2.1 Rounding

Numbers can be approximated (or rounded) to the nearest whole number, 10, 100 etc. or to a given number of decimal places or significant figures. These approximations can be used to find an estimate of a tricky calculation.

Rounding

Learning Objectives — Spec Ref N15:
- Round numbers to a specified degree of accuracy.
- Round numbers to a given number of decimal places or significant figures.

Numbers can be **rounded** to make them easier to work with. This can be very handy when solving equations that don't have nice neat answers. For example, a number like 5468.9 could be rounded to:

- the nearest **whole number** (= 5469)
- the nearest **ten** (= 5470)
- the nearest **hundred** (= 5500)
- the nearest **thousand** (= 5000)

Tip: Remember — a digit of 5 or more rounds up and a digit of 4 or less rounds down.

Be careful — if you round a number **too early** it can change the answer you get at the end. For example, if I had **£1.46** and rounded it to **the nearest 10p** I'd get **£1.50**. If I then rounded this to **the nearest pound** I'd get **£2**. But if I had rounded the original £1.46 to the nearest pound I'd get **£1**.

Exercise 1

Q1 Round the following to: (i) the nearest whole number (ii) the nearest ten (iii) the nearest hundred (iv) the nearest thousand

a) 672.48 b) 2536.13 c) 8499.3 d) 3822.8

Q2 At its closest, Jupiter is about 390 682 810 miles from Earth. Write this distance to the nearest million miles.

Decimal Places

You can also round to different numbers of **decimal places** (d.p.). The method is as follows:

1. **Identify** the position of the '**last digit**' that you want to keep. E.g. when rounding to 2 d.p., it's the digit in the hundredths place.
2. Look at the next digit to the **right** — called **the decider**.
3. If the **decider** is **5 or more**, then **round up** the last digit. If the **decider** is **4 or less**, then **leave** the last digit as it is.
4. There must be **no more digits** after the last digit (not even zeros).

Tip: When you're rounding up from 9 (to 10), replace the 9 with a 0 and **carry 1 to the left**. E.g. 4.98 rounded to 1 decimal place would be 5.0.

Example 1

Round the number 8.9471 to: a) 1 d.p. b) 2 d.p. c) 3 d.p.

a) The last digit is 9 and the decider is 4, so round down. **8.9**

b) The decider is 7, so round up. **8.95**

c) The decider is 1, so round down. **8.947**

Exercise 2

Q1 Round the following numbers to: (i) 1 d.p. (ii) 2 d.p. (iii) 3 d.p.

a) 2.6893 b) 0.3249 c) 5.6023 d) 0.0525

e) 6.2571 f) 0.35273 g) 0.07953 h) 0.96734

Q2 The mass of a field vole is 0.0384 kilograms. Round this mass to two decimal places.

Significant Figures

You can also round a value to a number of **significant figures** (s.f.). The method for significant figures is **identical** to the one for decimal places — except it can be a little harder to locate the **last digit**. There are a few key things to note:

- The **first significant figure** is the first digit that **isn't 0**, e.g. 2 is the first significant figure in 0.023.
- All the digits that follow are also **significant figures**, **regardless** of whether or not they're **0s**.
- Once you've **identified** your significant figures, **round** using the method on the previous page.
- After rounding the last digit, fill in all the places **up to the decimal point** with 0s. (You may need to **add a 0** after the decimal point to make up the correct number of significant figures — e.g. 32.0.)

Tip: If the last significant figure is **after** the decimal point, you don't add any extra zeros at the end.

Example 2

Round to three significant figures: a) 32 568 b) 0.00097151

a) 1. The first significant figure is 3. The decider is 6, so round up. 32 568

2. As the significant figures are before the decimal point, fill the remaining places with zeros. **32 600**

b) 1. The first significant figure is the first digit that isn't zero — this is 9. The decider is 5, so round up. 0.000 971 51

2. As the significant figures are after the decimal point, no more zeros are needed. **0.000972**

Exercise 3

Q1 Round the following numbers to: (i) 1 s.f. (ii) 2 s.f. (iii) 3 s.f.

a) 46.874 b) 5067 c) 35 722 d) 925 478

e) 0.08599 f) 0.10653 g) 0.00041769 h) 34.726

Q2 The speed of sound is 1236 km/h. Round this speed to two significant figures.

Estimates

Learning Objectives — Spec Ref N14/N6:
- Find approximate answers using estimates.
- Estimate the value of square roots.

Prior Knowledge Check: Be familiar with square numbers and their roots. See p.91.

Using rounded numbers in a **calculation** gives an **estimate** of the actual answer — by **simplifying** in this way you get an **approximate value**. You can use this method to **check** calculations — i.e. to see if your actual answer looks 'about right'. The symbol used in estimating is '≈' and means 'is approximately equal to'.

You can usually figure out if your answer will be an **over-estimate** or an **under-estimate**.

- **Addition** or **multiplication** — if both numbers are **rounded up** you'll get an **over-estimate** and if both numbers are **rounded down** you'll get an **under-estimate**.
- **Subtraction** or **division** — you'll get an **over-estimate** if you rounded the **1st number up** and the **2nd number down**, and an **under-estimate** if you rounded the **1st number down** and the **2nd number up**.

Example 3

Estimate the value of $\frac{9.7 \times 326}{1.823 \times 5.325}$ by rounding each number to one significant figure.

1. Round each number to 1 s.f. $\qquad \frac{9.7 \times 326}{1.823 \times 5.325} \approx \frac{10 \times 300}{2 \times 5}$
2. Work out the calculation in stages. $\qquad = \frac{3000}{10} = \mathbf{300}$

Tip: The actual value is 325.748..., so this is a good approximation.

To estimate a **square root**, find the two square numbers it lies between and decide which it's closer to.

Example 4

Estimate the value of $\sqrt{61}$ to 1 d.p.

1. Find the closest square number on either side of the number in the question (61).
   61 lies between 49 (= 7^2) and 64 (= 8^2).
2. Decide which is closer to 61, then make a sensible estimate of the digit after the decimal point.
   61 is much closer to 64, so a sensible estimate is $\sqrt{61} \approx \mathbf{7.8}$.

Exercise 4

Q1 Estimate each of the following by rounding each number to one significant figure.

a) 102.2 × 4.2 b) 288.7 × 7.8 c) 306.9 ÷ 6.4 d) 3.9 ÷ 5.1
e) 494.27 ÷ 5.05 f) 205.52 ÷ 8.44 g) 142.75 × 9.56 h) 8.31 ÷ 1.86

Q2 The following questions have three possible answers. Use estimation to decide which answer is correct.

a) 101 × 52 = (5252, 4606, 6304) b) 588 ÷ 12.4 = (75.78, 47.42, 69.86)
c) 0.79 × 1594.3 = (1259.50, 864.80, 679.57) d) 0.94 ÷ 3.68 = (0.124, 0.901, 0.255)

Q3 Estimate each of the following by rounding each number to one significant figure.

a) $\frac{9.9 \times 285}{18.7 \times 3.2}$ b) $\frac{174.3 \times 3.45}{162.8 \times 10.63}$ c) $\frac{432.4 \times 2.75}{233.39 \times 0.81}$ d) $\frac{176.65 \div 8.84}{564.36 \div 2.78}$

Q4 Estimate the following roots to the nearest whole number.

a) $\sqrt{18}$ b) $\sqrt{67}$ c) $\sqrt{51}$ d) $\sqrt{86}$

Q5 Estimate the following roots to 1 decimal place.

a) $\sqrt{11}$ b) $\sqrt{30}$ c) $\sqrt{20}$ d) $\sqrt{37}$

2.2 Upper and Lower Bounds

Rounding leaves a level of uncertainty between the actual number and rounded number. This is where bounds come in. Upper and lower bounds show the maximum and minimum actual values a rounded number can be.

Upper and Lower Bounds

Learning Objective — Spec Ref N16:
Understand and use upper and lower bounds.

Upper and lower bounds show you where the actual value of a rounded number can lie.

- The **lower bound** is the **smallest actual value** the number can be and still be **rounded up** to the number in question. The actual value is **greater than or equal to** the lower bound.
- The **upper bound** is the **biggest actual value** the number can be and still be **rounded down** to the number in question. The actual value is **strictly less than** the upper bound (if it was exactly equal to the upper bound it would round up to the next unit).

For any given rounding unit, the actual value is anything up to **half a unit bigger or smaller.** For example, if you round a number to 1 d.p., the **rounding unit** is 0.1 so the actual value is anything up to **0.05 either side**. If you round a figure to the nearest whole number, the rounding unit is 1 so the actual value can be 0.5 either side — e.g. a weight of 72 kg to the nearest kilogram has a **lower bound** of 71.5 kg and an **upper bound** of 72.5 kg.

Example 1

A length is given as 12 m when rounded to the nearest metre. State its lower and upper bounds.

1. The rounding unit is metres, so the smallest number that would round to 12 m is half a metre less: 12 – 0.5 = 11.5 m.

   Lower bound = **11.5 m**

2. The biggest number that would round down to 12 is 12.49999999... so, by convention, we say that the upper bound is 12.5 m.

   Upper bound = **12.5 m**

Exercise 1

Q1 The following figures have been rounded to the nearest whole number. State their lower and upper bounds.

a) 10 b) 34 c) 76 d) 102

e) 99 f) 999 g) 249 h) 2500

Q2 Give the lower and upper bounds of the following measurements.

a) 645 kg (measured to the nearest kg) b) 255 litres (measured to the nearest litre)

c) 800 g (measured to the nearest 100 g) d) 155 cm (measured to the nearest cm)

Q3 State the lower and upper bounds of the following prices.

a) £15 (when rounded to the nearest £1) b) £320 (when rounded to the nearest £10)

c) £76.70 (when rounded to the nearest 10p) d) £600 (when rounded to the nearest £50)

Calculating with rounded values will create a **discrepancy** between the **calculated value** and the **actual value**. This gives a **minimum** and **maximum value** of a calculation, found by using the lower and upper bounds.

If A and B are rounded numbers, then:

- **A + B:** Max = upper bound of A + upper bound of B, Min = lower bound of A + lower bound of B
- **A – B:** Max = upper bound of A – lower bound of B, Min = lower bound of A – upper bound of B

Example 2

Find the maximum and minimum perimeter of a rectangular garden measuring 38 m by 20 m to the nearest metre.

1. For the maximum perimeter, find the upper bounds of the garden's sides.
   38 + 0.5 = 38.5 m
   20 + 0.5 = 20.5 m
2. Use these to calculate the maximum possible perimeter.
   (38.5 × 2) + (20.5 × 2) = 77 + 41 = 118 m
   Maximum perimeter = **118 m**
3. For the minimum perimeter, find the lower bounds of the garden's sides.
   38 – 0.5 = 37.5 m
   20 – 0.5 = 19.5 m
4. Use these to calculate the minimum possible perimeter.
   (37.5 × 2) + (19.5 × 2) = 75 + 39 = 114 m
   Minimum perimeter = **114 m**

Exercise 2

Q1 Find the minimum and maximum possible perimeter of the following:

a) A rectangle with sides 5 cm and 6 cm measured to the nearest cm.

b) A square with sides of 4 m measured to the nearest m.

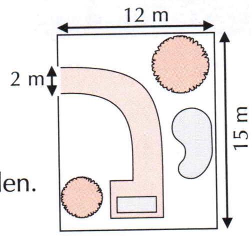
c) An equilateral triangle with side length 7 cm measured to the nearest cm.

d) A regular pentagon with sides 12.5 cm measured to the nearest mm.

Q2 Celia needs to sew a ribbon border onto a rectangular tablecloth. If the tablecloth measures 2.55 m by 3.45 m to the nearest cm, what is the longest length of ribbon that could be needed?

Q3 Last year Jack was 1.3 m tall, measured to the nearest 10 cm. This year he has grown 5 cm, to the nearest cm. Calculate the tallest height and the shortest height he could actually be this year.

Q4 Mr McGregor wants to build a fence around his garden, leaving a 2 m gap for the gate. The garden is a rectangle measuring 12 m by 15 m. All measurements are to the nearest metre.



a) (i) Give the minimum width of the gate.

(ii) Give the maximum length and width of the garden.

(iii) Hence find the maximum amount of fencing needed to fence the side of the garden with the gate on it. PROBLEM SOLVING

(iv) Calculate the maximum total length of fencing needed to fence the garden.

b) By finding the maximum width of the gate and the minimum dimensions of the garden, calculate the minimum total length of fencing needed.

You can also work out the maximum and minimum values for multiplication and division:

- **A × B:** Max = upper bound of A × upper bound of B, Min = lower bound of A × lower bound of B
- **A ÷ B:** Max = upper bound of A ÷ lower bound of B, Min = lower bound of A ÷ upper bound of B

Example 3

Find the maximum and minimum possible area (A) of a rectangular room that measures 3.8 m by 4.6 m to the nearest 10 cm.

1. Find the upper bounds. $3.8 + 0.05 = 3.85$ m
   $4.6 + 0.05 = 4.65$ m
2. Use these to work out the maximum possible area. $A = 3.85 \times 4.65 =$ **17.9025 m²**
3. Find the lower bounds. $3.8 - 0.05 = 3.75$ m
   $4.6 - 0.05 = 4.55$ m
4. Work out the minimum possible area. $A = 3.75 \times 4.55 =$ **17.0625 m²**

Example 4

Find the maximum and minimum possible speeds of an ostrich if it runs 17.2 km in 0.3 hours (both to 1 d.p.).

1. Find the upper and lower bounds of the distance (= d) and time (= t).
   Upper and lower bounds of 17.2 km are 17.25 km and 17.15 km, respectively.
   Upper and lower bounds of 0.3 hours are 0.35 hours and 0.25 hours, respectively.
2. Use the formulas from the previous page.
   $$\text{Maximum speed} = \frac{\text{upper bound of } d}{\text{lower bound of } t} = \frac{17.25}{0.25} = \mathbf{69 \text{ km/h}}$$
   $$\text{Minimum speed} = \frac{\text{lower bound of } d}{\text{upper bound of } t} = \frac{17.15}{0.35} = \mathbf{49 \text{ km/h}}$$

Exercise 3

Q1 Calculate the maximum and minimum possible volumes for a storage unit that measures 4.00 m by 3.00 m by 1.90 m to the nearest cm.

Q2 Michael drove in his car for a measured time of 13 minutes at 34 km/h. If his time was measured to the nearest minute, calculate the maximum possible distance that he could have driven.

Q3 A snail travels a distance of 5 m in 45 minutes. If the distance is measured to the nearest cm and the time to the nearest minute, what is the maximum possible speed of the snail in cm/s to 3 s.f.?

Q4 Max wants to paint a wall that measures 2.1 m by 5.2 m. A tin of paint states that it will cover 3.5 m². If the tin contains exactly 0.5 litres of paint, and all other measurements are correct to 1 d.p., find the maximum volume of paint needed to paint the wall. PROBLEM SOLVING

Representing Bounds Using Intervals

Learning Objectives — Spec Ref N15:

- Be able to truncate numbers.
- Use inequality notation to describe error intervals.

Tip: Computers use truncation when converting data.

You **truncate** a number by **chopping off decimal places**, so if the mass of a cake was 2.468 kg, truncated to 1 d.p. it would be 2.4 kg. When a measurement is truncated to a given unit, the actual measurement can be up to a **whole unit bigger but no smaller**.

Review Exercise

Q1 The length of a snake is 1.245 metres. Round this length to one decimal place.

Q2 The table on the right shows the mass (in kg) of some mammals. Round each mass to two significant figures.

Mammal	Mass (kg)
Common vole	0.0279
Badger	9.1472
Meerkat	0.7751
Red squirrel	0.1998
Shrew	0.00612
Hare	3.6894

Q3 Jade buys four items costing £1.35, £8.52, £14.09 and £17.93 from a shop. Estimate how much she spent by rounding each price to the nearest pound.

Q4 Estimate each of the following by rounding each number to one significant figure.

a) $\frac{64.4 \times 5.6}{17 \times 9.5}$ b) $\frac{310.33 \times 2.68}{316.39 \times 0.82}$ c) $\frac{13.7 \times 5.2}{12.3 \div 3.9}$ d) $\frac{173.64 \times 10.6}{64.44 \div 5.58}$

Q5 Estimate to 1 d.p. the value of:

a) $\sqrt{14}$ b) $\sqrt{77}$ c) $\sqrt{130}$ d) $\sqrt{56}$

Q6 Josie took 24 minutes, to the nearest minute, to walk from the cinema to the museum. State the upper and lower bounds of this time.

Q7 Find the maximum and minimum possible values of the following.

a) The area of a rectangle with sides given as 5 cm and 6 cm, measured to the nearest cm.

b) The volume of a cube with sides given as 6 cm, measured to the nearest cm.

c) The volume of a cuboid with sides given as 3.5 cm, 4.4 cm and 5.6 cm, measured to the nearest mm. Give your answer in cm^3, rounded to 2 d.p.

Q8 Kelly ran a 1500 m race in 260 s. The time was measured to the nearest second and the distance to the nearest 10 m. Calculate the maximum and minimum possible values of her average speed in m/s.

Q9 Truncate the following to 1 d.p.

a) 78.445 b) 32.510 c) 567.862 d) 999.999

Name	Height (m)
Lily	1.40
May	1.43
Isaac	1.60
Max	1.56
Daisy	1.28

Q10 A mum of 5 children has truncated her children's heights in metres to 2 decimal places, and written them down in the table on the left.

a) Write down the intervals within which each child's actual height lies.

b) Write down the minimum and maximum height difference between the tallest and shortest child. PROBLEM SOLVING