20.1 Angles on Lines and Around Points

When you have a cluster of angles along a straight line or around a single point, you can often find out the size of any unknown angles by using a simple fact: they always add up to a particular number.

Learning Objectives — Spec Ref G3:

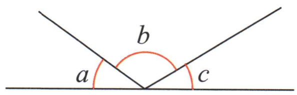
- Find angles that lie on a straight line.
- Find angles around a point.

Prior Knowledge Check: Be able to write and solve equations — see p.113-117.

Angles on a Straight Line

A group of angles on a **straight line** always **add up to 180°**:

$$a + b + c = 180°$$

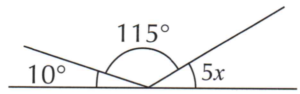


Tip: Angles aren't usually drawn accurately, so you need to know how to work out the size of them geometrically — you can't just measure them with a protractor.

The equation might have a different number of **variables** in it if there are a different number of angles on the line. To find missing angles on a straight line, use your **equation** and **rearrange** it so that the angle you want to find is the **subject**.

Example 1

Find the size of x in the diagram on the right.

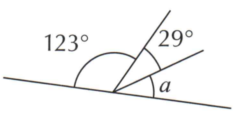


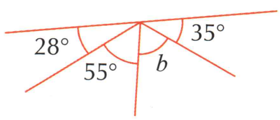
Not drawn accurately

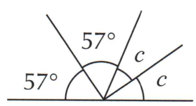
1. The angles lie on a straight line and so must add up to 180°. $10° + 115° + 5x = 180°$
2. Rearrange the equation and solve it for x. $5x = 180° - 10° - 115° = 55°$
   $x = 55° \div 5 = \mathbf{11°}$

Exercise 1

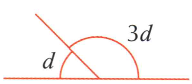
Q1 Find the value of each letter in the diagrams below. None of the angles are drawn accurately.

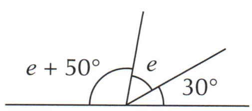
a) 

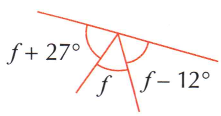
b) 

c) 

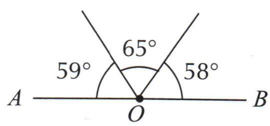
Q2 Find the value of each letter in the diagrams below. None of the angles are drawn accurately.

a) 

b) 

c) 

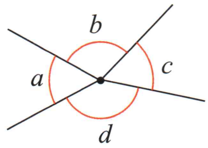
Q3 Explain why the line AOB on the right cannot be a straight line.



Angles Around a Point

A set of angles around a **point** always **add up to 360°**:

$$a + b + c + d = 360°$$



The equation might have a different number of **variables** in it if there are a different number of angles around the point. Use the **equation** to find any missing angles just like on the previous page.

Tip: You could draw a circle through all the angles round a point, and you know there are 360° in a circle.

Example 2

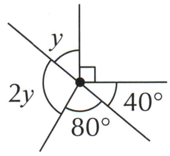
Find the value of y in the diagram on the right.

1. The angles are around a point so they must add up to 360°.

$$y + 2y + 80° + 40° + 90° = 360°$$

2. Rearrange the equation and solve it for y.

$$3y = 360° - 80° - 40° - 90°$$
$$= 150°$$
$$\mathbf{y = 50°}$$



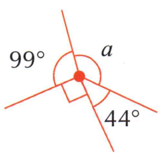
Not drawn accurately

Tip: The little square in the top-right of the diagram means that the angle is a right angle, i.e. it's 90°.

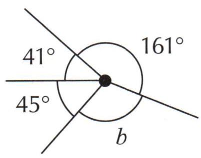
Exercise 2

Q1 Find the value of each letter in the diagrams below. None of the angles are drawn accurately.

a)



b)



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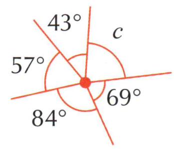
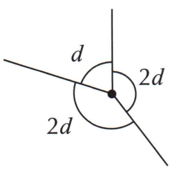
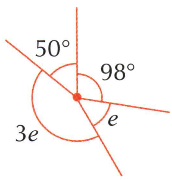


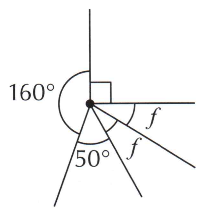
d)



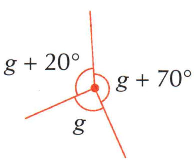
e)



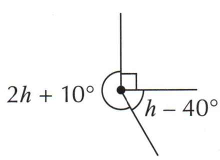
f)



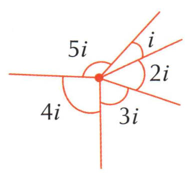
g)



h)



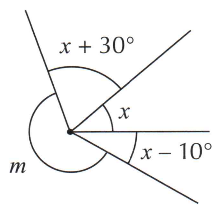
i)



Q2 a) All of the angles in the diagram on the right are larger than 1°. Bernie claims that $m < 307°$. Show that Bernie is right.

PROBLEM SOLVING

b) Given that $x = 40°$, find the value of m.



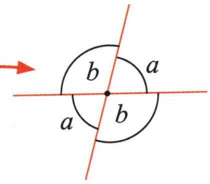
20.2 Parallel Lines

When a line crosses a pair of parallel lines, it produces special types of angles.

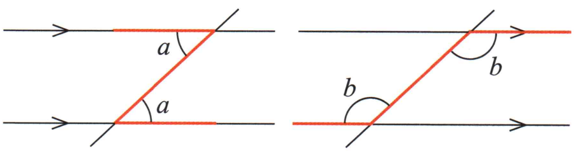
Vertically Opposite and Alternate Angles

Learning Objective — Spec Ref G3:
Find vertically opposite angles and alternate angles.

When **two lines intersect**, it produces **two pairs** of angles. The angles opposite one another are known as **vertically opposite angles**, and vertically opposite angles are always **equal**. In the diagram on the right, a and a are vertically opposite, as are b and b — so $a = a$ and $b = b$. Also, because the two **distinct** angles (i.e. a and b) form a straight line, they add up to **180°** — so $a + b = 180°$.



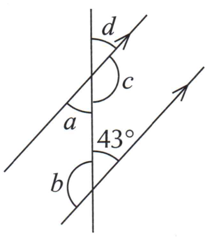
$a + b = 180°$



When a straight line crosses two **parallel** lines, it forms two pairs of **alternate angles** (in a sort of **Z-shape**). Alternate angles are always **equal**.

Example 1

Find the values of a, b, c and d in the diagram below.



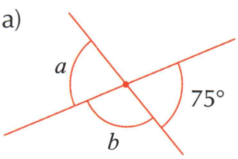
1. a and 43° are alternate angles, so they are equal. — $a = 43°$
2. b and the angle marked 43° lie on a straight line, so they add up to 180°. — $43° + b = 180°$, $b = 180° - 43° = \mathbf{137°}$
3. c and b are alternate angles, so they are equal. — $c = b = \mathbf{137°}$
4. a and d are vertically opposite angles, so they are equal. — $d = a = \mathbf{43°}$

Tip: The arrows on a pair of lines indicate those lines are parallel.

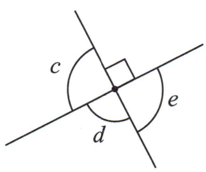
Exercise 1

Find the value of each letter in the diagrams below.

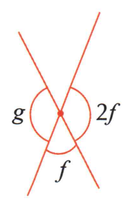
Q1 a)



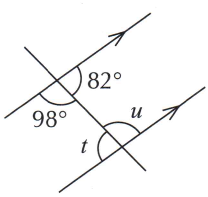
b)



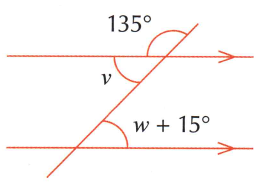
c)



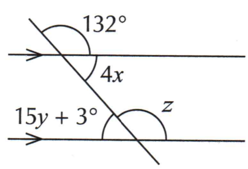
Q2 a)



b)



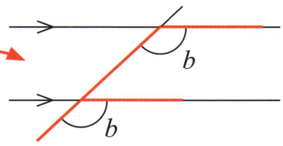
c)

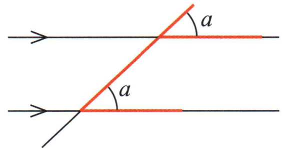


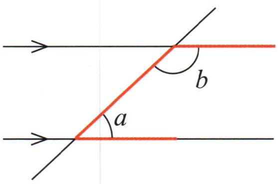
Corresponding and Allied Angles

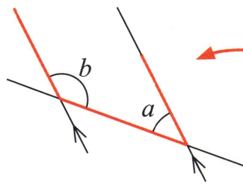
Learning Objective — Spec Ref G3:
Find corresponding and allied angles.

Corresponding angles form an **F-shape**. Corresponding angles are always **equal**.









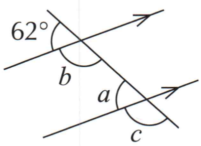
Allied angles form a **C-** or **U-shape**. They always **add up to 180°**.

$a + b = 180°$

Tip: Always state which rules you're using when solving geometry problems — e.g. say "because these are allied angles" or "as the angles all lie on a straight line". Make sure you use the proper terms — don't describe them as "angles in a Z-shape".

Example 2

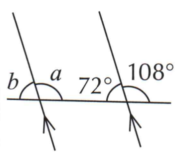
Find the values of *a*, *b* and *c* shown in the diagram below.

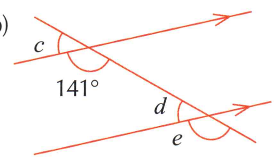


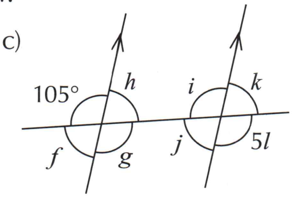
1. a and 62° are corresponding angles, so they are equal. **$a = 62°$**
2. b and the angle marked 62° lie on a straight line, so they add up to 180°. (Or you could say a and b are allied angles, so they add up to 180°.) $62° + b = 180°$, $b = 180° - 62° =$ **118°**
3. c and b are corresponding angles, so they're equal. $c = b =$ **118°**

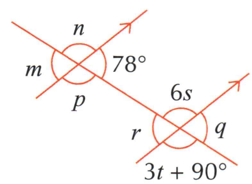
Exercise 2

Q1 Find the value of each letter in the diagrams below.

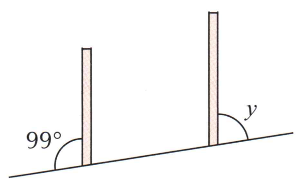
a)


b)


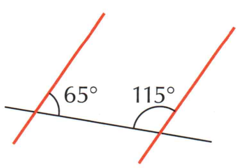
c)


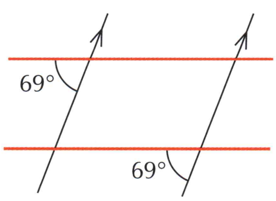
d)


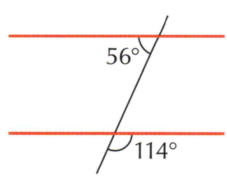
Q2 Two wooden posts stand vertically on sloped ground. The first post makes an angle of 99° with the downward slope, as shown. Find the angle that the second post makes with the upward slope, labelled y on the diagram.



Q3 Decide whether the pairs of orange lines below are parallel. Give reasons for your answers.

a)


b)


c)


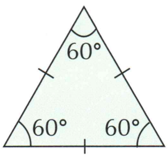
20.3 Triangles

Triangles are perhaps the simplest of the 2D shapes because they have so few sides — three. But they still come in plenty of different forms depending on the lengths of their sides and the sizes of their angles.

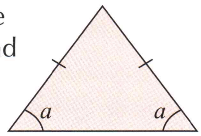
Learning Objectives — Spec Ref G3/G4:
- Know the properties of different types of triangles.
- Know and be able to prove that the angles in a triangle sum to 180°.
- Be able to find missing angles in triangles.

There are **different** types of triangles that you need to be familiar with. Make sure you know the defining features of each type.

An **equilateral** triangle has 3 equal sides and 3 equal angles (each of 60°).

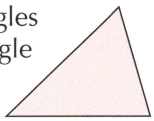


An **isosceles** triangle has 2 equal sides and 2 equal angles.

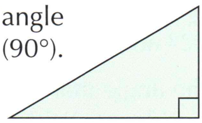


Tip: The little dashes on the sides of a shape mean those sides are of equal length.

The sides and angles of a **scalene** triangle are all different.



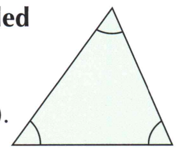
A **right-angled** triangle has 1 right angle (90°).



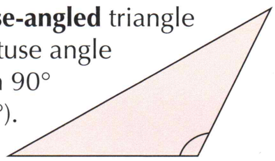
Tip: An isosceles right-angled triangle has one 90° angle and two 45° angles.

You might occasionally see triangles described by the size of their angles:

An **acute-angled** triangle has 3 acute angles (less than 90°).



An **obtuse-angled** triangle has 1 obtuse angle (between 90° and 180°).

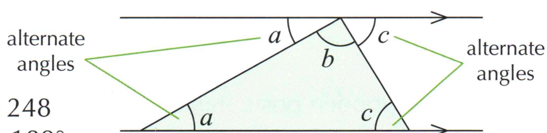


For any triangle, the angles inside (a, b and c) **add up to 180°**. You can use this to set up an **equation** that you can **solve** to find missing angles.

$$\boldsymbol{a + b + c = 180°}$$

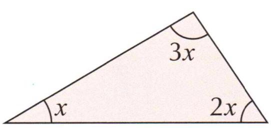
To prove this rule, draw parallel lines at the top and base of the triangle, as shown on the right. Then use the fact that **alternate** angles are equal from page 250.

Now, a, b and c lie on a straight line and you saw on page 248 that this means they must add up to 180° — so $a + b + c = 180°$.



Example 1

a) Find the value of x in the triangle below.



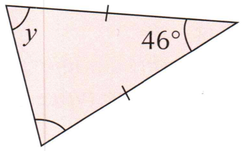
1. The angles in the triangle must add up to 180°, so set up an equation in x.
2. Solve your equation to find x.

$x + 2x + 3x = 180°$

$6x = 180°$

$x = 180° \div 6 = \mathbf{30°}$

b) Find the value of *y* in the isosceles triangle below.

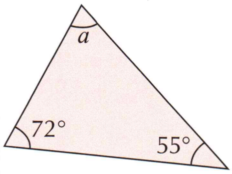


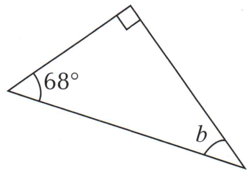
1. The triangle is isosceles so the unmarked angle must be equal to y.
2. All three angles must sum to 180°, so form an equation in y. $y + y + 46° = 180°$, $2y = 134°$
3. Solve your equation to find y. $y = 134° \div 2 = \mathbf{67°}$

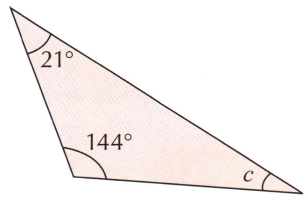
Exercise 1

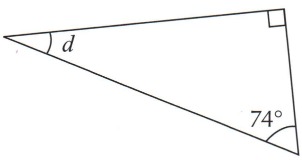
In Questions 1-4, the angles aren't drawn accurately, so don't try to measure them.

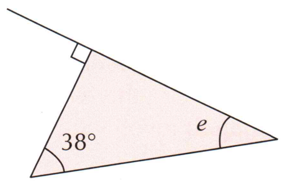
Q1 Find the missing angles marked with letters.

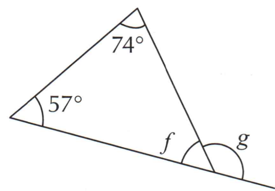
a)


b)


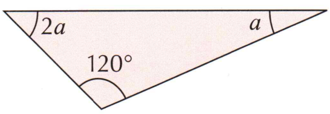
c)


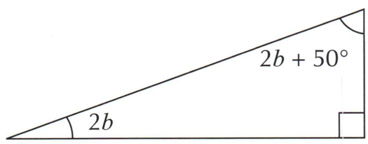
d)


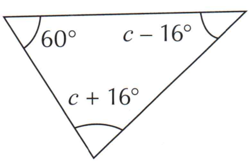
e)


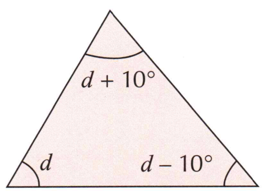
f)


Q2 Find the values of the letters shown in the following diagrams.

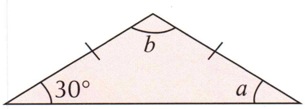
a)


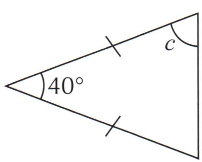
b)


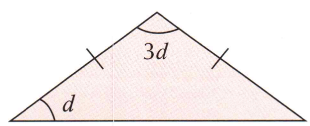
c)


d)


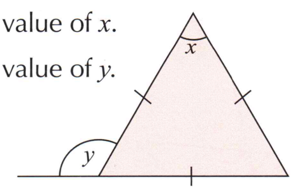
Q3 Find the values of the letters shown in these isosceles triangles.

a)


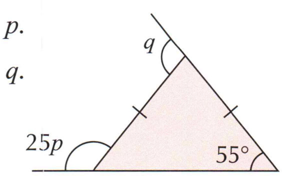
b)


c)


Q4 a) (i) Find the value of x.
(ii) Find the value of y.



b) (i) Find the value of p.
(ii) Find the value of q.



20.4 Quadrilaterals

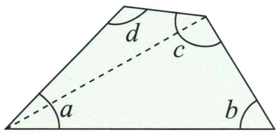
Quadrilaterals are shapes that have four sides. You're probably familiar with squares and rectangles, two of the simplest quadrilaterals, but the next few pages will introduce you to a whole host of other types.

Quadrilaterals

Learning Objectives — Spec Ref G3:
- Know that the angles in a quadrilateral sum to 360°.
- Be able to find missing angles in quadrilaterals.

The angles in a quadrilateral always **add up to 360°**:

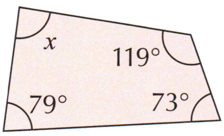
$$a + b + c + d = 360°$$



To prove this, **split** the quadrilateral into **two triangles** as shown by the dashed line on the left. Each triangle has angles that add up to **180°** and so the angles in **both triangles**, which is the same as the angles in the quadrilateral, add up to 180° + 180° = **360°**.

Example 1

Find the missing angle x in the quadrilateral below.



1. The angles in a quadrilateral add up to 360°. Use this to write an equation involving x.

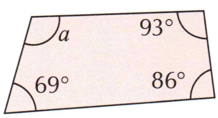
$$79° + 73° + 119° + x = 360°$$
$$271° + x = 360°$$

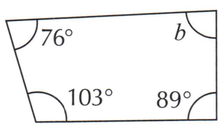
2. Then solve your equation to find the value of x.

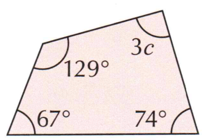
$$x = 360° - 271° = \mathbf{89°}$$

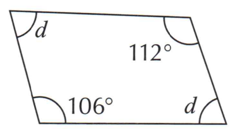
Exercise 1

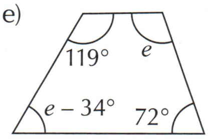
Q1 Find the values of the letters in the following quadrilaterals. They're not drawn accurately, so don't try to measure them.

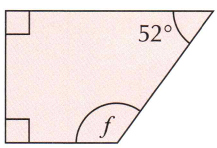
a) 

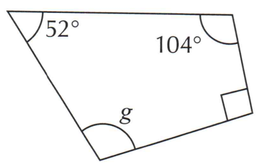
b) 

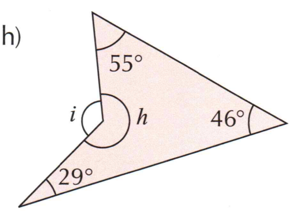
c) 

d) 

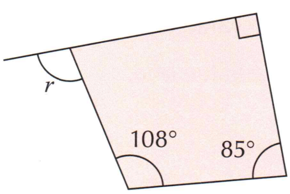
e) 

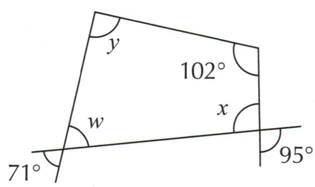
f) 

g) 

h) 

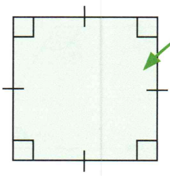
Q2 Find the values of the letters in the quadrilaterals below.

a) 

b) 

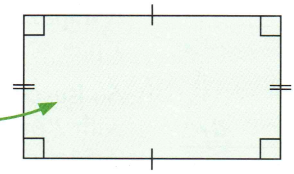
Squares, Rectangles, Parallelograms and Rhombuses

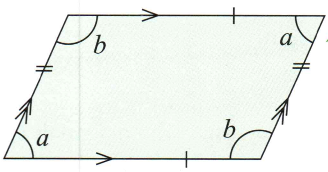
Learning Objective — Spec Ref G4:
Know the properties of squares, rectangles, parallelograms and rhombuses.



A **square** is a quadrilateral with 4 equal sides and 4 angles of 90°.

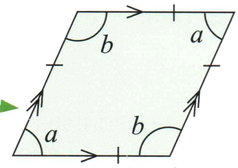
A **rectangle** is a quadrilateral with 4 angles of 90° and opposite sides of the same length.

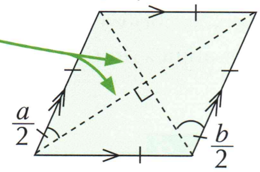




A **parallelogram** is a quadrilateral with 2 pairs of equal, parallel sides.

A **rhombus** is a parallelogram where all the sides are the same length. The **diagonals** of a rhombus **bisect** the angles (i.e. cut them in half) and cross at a **right angle**.



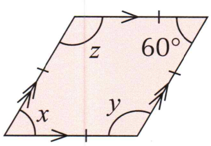


Opposite angles in parallelograms and rhombuses are **equal** and **neighbouring angles** always add up to **180°**: $a + b = 180°$

This is because the **parallel lines** that make up the sides of these shapes mean a and b are **allied angles** (page 251).

Example 2

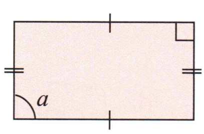
Find the size of the angles marked with letters in the rhombus below.

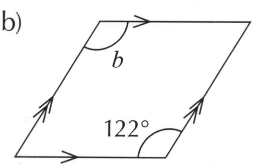


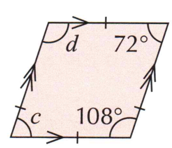
1. Opposite angles in a rhombus are equal. $\mathbf{x = 60°}$
2. Neighbouring angles in a rhombus add up to 180°. Use this fact to find angle y. $60° + y = 180°$
   $y = 180° - 60° = \mathbf{120°}$
3. Opposite angles in a rhombus are equal, so z is the same size as y. $\mathbf{z = 120°}$

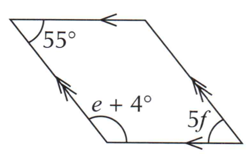
Exercise 2

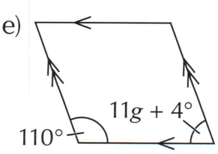
Q1 Calculate the values of the letters in these quadrilaterals.

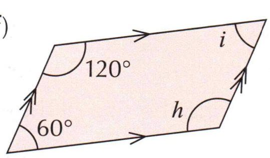
a) 

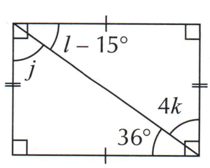
b) 

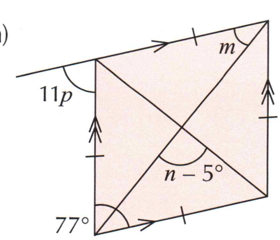
c) 

d) 

e) 

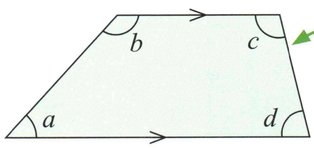
f) 

g) 

h) 

Trapeziums and Kites

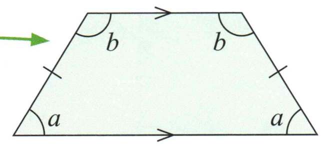
Learning Objective — Spec Ref G4:
Know the properties of trapeziums and kites.



$a + b = 180°$　　**$c + d = 180°$**

A **trapezium** is a quadrilateral with 1 pair of parallel sides.

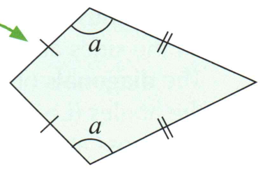
An **isosceles trapezium** is a trapezium with 2 pairs of equal angles and 2 sides of the same length.



$a + b = 180°$

Because of the **allied angles** created by the parallel sides, pairs of angles **add up to 180°** as shown.

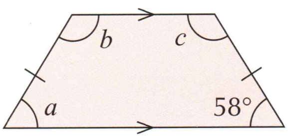
A **kite** is a quadrilateral with 2 pairs of equal sides and 1 pair of equal angles in opposite corners as shown on the diagram (the other pair of angles aren't generally equal).



Tip: The diagonals of a kite always cross at a right angle.

Example 3

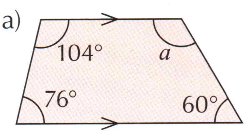
Find the size of the angles marked with letters in the isosceles trapezium below.

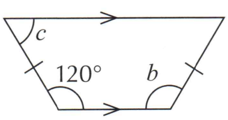


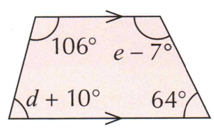
1. This is an isosceles trapezium, so a is the same size as the 58° angle. — $\boldsymbol{a = 58°}$
2. Angle c and the angle of 58° must add up to 180°. — $c + 58° = 180°$; $c = 180° - 58° = \mathbf{122°}$
3. It's an isosceles trapezium, so b is the same size as c. — $\boldsymbol{b = 122°}$

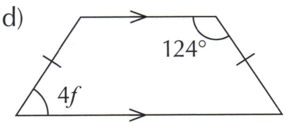
Exercise 3

Q1 Find the value of each letter in the trapeziums below.

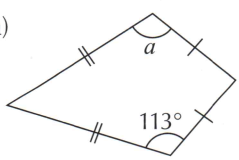
a)


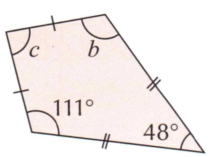
b)


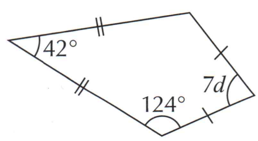
c)


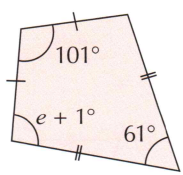
d)


Q2 Find the value of each letter in the kites below.

a)


b)


c)


d)


Q3 An isosceles trapezium has two angles of 53°. Find the size of the other two angles.

Q4 A kite has exactly one angle of 50° and exactly one angle of 90°. Find the size of the other two angles.

20.5 Polygons

You've met lots of polygons before — they're just 2D shapes with straight sides. Use the formulas on these pages to work out the number of sides and the size of the angles in polygon problems.

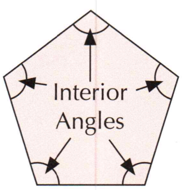
Interior Angles

Learning Objectives — Spec Ref G1/G3:
- Know the names of different types of polygons.
- Find the sum of interior angles in polygons.

A **polygon** is a 2D shape whose sides are all **straight** — the triangles and quadrilaterals on the previous pages were three- and four-sided polygons. The box on the right shows the names of some other polygons — their names depend on the **number of sides** they have.

pentagon = **5** sides	**oct**agon = **8** sides
hexagon = **6** sides	**non**agon = **9** sides
heptagon = **7** sides	**dec**agon = **10** sides

A **regular polygon** is one where the **sides** are all the **same length** and the **angles** are all **equal**. An **equilateral triangle** is a regular triangle and a **square** is a regular quadrilateral.



The **interior angles** of a polygon are the angles inside each vertex (corner). The interior angles of a regular pentagon are shown on the left.

The **sum** of a polygon's interior angles (S) and its **number of sides** (n) are related by this formula: $S = (n - 2) \times 180°$

By **rearranging** this formula, you can use it to find the number of sides (n) of a given polygon or the size of missing angles.

Tip: You can prove the formula by splitting the polygon into $n - 2$ triangles (see example below). The case $n = 4$ was done on p.254.

Example 1

a) A pentagon has four interior angles of 100°. Find the size of the fifth angle.

1. A pentagon has 5 sides, so substitute $n = 5$ into the formula for the sum of interior angles.

$$S = (n - 2) \times 180°$$
$$= (5 - 2) \times 180° = 540°$$

2. Write an equation for the size of the missing angle, x, and solve it to find x.

$$100° + 100° + 100° + 100° + x = 540°$$
$$400° + x = 540°$$
$$x = 540° - 400° = \mathbf{140°}$$

b) Calculate the size of an interior angle of a regular pentagon.

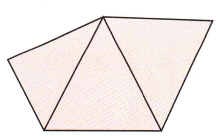
All the angles will be the same size if the pentagon is regular. So divide the sum of the angles by the number of angles.

The sum of the angles is 540° (from part a)).

So one angle will be 540° ÷ 5 = **108°**.

c) By splitting a pentagon into triangles, prove the formula for the sum of its interior angles.

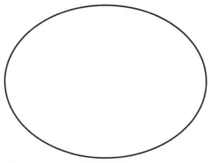
1. Split the pentagon into $5 - 2 = 3$ triangles.
2. Use the fact that the sum of the angles in a triangle is 180°.

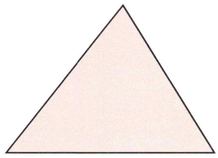


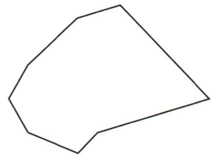
The angles in each triangle add up to 180°. There are three triangles so the angles in the whole shape add up to 180° + 180° + 180° = **540°**.

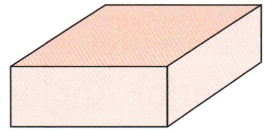
Exercise 1

Q1 For each of the shapes below, determine whether or not it is a polygon and, if so, state its name.

a) 

b) 

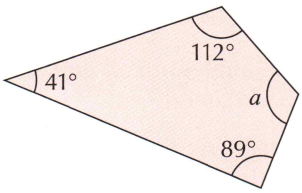
c) 

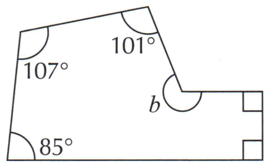
d) 

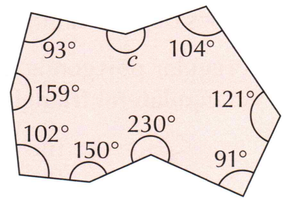
Q2 Find the sum of the interior angles of a polygon with:

a) 6 sides b) 10 sides c) 12 sides d) 20 sides

Q3 For each of the following shapes: (i) Find the sum of the interior angles.
(ii) Find the size of the missing angle.

a) 

b) 

c) 

Q4 Find the size of each of the interior angles in the following shapes.

a) Regular octagon b) Regular nonagon c) Regular decagon

Q5 a) Seven angles of an octagon are 130°. Find the size of the eighth angle.

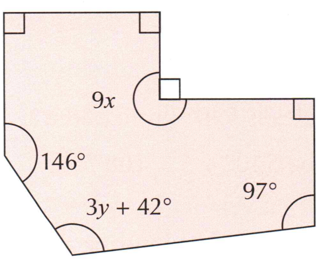
b) Is this a regular octagon? Give a reason for your answer.

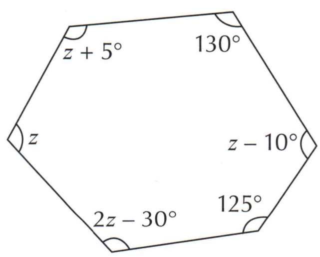
Q6 a) The interior angles of a shape add up to 2520°. Find the number of sides of this shape. PROBLEM SOLVING

b) Half of the angles in this shape are of size 95° and the other half are of size x. Find the value of x.

Q7 Find the number of sides of a regular polygon with interior angles of: a) 60° b) 150°

Q8 Find the values of x, y and z in the polygons below.

a) 

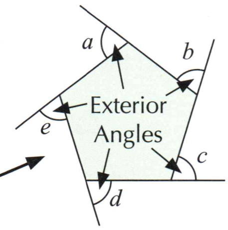
b) 

Q9 Draw a decagon. By dividing it into triangles, show that the sum of the interior angles of the decagon is 1440°.

Exterior Angles

Learning Objective — Spec Ref G3:
Find interior and exterior angles of regular and irregular polygons.

An **exterior angle** of a polygon is an angle between a **side** and a **line** that extends out from one of the **neighbouring sides**. For example, the exterior angles of a regular pentagon are marked on the right.



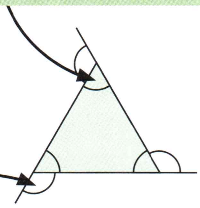
For any polygon, the exterior angles always **add up to 360°**. In the case of the pentagon, this is: $\boldsymbol{a + b + c + d + e = 360°}$

Since the exterior angle and the neighbouring interior angle lie on a straight line, they must **add up to 180°** (see page 248). So you can use this formula to find an interior angle given the exterior angle:

Interior angle = 180° − Exterior angle

For a **regular** polygon, the interior angles are all equal, and this means the **exterior angles are all equal** too. Then the formula for the size of an exterior angle for a regular n-sided polygon is:

$$\textbf{Exterior angle} = \frac{360°}{n}$$



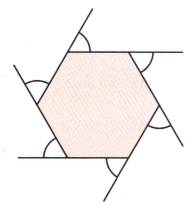
Example 2

Find the size of each of the exterior angles of a regular hexagon.

A hexagon has 6 sides.
The hexagon is regular so put $n = 6$ into the exterior angle formula.

$360° \div 6 = 60°$

So each exterior angle is **60°**.



Example 3

A regular polygon has exterior angles of 30°.
How many sides does the polygon have?

1. It's a regular polygon so put 30° into the exterior angle formula.

$30° = \frac{360°}{n}$

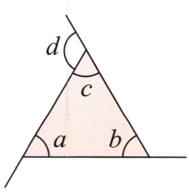
2. Solve the equation for n.

$n = 360° \div 30° = 12$
So the regular polygon has **12 sides**.

Tip: A 12-sided polygon is called a dodecagon.

Example 4

Prove that the exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.



1. You need to show that $a + b = d$.
2. The exterior angle and interior angle add up to 180°.

$d + c = 180° \Rightarrow c = 180° - d$

3. The angles in the triangle also add up to 180°.

$a + b + c = 180°$

$a + b + (180° - d) = 180°$

4. Rearrange the equation to get the result.

$\boldsymbol{a + b = d}$

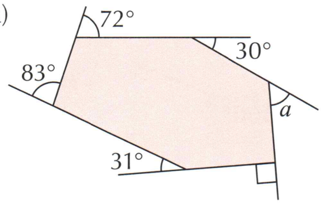
Exercise 2

Q1 Find the size of each of the exterior angles of the following polygons.

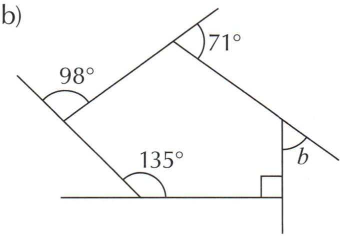
a) regular octagon b) regular nonagon c) regular heptagon

Q2 Find the size of the angles marked by letters in these diagrams.

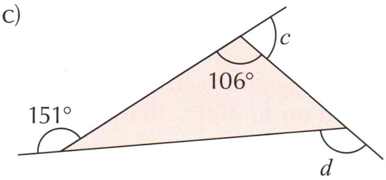
a)



b)



c)



Q3 Find the size of the unknown exterior angle in a shape whose other exterior angles are:

a) 100°, 68°, 84° and 55° b) 30°, 68°, 45°, 52°, 75° and 50°

c) 42°, 51°, 60°, 49°, 88° and 35° d) 19°, 36°, 28°, 57°, 101°, 57° and 22°

Q4 A regular polygon has exterior angles of 45°.

a) How many sides does the polygon have? What is the name of this polygon?

b) Sketch the polygon.

c) What is the size of each of the polygon's interior angles?

d) What is the sum of the polygon's interior angles?

Q5 The exterior angles of some regular polygons are given below. For each exterior angle, find:

(i) the number of sides the polygon has,

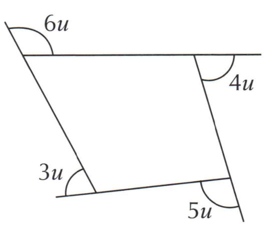
(ii) the size of each of the polygon's interior angles,

(iii) the sum of the polygon's interior angles.

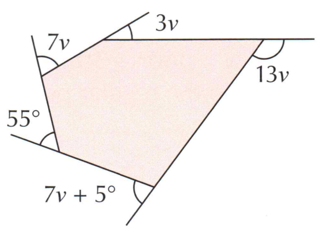
a) 40° b) 120° c) 3° d) 4.8°

Q6 Find the values of the letters in the following diagrams.

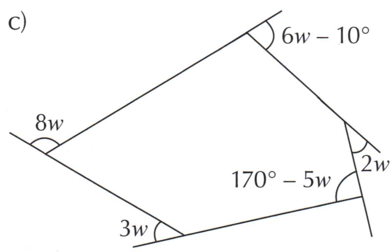
a)



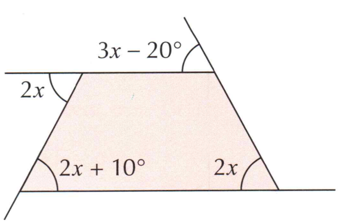
b)



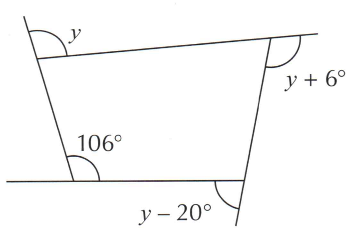
c)



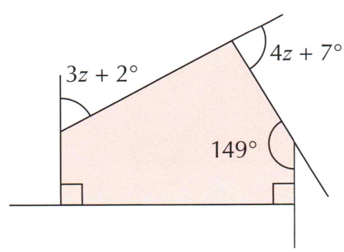
d)



e)



f)

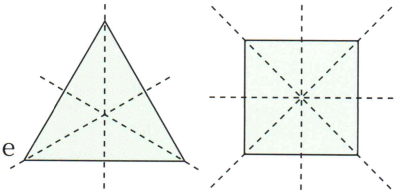


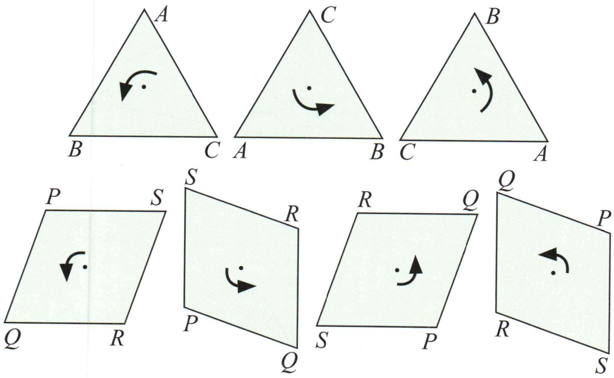
20.6 Symmetry

Symmetry is when a shape can be reflected or rotated and still look exactly the same afterwards. For instance, if you rotate a square by 90° then each of the vertices is in a different place but the square looks unchanged.

Learning Objective — Spec Ref G1:
Recognise lines of symmetry and rotational symmetry in 2D shapes.

A **line of symmetry** on a shape is a mirror line. Each side of the line of symmetry is a **reflection** of the other. For example, an equilateral triangle has three lines of symmetry and a square has four (shown on the right).





The **order of rotational symmetry** of a shape is the number of positions you can **rotate** (turn) the shape into so that it looks **exactly the same**. For example, an equilateral triangle has rotational symmetry of order 3 and a rhombus has order 2 (shown left). If a shape has **no rotational symmetry** then the order of rotational symmetry is **1**.

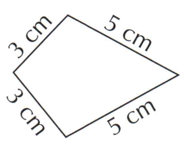
A **regular *n*-sided polygon** has ***n* lines of symmetry** and **rotational symmetry of order *n***.

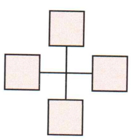
Different **quadrilaterals** have different symmetries. A **square** is regular so it has **4** lines of symmetry and rotational symmetry of order **4**. **Rectangles** and **rhombuses** have **2** lines of symmetry and rotational symmetry of order **2**, **kites** and **isosceles trapeziums** have **1** line of symmetry and rotational symmetry of order **1**, and **parallelograms** have **0** lines of symmetry and rotational symmetry of order **2**.

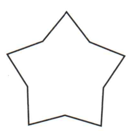
See pages 377 and 380 for how to perform reflections and rotations on a coordinate grid.

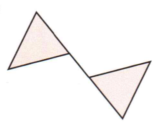
Exercise 1

Q1 For each of the shapes below, state: (i) the number of lines of symmetry,
(ii) the order of rotational symmetry.

a) 

b) 

c) 

d) 

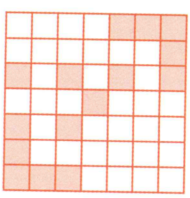
e) a rhombus

f) an isosceles trapezium

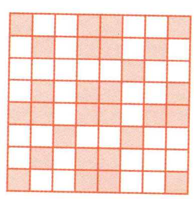
g) a regular pentagon

h) a regular decagon

Q2 a) Copy the diagram below, then shade two more squares to make a pattern with no lines of symmetry and rotational symmetry of order 2.

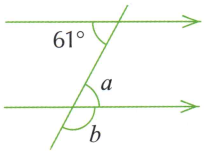


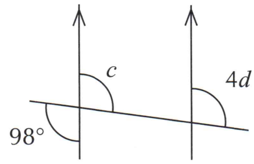
b) Copy the diagram below, then shade four more squares to make a pattern with 4 lines of symmetry and rotational symmetry of order 4.

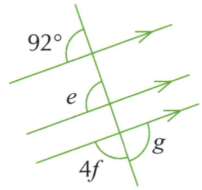


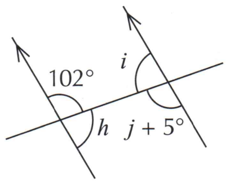
Review Exercise

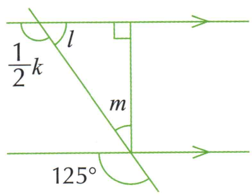
Q1 Find the value of each letter. The diagrams aren't drawn accurately.

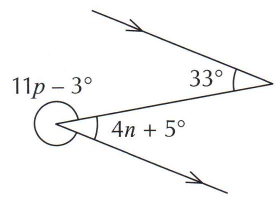
a)


b)


c)


d)


e)


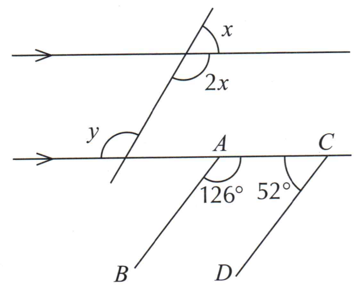
f)


Q2 Look at the system of lines on the right.

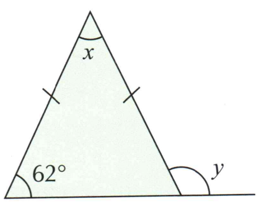
a) (i) Find the value of x.

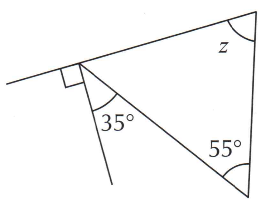
(ii) Hence find the value y.

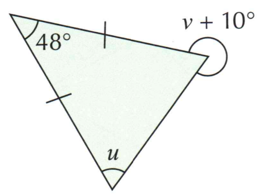
b) Are the lines AB and CD parallel to each other? Explain your answer.



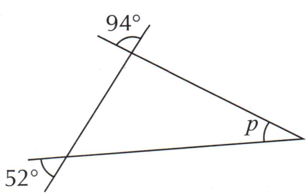
Q3 Find the value of each letter. The diagrams aren't drawn accurately.

a)


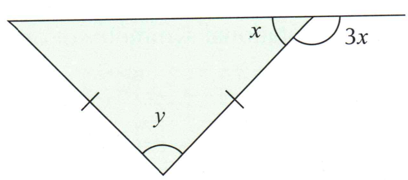
b)


c)


Q4 a) Find angle p in the diagram below.

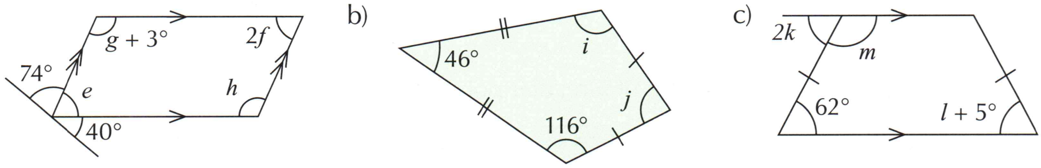


b) Find x and y in the diagram below.



Q5 Find the value of each letter. The diagrams aren't drawn accurately.

a) b) c)



Q6 Write down all the different types of quadrilaterals which satisfy each of the following properties.

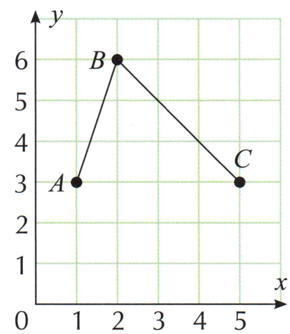
a) 4 equal sides
b) 4 angles of 90°
c) exactly 1 pair of equal angles
d) 2 pairs of parallel sides
e) at least 1 pair of parallel sides
f) exactly 1 pair of parallel sides

Q7 Three vertices of a quadrilateral, A, B and C, are plotted on the right. Give the coordinates of the fourth vertex, D, that would make the quadrilateral:

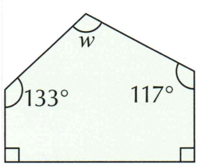
a) a parallelogram

b) a kite

c) an isosceles trapezium



Q8



a) (i) What is the name of the polygon on the left.

(ii) Is it regular? Explain your answer.

b) Calculate the sum of the interior angles of the polygon.

c) Use your answer to find the size of angle w.

Q9 Find the size of the exterior angles of a regular polygon with:

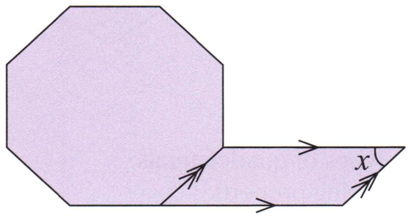
a) 10 sides
b) 12 sides
c) 15 sides
d) 25 sides

Q10 Copy and complete the table.

	equilateral triangle	parallelogram	isosceles trapezium	regular nonagon
No. of sides				
Lines of symmetry				
Order of rotational symmetry				
Sum of interior angles				

Exam-Style Questions

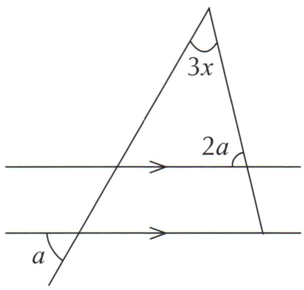
Q1 The shape below is made up of a regular polygon and a parallelogram.



Calculate the size of the angle x.

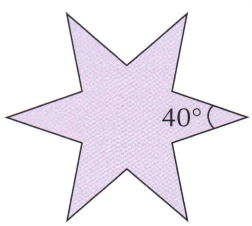
[2 marks]

Q2 Look at the diagram below. Show that $x = 60° - a$.



[3 marks]

Q3 Kendra wants to draw a star with 12 equal length sides.
She calculates that each of the acute interior angles needs to be 40°.



Calculate the size of each reflex interior angle.

[3 marks]

Q4 a) Using an appropriate calculation, find the sum of the interior angles of a heptagon.

[2 marks]

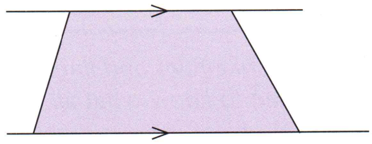
b) Prove that the sum of the interior angles of a triangle is 180°.

[3 marks]

c) Hence prove that your answer to part a) is correct.

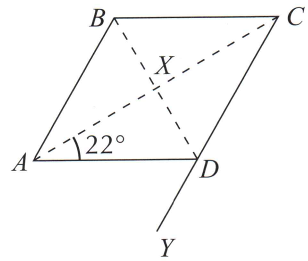
[2 marks]

Q5 Use the angle properties of parallel lines to prove that the interior angles of any trapezium sum to 360°. Use the diagram below to help you.



[2 marks]

Q6 A rhombus *ABCD* has diagonals which meet at *X*, as shown. The size of angle *DAX* is 22°.



Write down the size of:

a) angle *AXD*

[1 mark]

b) angle *ADX*

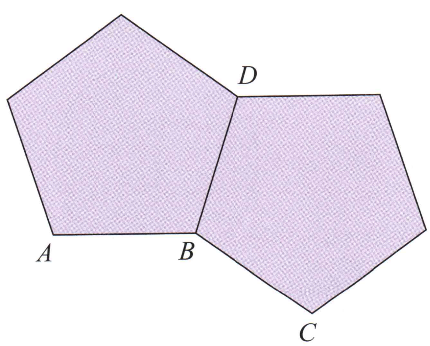
[1 mark]

Side *CD* is extended to a point *Y*.

c) Work out the size of angle *ADY*, giving reasons with your working.

[3 marks]

Q7 Two regular polygons have a common side *BD*, as shown in the diagram. A regular polygon can be made which has sides *AB* and *BC*. Work out the number of sides of this polygon. PROBLEM SOLVING



[4 marks]