Powers

1 Find an approximate value of $111^{\frac{1}{4}} \times 111^{\frac{1}{4}}$ to 3 significant figures.

.....

[Total 2 marks]

2 Find the exact value of $81^{\frac{3}{4}}$.

.....

[Total 2 marks]

3 Simplify fully $a^7 \times (25a^6b^{10}c^5)^{\frac{1}{2}}$.

.....

[Total 2 marks]

4 Find the value of $125^{\frac{1}{3}} \times 3^{-2}$. Give your answer as a recurring decimal.

.....

[Total 3 marks]

5 Evaluate $\left(\frac{64}{27}\right)^{-\frac{1}{3}}$. Give your answer as a fraction in its simplest form.

.....

[Total 2 marks]

6 Given that $\left(\frac{729}{8x}\right)^{\frac{1}{3}} = \frac{9}{4}$, find the value of x.

$x =$ .....

[Total 2 marks]

7 $a = 3b^3 + 2b^6$, where $b = (4c + 3)^{\frac{1}{3}}$

Express a in terms of c, simplifying your answer as much as possible.

.....

[Total 3 marks]

8 Find the values of x and y, given that $\left(\frac{64}{49}\right)^{-\frac{x}{y}} = \frac{343}{512}$.

$x =$ ..... , $y =$ .....

[Total 3 marks]

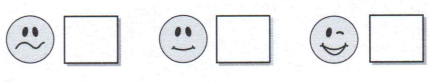
9 Evaluate $\dfrac{\left(2\frac{7}{9}\right)^{-\frac{1}{2}} \times 3\frac{1}{3}}{2^{-2}}$.

Start by turning the mixed numbers into improper fractions.

.....

[Total 5 marks]

Score: ____ / **24**



Surds

1 Simplify the expression $\sqrt{2}+(\sqrt{2})^2+(\sqrt{2})^3+(\sqrt{2})^4+(\sqrt{2})^5$.

.....

[Total 3 marks]

2 Write $\sqrt{343}+\frac{21}{\sqrt{7}}-4\sqrt{252}$ in the form $a\sqrt{b}$, where a and b are integers.

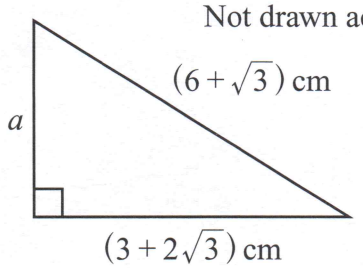
.....

[Total 4 marks]

3 Find the exact length of side a in the triangle on the right.

Give your answer in its simplest form.

Not drawn accurately



a = ... cm

[Total 4 marks]

4 Expand and simplify $(\sqrt{5}-6)^3$. Give your answer in the form $a+b\sqrt{5}$.

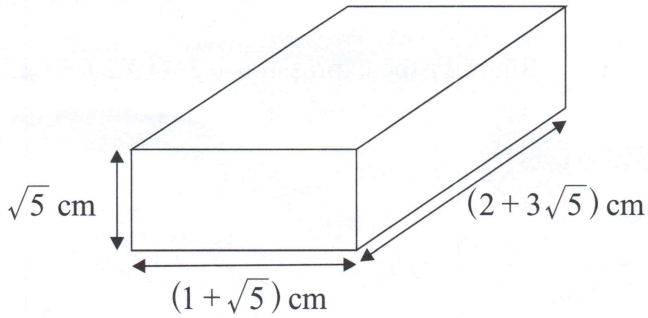
.....

[Total 3 marks]

5 Find the exact volume of the cuboid on the right.

Not drawn accurately

Give your answer in the form $a + b\sqrt{5}$, where a and b are integers.



..... cm^3

[Total 4 marks]

6 Simplify $\dfrac{2\sqrt{3}}{3+\sqrt{3}} + \dfrac{2+\sqrt{3}}{2-\sqrt{3}}$.

You should start by rationalising the denominator of each fraction.

.....

[Total 5 marks]

7 Simplify $\dfrac{(1+2\sqrt{2})^2}{\sqrt{2}-1}$.

.....

[Total 4 marks]

Exam Practice Tip

No doubt about it — surds are tricksy little devils. Always remember to simplify your answers as much as possible — if you have a big number in a surd, see if it'll divide by a square number to simplify it some more. The smaller the surd, the easier it is to deal with — especially if you're trying to combine a few different surds.

Score

27

Quadratic Equations

1 The surface area of a sphere is $36\pi x^2 + 48\pi x + 16\pi$ cm^2, where x is positive. Find the radius of the sphere in terms of x.

The surface area of a sphere is given by $4\pi r^2$.

..... cm

[Total 3 marks]

2 Look at the quadratic equation $2x^2 - 3x - 35 = 0$.

a) Fully factorise the expression $2x^2 - 3x - 35$.

.....

[2]

b) Use your answer to part a) to solve the equation $2(2x - 1)^2 - 3(2x - 1) - 35 = 0$.

$x =$ ..... or $x =$ .....

[3]

[Total 5 marks]

3 Solve the equation $\frac{x}{2x+1} - \frac{x+3}{x-1} = 2$.

$x =$ ..... or $x =$ .....

[Total 4 marks]

4 Look at the quadratic equation $3x^2 - 14x - 24 = 0$.

a) Fully factorise the expression $3x^2 - 14x - 24$.

.....

[2]

b) Use your answer to part a) to solve the equation $3x^2 - 14x - 24 = (3x + 4)^2$.

Remember — you're looking for two solutions here.

$x =$ ..... or $x =$ .....

[4]

[Total 6 marks]

5 The surface area of a cylinder with height 1 m is 31π m^2.
Find r, the exact radius of the cylinder in its simplest form.

The surface area of a cylinder is given by $2\pi rh + 2\pi r^2$.

$r =$ ..... m

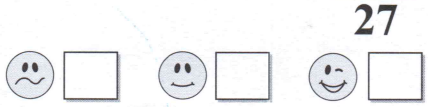
[Total 4 marks]

6 Calculate the positive value of $\frac{1}{1-3x}$ if $\frac{1}{x} + \frac{6}{x+2} = 5$.

$x =$ .....

[Total 5 marks]

Score: ☐/**27**



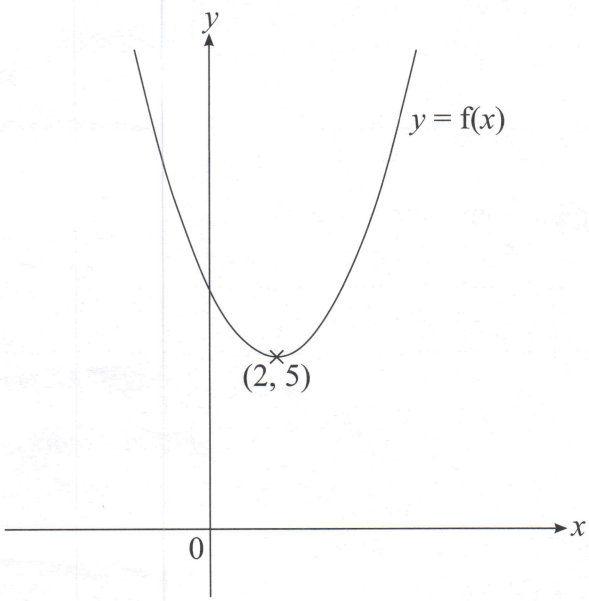
Completing the Square

1 Write the expression $x^2 + 7x + 11$ in the form $(x + a)^2 + b$.

.....

[Total 3 marks]

2 $f(x) = x^2 + px + q$. The graph of $y = f(x)$ has a turning point at (2, 5), as shown below.



a) Find the values of p and q.

p = ... and q = ...

[3]

b) Hence, or otherwise, find the turning point of the graph of $y = f(x - 2) - 3$.

Use the completed square form of f(x).

.....

[2]

c) On the axes above, sketch the graph of $y = f(x - 2) - 3$, labelling the coordinates of the turning point.

[2]

[Total 7 marks]

3 $3x^2 + sx + 29$ can be written in the form $r(x + 4)^2 + t$, where r, s and t are integers.

By finding the values of r, s and t, work out the coordinates of the turning point of the curve $y = 3x^2 + sx + 29$.

.....

[Total 4 marks]

4 A curve has equation $y = x^2 + px + q$. The curve has a minimum point at (4, 7).

Show that the curve passes through the point (11, 56).

[Total 2 marks]

5 A curve has equation $y = 3x - x^2 + 5$.

a) Find the turning point of the curve.

.....

[3]

b) Is this turning point a maximum or minimum? Explain your answer.

.....

.....

[1]

[Total 4 marks]

6 Look at the quadratic equation $5x^2 + 20x + 12 = 0$.

a) Write the expression $5x^2 + 20x + 12$ in the form $u(x + v)^2 + w$.

.....

[4]

b) Hence find the solutions of $5x^2 + 20x + 12 = 0$. Give your answers to 3 significant figures.

$x =$ ..... or $x =$ .....

[2]

[Total 6 marks]

Exam Practice Tip

Completing the square can be really nasty — but remember, you can always check your answer by expanding your completed square form and checking that you end up with the original equation. It's easy to make mistakes when there are a load of awkward fractions flying around, so take your time and don't rush or panic.

Score

26

Algebraic Fractions

1 Simplify fully $\frac{2v^2 - 18}{v^2 + 3v} \times \frac{v^2 - v}{v^2 + 8v - 9}$.

.....

[Total 5 marks]

2 Let $a = 5x^2 - 80y^2$ and $b = 40y - 10x$. Find an expression for $\frac{1}{a} \div \frac{1}{b}$.

Give your answer in its simplest form.

.....

[Total 4 marks]

3 Write $\frac{3}{x} + \frac{2x}{x+4}$ as a single fraction.

.....

[Total 3 marks]

4 Simplify fully $\frac{x+7}{x^2} \times \frac{x^2 + 2x}{x^2 - 49} \times \frac{6x - 42}{3x + 6}$.

.....

[Total 5 marks]

5 Write $\frac{1}{x^2}+\frac{x+3}{x-2}-\frac{4}{x}$ as a single fraction.

.....

[Total 4 marks]

6 Simplify fully $\frac{x^2-5}{2x^2-7x-4}\times\frac{2x+1}{x-\sqrt{5}}$.

Watch out — $x^2 - 5$ is a difference of two squares.

.....

[Total 3 marks]

7 Prove that $\frac{14x-35}{2x^2+x-15}\div\frac{4xy-12y}{2x^2y-18y}\equiv k$, where k is a number to be found.

[Total 6 marks]

Score: ____ / **30**

Sequences

1 The nth term of a sequence is $\left(\frac{1}{2}\right)^n$.

Find the difference between the 5th and 8th terms in the sequence.

.....

[Total 2 marks]

2 Imogen and Justin each think of a sequence. Imogen's sequence is an arithmetic sequence with nth term $93 - 6n$. Justin's sequence is quadratic and starts 3, 9, 21, 39, ...

a) Find an expression for the nth term of Justin's sequence.

.....

[3]

b) Find the only term that is the same in both sequences.

.....

[3]

[Total 6 marks]

3 A quadratic sequence has nth term $3n^2 - 4n + 1$.
The sum of two consecutive terms in the sequence is 581.

What are the two terms?

..... and .....

[Total 5 marks]

4 The first, second and third terms of an arithmetic sequence are $6x + 1$, $8x - 29$ and $5x + 6$, where x is an integer.

Find the 20th term in the sequence.

In an arithmetic sequence, the difference between each consecutive pair of terms is the same.

.....

[Total 6 marks]

5 A sequence has nth term $50 - \frac{1}{2}n^2$. Find the value of the first term in the sequence that is less than 0.

Don't use trial and error here — set up and solve a quadratic inequality.

.....

[Total 3 marks]

6 Kim and Alex each think of a sequence.

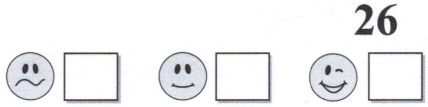
Kim's geometric sequence:	Alex's quadratic sequence: First three terms are 18, 21, 27
nth term $= (\sqrt{3})^n$	

Show that the sum of the 8th term of Kim's sequence and the 6th term of Alex's sequence is a square number.

You don't have to work out the nth term of Alex's sequence as you only need to go as far as the 6th term.

[Total 4 marks]

Score: ☐ / **26**



Quadratic Inequalities

1 Solve the inequality $x^2 + x - 56 < 0$.

.....

[Total 3 marks]

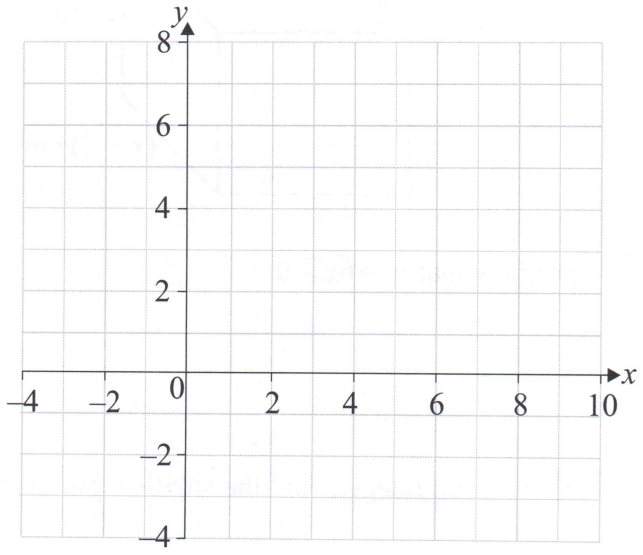
2 Look at the grid on the right.

On the grid, shade the region(s) that satisfies the inequalities below:

$y \leq 3$

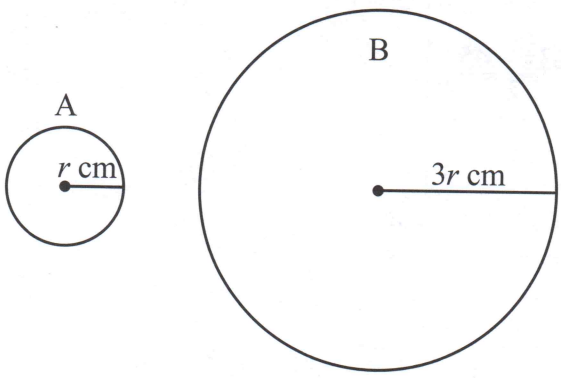
$y + x \leq 5$

$4x - x^2 \leq 0$



[Total 4 marks]

3 The sum of the areas of the circles below is greater than 160π cm^2.



Given that r is an integer, find the smallest possible radius of each circle.

radius of circle A = ... cm, radius of circle B = ... cm

[Total 3 marks]

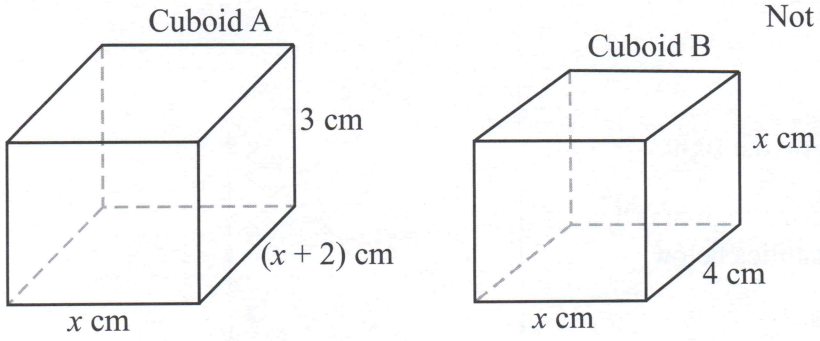
4 Solve the inequality $3x^2 - x - 90 \geq 5x + 15$.

.....

[Total 4 marks]

5 Look at the two cuboids below. The volume of cuboid B is greater than the volume of cuboid A.

Not drawn accurately



a) Show that $x^2 - 6x > 0$.

[3]

b) If x is an integer, find the smallest possible volume of cuboid B.

..... cm^3

[4]

[Total 7 marks]

6 Solve the inequality $x^2 \leq \frac{23x - 45}{2}$.

.....

[Total 4 marks]

Exam Practice Tip

It's always a good idea to very quickly sketch the quadratic graph to work out which bit of it you want (either above or below the x-axis, depending on the inequality sign). This will tell you whether you want a solution within an enclosed region (e.g. $-1 < x < 1$) or a solution in two separate bits (e.g. $x < -1$ and $x > 1$).

Score / **25**

Iterative Methods

1 The graphs of $y = (x - 1)^2(2x + 5)$ and $y = 10x^2 - 8x - 2$ intersect three times.

a) Show that one of the points of intersection occurs at $x = 1$.

[2]

b) Show that, at the points of intersection, $2x^3 - 9x^2 + 7 = 0$.

[3]

c) Hence show that, for one of the points of intersection, the value of x lies between 4 and 5.

[2]

d) By filling in the table below, find an approximation to the solution of $2x^3 - 9x^2 + 7 = 0$ to 1 d.p. You might not need to use all the rows.

x	$2x^3 - 9x^2 + 7$	
4	–9	
5		Positive
4.1	–6.448	
4.2		

$x =$ .....

[4]

[Total 11 marks]

2 The equation $x^3 + 3x^2 - 5 = 0$ has one solution.

a) Show that the solution of $x^3 + 3x^2 - 5 = 0$ lies between 1 and 1.5.

[2]

b) The formula $x_{n+1} = x_n - \dfrac{x_n^3 + 3x_n^2 - 5}{3x_n^2 + 6x_n}$ can be used to solve $x^3 + 3x^2 - 5 = 0$.
Starting with $x_0 = 1$, find the solution to the equation $x^3 + 3x^2 - 5 = 0$ to 5 decimal places.

Put 1 into your calculator and press =.
Then if you enter ANS $- \dfrac{\text{ANS}^3 + 3\text{ANS}^2 - 5}{3\text{ANS}^2 + 6\text{ANS}}$
(this might be slightly different for your calculator) and keep pressing =, you won't have to enter the calculation each time.

$x =$ .....

[3]

[Total 5 marks]

3 The equation $x^3 - 3x^2 - 4x + 10 = 0$ has three solutions.

a) The equation can be rearranged as $x = \pm\sqrt{\dfrac{ax - 10}{x - b}}$.
Show that $a = 4$ and $b = 3$.

[3]

b) The formula $x_{n+1} = \sqrt{\dfrac{4x_n - 10}{x_n - 3}}$ can be used to solve $x^3 - 3x^2 - 4x + 10 = 0$.
Starting with $x_0 = 1.5$, find one solution to the equation $x^3 - 3x^2 - 4x + 10 = 0$ to 3 significant figures.

$x =$ .....

[3]

[Total 6 marks]

Score: ☐ / **22**

Simultaneous Equations

1 Solve the following pair of simultaneous equations.

$x^2 + 4y^2 = 37$
$2x - y = x + 4$

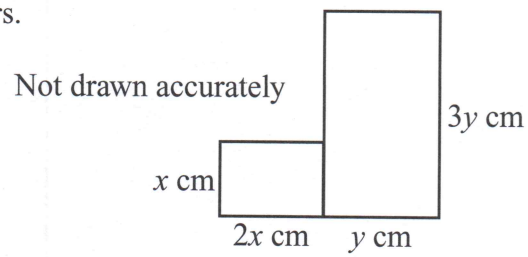
$x =$ ..... , $y =$ .....

and $x =$ ..... , $y =$ .....

[Total 5 marks]

2 The shape below is made up of two rectangles with dimensions as shown. The total area of the shape is 83 cm^2, and the total base length is 9 cm.

Find the values of x and y, given that they are both integers.



$x =$ ..... , $y =$ .....

[Total 5 marks]

3 The line $7x - y = 25$ intersects the circle $x^2 + y^2 = 25$ at two points, A and B. Find the exact length of the line AB. Give your answer in its simplest form.

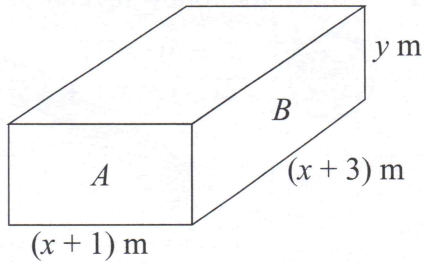
.....

[Total 6 marks]

4 The cuboid below has a weight of 120 N. When the cuboid rests on face A, the pressure exerted is 10 N/m^2. When the cuboid rests on face B, the pressure exerted is 7.5 N/m^2.

Find the volume of the cuboid.

Not drawn accurately

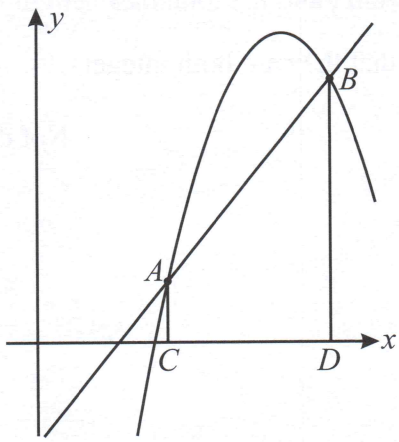


Use the formula pressure = force ÷ area to form two simultaneous equations.

..... m^3

[Total 6 marks]

5 The line $y = 2x - 5$ intersects the curve $y = -x^2 + 15x - 41$ at the points A and B, as shown.



C and D are points on the x-axis. The lines CA and DB are parallel to the y-axis.
Calculate the area of the quadrilateral $ABDC$.

..... units^2

[Total 6 marks]

Exam Practice Tip

Remember, you can always check your x- and y-values by putting them back into the original equations and checking they produce the numbers given in the question. If they don't, you've gone wrong somewhere. Check your factorising — it's easy to make a mistake (especially with tricky numbers).

Score

28

Proof

1 Prove that $(2n + 1)^3 - 1 \equiv 2n(4n^2 + 6n + 3)$.

[Total 3 marks]

2 Prove that the product of three rational numbers is always rational.

Remember, a rational number is one that can be written as $\frac{a}{b}$, where a and b are both integers and $b \neq 0$.

[Total 2 marks]

3 If a and b are both odd, prove that $(a + b)^{40}$ is even.

[Total 2 marks]

4 Prove that the sum of any three consecutive cube numbers is a multiple of 3.

[Total 4 marks]

5 Max thinks of a whole number that is one more than a multiple of 5.
Samira thinks of the number that is four less that Max's number.

Prove that the difference in the squares of their values is a multiple of 8.

[Total 5 marks]

6 Show that $3^8 - 7^4$ is a multiple of 13.

[Total 3 marks]

7 Show that the sum of 15^{12} and 12^{16} is a multiple of 9.

Start by writing 15^{12} and 12^{16} as products of their prime factors.

[Total 3 marks]

Score: ____ / **22**

Functions

1 $f(x) = \sqrt{2x - 8}$ $(x \geq 4)$ and $g(x) = x^2 + 4$.

a) Find the exact value of fg(4). Give your answer in its simplest form.

.....

[2]

b) Find gf(x).

gf(x) = ...

[2]

c) Find the inverse function $f^{-1}(x)$.

$f^{-1}(x)$ = ...

[3]

[Total 7 marks]

2 $f(x) = \frac{x+5}{2}$ and $g(x) = 3x - 10$.

Find the value of x for which $f^{-1}(x) = g^{-1}(x)$.

x = ...

[Total 6 marks]

3 $f(x) = 2x - 1$ and $g(x) = \sin x$.

Find both solutions to the equation $fg(x) = 0$ for $0° \leq x \leq 360°$.

A quick sketch of the graph of $y = \sin x$ will help you find the second x-value.

x = ...° and x = ...°

[Total 4 marks]

4 $f(x) = x^2 + 4x + 3$ and $g(x) = x + 2$.

a) Find $fgg(x)$.

$fgg(x) =$ .....

[3]

b) Solve the equation $fgg(x) = 0$.

$x =$ ..... or $x =$ .....

[2]

[Total 5 marks]

5 $f(x) = 3x + 1$, $g(x) = x^2 + 3x$ and $h(x) = x^3$.

Find $fgh^{-1}(x)$.

$fgh^{-1}(x) =$ .....

[Total 4 marks]

6 $f(x) = \frac{4x}{x+9}$ $(x \neq -9)$ and $g(x) = 2x + 1$.

Solve the equation $fg(x) = x$.

$x =$ ..... or $x =$ .....

[Total 5 marks]

Exam Practice Tip

Don't be put off if you have to solve things in function questions. Just put in the expression you know for the function (you might have to work it out first if it's a composite or inverse function), then solve it like a normal equation. And in composite functions, always remember to do the function closest to x first.

Score

31