Algebra Basics

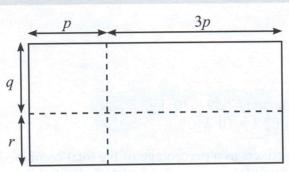
1 Leah is tiling a section of her bathroom wall. The tiles are a cm wide and b cm tall and she needs 20 tiles in total



Find an expression for the area of the wall she is tiling in terms of a and b.

[Total 1 mark]

On the diagram below, shade the area represented by pq + 3pr. (3) 2



[Total 1 mark]

Peter is making a sculpture using different pieces of metal tubing. 3 He makes a tower by stacking 7 pieces that are (f+g) cm tall, 9 pieces that are (h-g) cm tall and 5 pieces that are 2h cm tall on top of each other.

Find a simplified expression for the height of the tower in terms of f, g and h.

[Total 2 marks]

The diagram below shows a rectangle with sides that are 4x + 3 cm and 5x - 9 cm long.



Find an expression in terms of x for the side length of a regular hexagon with the same perimeter as the rectangle.

5x - 9 cm 4x + 3 cm

Diagram not accurately drawn

..... cm [Total 3 marks]

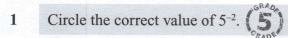






Section Two — Algebra

Powers and Roots





-25

2.5

 $\frac{1}{25}$

[Total 1 mark]

2 Show that
$$8^{\frac{4}{3}} = 16$$
.

$$8^{\frac{4}{3}} = (8^{\frac{1}{3}})^4 = (\dots)^4 = \dots$$

[Total 2 marks]

For values of $y \ge 2$, write the following expressions in order from smallest to largest. 3



 $v^{\frac{1}{3}}$

[Total 2 marks]

Estimate the value of each of the following to 1 decimal place: (6)



a)
$$x$$
, where $x = \sqrt{70}$



b) *y*, where
$$3^y = 20$$



[Total 4 marks]

Completely simplify the expression below.



$$(9a^4)^{\frac{1}{2}} \times \frac{2ab^2}{6a^3b}$$

[Total 3 marks]











Multiplying Out Brackets

Expand the brackets in the following expressions. 1 Simplify your answers as much as possible.





a) 5p(6-2p)

b) (2t-5)(3t+4)

[Total 4 marks]

121

a, b and c are integers such that 4(5x-7)+6(4-2x)=a(bx+c), 2 and a > 0. Find the values of a, b and c.

 $a = \dots, b = \dots, c = \dots$

[Total 3 marks]

Write an expression for the area of the triangle below. 3 Simplify your expression as much as possible.



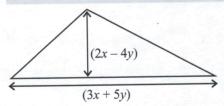


Diagram not accurately drawn

[Total 3 marks]

Expand and simplify (x-1)(2x+3)(2x-3).

[Total 3 marks]

Exam Practice Tip

If you're struggling with double brackets in the exam, don't forget you can always use the FOIL method – multiply the Eirst term in each bracket together, then multiply the $\underline{ extstyle O}$ utside terms together, then the Inside terms, and finally multiply together the Last term in each bracket... easy.



Factorising

Factorise the following expressions fully. 1





a) $7y - 21y^2$

[2]

b) $2v^3w + 8v^2w^2$

[2]

[Total 4 marks]

Factorise the following expressions fully. 6 2





a) $x^2 - 16$

b) $9n^2 - 4m^2$

c) $3y^2 - 15$

[2]

[2]

[Total 5 marks]

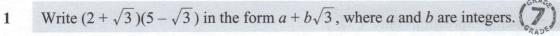
Fully factorise $x^3 - 25x$. (6)

[Total 3 marks]





Manipulating Surds







[Total 2 marks]

Write $2\sqrt{50} - (\sqrt{2})^3$ in the form $a\sqrt{b}$, where a and b are integers.





[Total 2 marks]

Express $\sqrt{396} + \frac{22}{\sqrt{11}} - \frac{220}{\sqrt{44}}$ in the form $a\sqrt{11}$, where a is an integer.





[Total 4 marks]

Express $\frac{1+\sqrt{7}}{3-\sqrt{7}}$ in the form $a+b\sqrt{7}$, where a and b are integers.





[Total 4 marks]







Solving Equations

Poppy, Felix and Alexi sell 700 raffle tickets between them.
Poppy sells twice as many tickets as Felix, and Alexi sells 25 more tickets than Poppy.



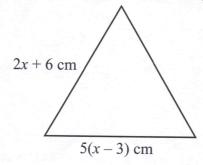
How many tickets did each of them sell?

Poppy	
Felix	
Alevi	

[Total 5 marks]

2 The diagram below shows an equilateral triangle.





Find the length of one side of the equilateral triangle.

..... cm [Total 4 marks]

Solve the equation $\frac{5}{4}(2c-1) = 3c-2$



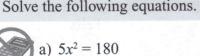
 $c = \dots$ [Total 3 marks]

4 Liam and Neil want to buy a new games console, which costs £360. They both get weekend jobs, where they each get paid £4.50 per hour. When they can afford the games console, Liam has worked 30 more hours than Neil.

How many hours did each boy work?

Liam:	 hours,	Neil:	hours
			[Total 3 marks]

Solve the following equations.



1 \	8 - 2x	$+\frac{2x+4}{2}$	_ 10
b)	3	9	- 12

 $\chi = \dots$ [2]

[4]

[Total 6 marks]

Hassan thinks of two different positive integers. 6 Their product is 147, and one number is three times the other number.

What are the two numbers Hassan is thinking of?

[Total 3 marks]

Exam Practice Tip

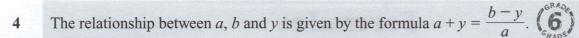
It's a good idea to check your solutions by substituting them back into the equation and checking that everything works out properly. If the unknown appears on both sides of the equation, make sure you work out the value on each side. It certainly beats sitting and twiddling your thumbs or counting sheep to kill time.



Formulas

1 Neela is on holiday in New York. The local weather forecast says that the temperature tomorrow will be 41 °F. Neela wants to know what this temperature is in °C. The formula for converting temperatures in °C to °F is: $F = \frac{9}{5}C + 32$. a) Rearrange the formula to make C the subject. [2] b) What will the temperature be in New York tomorrow in °C? [2] [Total 4 marks] A result used in physics is $P = \frac{V^2}{R}$, where P is the power in watts (W), 2 V is the voltage in volts (V) and R is the resistance in ohms (Ω). a) Calculate the power of an electrical circuit with a voltage of 12 V and a resistance of 16 Ω . [1] b) In a different circuit, the power is 25W when the voltage is 20V. Find the resistance in ohms. [Total 3 marks] The formula for the displacement, s, of a dropped object in free fall is $s = \frac{1}{2}gt^2$, where g is the constant acceleration due to gravity and t is time taken. 3 Rearrange the formula to make t the subject.

[Total 3 marks]





a) Rearrange this formula to make y the subject.

•	•	•	•	•	•	•	•	•	•	•	•		•	•			•	•	•	•	•	•	•	•	•	•	•	•		•	•	•	•		•	,
																																1	7	1	1	Ī

b) Find the value of y when a = 3 and b = 6.

$$y = \dots$$
 [1]

Rearrange the formula below to make n the subject.



$$x = \sqrt{\frac{(1+n)}{(1-n)}}$$

	•			•			•		•		•						•							•	•	•	•	•	•	•	•	•
										1	1	7	T	-)	+	1	7	1	4	5	v	v	7	1	7	v	.1	Į		,	7

6
$$x = \frac{1-y}{x}$$
, where $x > 1$. Decide whether y is positive or negative.



[Total 3 marks]

7
$$b = \sqrt{2a-1}$$
 and $c = 2b^4 + 4b^2$.

Find a formula for a in terms of c.

Start by finding
$$b^2$$
 = and b^4 in terms of a .

[Total 4 marks]

Score:







Factorising Quadratics

1 The product of two consecutive positive even numbers is 288. By forming and solving an equation, find the larger of the two numbers.





[Total 4 marks]

The expression $5x^2 - 19x + 18$ is an example of a quadratic expression.





a) Fully factorise the expression $5x^2 - 19x + 18$.

[2]

b) Use your answer to part a) to factorise the expression $5(x-1)^2 - 19(x-1) + 18$.

[2]

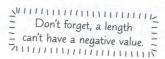
[Total 4 marks]

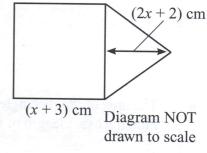
3 The shape on the right is made from a square and a triangle.



The sides of the square are (x + 3) cm long and the height of the triangle is (2x + 2) cm. The area of the whole shape is 60 cm^2 .

Find the value of x.





 $x = \dots$ [Total 7 marks]

Score:









The Quadratic Formula

Solve the quadratic equation $x^2 + 5x + 3 = 0$, giving your answers to 2 decimal places.



$$a =$$
 $b =$ and $c =$

$$x =$$
..... or $x =$

[Total 3 marks]

Solve the equation $2x^2 - 7x + 2 = 0$. Give your answers correct to 2 decimal places.



$$x =$$
 or $x =$

[Total 3 marks]

Solve the equation $3x^2 - 2x - 4 = 0$. Give your answers in simplified surd form. 3

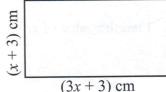


x = or x =

[Total 3 marks]

The area of the rectangle on the right is 30 cm². 4 Find the exact length of the longer side of the rectangle. \$\(\frac{1}{6} \)





1555
(A)

You're being asked for the = leave the surds in your answer.

> cm [Total 5 marks]

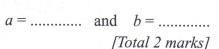
Exam Practice Tip

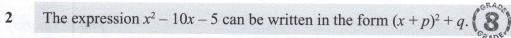
One thing you really need to watch out for when it comes to using the quadratic formula are those pesky minus signs — especially if a, b or c are negative. Just take your time when you're putting them into the formula — you don't want to throw away easy marks simply because you've messed up your minuses.



Completing the Square

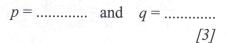
		GRAO.
1	Given that $x^2 + ax + b = (x + 2)^2 - 9$, work out the values of a and b.	(6)
		GRADE







a) Find the values of p and q.



b) Use your answer to solve the equation $x^2 - 10x - 5 = 0$. Leave your answer in surd form.

$$x =$$
 or $x =$ [2]

[Total 5 marks]

A curve has equation $y = 2x^2 - 8x + 19$.



a) Write the expression $2x^2 - 8x + 19$ in the form $a(x + b)^2 + c$.

[4]

b) Find the coordinates of the minimum point of the graph.

[1]

c) State if and where the graph of the equation crosses the x-axis.

Think about the minimum = value of the graph.

......[1]

[Total 6 marks]

Exam Practice Tip

Completing the square is pretty tough stuff. If you're struggling to get your head around it, just remember... when the quadratic expression is in the form $x^2 + bx + c$, the number in the brackets is always $b \div 2$ and the number outside the brackets is always $c - (b \div 2)^2$.

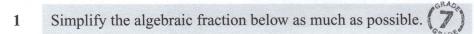








Algebraic Fractions



$$\frac{4x^2 + 10x - 6}{16x^2 - 4}$$

[Total 3 marks]

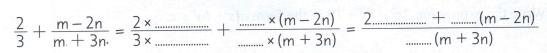
Simplify the calculation below as much as possible. (8) 2



$$\frac{2a-8}{a^2-9} \div \frac{a^2-2a-8}{a^2+5a+6} \times (2a^2-a-15)$$

[Total 5 marks]

Write $\frac{2}{3} + \frac{m-2n}{m+3n}$ as a single fraction.



[Total 3 marks]

Write $\frac{1}{x-5} + \frac{2}{x-2}$ as a single fraction.



[Total 3 marks]







Section Two — Algebra

Sequences

1	The first four terms in a sequence are 3, 8, 13, 18,	
	a) Find the <i>n</i> th term of the sequence. $\binom{n}{n}$	
	e i e a brita de la Darrigi e di mante di la contaction mantenane mili end adella d	[2]
	b) Using your answer to part a), find an expression for the product of the <i>n</i> th a terms of the sequence. Simplify your answer as much as possible.	
		[2]
		[Total 4 marks]
2	This question is about the sequence 3, 7, 11, 15, 19	
	a) Find the <i>n</i> th term of the sequence.	
	b) Explain why 502 cannot be a term in this sequence.	[2]
		F27
		[2] [Total 4 marks]
3	The first four terms in a sequence are $\sqrt{2}$, 2, $2\sqrt{2}$, 4	
	a) Find the next two terms in the sequence.	
	to a compare with longer or dimension for the	
	b) Circle the expression for the <i>n</i> th term of the sequence.	[2]
	$\sqrt{2n} \qquad \qquad n\sqrt{2} \qquad \qquad (\sqrt{2})^n \qquad \qquad n(\sqrt{2})^2$	
		[1]
		[Total 3 marks]

The term-to-term rule of a sequence is $u_{n+1} = 2u_n + 1$.
a) If $u_1 = 0.5$, find the values of the next two terms in the sequence.
[2]
b) A different sequence has the same term-to-term rule, but $u_1 = 1.5$. Find u_2 , u_3 and u_4 .
c) What do you notice if you start with $u_1 = -1$?
[1] [Total 6 marks]
and the same of th
A quadratic sequence begins 2, 6, 12, 20,
a) Write down the next term in the sequence.
Essentes out at a material and built in [2]
b) Find an expression for the <i>n</i> th term of the sequence.
[3]

6	The term-to-term rule of a sequence is $u_{n+1} = \frac{-1}{2u_n}$.	
	a) If $u_1 = 2$, find the values of the next three terms in the sequence.	
	b) Write down the value of u_{50} .	[2]
	[Total .	[1] 3 marks]
7	The patterns below are made up of grey and white squares.	
	Pattern 1 Pattern 2 Pattern 3 Pattern 4	
	a) Find an expression for the number of grey squares in the n th pattern.	
	b) Giles makes two consecutive patterns in the sequence. He uses 414 grey squares in total. Which 2 patterns has he made?	[2]
	c) Find an expression for the total number of squares in the <i>n</i> th pattern. $\binom{8}{6}$	[3]
	[Total 8	[3] marks]
Exa	am Practice Tip	Score
acros	uence questions are all about spotting the pattern — don't be put off if it's one you haven't come poss before (examiners like to try and catch you off guard by throwing in things like roots and fractions). might even come across a sequence where a numerator and denominator each follow a different rule.	33

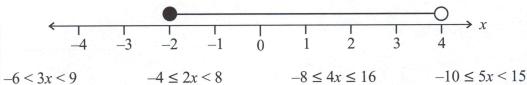




Inequalities

Circle the inequality that is shown on the number line below. 1





[Total 1 mark]

Solve the inequality 4x + 1 > x - 5. 2



[Total 2 marks]

Find the integer values that satisfy both of the following inequalities: 3



 $5n - 3 \le 17$ and 2n + 6 > 8Give your answer using set notation.

[Total 3 marks]

Find the largest three consecutive even numbers that sum to less than 1000.



[Total 3 marks]

Solve the following inequalities. 5





a) $5x^2 < 80$

[3]

b) $x^2 + 1 < x + 7$

[3]

[Total 6 marks]



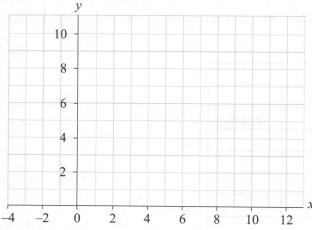




Graphical Inequalities

Look at the grid below. (6) 1





Use the grid to draw the graphs of 2x + y = 10 and y = x + 2.

[2]

Shade and label, using the letter S, b) the area represented by the inequalities $x \ge 1$, $2x + y \le 10$, $y \ge x + 2$.

[2]

[Total 4 marks]

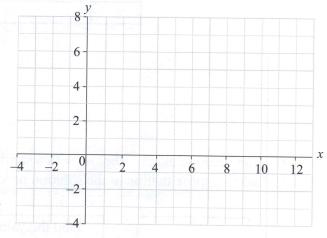
2 Look at the grid on the right.



On the grid, shade the region that represents these inequalities:

$$y \ge -2$$

$$y - x \le 1$$



[Total 4 marks]

Look at the grid on the right. 3

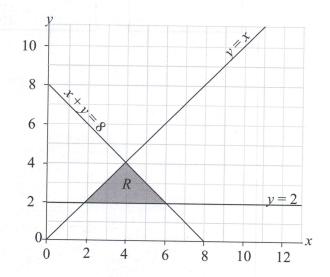


The shaded region *R* is bounded by the lines y = 2, y = x and x + y = 8.

Write down three inequalities which define R.



[Total 3 marks]



Exam Practice Tip

You need to pay close attention to whether the symbol is just < or > or whether it's \le or \ge . If it's < or >, draw a dashed line on the graph. If it's \leq or \geq you need to use a solid line. If you're not sure which side of the line you want, pick a point with coordinates that satisfy the inequality and see which side of the line it lies.







Iterative Methods

The equation $x^3 - 4x + 2 = 0$ has a solution between x = -2 and x = -3.

-3. (7)

By filling in the table below, find an approximation to the solution of $x^3 - 4x + 2 = 0$ to 1 d.p. You might not need to use all the rows.

x	$x^3 - 4x + 2$	
-2	2	
-3	SW 301	Negative
-2.1 -2.2	1.139	
-2.2		
		5

 $x = \dots$ [Total 4 marks]

Use the iteration machine below with a starting value of $x_0 = 1$ to find an approximation to the solution of $5x^3 + 3x - 6 = 0$ to 5 d.p.



Start with x_n Find the value of x_{n+1} by using the formula $x_{n+1} = \frac{10x_n^3 + 6}{15x_n^2 + 3}$ When

If $x_{n+1} = x_n$ when rounded to 5 d.p. then stop. If $x_{n+1} \neq x_n$ when rounded to 5 d.p. start again using x_{n+1} .

Type in '1 =' on your calculator. Now, if you enter

(10ans³ + 6) \div (15ans² + 3) and keep pressing '=', you'll get the

values of x_1 , x_2 etc. without having to put the calculation in again.

x =

[Total 3 marks]

The equation $3x - 2x^3 + 5 = 0$ has one solution. 3



a) Show that this solution lies in the interval 1.5 < x < 2.

[2]

b) Show that $3x - 2x^3 + 5 = 0$ can be written as:

$$x = \sqrt[3]{\frac{3x+5}{2}}$$

[3]

c) Use the iteration $x_{n+1} = \sqrt[3]{\frac{3x_n + 5}{2}}$ to find the solution to $3x - 2x^3 + 5 = 0$ to 5 d.p. Use a starting value of $x_0 = 2$.

[3]

[Total 8 marks]

Exam Practice Tip

Don't panic — the exam questions will guide you through the iteration and tell you which method to use. It wouldn't just say 'use iteration to find the solution to this equation' as that would just be mean. Be careful with decimal places — you might have to find a root to 2, 3, 4 or even 5 d.p.

Score







Simultaneous Equations

Solve this pair of simultaneous equations. (5) 1



$$x + 3y = 11$$
$$3x + y = 9$$

x =		<i>y</i> =
	[7	Total 3 marks]

Solve this pair of simultaneous equations. 2

$$2x + 3y = 12$$
$$5x + 4y = 9$$

$$x = \dots y = \dots$$
[Total 4 marks]

A sweet shop sells bags of pick 'n' mix. A bag that contains 4 chocolate frogs and 3 sugar 3 mice costs £3.69. A bag that contains 6 chocolate frogs and 2 sugar mice costs £3.96.



How much would a bag that contains 2 chocolate frogs and 5 sugar mice cost? Show your working.

[Total 4 marks]

4 Solve the following pair of simultaneous equations.





$$x^2 + y = 4$$
$$y = 4x - 1$$

	x =	••••••	,	y =	•••••	••••
and	x =		,	<i>y</i> =		
		[Total 5 marks]				

5 Solve the following pair of simultaneous equations.





$$2x^2 + y^2 = 51$$
$$y = x + 6$$

	$\chi = \dots,$	y =
and	$x = \dots,$	<i>y</i> =
		Total 5 marks]

The lines $y = x^2 + 3x - 1$ and y = 2x + 5 intersect at two points. The line joining the two points has length $k\sqrt{5}$. Find the value of k.



Use Pythagoras' theorem to find the distance between the two points.

k =[Total 6 marks]

Exam Practice Tip

When you're solving simultaneous equations in the exam, it's always a good idea to check your answers at the end. Just substitute your values for x and y back into the original equations and see if they add up as they should. If they don't then you must have gone wrong somewhere, so go back and check your working.









Proof

Prove that $(3n+2)^2 - (n+2)^2 \equiv 8n(n+1)$.

[Total 2 marks]

Jake says "If a < b < c < d (where b and d are not zero), then $\frac{a}{b} < \frac{c}{d}$ ". (68.40)

Is he correct? Explain your answer.

[Total 3 marks]

Prove that the difference between the squares of two consecutive even numbers is always a multiple of 4.

[Total 3 marks]

4 Show that the number $2^{64} - 1$ is not prime.

[Total 3 marks]

Score:







Functions

- f is a function such that $f(x) = \frac{3}{2x+5}$.
 - a) Find f(7.5) (6)

[1]

b) Find the inverse function f^{-1} in the form $f^{-1}(x)$. Show your working clearly.



 $f^{-1}(x) = \dots [3]$

c) Show that $ff^{-1}(x) = x$. $\binom{\mathbb{R}^{R_{A_0}}}{\mathbb{R}^{R_0}}$

[3]

[Total 3 marks]

- f and g are functions such that $f(x) = 2x^2 + 3$ and $g(x) = \sqrt{2x 6}$.
 - a) Find g(21) (${}^{G}_{G_{AAD}}$)

[1]

b) Find gf(x)Remember to do the

function closest to x first.

 $gf(x) = \dots [2]$

c) Solve fg(a) = 7

a =

[3]

[Total 6 marks]

Score:





