a constant speed of 80 mph.
- Corey sets off from home at 09:00 and drives 90 miles at a constant speed of 60 mph. After 45 minutes at his destination, he drives back home at a constant speed of 40 mph.

Q5 Estimate the speed of the object represented by each of these distance-time graphs at time = 1 hour.





## 23.3 Velocity-Time Graphs

Velocity is speed measured in a particular direction. For example, objects with velocities of 5 m/s and -5 m/s are travelling at the same speed but in opposite directions. A velocity-time graph shows the speed and direction of an object over a period of time.

### Calculating Acceleration from a Velocity-Time Graph

#### Learning Objectives — Spec Ref A15/R14:

- Draw and interpret velocity-time graphs.
- Find acceleration from a velocity-time graph.

**Prior Knowledge Check:**  
Be able to find the gradient of a line. See p.180.

**Acceleration** is the **rate of change** of **velocity** over time, i.e. the rate at which an object is speeding up or slowing down. The **units** for acceleration are a combination of the units for velocity and time, e.g.  $\text{m/s}^2$ . The **formula** for acceleration is:

$$\text{Acceleration} = \frac{\text{Change in Velocity}}{\text{Change in Time}}$$

The **gradient** of a velocity-time graph shows the **acceleration** (or **deceleration**) of an object — a **positive slope** is acceleration while a **negative slope** is deceleration. The **steeper** the graph, the **greater** the acceleration (or deceleration) of the object.

A **straight** line shows that an object has a **constant acceleration**, a **horizontal** line means there is **no acceleration** so the object is moving at a **constant velocity** and a **curved** line means the rate of acceleration is **changing**.

To **estimate** the acceleration from the gradient of a curved graph, draw a **tangent** to the line at the point at which you want to find the acceleration and work out the gradient of the tangent (see p.223).

**Tip:** A negative acceleration is the same as a deceleration.  
E.g. an acceleration of  $-5 \text{ m/s}^2$  is the same as a deceleration of  $5 \text{ m/s}^2$ .

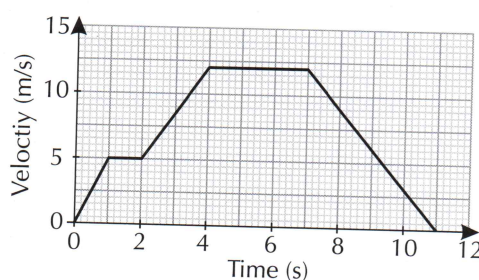
#### Example 1

The graph shows the velocity of a taxi during a journey. Calculate the acceleration of the taxi:

a) 6 seconds into the journey

At 6 seconds, the graph is horizontal. This means there is no acceleration, i.e. velocity is constant.

Acceleration at 6 seconds =  $0 \text{ m/s}^2$



b) 10 seconds into the journey

- At 10 seconds, the graph is straight and sloping down so the acceleration is constant and negative — i.e. the taxi is decelerating.

- Use the acceleration formula.  
From 7 seconds to 11 seconds the velocity changed from 12 m/s to 0 m/s.

$$\text{Acceleration} = \frac{\text{Change in Velocity}}{\text{Change in Time}}$$

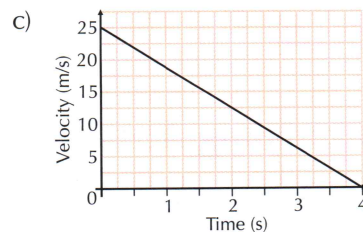
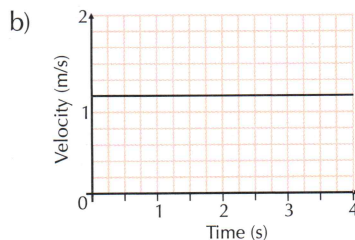
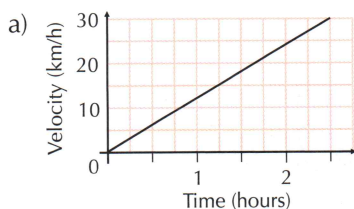
$$\text{Change in Velocity} = 0 - 12 \text{ m/s} = -12 \text{ m/s}$$

$$\text{Change in Time} = 11 - 7 = 4 \text{ seconds}$$

$$\text{Acceleration} = \frac{-12 \text{ m/s}}{4 \text{ seconds}} = -3 \text{ m/s}^2$$

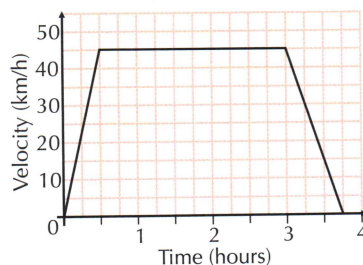
## Exercise 1

Q1 Calculate the acceleration shown by each of the graphs below.



Q2 The diagram shows the velocity of a train travelling between two stations.

- What is the maximum velocity of the train during the journey?
- Calculate the initial acceleration of the train.
- Calculate the deceleration of the train at the end of its journey.



### Example 2

Look at the velocity-time graph for a tram on the right.

- a) Describe the acceleration of the tram in the first second of the journey.

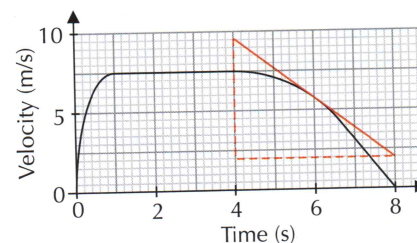
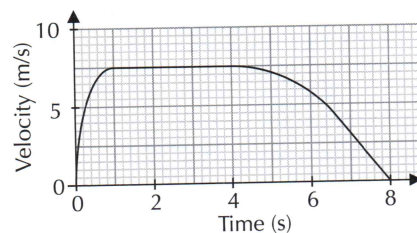
The gradient starts off steep and then decreases. This means the tram's acceleration also decreases.

The acceleration of the tram **decreases** over time.

- b) Find the acceleration of the tram at 6 seconds.

- Draw a tangent to the curve at 6 seconds.
- The tangent goes through points (4, 9.5) and (8, 2). Work out the gradient of the line — the velocity is decreasing, so it'll be a negative acceleration.

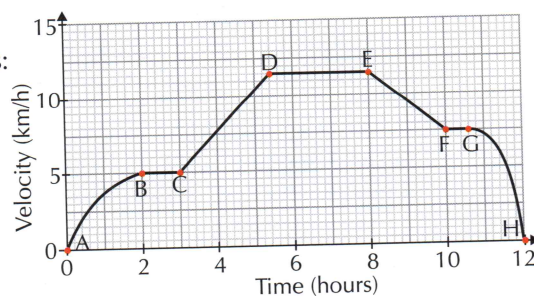
$$\text{Gradient of tangent} = \frac{2 - 9.5}{8 - 4} = \frac{-7.5}{4} = -1.875 \text{ m/s}^2$$



## Exercise 2

Q1 Look at the velocity-time graph on the right.

- Describe the velocity of the object between points:
  - A and B
  - D and E
  - G and H
- Describe the acceleration of the object between points:
  - A and B
  - D and E
  - G and H
- Calculate the acceleration between points:
  - C and D
  - E and F
- Estimate the acceleration of the object after:
  - 1 hour
  - 11 hours





# Calculating Distance from a Velocity-Time Graph

## Learning Objective — Spec Ref A15:

Find the area under a velocity-time graph.

## Prior Knowledge Check:

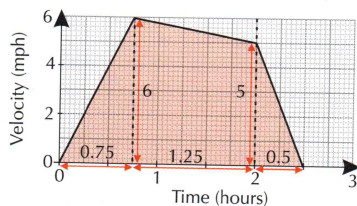
Find the area of 2D shapes. See p.349-352.

The **area** under a velocity-time graph is equal to the **distance** travelled. When a graph only has **straight lines**, you can **calculate** the **exact distance travelled** by splitting the area under the graph into triangles, rectangles and trapeziums. Then find the **area** of each shape and **add them up**.

### Example 3

The graph on the right shows the velocity of a runner. Calculate the total distance the runner travelled.

1. Split the area into two triangles and a trapezium and find the dimensions of the shapes using the axes.



2. Find the area of each section of the graph. Then add them up to find the total area.

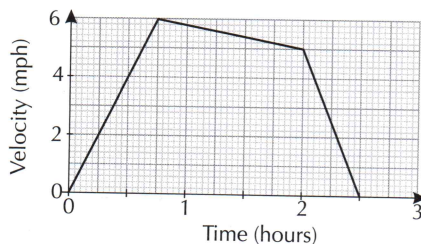
$$\text{Area of 1st triangle} = \frac{1}{2}(6 \times 0.75) = 2.25$$

$$\text{Area of trapezium} = \frac{1}{2}(6 + 5) \times 1.25 = 6.875$$

$$\text{Area of 2nd triangle} = \frac{1}{2}(5 \times 0.5) = 1.25$$

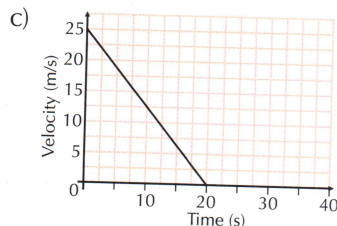
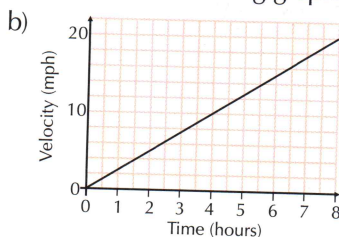
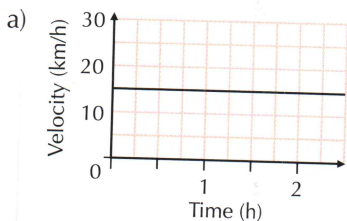
$$\text{Total area} = 2.25 + 6.875 + 1.25 = 10.375$$

So the total distance travelled is **10.375 miles**.



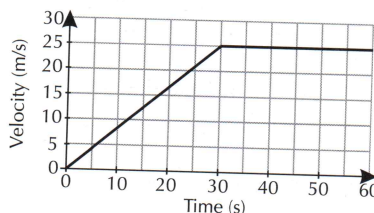
### Exercise 3

Q1 Calculate the distance travelled for each of the following graphs.



Q2 The diagram shows the velocity of a train for the first 60 seconds of a journey.

- Calculate the total distance travelled during this part of the train's journey.
- Calculate the average velocity of the train.



Q3 A car accelerates for 8 seconds, causing its velocity to increase from 0 m/s to 48 m/s. Then it travels at a constant velocity of 48 m/s for 20 seconds, before decelerating to 23 m/s over 5 seconds.

- Draw a velocity-time graph to show the car's velocity during this part of the journey.
- Calculate both the acceleration and deceleration of the car.
- Calculate the distance travelled by the car.



When a velocity-time graph is **curved**, you can **estimate** the **total distance** travelled by **roughly** splitting the area under the graph into triangles, rectangles and trapeziums. There are often **different ways** to split the graph up and you'll get a **different estimate** depending on how you do it. A common way to split graphs up is into strips of **equal width**, but this doesn't always give the most accurate estimate.

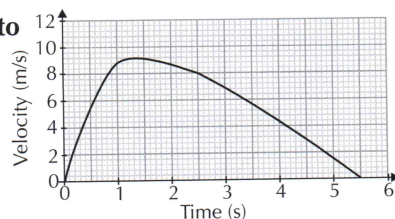
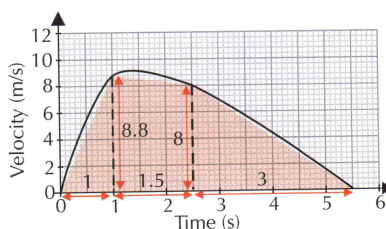
**Tip:** The greater the number of sections you split the graph into, the more accurate your estimate is likely to be.

If the shapes cover **less than** the whole area under the graph, the distance will be an **underestimate**. If the shapes cover **more than** the whole area under the graph, the distance will be an **overestimate**.

#### Example 4

The graph on the right shows the velocity of a ball after it is kicked into the middle of a field. Estimate the total distance the ball travelled.

1. Roughly split the area under the curve into simple shapes. Here the area has been split into two triangles and a trapezium. Then use the axes to work out the dimensions of the shapes.



2. Find the area of each section of the graph. Then add them up to find the total area. All of the shapes are under the graph so the answer will be an underestimate of the total area.
3. The units on the graph are m/s and seconds so the total distance will be in metres.

$$\text{Area of 1st triangle} = \frac{1}{2}(8.8 \times 1) = 4.4$$

$$\text{Area of trapezium} = \frac{1}{2}(8.8 + 8) \times 1.5 = 12.6$$

$$\text{Area of 2nd triangle} = \frac{1}{2}(8 \times 3) = 12$$

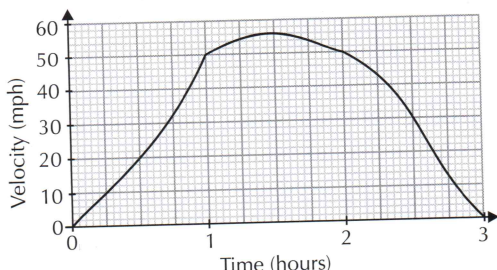
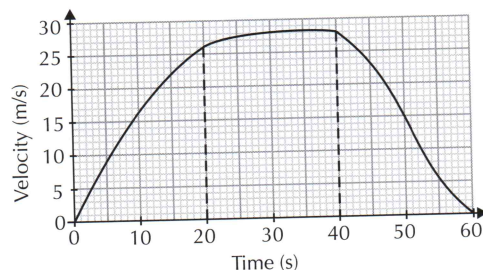
$$\text{Total area} = 4.4 + 12.6 + 12 = 29$$

So an estimate for the total distance the ball travelled is **29 m**.

#### Exercise 4

Q1 The diagram shows the velocity of an object over 60 seconds. It has been split up into 20 second intervals.

- a) Estimate the distance travelled during the first 20 seconds of the journey.
- b) Estimate the distance travelled during the last 20 seconds of the journey.
- c) By splitting the middle 20 seconds into two intervals of 10 seconds each, estimate the total distance the object travelled in 60 seconds.



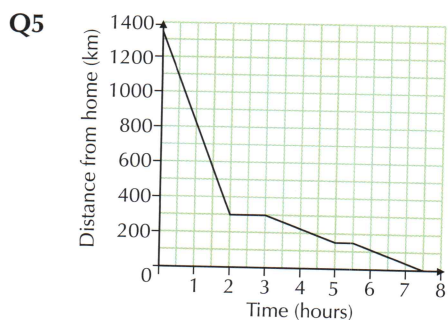
Q2 The graph on the left shows the velocity of an object during a 3 hour journey.

- a) Estimate the total distance travelled by the object.
- b) Estimate the average velocity of the object over the 3 hour journey.



# Review Exercise

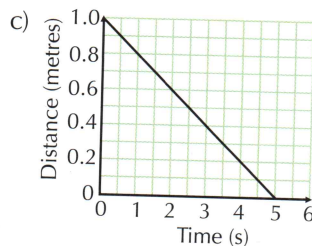
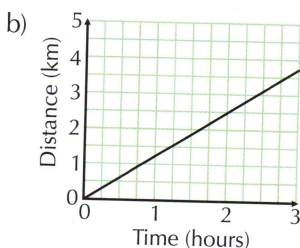
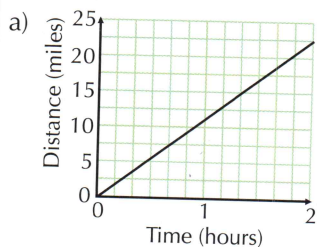
- Q1** It takes a high-speed train 25 minutes to travel 240 km. Calculate the average speed of the train in kilometres per hour to 3 significant figures.
- Q2** If a skydiver falls at a terminal velocity of 120 mph for 16 seconds, how many miles do they fall? Give your answer to 2 significant figures.
- Q3** For each of the following, use the formula for density, mass and volume to find the missing value.
- a) mass = 642 kg, volume =  $0.05 \text{ m}^3$
  - b) mass = 0.06 kg, volume =  $0.025 \text{ m}^3$
  - c) density =  $42 \text{ kg/m}^3$ , volume =  $6.2 \text{ m}^3$
  - d) density =  $120 \text{ g/cm}^3$ , mass = 4.8 g
- Q4** A cereal box is resting with its base on a horizontal surface. The weight of the box is 1.5 N and it exerts a pressure of 150 Pa on the surface. What is the area of the base of the box in  $\text{cm}^2$ ?



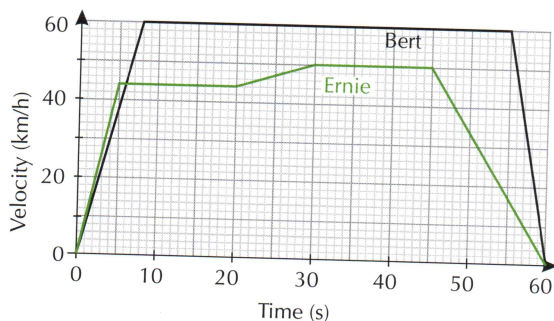
Elsie and Aggie travel home from Vienna to York. They take a flight from Vienna to London, go through passport control, then drive from London to York, stopping once for a break.

- a) How long was the flight?
- b) How long was their break during the driving section of the journey?
- c) How far did they drive?

- Q6** Find the speed of the object represented by each of the following distance-time graphs.



- Q7** The velocity-time graph on the right shows the velocity of two ostriches as they run.
- a) How far did each ostrich run?
  - b) Calculate the average velocity of Bert.
  - c) Work out the initial acceleration of Ernie.
  - d) Without any further calculation, state which ostrich had the greatest initial acceleration. Explain your answer.



# Exam-Style Questions

- Q1** 36 cm<sup>3</sup> of copper and 4 cm<sup>3</sup> of tin are melted down and mixed to make a bronze medal. The densities of copper and tin are 9 g/cm<sup>3</sup> and 7 g/cm<sup>3</sup> respectively. Work out the mass of the medal.

[2 marks]

- Q2** A skip exerts a pressure of 625 Newtons per square metre (N/m<sup>2</sup>) over a ground area of 40 000 cm<sup>2</sup>. Calculate the force in Newtons (N) exerted by the skip on to the ground.

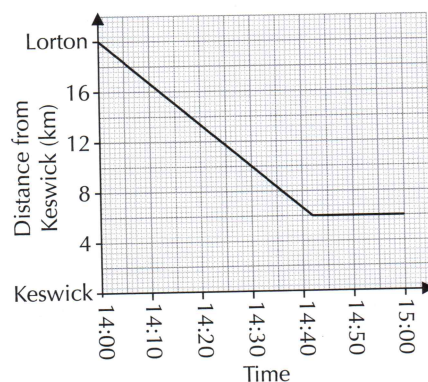
[3 marks]

- Q3** Two cyclists, Michael and Nigel, plan to meet for a coffee. Michael starts his ride from Keswick and Nigel from Lorton. Nigel's journey is shown by the graph on the right.

Michael cycles at a constant speed that is half that of Nigel's. He arrives at the meeting point 2 minutes earlier than Nigel, where he waits. What time did Michael set off?



[4 marks]



- Q4** The density of the metal mercury is 13.6 g/cm<sup>3</sup>. Work out the density of mercury in kg/m<sup>3</sup>.

[3 marks]

- Q5** The graph on the right shows the velocity of a bus in m/s for the first minute of a journey.

a) Explain what the gradient of the graph represents.

[1 mark]

b) By using three strips of equal width, estimate the distance travelled by the bus during this minute.

[3 marks]

c) Is your answer to part b) an underestimate or an overestimate for the actual distance travelled? You must give a reason for your answer.

[1 mark]

