

19.1 Sets

A set is a collection of things — it can be considered as an object in its own right.
Set notation can look quite complicated, but don't worry — it's not as scary as it first appears.

Learning Objectives — Spec Ref P6:

- Understand and use set notation.
- Be able to list elements of sets.

Prior Knowledge Check:

Be familiar with algebraic expressions (see Section 6) and inequalities (see Section 13).

A **set** is a group of items or numbers. Sets are written in pairs of **curly brackets** $\{ \}$.

- Each item in a set is called an **element** or **member** of the set.
- You can describe a set by **listing every element** in that set (e.g. $\{2, 4, 6\}$) or by **giving a rule** that all elements must follow (e.g. $\{\text{all red objects}\}$).
- You usually use a **capital letter** to represent the set (e.g. A), so $A = \{\text{all red objects}\}$ means 'A is the set of all red objects'. **Lower case letters** represent elements within the set (e.g. a might be 'a tomato').

Here's some of the **set notation** you'll need to be familiar with:

$A = \{...\}$	A is the set of ...
$x \in A$	x is an element of A
$y \notin A$	y is not an element of A
$n(A)$	the number of elements in A

Tip: To find $n(A)$, just count the number of elements in the set.

Example 1

- List the members of set A, where $A = \{\text{days of the week with the letter u in their name}\}$.
- Write down the number of elements in set A.
- Is Monday a member of the set?

a) The days that contain the letter u are Tuesday, Thursday, Saturday and Sunday.

$A = \{\text{Tuesday, Thursday, Saturday, Sunday}\}$

b) There are 4 elements in set A.

$n(A) = 4$

c) Monday doesn't have the letter u in it, so it's not a member of set A.

$\text{Monday} \notin A$

Exercise 1

Q1 List the elements of the following sets:

- $A = \{\text{months of the year with fewer than 31 days}\}$
- $B = \{\text{months of the year with fewer than 4 letters in their name}\}$
- $C = \{\text{months of the year with the letter a in their name}\}$

Q2 List the elements of the following sets:

- $A = \{\text{even numbers between 11 and 25}\}$
- $B = \{\text{prime numbers less than 20}\}$
- $C = \{\text{square numbers less than 200}\}$
- $D = \{\text{factors of 30}\}$

There are two special types of set: the **empty set** and the **universal set**.

\emptyset or $\{\}$	the empty set
ξ	the universal set

Tip: ξ is the lowercase Greek letter xi.

- The **empty set** is a set that has **no members**. E.g. if set $B = \{\text{square numbers between 50 and 60}\}$, then $B = \emptyset$ because there are no square numbers between 50 and 60.
- The **universal set** is the group of things **under consideration** — the group that members of a set are **selected** from. The universal set is often a set of **integers**, e.g. $\xi = \{\text{positive integers between 1 and 20}\}$.
- Some sets are described using an **algebraic expression** — e.g. $A = \{x : x + 2 < 5\}$ is read as 'A is the set of numbers x , such that $x + 2$ is less than 5'. To list the elements of a set like this, you need to know the **universal set**. E.g. for $A = \{x : x + 2 < 5\}$, if $\xi = \{\text{positive integers}\}$, the elements in set A would be $\{1, 2\}$.

Example 2

List the members of $A = \{x : x < 15\}$ if $\xi = \{x : x \text{ is a square number}\}$.

- Firstly, work out what the symbols and numbers mean:
A is the set of numbers x , such that x is less than 15.
 ξ is the universal set — it contains all square numbers.
- Elements of A can only come from the universal set ξ ,
so here $A = \{\text{square numbers less than 15}\}$.

Tip: The colon just means 'such that'.

$$A = \{1, 4, 9\}$$

Exercise 2

Q1 a) If $\xi = \{x : x \text{ is a positive integer, } x \leq 10\}$, list all the elements of:

- | | |
|--|--|
| (i) $A = \{x : x \text{ is odd}\}$ | (ii) $B = \{x : x \text{ is a factor of 16}\}$ |
| (iii) $C = \{x : x \text{ is a square number}\}$ | (iv) $D = \{x : x \text{ is a factor of 30}\}$ |

b) If $\xi = \{x : x \text{ is an integer, } 20 \leq x \leq 30\}$, list all the elements of:

- | | |
|--|---|
| (i) $A = \{x : x \text{ is odd}\}$ | (ii) $B = \{x : x \text{ is even}\}$ |
| (iii) $C = \{x : x \text{ is prime}\}$ | (iv) $D = \{x : x \text{ is a multiple of 3}\}$ |

Q2 For all the sets in this question, $\xi = \{x : x \text{ is a positive integer, } x < 30\}$

$A = \{\text{prime numbers}\}$ $B = \{\text{square numbers}\}$ $C = \{\text{cube numbers}\}$ $D = \{\text{multiples of 4}\}$

a) List the members of A, B, C and D.

b) Find (i) $n(A)$ (ii) $n(B)$ (iii) $n(C)$ (iv) $n(D)$

c) Find (i) $n(E)$ where $E = \{x : x \in \text{both A and B}\}$

(ii) $n(F)$ where $F = \{x : x \in B \text{ but not } D\}$

(iii) $n(G)$ where $G = \{x : x \notin A, x \notin B, x \notin C, x \notin D\}$

Q3 $A = \{1, 2, 3, 4\}$ $B = \{0, 2, 4\}$

a) List the elements of the following sets:

(i) $C = \{x : x \in \text{either A or B or both}\}$

(ii) $D = \{x : x \in A \text{ but not } B\}$

(iii) $E = \{x : x = a - b, a \in A, b \in B, x < 1\}$

b) Each element of the following sets is a pair of coordinates. List the elements of each set.

(i) $F = \{(a, b) : a \in A, b \in B, a + b < 5\}$

(ii) $G = \{(a, b) : a \in A, b \in B, a \times b > 6\}$

19.2 Venn Diagrams

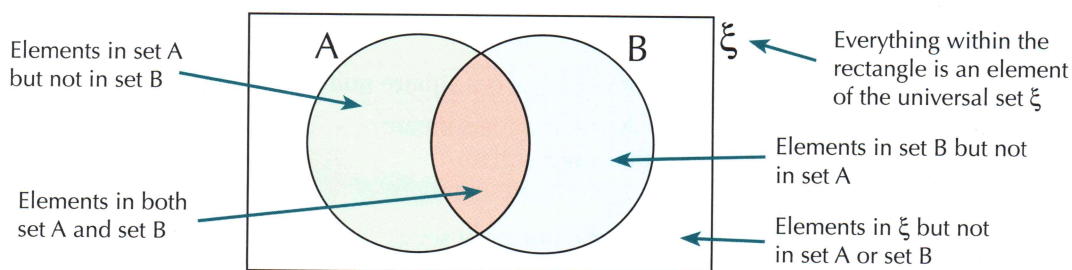
Venn diagrams can be used to display sets. They're great at showing the overlap between sets.

Representing Sets Using Venn Diagrams

Learning Objective — Spec Ref P6:

Use Venn diagrams to show sets.

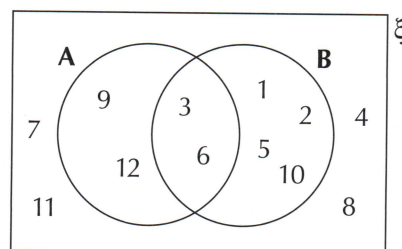
Venn diagrams use **circles** to represent sets — the **space inside** the circle represents everything in the set. Each **circle** is labelled with a **letter** — this tells you **which set** the circle represents. The **number inside a circle** can tell you either the **number of members** of that set or **actual elements** of the set.



Example 1

Given that $\xi = \{\text{positive integers less than or equal to } 12\}$, draw a Venn diagram to show the elements in the sets $A = \{x : x \text{ is a multiple of } 3\}$ and $B = \{x : x \text{ is a factor of } 30\}$.

- Write out sets A and B. $A = \{3, 6, 9, 12\}$
 $B = \{1, 2, 3, 5, 6, 10\}$
- Look for the elements that appear in both sets. Here, 3 and 6 are in both A and B, so they go in the overlap between the circles.
- The other elements of each set go in the circles for A and B, but not in the overlap.
- The elements of ξ that aren't in either set go outside the circles.

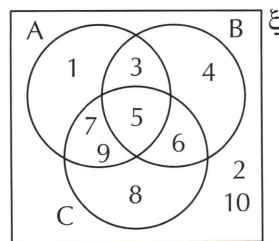


Exercise 1

- Q1 Draw Venn diagrams to show the elements in the following pairs of sets, where $\xi = \{x : x \text{ is a positive integer, } x \leq 10\}$ in each case.
- $A = \{1, 3, 5\}$ $B = \{1, 3, 7\}$
 - $A = \{2, 3, 4, 5\}$ $B = \{1, 3, 5, 7, 9\}$
 - $A = \{2, 6, 10\}$ $B = \{1, 3, 6, 9\}$
- Q2 Draw Venn diagrams to show the elements in the following pairs of sets, where $\xi = \{x : x \text{ is an integer, } 20 \leq x \leq 30\}$ in each case.
- $A = \{x : x \text{ is odd}\}$ $B = \{x : x \text{ is a multiple of } 5\}$
 - $A = \{x : x \text{ is a multiple of } 3\}$ $B = \{x : x \text{ is a multiple of } 4\}$
 - $A = \{x : x \text{ is even}\}$ $B = \{x : x \text{ is a factor of } 100\}$

Q3

The Venn diagram on the right represents sets A, B and C.



- List the elements of set A.
- Which elements are in both set A and set C?
- What is the value of $n(B)$?
- List the elements which are in both set A and set B, but not in set C.
- Which elements of set C are not also in set B?
- List the elements which are neither in set A nor set B.

Solving Problems with Venn Diagrams

Learning Objective — Spec Ref P6:

Use Venn diagrams to solve problems.

Prior Knowledge Check:

Be able to set up and solve algebraic equations. See Section 9.

When a Venn diagram is labelled with the **number of elements**, it can be used to solve problems involving the numbers of elements in sets. With this information you can **set up equations** and **solve** them to find missing values.

Tip: $n(\xi)$ is the total number of elements in the whole Venn diagram.

Example 2

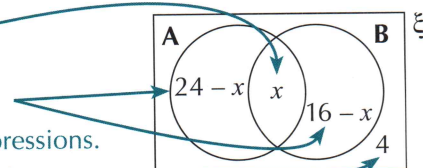
A and B are sets. $n(A) = 24$, $n(B) = 16$ and $n(\xi) = 32$. 4 elements of ξ are neither in A nor B.

a) Draw a Venn diagram to show sets A and B. Label each part of the diagram with a number or expression representing the number of elements in that part.

b) Find the number of elements that are in both set A and set B.

1. Call the number of elements that are in both A and B x , then write this in the overlap between the circles.
2. You can now write the number of elements in A only as $n(A) - x = 24 - x$, and the number of elements in B only as $n(B) - x = 16 - x$. Label the two circles with these expressions.
3. 4 elements are in neither set, so this goes outside the circles.

b) Use $n(\xi) = 32$ to write and solve an equation for x .



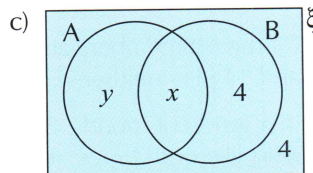
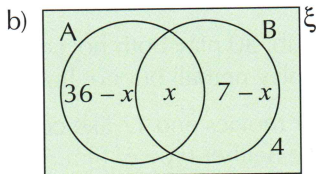
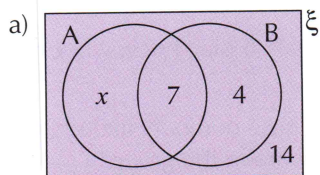
$$n(\xi) = (24 - x) + x + (16 - x) + 4$$

$$32 = 44 - x \Rightarrow x = 12$$

So **12 elements** are in both A and B.

Exercise 2

Q1 Below are the Venn diagrams a)-c). They are labelled with the number of elements in each part of the diagram.



- For a), $n(\xi) = 30$. Find: (i) x (ii) $n(A)$ (iii) $n(\text{not } B)$
- If the number of elements in A or B or both is 40 in b), find: (i) x
- For c), $n(A) = 50$ and $n(B) = 40$. Find: (i) x and y (ii) $n(\xi)$

- (ii) $n(A)$ (iii) $n(\xi)$
- (iii) $n(A \text{ or } B \text{ or both})$

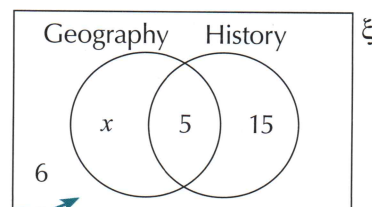
Q2

Draw a Venn diagram to represent each situation below, and use it to answer the questions.

- a) Set A contains 26 elements in total. 18 of the elements of set A are not in set B.
10 of the elements of set B are not in set A. 12 elements of ξ are neither in A nor B. Find:
(i) $n(B)$ (ii) $n(\xi)$
- b) 9 of the elements of set A are not in set B. 17 of the elements of set B are not in set A.
4 elements are in both set A and set B. $n(\xi) = 40$.
Find the number of elements of ξ that are: (i) not in A (ii) not in B
- c) 3 elements are in both set A and set B. 11 elements of ξ are not in A or B. The number of elements in B but not A is 4. $n(\xi) = 25$. Find the number of elements:
(i) in B (ii) not in B (iii) in A (iv) in A but not B
- d) 37 of the elements of set B are not in set A. If the number of elements that are in both set A and set B is x , $n(A) = 34 + x$. 4 elements of ξ are neither in A nor B. $n(\xi) = 88$.
Find the total number of elements in: (i) both A and B (ii) B

Example 3

In a class of 30 students, 5 study both geography and history, 15 study history but not geography and 6 study neither subject.



a) Draw a Venn diagram to show this information.

b) Find how many students in total study geography.

- a) 1. As 5 students study both subjects, this goes in the overlap.
2. 15 study history only, so this goes in the history circle.
3. 6 students study neither, so this goes outside the circles.
4. Label the number of students who study geography only x .

b) Use the Venn diagram to set up an equation and solve it, using the fact that there are 30 students in total.

$$n(\xi) = 30 \Rightarrow 30 = 6 + 5 + 15 + x \Rightarrow x = 30 - 26 = 4$$

4 students study geography only,
so $4 + 5 = 9$ students in total study geography.

Exercise 3



For each question, draw a Venn diagram to represent the situation and use it to answer the question.

- Q1 In a class of pupils, 6 play the guitar and piano, 8 play the guitar only, 4 play the piano only and 12 play neither instrument. How many pupils are in the class?
- Q2 36 children were asked about their pets. 8 owned geese only, 12 owned ducks only and 12 owned ducks and geese. How many owned neither geese nor ducks?
- Q3 Of the 60 members at a sports club, 30 play both hockey and netball, 6 play hockey but not netball and 14 play neither. How many play netball but not hockey?
- Q4 In a group of friends, 8 like horror movies and 12 like comedies. These numbers include 6 people who like both types of film. 3 like neither type of film. How many are in the group?
- Q5 Of the 55 pupils entered for English and maths tests, everybody passed at least one of English and maths. Including those who passed both subjects, 44 passed maths and 50 passed English. How many passed exactly one of English and maths?

19.3 Unions and Intersections

Combinations of sets can be described using the terms 'union' and 'intersection' — they represent different regions on a Venn diagram.

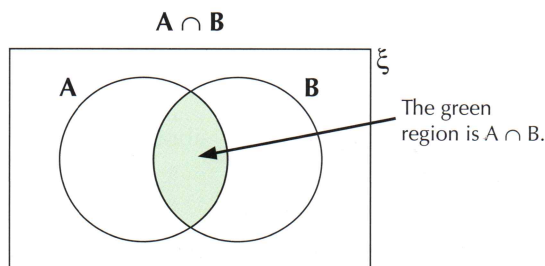
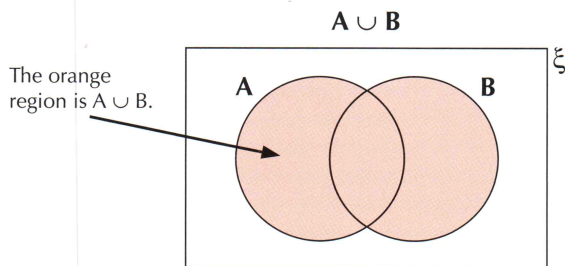
Learning Objective — Spec Ref P6:

Find unions and intersections of sets.

The **union** of two sets contains **all the members** that are in **either set**. On a Venn diagram, the union is **everything inside the circles**. You write 'the union of set A and set B' as $A \cup B$.

The **intersection** of two sets **only** contains objects that are members of **both sets**. This is **everything in the overlap** between the circles on a Venn diagram. You write 'the intersection of set A and set B' as $A \cap B$.

Tip: Read $A \cup B$ as "A union B" and $A \cap B$ as "A intersection B".



For example, consider the sets $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$. $A \cup B = \{1, 2, 3, 4, 5, 6\}$ as these are all the elements in set A or set B or both, and $A \cap B = \{3\}$ as this is the only element that is in both sets.

Example 1

For the sets represented on the Venn diagram on the right, list the elements of:

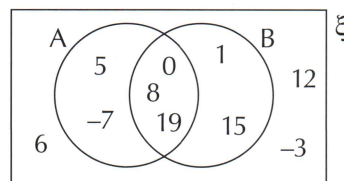
a) $A \cap B$ b) $A \cup B$

a) $A \cap B$ means "everything that's a member of both A and B". List the elements in the part of the diagram where the circles overlap.

$A \cap B = \{0, 8, 19\}$

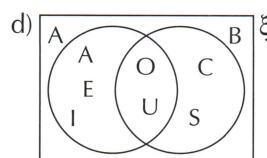
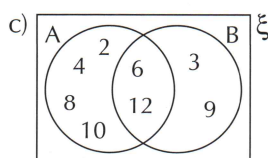
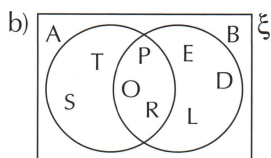
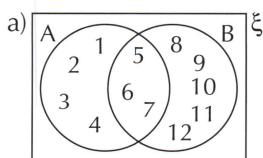
b) $A \cup B$ means "everything that's in A or B or both", so list everything that's in either circle.

$A \cup B = \{-7, 0, 1, 5, 8, 15, 19\}$



Exercise 1

Q1 For each of the following, list the elements of: (i) $A \cap B$ (ii) $A \cup B$



Q2 Describe each of the following:

- $A \cap B$ when $A = \{\text{blue cars}\}$ and $B = \{\text{four-wheel-drive cars}\}$
- $A \cup B$ when $A = \{\text{boys' names}\}$ and $B = \{\text{girls' names}\}$
- $A \cap B$ when $A = \{\text{towns in France}\}$ and $B = \{\text{seaside towns}\}$
- $A \cup B$ when $A = \{\text{countries in Europe}\}$ and $B = \{\text{countries not in Europe}\}$
- $A \cap B$ when $A = \{\text{right-handed people}\}$ and $B = \{\text{people with fair hair}\}$

Q3 For each of the following sets of numbers, list the elements of: (i) $A \cap B$ (ii) $A \cup B$

- $A = \{\text{positive even numbers less than 30}\}$ $B = \{\text{positive integers less than 20}\}$
- $A = \{\text{positive even numbers less than 20}\}$ $B = \{\text{positive odd numbers less than 20}\}$
- $A = \{\text{square numbers less than 70}\}$ $B = \{\text{cube numbers less than 70}\}$
- $A = \{\text{prime numbers less than 10}\}$ $B = \{\text{multiples of 3 less than 10}\}$

Q4 Find $A \cap B$ for each of the following, where the universal set is the set of all real numbers.

- $A = \{x : 0 < x < 50\}$, $B = \{x : 30 < x < 100\}$
- $A = \{x : 20 < x \leq 30\}$, $B = \{x : 30 \leq x < 100\}$
- $A = \{x : x \leq 100\}$, $B = \{x : x \leq 50\}$
- $A = \{x : x < 50\}$, $B = \{x : x > 60\}$

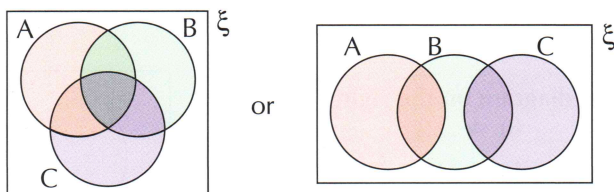
Q5 a) Assuming neither A nor B are empty, draw diagrams to show the following:

- $A \cap B = \emptyset$
- $A \cap B = A$
- $A \cup B = B$

b) What can you say about A and B if $A \cap B = A \cup B$?

Three Sets on a Venn Diagram

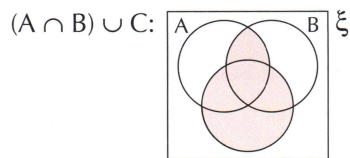
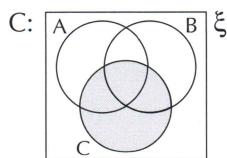
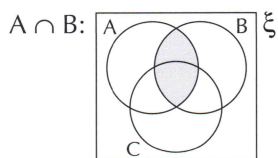
You can use Venn diagrams to show **three sets** in a similar way — just add an **extra circle** to represent the third set. For example:



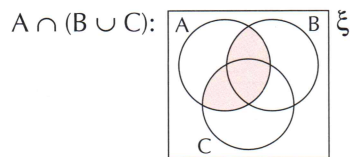
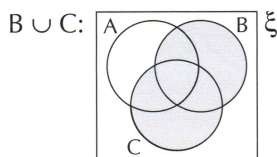
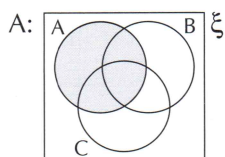
Tip: In the right-hand diagram, $A \cap C = \emptyset$.

You can have **unions** and **intersections** between all three sets, which might use **brackets** in the notation. To show unions and intersections, **shade** the relevant regions on **separate diagrams** first, then **compare** the diagrams. For **unions**, you want the area that's shaded on **any** diagram, and for **intersections**, you want the area that's shaded on **all** of them.

- $(A \cap B) \cup C$ means 'in both A and B , or in C '.

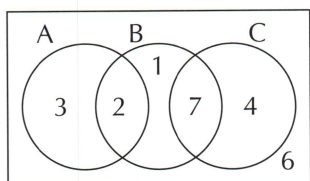


- $A \cap (B \cup C)$ means 'in A , and in either B or C or both'.

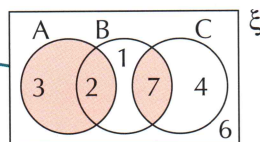


Example 2

The Venn diagram below is labelled with the number of elements in each region. Find $n(A \cup (B \cap C))$.



1. $A \cup (B \cap C)$ means 'either in A or in both B and C'. Shade the area that represents this.
2. Find the total number of elements in the shaded regions.

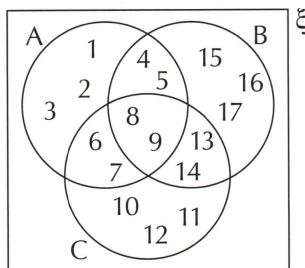


$$n(A \cup (B \cap C)) = (3 + 2) + 7 = 12$$

Exercise 2

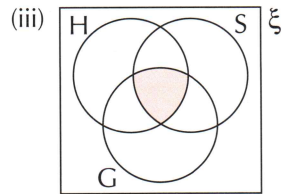
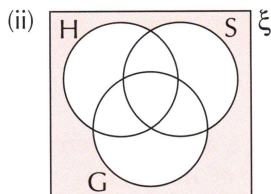
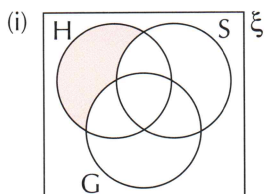
Q1 For the sets A, B and C shown on the right, list the elements of:

- $A \cap C$
- $A \cap B$
- $A \cap B \cap C$
- $B \cap C$
- $(A \cap C) \cup (B \cap C)$



Q2 On a day in winter, members of a class arrived at school wearing hats (H), scarves (S) and gloves (G).

a) Describe what each of the shaded areas represents.

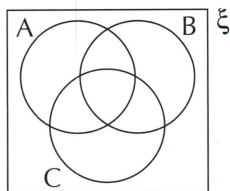
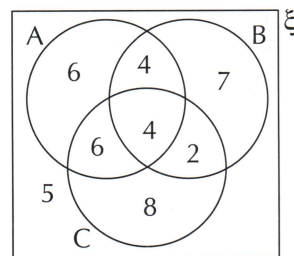


b) Draw diagrams to show the following groups of students.

- Those wearing scarves only.
- All those wearing gloves.
- Those wearing exactly two of hat, scarf and gloves.

Q3 The Venn diagram on the right is labelled with the number of elements in each region. Find:

- $n(A)$
- $n(A \cap B)$
- $n(B \cup C)$
- $n((A \cup B) \cap C)$
- $n(A \cup (B \cap C))$



Q4

- On separate copies of the Venn diagram on the left, shade the following regions.
 - $(A \cup C) \cap B$
 - $(A \cap B) \cup (B \cap C)$
- What do your diagrams show about $(A \cup C) \cap B$ and $(A \cap B) \cup (B \cap C)$?
- Use diagrams to show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

19.4 Complement of a Set

The complement of a set refers to everything outside the set. It's another way of writing 'the elements that are not in set A'.

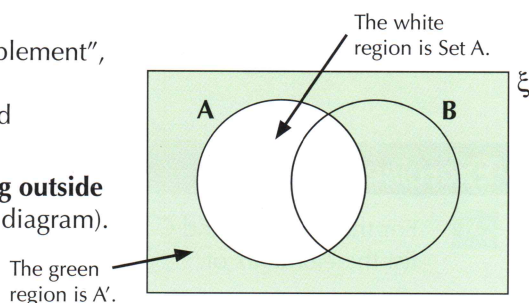
Learning Objective — Spec Ref P6:

Find the complement of a set.

The **complement** of a set A, written as A' and read as "A complement", is all the members of the **universal set** that **aren't** in set A.

For example, if the universal set is $\xi = \{\text{integers between 1 and 10 inclusive}\}$ and $A = \{2, 4, 6, 8, 10\}$, then $A' = \{1, 3, 5, 7, 9\}$.

- On a Venn diagram, the complement of a set is **everything outside the circle** representing that set (the shaded region on this diagram).
- If you know $n(\xi)$ and $n(A)$, then $n(A') = n(\xi) - n(A)$.



Example 1

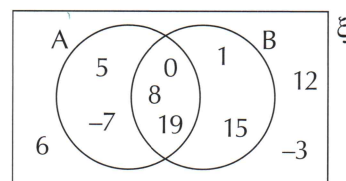
For the sets represented on the Venn diagram on the right, list the elements of: a) A' b) $(A \cup B)'$

- a) A' is everything that's not in A, so list everything that's outside circle A.

$$A' = \{-3, 1, 6, 12, 15\}$$

- b) $(A \cup B)'$ is everything that's not in $A \cup B$, so list everything that's outside the circles.

$$(A \cup B)' = \{-3, 6, 12\}$$



Example 2

The Venn diagram on the right shows sets A and B. Given that $n(A \cap B)' = 15$ and $n(A) = 12$, find: a) x b) y c) $n(A' \cup B')$

- a) $(A \cap B)'$ means everything that's not in $A \cap B$ — i.e. everything that's not in the intersection.

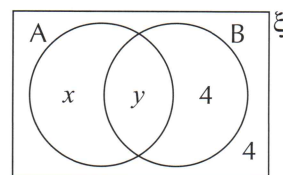
$$\begin{aligned} n(A \cap B)' &= 4 + 4 + x \\ 15 &= 8 + x \\ x &= 15 - 8 = 7 \end{aligned}$$

- b) $n(A) = x + y$, and, from part a), $x = 7$.

$$\begin{aligned} n(A) &= 12 = 7 + y \\ y &= 12 - 7 = 5 \end{aligned}$$

- c) $(A' \cup B')$ means everything that's not in set A together with everything that's not in set B.

$$n(A' \cup B') = 4 + 4 + 7 = 15$$



Tip: Notice that $n(A' \cup B') = n(A \cap B)'$.

Exercise 1

Q1 For each of the following, describe or list the elements of A' , the complement of set A.

- a) $\xi = \{\text{polygons with fewer than 5 sides}\}$

$A = \{\text{quadrilaterals}\}$

- b) $\xi = \{\text{months of the year with fewer than 30 days}\}$

$A = \{\text{February}\}$

- c) $\xi = \{\text{factors of 18}\}$

$A = \{\text{multiples of 2}\}$

- d) $\xi = \{\text{books in a library}\}$

$A = \{\text{paperback books}\}$

- e) $\xi = \{\text{cars}\}$

$A = \{\text{cars with an automatic gearbox}\}$

Q2 For each of the following, list the elements of B' , the complement of set B .

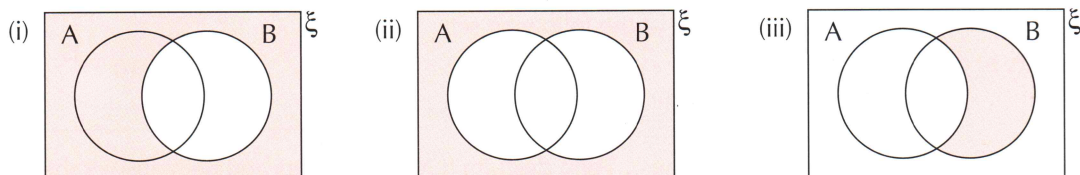
- a) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $B = \{\text{even numbers}\}$
 b) $\xi = \{\text{prime numbers}\}$ $B = \{\text{odd numbers}\}$
 c) $\xi = \{x : x \text{ is an even number, } 0 < x \leq 30\}$ $B = \{\text{factors of } 100\}$
 d) $\xi = \{\text{factors of } 120\}$ $B = \{x : x < 20\}$

Q3 $A = \{\text{multiples of } 3\}$ and $B = \{\text{multiples of } 4\}$, where the universal set is $\xi = \{x : x \text{ is an integer, } 1 \leq x \leq 20\}$.

- a) List the elements of $A \cup B$ b) List the elements of $(A \cup B)'$

Q4 a) Shade Venn diagrams to show the following sets: (i) $(A \cap B)'$ (ii) $A \cap B'$ (iii) $A \cup B'$

- b) Name the sets shaded in each of the following diagrams.

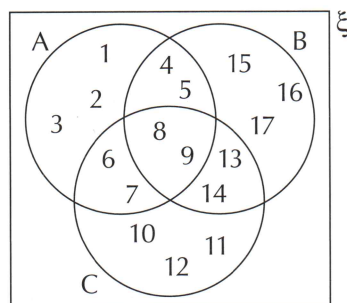


Q5 A group of people were asked to try two new biscuits. If J is the set of people who liked the Chocolate Jamborees and F is the set of people who liked the Apricot Fringits, what do the following sets represent?

- a) F' b) $(J \cup F)'$ c) $J' \cup F$

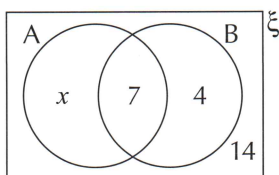
Q6 For the Venn diagram shown on the right, list the members of the following sets.

- a) $A' \cap B$
 b) $(A \cup B \cup C)'$
 c) $(A \cup B) \cap C'$
 d) $A \cap B' \cap C$
 e) $(B \cup C)'$



Q7 Given that $n(B') = 20$, find:

- a) x
 b) $n(A \cup B')$



Draw Venn diagrams to help solve Questions 8-10.

Q8 Given that $n(L \cap M) = 12$, $n(L \cup M)' = 8$, $n(L') = 10$ and $n(M') = 14$, find:

- a) $n(L)$ b) $n(\xi)$ c) $n(L \cap M)'$ d) $n(L \cup M)'$

Q9 $\xi = \{x : x \text{ is an integer, } 11 \leq x \leq 24\}$ is the universal set for the sets $A = \{11, 14, 16, 20, 21, 24\}$, $B = \{11, 13, 16, 19, 22, 24\}$ and $C = \{13, 14, 15, 16, 21, 22, 23, 24\}$.

List the elements of each of the following:

- a) $A' \cap B$ b) $A \cap B' \cap C'$ c) $B \cap (A' \cup C')$ d) $(A \cap B') \cup C$

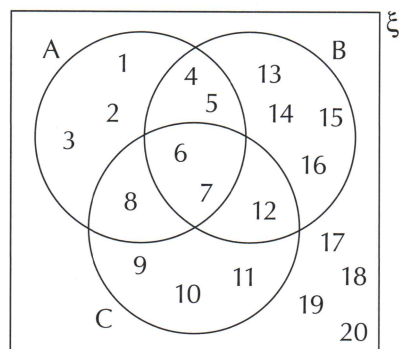
Q10 If $A = \{\text{multiples of } 2\}$, $B = \{\text{multiples of } 5\}$ and $C = \{\text{multiples of } 3\}$, where $\xi = \{x : x \text{ is an integer, } 1 \leq x \leq 30\}$, list the elements of:

- a) $(A \cap C) \cup B'$ b) $A' \cap B' \cap C$ c) $(A' \cap C') \cup B$

Review Exercise

Q1 For the sets shown in the diagram:

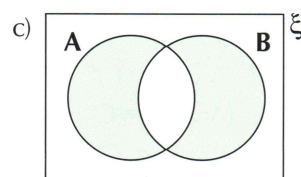
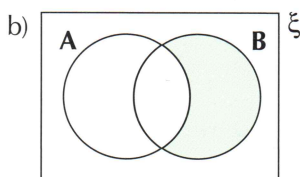
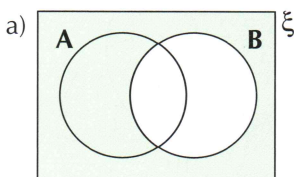
- a) State whether each of the following is true or false.
For those that are false, change the right hand side of the expression to make a correct statement.
- (i) $n(A) = 7$ (ii) $n(A \cap B) = 4$
(iii) $20 \in B'$ (iv) $C \cap B' = \{9, 10, 11\}$
- b) List the members of each of the following sets.
- (i) $A \cap C$ (ii) $A \cap B \cap C$ (iii) $(B \cap C)$
(iv) $A' \cap C$ (v) $(A \cup B)'$ (vi) $(A \cap C) \cup (B \cap C)$



Q2 For each of the following, draw a Venn diagram to show the sets and help answer the questions.

- a) $\xi = \{\text{positive integers less than 20}\}$ $T = \{\text{multiples of 2}\}$ $F = \{\text{multiples of 5}\}$
Find: (i) $n(T \cap F)$ (ii) $n(T \cup F)'$
- b) $\xi = \{\text{positive integers less than 20}\}$ $T = \{\text{multiples of 2}\}$ $S = \{\text{square numbers}\}$
Find: (i) $T \cap S$ (ii) $n(T')$

Q3 Name the sets corresponding to the shaded area in each of the following diagrams.



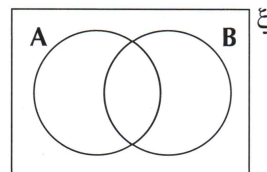
Q4 50 families were asked about the children they had. 30 of those asked said they had girls in the family while 38 said they had boys. 23 families had both boys and girls.



- a) Draw a Venn diagram to represent this.
b) How many families had no children?

Q5 On separate copies of the Venn diagram on the right, shade the areas corresponding to the following sets:

- a) $A' \cup B$
b) $(A \cup B)'$
c) $A' \cap B'$



Q6 A group of 100 people were asked whether they had in their pockets any of three items. 72 had all three of keys, crayons and a magic ring. Including those with all three items, 74 had both keys and crayons, 80 had both keys and a magic ring and 78 had both crayons and a magic ring. Of those with exactly one item, 3 had just keys, 5 just crayons and 3 just a magic ring.



- a) Show the results of this survey on a Venn diagram.
b) Of those asked, how many had: (i) none of the items, (ii) exactly two of the items?

Exam-Style Questions

Q1 In a class of 36 students, 28 are taking GCSE French and 18 are taking GCSE Spanish. 13 take both subjects.

a) Represent this information on a Venn diagram.

[1 mark]

b) How many in the class take neither subject?

[1 mark]

Q2 A company produces 120 different products. 32 of the products are made using grommets but not widgets. 20 of the products are made using widgets but not grommets. 46 of the products require neither widgets nor grommets. How many products are made using widgets?

[2 marks]

Q3 100 members of a health spa were asked whether they used the gym, pool or sauna. The replies were as follows:



A total of 52 used the gym, 30 used the pool and 65 used the sauna. 17 used the gym and the pool, 18 used the pool and the sauna, 30 used the gym and sauna. 15 used all three.

a) Draw a Venn diagram showing this information.

[2 marks]

b) Find how many people: (i) don't use the gym, the pool or the sauna,

[1 mark]

(ii) use only one of the gym, pool and sauna,

[1 mark]

(iii) use at least two out of the gym, pool and sauna.

[1 mark]

Q4 A and B are sets such that $n(\xi) = 20$, $n(A' \cap B') = 2$, $n(A') = 5$, $n(A \cap B) = x$ and $n(A \cap B') = y$. Use a Venn diagram to prove that $y = 15 - x$.

[3 marks]

Q5 If $\xi = \{\text{positive integers less than or equal to } 20\}$, $X = \{x : 10 < x < 20\}$, $Y = \{\text{prime numbers}\}$, $Z = \{\text{multiples of } 3\}$, find:

a) $X \cap Y$

[1 mark]

b) $(X' \cap Y) \cup Z$

[2 marks]