

22.1 Metric Units — Length, Mass and Volume

When comparing, adding or subtracting measurements, it's much easier to work with them if they all have the same units — so being able to convert between units is a really useful skill.

Learning Objective — Spec Ref NI3/RI/G14:

Convert between different metric units for length, mass and volume.

The **metric units** for length, mass and volume are as follows.

- **Length** is measured in **millimetres** (mm), **centimetres** (cm), **metres** (m) and **kilometres** (km).
- **Mass** is measured in **milligrams** (mg), **grams** (g), **kilograms** (kg) and **tonnes**.
- **Volume** is measured in **millilitres** (ml), **litres** (l) and **cubic centimetres** (cm³).

Tip: Metric units increase by powers of 10 to make larger numbers easier to deal with.

To **convert** between different units of length, mass or volume, you **multiply** or **divide** by a **conversion factor**. The most commonly used conversion factors are shown below.

Length

$$1 \text{ cm} = 10 \text{ mm}$$

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ km} = 1000 \text{ m}$$

Mass

$$1 \text{ g} = 1000 \text{ mg}$$

$$1 \text{ kg} = 1000 \text{ g}$$

$$1 \text{ tonne} = 1000 \text{ kg}$$

Volume

$$1 \text{ litre (l)} = 1000 \text{ ml}$$

$$1 \text{ ml} = 1 \text{ cm}^3$$

When converting from small units to **bigger units** (e.g. cm to m), you **divide** by the conversion factor. When converting from big units to **smaller units**, you **multiply** by the conversion factor. Always **check** your answers to make sure they seem **reasonable** — e.g. if you converted an elephant's height from m to cm and got 0.025 cm, you'd know you'd gone wrong. You'd expect **more** small units than big units — there are 100 cm in 1 m, so you'd expect there to be more centimetres than metres.

Example 1

What is 0.035 km in cm?

1. Convert km to m — a smaller unit, so multiply by the conversion factor.
 $1 \text{ km} = 1000 \text{ m}$
 $0.035 \text{ km} = 0.035 \times 1000 = 35 \text{ m}$
2. Then convert to cm — a smaller unit, so multiply by the conversion factor.
 $1 \text{ m} = 100 \text{ cm}$
 $35 \text{ m} = 35 \times 100 = 3500 \text{ cm}$

Tip: You could convert km into cm in one step by multiplying by $100 \times 1000 = 100\,000$.

Example 2

Find the total of 0.2 tonnes, 31.8 kg and 1700 g. Give your answer in kg.

1. Convert tonnes to kg — a smaller unit, so multiply by the conversion factor.
 $1 \text{ tonne} = 1000 \text{ kg}$
 $0.2 \text{ tonnes} = 0.2 \times 1000 = 200 \text{ kg}$
2. Convert g to kg — a bigger unit, so divide by the conversion factor.
 $1 \text{ kg} = 1000 \text{ g}$
 $1700 \text{ g} = 1700 \div 1000 = 1.7 \text{ kg}$
3. The masses are now in kg so add them together.
 $200 \text{ kg} + 31.8 \text{ kg} + 1.7 \text{ kg} = 233.5 \text{ kg}$

Example 3

Liam buys a 2.5 litre bottle of lemonade and pours out 175 ml. How much is left in the bottle?
Give your answer in cm^3 .

1. Convert litres to ml — a smaller unit,
so multiply by the conversion factor.

$$1 \text{ litre} = 1000 \text{ ml}$$

$$2.5 \text{ litres} = 2.5 \times 1000 = 2500 \text{ ml}$$

2. Subtract 175 ml from 2500 ml.

$$2500 \text{ ml} - 175 \text{ ml} = 2325 \text{ ml}$$

3. Swap ml for cm^3 , as $1 \text{ cm}^3 = 1 \text{ ml}$.

$$2325 \text{ ml} = \mathbf{2325 \text{ cm}^3}$$

Exercise 1

Q1 Convert each measurement into the units given.

a) 3000 kg into tonnes

b) 0.4 g into mg

c) 123 ml into litres

d) 5116 g into kg

e) 12.6 kg into tonnes

f) 2.7165 m into cm

Q2 Convert each measurement into the units given.

a) 0.15 kg into mg

b) 1532 g into tonnes

c) 1005 cm into km

d) 3023 mg into kg

e) 3 mm into km

f) 49 tonnes to g

Q3 Hafsa is having a party for 32 guests. Her glasses have a capacity of 400 ml each.

If she wants everyone to have a glass of juice, how many 2 litre bottles of juice should she buy?

Q4 A go-kart has a 5 litre petrol tank. It uses 10 ml of petrol per lap of a 400 m track.

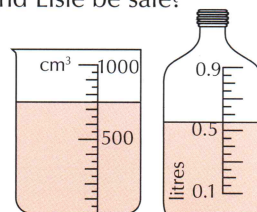
a) If James fills up the tank, how many laps of the track can he do?

b) How many km can James travel on each full tank of petrol?

Q5 Sharon, Leuan and Elsie get into a cable car while skiing. Sharon weighs 55.2 kg, Leuan weighs 78.1 kg and Elsie weighs 65.9 kg. Their skis weigh 10 000 g a pair. The cable car is unsafe when carrying a mass of over half a tonne. Will Sharon, Leuan and Elsie be safe?

Q6 Milly runs a 1500 m fun run, a 50 m sprint and a 13.2 km race. How many km has she run in total?

Q7 Find the difference in the amounts of liquid held by the two containers shown on the right. Give your answer in cm^3 .



Q8 A reservoir contains 600 000 litres of water. During a period of heavy rain, the volume of water in the reservoir increases by 750 000 ml every day. The reservoir can only hold 800 000 litres of water. If the rain continues at this rate, calculate the value of N , the number of whole days that will pass before the reservoir overflows.



Q9 A lasagne recipe requires 0.7 kg of minced beef, 400 g of tomato sauce, 300 g of cheese sauce, 0.2 kg of lasagne sheets and 2500 mg of herbs and spices.

a) How many kg do these ingredients weigh in total?

b) 0.2 kg of ingredients are needed for each person. How many people can be fed with this recipe?

22.2 Metric Units — Area and Volume

When converting between units of area and volume, you don't just multiply by the conversion factor.

Learning Objective — Spec Ref R1/G14:

Convert between different metric units for area and volume.

The **area** of a shape is found by multiplying **two lengths** — so the units for area are the units for length **squared** (e.g. $\text{m} \times \text{m} = \text{m}^2$). To **convert** between different units of area, you need to multiply or divide by the **square** of the 'length' conversion factor. For example, to convert 1 m^2 to cm^2 , you need to multiply by $100 \times 100 = 10\,000$ — you're multiplying by the conversion factor **twice**, once for each dimension.

The **volume** of an object is found by multiplying **three lengths** — so the units for volume are the units for length **cubed** (e.g. $\text{m} \times \text{m} \times \text{m} = \text{m}^3$). To **convert** between different units of volume, you need to multiply or divide by the **cube** of the 'length' conversion factor. For example, to convert 1 m^3 to cm^3 , you need to multiply by $100 \times 100 \times 100 = 1\,000\,000$ — you're multiplying by the conversion factor **three times**, once for each dimension.

Here are some commonly used **conversion factors** for area and volume:

Area

$$1 \text{ cm}^2 = 10^2 \text{ mm}^2 = 100 \text{ mm}^2$$

$$1 \text{ m}^2 = 100^2 \text{ cm}^2 = 10\,000 \text{ cm}^2$$

$$1 \text{ km}^2 = 1000^2 \text{ m}^2 = 1\,000\,000 \text{ m}^2$$

Volume

$$1 \text{ cm}^3 = 10^3 \text{ mm}^3 = 1000 \text{ mm}^3$$

$$1 \text{ m}^3 = 100^3 \text{ cm}^3 = 1\,000\,000 \text{ cm}^3$$

$$1 \text{ km}^3 = 1000^3 \text{ m}^3 = 1 \times 10^9 \text{ m}^3$$

Tip: Use standard form (p.96) if the numbers are very large or very small — e.g. $1 \times 10^9 = 1\,000\,000\,000$.

As on p.276, you **divide** by the conversion factor when converting from small units to **bigger units** (e.g. cm^2 to m^2) and you **multiply** by the conversion factor when converting from big units to **smaller units**.

Example 1

Convert an area of 0.06 m^2 to cm^2 .

1. Work out the conversion factor from cm^2 to m^2 .
2. You're converting to a smaller unit, so multiply by the conversion factor.

$$\begin{aligned} 1 \text{ m} &= 100 \text{ cm}, \\ \text{so } 1 \text{ m}^2 &= 100 \times 100 = 10\,000 \text{ cm}^2 \end{aligned}$$

$$0.06 \text{ m}^2 = 0.06 \times 10\,000 = \mathbf{600 \text{ cm}^2}$$

Example 2

Convert a volume of $382\,000 \text{ cm}^3$ to m^3 .

1. Work out the conversion factor from m^3 to cm^3 .
2. You're converting to a bigger unit, so divide by the conversion factor.

$$\begin{aligned} 1 \text{ m} &= 100 \text{ cm} \\ \text{so } 1 \text{ m}^3 &= 100 \times 100 \times 100 = 1\,000\,000 \text{ cm}^3 \end{aligned}$$

$$382\,000 \text{ cm}^3 = 382\,000 \div 1\,000\,000 = \mathbf{0.382 \text{ m}^3}$$

Example 3

A cuboid has a width of 55 mm and a height of 40 mm.

a) What is the area of the front face of the cuboid in cm^2 ?

- | | |
|--|---|
| 1. Work out the area in mm^2 . | $55 \times 40 = 2200 \text{ mm}^2$ |
| 2. Work out the conversion factor from mm^2 to cm^2 . | $1 \text{ cm} = 10 \text{ mm}$
so $1 \text{ cm}^2 = 10 \times 10 = 100 \text{ mm}^2$ |
| 3. You're converting to a bigger unit, so divide by the conversion factor. | $2200 \text{ mm}^2 = 2200 \div 100 = \mathbf{22 \text{ cm}^2}$ |

Tip: You could also have converted the dimensions into cm first.

b) If the cuboid has a length of 6.8 cm, what is its volume in mm^3 ?

- | | |
|---|--|
| 1. Work out the volume in cm^3 . | $22 \times 6.8 = 149.6 \text{ cm}^3$ |
| 2. Work out the conversion factor from cm^3 to mm^3 . | $1 \text{ cm} = 10 \text{ mm}$
so $1 \text{ cm}^3 = 10 \times 10 \times 10 = 1000 \text{ mm}^3$ |
| 3. You're converting to a smaller unit, so multiply by the conversion factor. | $149.6 \text{ cm}^3 = 149.6 \times 1000 = \mathbf{149\ 600 \text{ mm}^3}$ |

Exercise 1

Q1 Convert each of these measurements into the units given.



- | | | |
|---|---|--|
| a) 84 mm^2 into cm^2 | b) 1750 cm^2 into m^2 | c) $29\ 000 \text{ mm}^2$ into cm^2 |
| d) 0.001 km^3 into m^3 | e) 15 cm^3 into mm^3 | f) 0.2 m^3 into cm^3 |
| g) $3\ 150\ 000 \text{ m}^2$ into km^2 | h) 8500 mm^2 into cm^2 | i) 1700 cm^2 into m^2 |
| j) 0.435 km^3 into m^3 | k) 6.7 km^3 into m^3 | l) 0.00045 cm^3 into mm^3 |

Q2 Sandeesh wants to carpet two rectangular floors. One of the floors measures 1.7 m by 3 m, while the other is 670 cm by 420 cm. How many square metres of carpet will she need?

Q3 25 cm^3 of squash must be diluted with 0.5 litres of water to make one glass.

- a) How many glasses can you make from a 1 litre bottle of squash?
b) What is the total volume of one glass in mm^3 ?

Q4 A swimming pool is 3 m deep and has a base with area 375 m^2 .

- a) Find the volume of the pool in cm^3 . b) How many litres of water can the pool hold?

Q5 A brand of coffee powder is sold in cuboid packets with dimensions 20.7 cm by 25.5 cm by 10 cm.

- a) A volume of 0.003 m^3 of coffee powder has already been used. What volume (in m^3) is left?
b) Find the total surface area of the packet of coffee powder. Give your answer in mm^2 .

Q6 Convert each of these measurements into the units given.

- | | | |
|---|--|---|
| a) 1.2 m^2 into mm^2 | b) 0.001 km^2 into cm^2 | c) 50 million mm^2 into m^2 |
| d) 3 million mm^3 into km^3 | e) 0.0006 m^3 into mm^3 | f) 999 cm^3 into km^3 |

22.3 Metric and Imperial Units

Unlike metric units, imperial units are not based on powers of 10 — e.g. there are 16 ounces in a pound and 14 pounds in a stone.

Learning Objective — Spec Ref N13/RI/G14:

Convert between metric and imperial units.

The **imperial units** for length, mass and volume are as follows:

- **Length** is measured in **inches** (in), **feet** (ft), **yards** and **miles**.
There are **12 inches** in 1 foot, **3 feet** in 1 yard and **1760 yards** in 1 mile.
- **Mass** is measured in **ounces** (oz), **pounds** (lb) and **stones**.
There are **16 ounces** in 1 pound and **14 pounds** in a stone.
- **Volume** is measured in **pints** and **gallons**. There are **8 pints** in 1 gallon.

Tip: As 1 foot = 12 in, the conversion factor from feet to inches is 12.

To write small imperial units as a **mixture of big and small units** (e.g. writing inches as feet and inches), you **divide** by the conversion factor and keep the **remainder** in the smaller units.

To write a **mixture** of big and small units in **smaller units** (e.g. writing feet and inches as inches), you **multiply** the big unit by the conversion factor and **add on** the remaining small units.

There are **approximate conversion factors** to switch between metric and imperial units — e.g. there are approximately 2.5 cm in 1 inch. The symbol '≈' means 'approximately equal to'. To **convert** between metric and imperial units, just **multiply** or **divide** by the conversion factors below.

Length

1 inch ≈ 2.5 cm
1 foot ≈ 30 cm
1 yard ≈ 90 cm
1 mile ≈ 1.6 km

Mass

1 ounce ≈ 28 g
1 pound ≈ 450 g
1 stone ≈ 6400 g
1 kg ≈ 2.2 pounds

Volume

1 pint ≈ 0.57 litres
1 gallon ≈ 4.5 litres

Tip: You'll be given metric to imperial conversion factors in an exam if they're needed. Don't be surprised if they're not exactly the same as these ones.

Example 1

Convert 65 cm into feet and inches.

1. Divide by the conversion factor for cm to inches. 1 inch ≈ 2.5 cm,
so 65 cm ≈ $65 \div 2.5 = 26$ inches
2. Convert 26 inches into feet and inches. 1 foot = 12 inches,
 $26 \div 12 = 2$ remainder 2
3. Keep the remainder in inches. So 65 cm ≈ **2 feet 2 inches**

Tip: 1 inch is bigger than 1 cm so you divide by the conversion factor.

Example 2

Convert 6 pounds and 4 ounces into kilograms.

1. Write the whole mass using the same unit. 1 pound = 16 ounces,
so 6 pounds and 4 ounces = $(6 \times 16) + 4 = 100$ ounces
2. Convert this into g using the conversion factor for ounces to g. 1 ounce ≈ 28 g,
100 ounces ≈ $100 \times 28 = 2800$ g
3. Then convert the result from g to kg. $2800 \text{ g} = 2800 \div 1000 = \mathbf{2.8 \text{ kg}}$

Example 3

Convert 11 400 ml into both pints and gallons.

1. Convert into litres. $1 \text{ litre} = 1000 \text{ ml}$,
so $11\,400 \text{ ml} = 11\,400 \div 1000 = 11.4 \text{ litres}$
2. Divide by the conversion factor for litres to pints. $1 \text{ pint} \approx 0.57 \text{ litres}$,
so $11.4 \text{ litres} \approx 11.4 \div 0.57 = \mathbf{20 \text{ pints}}$
3. Convert into gallons $1 \text{ gallon} = 8 \text{ pints}$,
so $20 \text{ pints} = 20 \div 8 = \mathbf{2.5 \text{ gallons}}$

Tip: A pint is less than a litre, so you'd expect more pints than litres.

Exercise 1

In these questions, use the conversion factors given on the previous page.

Q1 Convert each of the following measurements into the units given.

- | | | |
|-------------------------------|-------------------------------|---------------------------------|
| a) 3 ft 7 in to inches | b) 12 ft 5 in to inches | c) 5 lb 2 oz to ounces |
| d) 280 in to feet and inches | e) 1001 in to feet and inches | f) 72 oz to lb and oz |
| g) 70 lb to stones and pounds | h) 200 oz to lb and oz | i) 5.5 yards to feet and inches |
| j) 4.75 ft to feet and inches | k) 2.5 stone to lb and oz | l) 8.25 stone to lb and oz |

Q2 Convert each of the following masses into pounds and ounces.

- | | | | |
|-----------|----------|-----------|-----------|
| a) 1904 g | b) 840 g | c) 2688 g | d) 4.9 kg |
|-----------|----------|-----------|-----------|

Q3 Convert each of the following into feet and inches.

- | | | | |
|--------|------------|-----------|----------|
| a) 2 m | b) 52.5 cm | c) 1.5 km | d) 50 mm |
|--------|------------|-----------|----------|

Q4 State which is the greater amount in each of the following pairs.

- | | | |
|-------------------------|---------------------------|----------------------|
| a) 10 feet or 3.5 m | b) 1 stone or 7 kg | c) 10 miles or 12 km |
| d) 15 pints or 9 litres | e) 3 lb or 1.5 kg | f) 5 stone or 31 kg |
| g) 160 stone or 1 tonne | h) 2 gallons or 10 litres | i) 16 lb or 7 kg |

Q5 A ride at a theme park states you must be 140 cm or over to ride. Maddie is 4 feet 5 inches and Lily is 4 feet 9 inches. Who can go on the ride — Maddie, Lily, both or neither of them?



Q6 A running track is 400 m. How many laps of the track make one mile?



Q7 Jamie and Oliver are cooking. They need 1 pound 12 ounces of meat for their recipe. They buy 750 g of meat in the supermarket. Will this be enough?



Q8 A large box of juice holds 3 litres.



- How many whole pint jugs can be filled from the box?
- Approximately how many litres of juice will be left in the box after filling the jugs?

Q9 Marion is on holiday in Spain. Her car measures speed in mph but the road signs are in km/h. The speed limit is 90 km/h. What is this in mph? Give your answer to the nearest mph.



22.4 Estimating in Real Life

When it's not possible to measure something, you might have to estimate its size instead.

Learning Objective — Spec Ref N14:

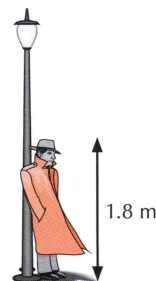
Estimate the dimensions of real-life objects by comparison.

You can **estimate** the **dimensions** (e.g. length, mass or volume) of something by **comparing** it with something else you already know the size of. An estimate does not have to be completely accurate, but make sure your answer seems **realistic** and is given in **suitable units**.

Example 1

Estimate the height of this lamp post.

- | | |
|---|---|
| 1. Estimate the height of the man. | Average height of a man ≈ 1.8 m. |
| 2. Compare the height of the lamp post and the man. | The lamp post is roughly twice the height of the man. |
| 3. Multiply the height of the man by 2. | Height of the lamp post $\approx 2 \times 1.8$ m
$= 3.6$ m |



Example 2

Give a sensible metric unit for measuring the height of a room.

- | | |
|--|------------------------------------|
| 1. Most rooms are taller than an average person but of the same order of magnitude — at roughly 2 to 3 metres. | |
| 2. So it makes sense to measure the height of a room in the same metric unit as a person's height. | A sensible unit is metres . |

Tip: Feet would be an acceptable answer if you were asked for an imperial unit.

Exercise 1

- Q1 For each of the following, suggest a sensible unit of measurement, using
- | | | |
|-----------------------|-----------------------|----------------------------|
| (i) metric units | (ii) imperial units. | |
| a) length of a pencil | b) mass of a tomato | c) length of an ant |
| d) mass of a bus | e) volume of a teacup | f) distance between cities |
- Q2 Estimate each of the following, using sensible metric units.
- | | |
|------------------------------|---------------------------------|
| a) height of your bedroom | b) length of an average car |
| c) height of a football goal | d) arm span of an average adult |
| e) diameter of a football | f) volume of a typical bath tub |



Review Exercise

Q1 Convert each measurement into the units given.

- | | | |
|---------------|---------------------|--------------------------------|
| a) 10 kg to g | b) 14 cm to mm | c) 4.6 litres to cm^3 |
| d) 22 g to kg | e) 150 ml to litres | f) 6900 cm to m |

Q2 Convert each measurement into the units given.

- | | | |
|-----------------------|------------------|---------------------|
| a) 0.006 m to mm | b) 0.57 kg to mg | c) 1.2 km to cm |
| d) 8 000 000 mg to kg | e) 12 cm to km | f) 1101 g to tonnes |

Q3 Convert each of these areas into the units given.

- | | | |
|---|---|--|
| a) 10 cm^2 to m^2 | b) 18 km^2 to m^2 | c) 8.6 m^2 to mm^2 |
| d) 673 000 000 cm^2 into km^2 | e) 60 500 mm^2 into m^2 | f) 0.000005 km^2 into mm^2 |

Q4 Convert each of these volumes into the units given.

- | | | |
|---|---|--|
| a) 0.005 m^3 to cm^3 | b) 69 mm^3 to cm^3 | c) 720 cm^3 to m^3 |
| d) 19 cm^3 into m^3 | e) $17\,440 \text{ mm}^3$ into m^3 | f) 0.00345 km^3 into mm^3 |

Q5 The volume of a beachball is $49\,900 \text{ cm}^3$. What is the volume of the beachball in m^3 ?

For Q6-8, use the approximate conversion factors given on page 280.

Q6 Complete each of these calculations. Give each of your answers to two significant figures.

- | | |
|--|--|
| a) $681 \text{ cm} + 12.4 \text{ yards} \approx \square \text{ m}$ | b) $16.49 \text{ km} + 21.5 \text{ miles} \approx \square \text{ km}$ |
| c) $3 \text{ kg} + 1 \text{ stone} + 1.5 \text{ kg} \approx \square \text{ kg}$ | d) $100 \text{ cm} + 2 \text{ yards} + 15 \text{ feet} \approx \square \text{ cm}$ |
| e) $3 \text{ l} + 4.5 \text{ pints} + 250 \text{ ml} \approx \square \text{ ml}$ | f) $7 \text{ m} + 8 \text{ inches} + 35 \text{ mm} \approx \square \text{ cm}$ |

Q7 Jakov's mother asks him to buy 4 pints of milk. The shop only sells milk in 1 litre, 2 litre and 4 litre cartons. Which of these amounts is closest to what his mother asked for?

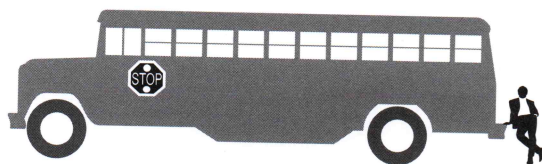


Q8 The weights of 6 people getting into a lift are shown below. The lift has a weight limit of 0.5 tonnes. Will the total weight of the 6 people exceed this limit?



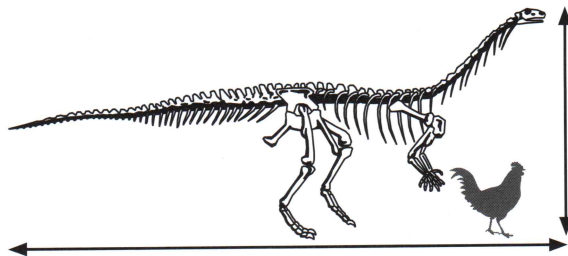
10 stone, 8 stone, 14 stone 1 pound, 9 stone 9 pounds, 12 stone, 17 stone

Q9 Estimate the length and height of the bus. Give your answer in sensible metric units.



Exam-Style Questions

- Q1** Estimate the height and length of this dinosaur in metric units by comparing it with a chicken.



[2 marks]

- Q2** Jack buys 1.2 kg of flour. How many pizzas can he make if each pizza needs 300 g of flour?



[1 mark]

- Q3** What is the volume of a cube with sides of 150 cm? Give your answer in m^3 .

[2 marks]

- Q4** Two nature reserves are approximately rectangular, measuring 2.9 km by 3.3 km and 2700 m by 4100 m. Which nature reserve has the largest area?

[2 marks]

- Q5** Elsie adds 16 inches of ribbon to the hats she makes. Ribbon cost £1.50 per metre. How many hats could Elsie make with £33 worth of ribbon? Use 1 inch \approx 2.5 cm.



[3 marks]

- Q6** A cup of tea contains 0.25 litres of liquid, of which 25 cm^3 is milk and the rest is water. How much water is needed, in ml, for 5 cups of tea?

[3 marks]

- Q7** Lotte's car travels 55 miles per gallon of petrol. Her car contains 45 litres of petrol.
- a) How many whole miles can she travel? Use the approximation 1 gallon \approx 4.5 litres.

[2 marks]

- b) If petrol costs £5 per gallon, how much did the petrol in her car cost?

[1 mark]