17.1 Interpreting Real-Life Graphs

Sometimes, graphs show something more interesting than just how y changes with x. They can be used to illustrate motion (such as in distance-time graphs), unit conversions (such as changing between temperatures in °C and °F), and many other connections between real-life quantities.

Learning Objective — Spec Ref A14:
Understand and interpret graphs that represent real-life situations.

Prior Knowledge Check: Be able to read values off the graphs of straight lines (Section 15) and other functions (Section 16).

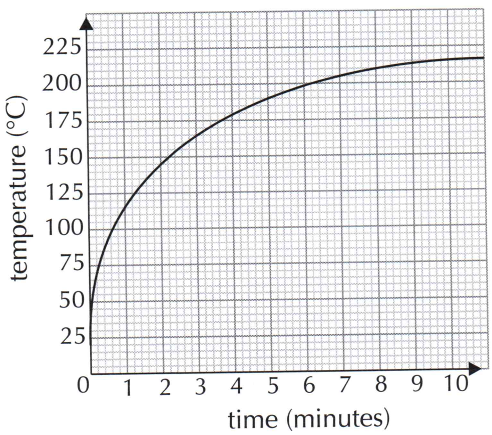
Real-life graphs show how one thing changes in relation to another. When **describing** real-life graphs, look at the following **features** and **interpret** them in the **context** of the graph.

- The **direction** of the graph — i.e. is the variable on the vertical axis **increasing** or **decreasing** as the other increases?
- The **gradient** (steepness) — this shows the **rate of change** of one variable with the other. E.g. on a distance-time graph, a **steep gradient** represents a **high speed** because a large distance (on the vertical axis) is being covered in a short time (on the horizontal axis).
- The **change** in **direction** or **gradient** — e.g. a distance-time graph that is initially steep and then levels off shows that an object is moving fast at first then slowing down over time.

Tip: See page 180 for how to find the gradient of a straight line and page 223 for working it out for curves.

Example 1

The graph shows the temperature of an oven as it heats up. Describe how the temperature of the oven changes during the first 10 minutes shown on the graph.



1. Look at the direction of the graph — temperature is increasing with time.
2. The gradient of the graph is steep initially and quite flat towards the end.
3. The graph doesn't change direction (it keeps increasing) but the gradient decreases over time.
4. Relate these features to the context:

   The temperature of the oven **rises** for the entire 10 minutes. This rise is **rapid at first** but then **becomes slower** as the oven heats up.

To **read off** values from a graph:

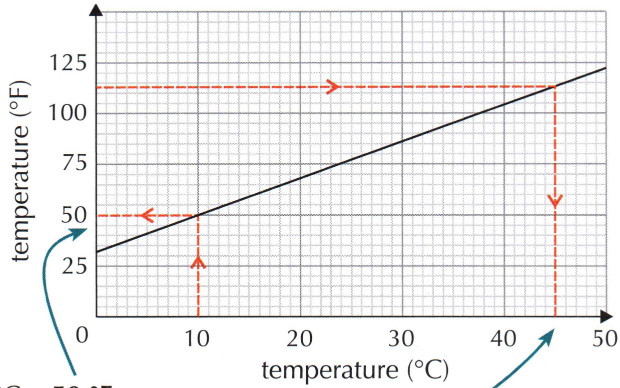
- Draw a **straight line** from **one axis** to the **graph**.
- Draw **another** straight line at a **right angle** to the first from the **graph** to the **other axis**.
- **Read off** the **value** from this axis, including any **units**.

Example 2

The conversion graph shows the connection between temperatures in °C and °F.

a) Convert a temperature of 10 °C to °F.

b) On a distant planet, the average daytime temperature is 80 °F higher than the average night-time temperature. If the night-time temperature is 33 °F, what is the daytime temperature in °C?



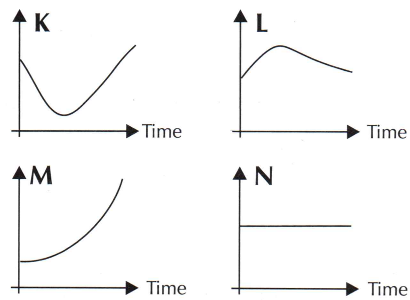
a) Read up from 10 °C and then across. 10 °C = **50 °F**

b) First work out the daytime temperature in °F. Then use the graph to convert this into °C.

33 °F + 80 °F = 113 °F
= **45 °C**

Exercise 1

Q1 Each statement below describes one of the graphs on the right. Match each statement to the correct graph.

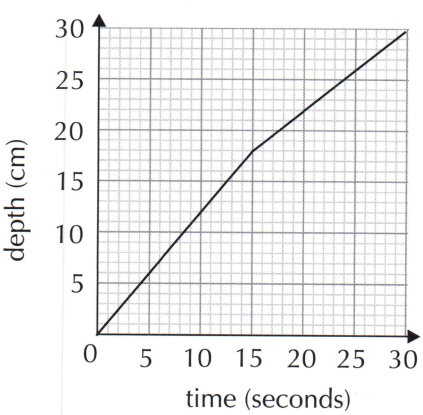


a) The temperature rose quickly, and then fell again gradually.

b) The number of people who needed hospital treatment stayed at the same level all year.

c) The cost of gold went up more and more quickly.

d) The temperature fell overnight, but then climbed quickly again the next morning.

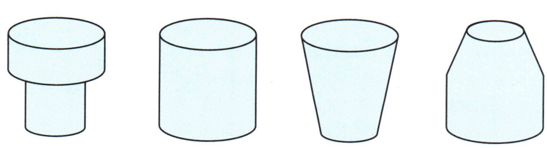


Q2 A vase that is 30 cm tall is filled using a tap flowing at a steady rate. The graph on the left shows how the depth of water varies in the vase over time.

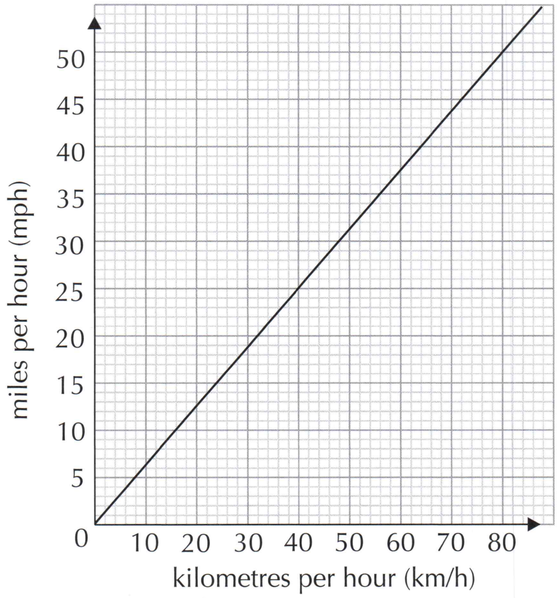
a) Describe how the depth of water in the vase changes over time.

b) How long did it take for the water depth to be half the height of the vase?

c) Which of the diagrams A-D shown below best matches the shape of the vase? PROBLEM SOLVING



A B C D



Q3 The graph on the left can be used to convert between speeds in kilometres per hour and miles per hour.

a) Convert 38 km/h into miles per hour, to the nearest 1 mph.

b) (i) Convert 25 mph into km/h.

(ii) Use your answer to convert 75 mph into km/h.

c) The speed limit on a particular road is 30 mph. A driver travels at 52 km/h. By how many miles per hour is the driver breaking the speed limit?

d) The maximum speed limit in the UK is 70 mph. The maximum speed limit in Spain is 120 km/h. Which country has the greater speed limit, and by how much?

Q4 The graph shows the depth of water in a harbour between 08:00 and 20:00.

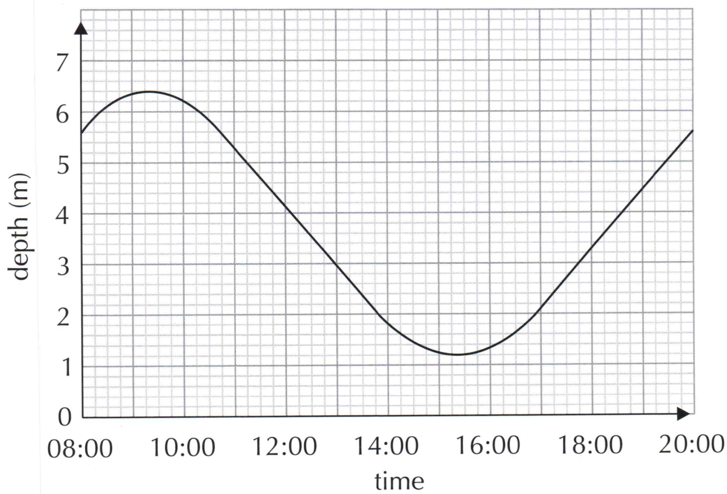
a) Describe how the depth of water changed over this time period.

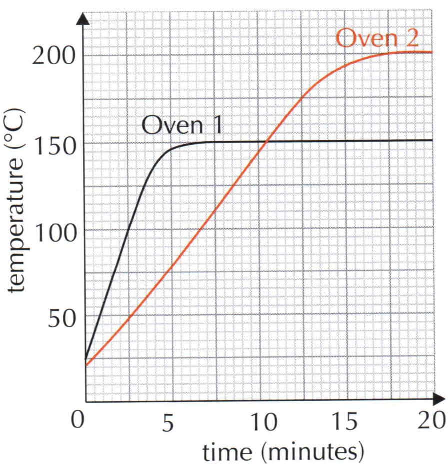
b) At approximately what time was the depth of water the greatest?

c) What was the minimum depth of water during this period?

d) At approximately what times was the water 3 m deep?

e) Mike's boat floats when the depth of the water is 1.6 m or over. Estimate the amount of time that his boat was not floating during this period.





Q5 The graph on the left shows the temperature in two ovens as they warm up.

a) Which oven reaches 100 °C more quickly?

b) Which oven reaches a higher maximum temperature?

c) How long does it take Oven 2 to reach its maximum temperature?

d) (i) After how many seconds are the two ovens at the same temperature?

(ii) What is the temperature at this time?

e) Calculate the rate at which the temperature of Oven 1 changes in the first 3 minutes after being switched on. PROBLEM SOLVING

17.2 Drawing Real-Life Graphs

Now you know how to interpret real-life graphs, it's time to have a go at drawing them.

Learning Objective — Spec Ref A14:
Draw graphs that represent real-life situations.

To **draw** a graph of the connection between two **real-life variables**, you'll need a **table of values**. Decide which of the variables should go on the horizontal axis and which on the vertical axis. Generally, the one that **depends** on the other should go on the **vertical axis**. Then **plot** the pairs of values as **coordinates** and carefully draw a **smooth line or curve** through them.

Example 1

A plumber charges customers a standard fee of £40, plus £30 per hour for all work carried out.

a) Draw a graph to show how the plumber's fee varies with the amount of time the job takes.

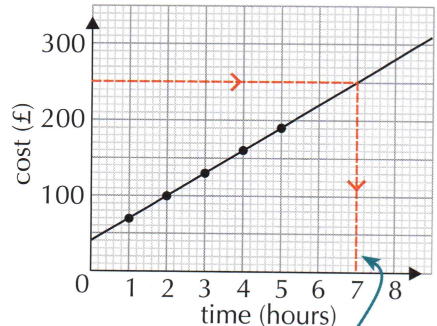
b) Use the graph to estimate the amount of time a job costing £250 would have taken.

a) 1. Make a table of values showing the fee for different numbers of hours.
A 1-hour job will cost £40 + £30 = £70.
A 2-hour job will cost £40 + (2 × £30) = £100 etc.

Time (hours)	1	2	3	4	5
Fee (£)	70	100	130	160	190

2. Draw your axes. The cost of the job depends on the time it takes to complete so the fee goes on the vertical axis and time on the horizontal axis. Make sure to label them and choose a scale that makes the graph easy to read.

3. Plot the values and join the points to draw the graph. For each extra hour, the fee increases by £30, so this is a straight-line graph.



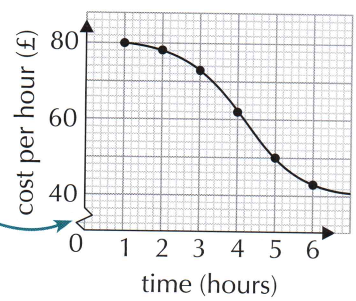
b) Draw a straight line from £250 on the vertical axis and then down to the horizontal axis to find the correct time... So it would have taken **7 hours.**

Example 2

A different plumber charges less per hour for longer jobs. Some of his fees are shown in the table. Draw a graph to illustrate this plumber's fees. Join your points with a smooth curve.

Time (hours)	1	2	3	4	5	6
Fee per hour (£)	80	78	73	62	50	43

1. Draw the axes — again, the cost goes on the vertical axis and time on the horizontal axis. The cost per hour values are between £43 and £80, so you can cut some of the vertical axis from the bottom of the scale.
2. Plot the values in the table as coordinates.
3. Draw a smooth curve through the points. There shouldn't be any sudden changes of direction or sharp kinks.



Exercise 1

Q1 The instructions for cooking different weights of chicken are as follows:

'Cook for 35 mins per kg, plus an extra 25 minutes.'

a) Copy and complete the table below to show the cooking times for different weights of chicken.

Weight (kg)	1	2	3	4	5
Time (minutes)					

b) Use the values from your table to draw a graph showing the cooking times for different weights of chicken.

c) A chicken cooks in 110 minutes. What is the weight of the chicken?

Q2 The table below shows how the fuel efficiency of a car in miles per gallon (mpg) varies with the speed of the car in miles per hour (mph).

Speed (mph)	55	60	65	70	75	80
Fuel Efficiency (mpg)	32.3	30.7	28.9	27.0	24.9	22.7

a) Plot the points from the table on a graph and join them up with a smooth curve.

b) Use your graph to predict the fuel efficiency of the car when it is travelling at 73 mph.

Q3 Helena is a baby girl. A health visitor records the weight of Helena every two months. The measurements are shown in the table below.

Age (months)	0	2	4	6	8	10	12	14	16
Weight (kg)	3.2	4.6	5.9	7.0	7.9	8.7	9.3	9.8	10.2

a) Draw a graph to show this information. Join your points with a smooth curve.

b) Keira is 9 months old and has a weight of 9.1 kg.
Use your graph to estimate how much heavier Keira is than Helena was at the same age.

Q4 The number of bacterial cells, N, in a sample after d days is $N = 5000 \times 1.2^d$.

a) Copy and complete the table to show the number of bacterial cells in the sample for the values of d given.

d	0	1	2	3	4
N					

b) Draw a graph to show this information.

c) Use your graph to estimate how many days it takes for there to be 9000 bacterial cells.

Q5 Alfred sells high-tech 'stealth fabric' on his market stall.
The cost of the fabric is £80 per metre for the first 3 metres, then £50 per metre after that.

a) Draw a graph showing how the cost of the stealth fabric varies with the amount purchased.

b) Use your graph to find how much it would cost to buy 6.5 metres of stealth fabric.

c) Bruce bought some fabric to make a stealth cape.
If he was charged £480, how much fabric did he buy?

Q6 The conical flask shown on the right is filled from a steadily running tap. Sketch a graph to show the depth of water in the flask t seconds after it has started to be filled.

PROBLEM SOLVING

17.3 Solving Simultaneous Equations Graphically

You first met simultaneous equations in Section 12, where you found the solutions algebraically. You can also solve them using their graphs.

Learning Objective — Spec Ref A19:
Solve simultaneous equations using graphs.

Prior Knowledge Check:
Be able to draw the graphs of straight lines (p.177-179) and quadratics (p.194-195).

If you have two equations and you draw their corresponding graphs, the points where the graphs **intersect** will be the **solutions** to **both** equations. So, to solve a pair of **simultaneous equations**, draw their graphs and read off the **intersection points**.

Example 1

Solve the following simultaneous equations graphically: $x + y = 8$ and $y = 2x + 2$.

1. Both equations are straight lines — they can be written in the form $y = mx + c$. Find three pairs of x- and y-values for each equation and plot them.

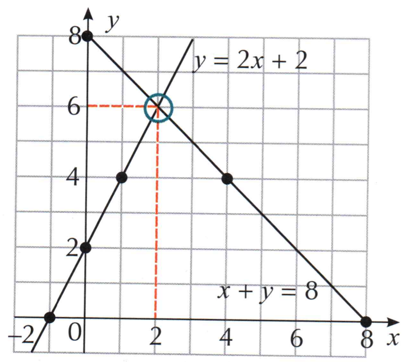
$x + y = 8$

x	0	4	8
y	8	4	0

$y = 2x + 2$

x	–1	0	1
y	0	2	4

2. Draw a straight line through each set of points.

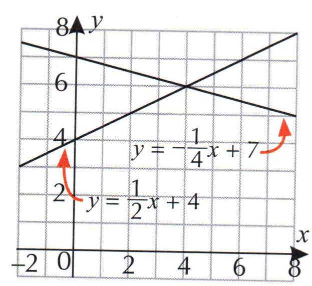


3. Read off the x- and y-values of the point where the graphs intersect. These values are the solution to the pair of equations.

The graphs cross at (2, 6), so the solution is **$x = 2$ and $y = 6$.**

Exercise 1

Q1 The graphs of $y = \frac{1}{2}x + 4$ and $y = -\frac{1}{4}x + 7$ are shown on the right. Use the graphs to find the solution to the simultaneous equations $y = \frac{1}{2}x + 4$ and $y = -\frac{1}{4}x + 7$.



Q2 Solve the following simultaneous equations by drawing graphs.

a) $x + y = 7$ and $y = x - 3$
b) $x + y = 8$ and $y = x$
c) $x + y = 9$ and $y = 2x$
d) $x + y = 1$ and $y = 2x - 7$
e) $x + \frac{1}{2}y = -4$ and $y = 2x$
f) $y = 2x + 3$ and $y = 4x + 2$
g) $y = 2x - 5$ and $2x + y = 1$
h) $y = \frac{1}{4}x$ and $x + 2y = 3$
i) $y = \frac{1}{2}x$ and $x + 2y = 6$
j) $y = 2x$ and $y = 4x + 3$

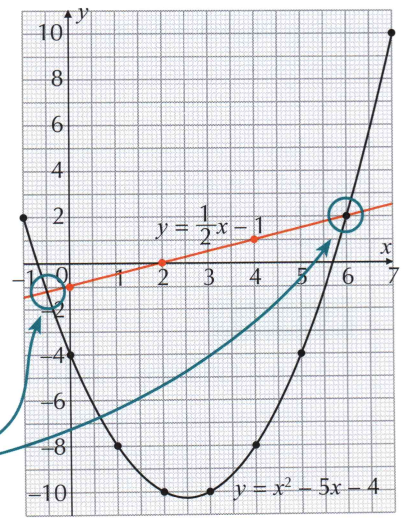
Q3 a) Draw the graphs of $y = x + 3$ and $y = x - 2$.
b) Explain how this shows that the simultaneous equations $y = x + 3$ and $y = x - 2$ have no solutions.

The **same method** is used for simultaneous equations where one or more of the equations is **non-linear**. If one equation is a **quadratic** and the other is linear, draw the quadratic **curve** and the straight **line** and see where they **intersect**. There could be **up to two points** of intersection which give different **solutions** to the pair of equations.

Example 2

a) Draw the graphs of $y = \frac{1}{2}x - 1$ and $y = x^2 - 5x - 4$ for $-1 \le x \le 7$.

b) Solve the simultaneous equations $y = \frac{1}{2}x - 1$ and $y = x^2 - 5x - 4$.



a) Work out pairs of x- and y-values for each equation and plot them on the same axes. You'll need three pairs for a straight line but more for a quadratic.

$y = \frac{1}{2}x - 1$

x	0	2	4
y	–1	0	1

$y = x^2 - 5x - 4$

x	–1	0	1	2	3	4	5	6	7
y	2	–4	–8	–10	–10	–8	–4	2	10

b) The points where the graphs cross are the solutions to the simultaneous equations.

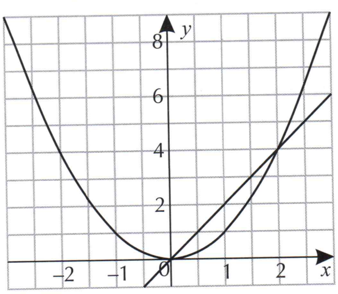
The graphs intersect at (6, 2) and (–0.5, –1.25).
So the solutions are $\boldsymbol{x = 6}$, $\boldsymbol{y = 2}$ and $\boldsymbol{x = -0.5}$, $\boldsymbol{y = -1.25}$.

Tip: There are two points of intersection so there are two solutions.

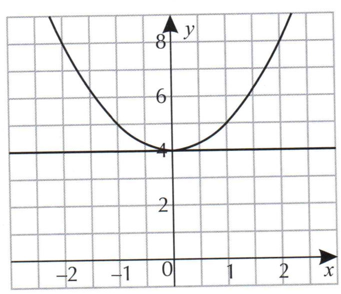
Exercise 2

Q1 Each of the following diagrams shows the graphs of two equations.
Use the graphs to solve the equations simultaneously.

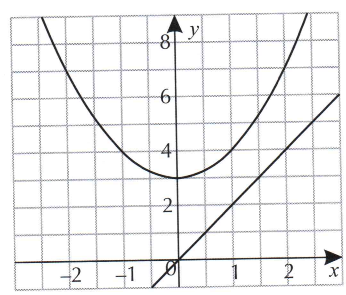
a)



b)



c)



Q2 For each of the following (i) Draw the graphs for the x-value range given.
(ii) Use these graphs to solve the pair of simultaneous equations.

a) $y = x - 1$ and $y = x^2 - 3$ $\quad -4 \le x \le 4$

b) $2x + y = 8$ and $y = x^2$ $\quad -4 \le x \le 4$

c) $y = x + 2$ and $y = x^2 + x + 1$ $\quad -4 \le x \le 4$

d) $x + y = 3$ and $y = x^2 - 3x$ $\quad -3 \le x \le 5$

e) $x + y = 1$ and $y = x^2 - 1$ $\quad -4 \le x \le 4$

f) $y = x + 6$ and $y = x^2 + 5x + 1$ $\quad -6 \le x \le 3$

g) $y = \frac{1}{2}x$ and $y = x^2 - 3x - 2$ $\quad -3 \le x \le 5$

h) $y = \frac{1}{2}x + 1$ and $y = \frac{1}{2}x^2 - 5$ $\quad -6 \le x \le 6$

17.4 Solving Quadratics Graphically

Quadratic graphs of the form $y = ax^2 + bx + c$ can cross the x-axis in up to two places. These points are the solutions to the quadratic equation $ax^2 + bx + c = 0$ — also known as the 'roots' of the quadratic.

Learning Objective — Spec Ref A18:
Solve quadratic equations using graphs.

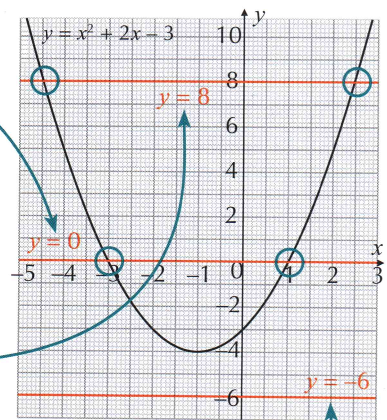
Prior Knowledge Check:
Be able to draw the graphs of straight lines (see p.177-179) and quadratics (see p.194-195).

The **solutions** (or **roots**) of a quadratic equation $ax^2 + bx + c = 0$ are the **x-intercepts** of the graph of $y = ax^2 + bx + c$, i.e. the intersection of $y = ax^2 + bx + c$ and $y = 0$. Similarly, the solutions of $ax^2 + bx + c = k$ are the x-values of the intersection points of $y = ax^2 + bx + c$ and $y = k$. You can **rearrange** quadratic equations into either of these forms to make it easier to **plot graphs** or read off solutions from graphs that you **already have**.

Example 1

The graph on the right shows $y = x^2 + 2x - 3$ for $-5 \le x \le 3$.

a) Use the graph to solve: (i) $x^2 + 2x - 3 = 0$ (ii) $x^2 + 2x - 11 = 0$



(i) The roots of a quadratic are at the points where the graph meets the x-axis.

The graph cuts the x-axis twice so there are two solutions: $\mathbf{x = -3}$ and $\mathbf{x = 1}$

(ii) 1. Rearrange $x^2 + 2x - 11 = 0$ so that $x^2 + 2x - 3$ is on the left-hand side and a constant k is on the right-hand side.

$x^2 + 2x - 11 = 0$
$\Rightarrow x^2 + 2x - 11 + 11 - 3 = 11 - 3$
$\Rightarrow x^2 + 2x - 3 = 8$

2. Draw the graph of $y = k$. The solutions are where this intersects the curve.

The line $y = 8$ intersects the curve at $\mathbf{x = 2.5}$ and $\mathbf{x = -4.5}$ (to 1 d.p.)

b) Explain how the graph shows that the equation $x^2 + 2x + 3 = 0$ has no solutions.

Rearrange $x^2 + 2x + 3 = 0$ to get $x^2 + 2x - 3$ on the left-hand side and a constant k on the right-hand side. Draw the graph of $y = k$ — this time it never meets the curve.

$x^2 + 2x + 3 = 0 \Rightarrow x^2 + 2x - 3 = -6$

The line $y = -6$ **does not cross** the graph of $y = x^2 + 2x - 3$, so $x^2 + 2x + 3 = 0$ has **no solutions**.

Exercise 1

Q1 a) Draw the graph of $y = x^2 + 2x$ for $-5 \le x \le 3$.

b) Use your graph to solve the following equations to 1 decimal place (where appropriate).

(i) $x^2 + 2x = 0$ (ii) $x^2 + 2x = 10$ (iii) $x^2 + 2x = 7$

Q2 a) Draw the graph of $y = x^2 - 3x + 1$ for $-2 \le x \le 5$.

b) Use your graph to solve the following equations to 1 decimal place (where appropriate).

(i) $x^2 - 3x + 1 = 0$ (ii) $x^2 - 3x + 1 = 3$ (iii) $x^2 - 3x + 1 = -0.5$

Q3 a) Draw the graph of $y = x^2 + 4x - 7$ for $-7 \leq x \leq 3$.

b) Use the graph to find the roots of the following equations to 1 decimal place.

(i) $x^2 + 4x - 7 = 0$ (ii) $x^2 + 4x - 10 = 0$ (iii) $x^2 + 4x - 3 = 0$ (iv) $x^2 + 4x + 2 = 0$

Q4 a) Draw the graph of $y = 6 + x - x^2$ for $-4 \leq x \leq 5$.

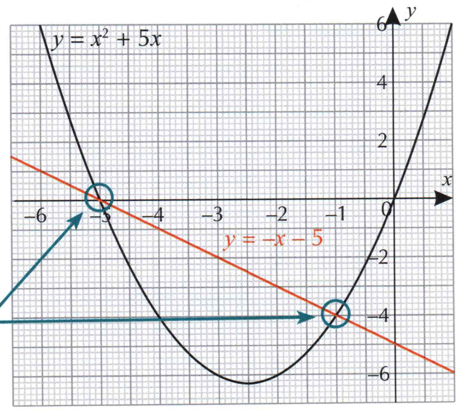
b) Use the graph to find the roots of the following equations to 1 decimal place.

(i) $5 + x - x^2 = 0$ (ii) $16 + x - x^2 = 0$ (iii) $3 + x - x^2 = 0$ (iv) $10 + x - x^2 = 0$

Given a graph of $y = ax^2 + bx + c$, you can also solve $ax^2 + ex + d = 0$ (i.e. where both the **x** and **constant** terms are different). Rearrange to get $\boldsymbol{ax^2 + bx + c}$ on one side and $\boldsymbol{kx + l}$ on the other. Plot the graph of $y = kx + l$ and find the solutions where it meets with $y = ax^2 + bx + c$.

Example 2

The graph of $y = x^2 + 5x$ is plotted on the right. By drawing a suitable straight line, use the graph to find the solutions to $x^2 + 6x + 5 = 0$.



1. Rearrange $x^2 + 6x + 5 = 0$ so it has $x^2 + 5x$ on one side and $kx + l$ on the other.

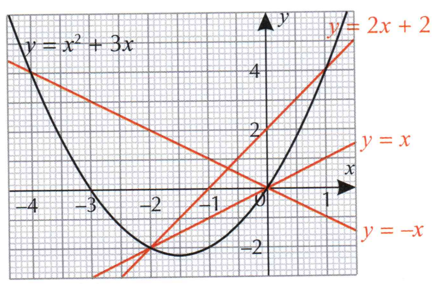
$x^2 + 6x + 5 = 0$
$\Rightarrow x^2 + 6x + 5 - 6x - 5 + 5x$
$= -6x - 5 + 5x$
$\Rightarrow x^2 + 5x = -x - 5$

2. Plot $y = kx + l$ (this is the 'suitable straight line'). The solutions to $x^2 + 6x + 5 = 0$ are found where this intersects $y = x^2 + 5x$.

The graph of $y = -x - 5$ intersects $y = x^2 + 5x$ at $\boldsymbol{x = -1}$ **and** $\boldsymbol{x = -5}$.

Exercise 2

Q1 The diagram on the right shows the graphs of $y = x^2 + 3x$, $y = x$, $y = 2x + 2$ and $y = -x$.



Use the diagram to solve the following equations graphically:

a) $x^2 + 4x = 0$ b) $x^2 + x - 2 = 0$ c) $x^2 + 2x = 0$

Q2 a) Draw the graph of $y = x^2 - 2x$ for $-3 \leq x \leq 5$.

b) Rearrange $x^2 - 4x - 1 = 0$ into the form $x^2 - 2x = mx + c$.

c) By drawing a suitable straight line, use your graph to find the solutions to the quadratic equation $x^2 - 4x - 1 = 0$ to 1 decimal place.

Q3 a) Draw the graph of $y = x^2 + 2x + 1$ for $-5 \leq x \leq 3$.

b) Use your graph to find the solutions to the quadratic equation $x^2 + 3x + 1 = 0$ to 1 decimal place.

Q4 a) Draw the graph of $y = 4x - x^2$ for $-2 \leq x \leq 5$.

b) Use your graph to find the solutions to the quadratic equation $5x - x^2 - 5 = 0$ to 1 decimal place.

17.5 Gradients of Curves

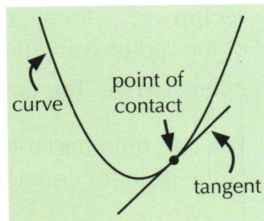
Working out the gradient of a curve can be tricky as it's not constant — it changes as you go along the curve.

Gradients of Curves

Learning Objective — Spec Ref A15:
Find the gradient of a curve at a given point.

Prior Knowledge Check:
Be able to find the gradient of a line (see page 180) and draw other types of graph (see Section 16).

A straight line has the **same gradient** everywhere — the steepness/slope never changes. But the gradient of a **curve** is **different** at different points on the curve. For example, a **quadratic graph** gets **less steep** as you move towards the turning point and then **steeper** as you move away from it.

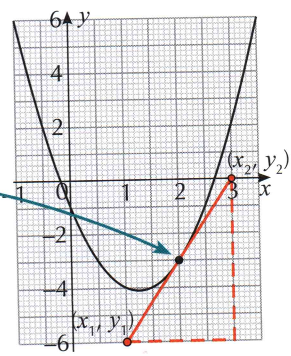


To find the **gradient at a point** on a curve, you can find the gradient of the **tangent** at this point. A tangent is a **straight line** that **just touches** the curve — at this point of contact the **gradient** of the line and the curve are **the same**.

Example 1

The graph of $y = 2x^2 - 5x - 1$ is shown here. Find the gradient of the curve at $x = 2$.

1. Draw a tangent to the curve at $x = 2$. It should be a line that just touches the curve with the same slope as the curve.
2. Find the gradient of this line using two points, (x_1, y_1) and (x_2, y_2), on the line.



Using the points (1, –6) and (3, 0) the gradient of the tangent is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-6)}{3-1} = \frac{6}{2} = 3$. So the gradient of the curve at $x = 2$ is also **3**.

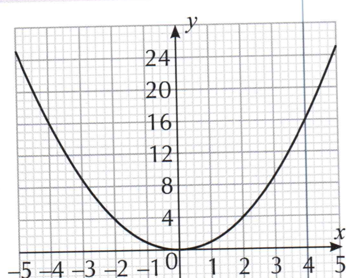
Tip: The gradient of a straight line is $\frac{\text{change in } y}{\text{change in } x}$ (see p.180).

Exercise 1

Q1 The graph $y = x^2$ is shown on the right.

a) Draw tangents to find the gradient at: (i) $x = 4$ (ii) $x = -4$

b) What do you notice about the gradients at these two points?



Q2 a) Copy and complete the table below for $y = \frac{1}{x}$.

x	0.1	0.2	0.5	1	1.5	2	3
y							

b) Draw the graph of $y = \frac{1}{x}$ for $0 < x \leq 3$.

c) Find the gradient of the graph at: (i) $x = 2$ (ii) $x = 0.2$ (iii) $x = 0.6$

Q3 a) Draw the graph of $y = x^3 + x^2 - 6x$ for $-4 \leq x \leq 3$.

b) Find the gradient of the graph at: (i) $x = -1$ (ii) $x = 2$

c) Use your graph to estimate the x-values at which the gradient of the graph is zero.

Tangents to a Circle

Learning Objective — Spec Ref A16:
Find the equation of a tangent to a circle.

Prior Knowledge Check:
Be able to find the equation of a straight line (p.183-184) and understand the equation of a circle (p.203).

A curve with equation $x^2 + y^2 = r^2$ is a **circle** centred at the **origin** with radius r.
The **tangent to a circle** is the same as the tangent to any other curve — a **straight line** that just **touches** the circle at a point and has the **same gradient** as the circle at that point.

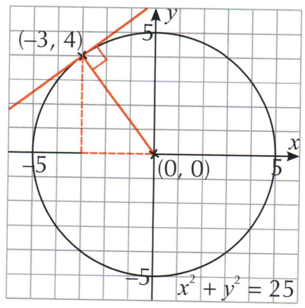
The tangent to a circle at a point is **perpendicular** to the **radius** of the circle **at that point**. So the tangent's **gradient** is the **negative reciprocal** of the gradient of the radius. Find the gradient, m, of the radius using the **centre** of the circle and the point you're interested in. The gradient of the tangent at that point is $-\frac{1}{m}$.

Tip: See page 186 for more on perpendicular lines and their gradients.

You can then find the **equation** of the tangent using the gradient and the coordinates of the point of contact with the circle in the form $y = mx + c$ (see page 183).

Example 2

The graph of $x^2 + y^2 = 25$ is shown on the right.
Find the equation of the tangent at the point (–3, 4).



1. Find the gradient of the radius of the circle between (–3, 4) and (0, 0).

   Gradient of radius $= \frac{4-0}{-3-0} = -\frac{4}{3}$

2. The tangent is perpendicular to the radius, so its gradient is –1 ÷ gradient of radius.

   Gradient of tangent $= -1 \div -\frac{4}{3}$
   $= \frac{3}{4}$

3. Find the equation of the tangent by substituting (–3, 4) into $y = mx + c$, where m is the gradient $\frac{3}{4}$.

   $4 = \frac{3}{4} \times -3 + c \Rightarrow c = 4 + \frac{9}{4} = \frac{25}{4}$

   So the equation of the tangent is $y = \frac{3}{4}x + \frac{25}{4}$.

Exercise 2

Q1 a) Find the radius of the circle with equation $x^2 + y^2 = 169$ and hence sketch the graph of the circle.
b) Find the gradient of the radius between the centre of the circle and the point (5, 12) as a fraction.
c) Hence, write down the gradient of the tangent to the circle at the point (5, 12).
d) Find the equation of this tangent.

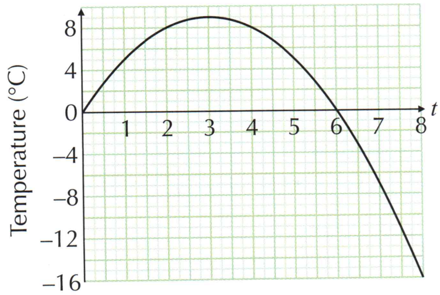
Q2 a) Sketch the graph of $x^2 + y^2 = 100$.
b) Find the equations of the tangents to the graph at these points:
(i) (6, 8) (ii) (8, 6) (iii) (–8, –6) (iv) (–6, 8)

Q3 a) Sketch the graph of $x^2 + y^2 = 625$.
b) Find the equations of the tangents to the graph at these points:
(i) (15, 20) (ii) (–7, 24) (iii) (–15, –20) (iv) (24, –7)

Q4 a) Find the equation of the tangent to $x^2 + y^2 = 16$ at the point $(-2\sqrt{2}, 2\sqrt{2})$.
b) Find the equation of the tangent to $x^2 + y^2 = 16$ at the point $(2\sqrt{2}, -2\sqrt{2})$.
c) What do you notice about the two tangents from parts a) and b)?

Review Exercise

Q1 A scientist is conducting an experiment.
The graph on the right shows the temperature of the experiment after t seconds.



a) Give a brief description of how the temperature changes in the first 8 seconds of the experiment.

b) State the maximum temperature that it reaches.

c) A temperature of 8 °C was recorded twice.
At what times was the temperature 8 °C?

Q2 A farm allows people to pick their own Brussels sprouts.
They charge 60p per kilogram of sprouts picked, plus an admin fee of £2.40 per customer.

a) Draw a graph showing how the total cost per customer varies with the mass of sprouts picked for the range 0 kg to 8 kg.

b) Use your graph to find the cost of picking 4.5 kg of sprouts.

c) What mass of sprouts did a customer pick if she was charged £6.60?

Q3 a) Draw the graph of $x + y = 5$.

b) On the same axes, draw the graph of $y = x - 3$.

c) Use your graphs to solve the simultaneous equations $x + y = 5$ and $y = x - 3$.

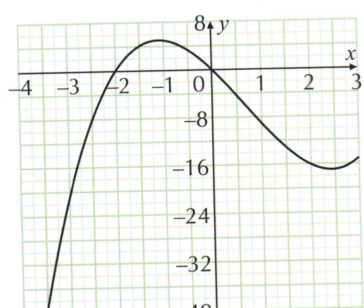
Q4 a) Draw the graph of $y = 2x^2 - 3x - 1$ for $-2 \leq x \leq 4$.

b) Use the graph to solve the following equations to 1 decimal place (where appropriate).

(i) $2x^2 - 3x - 1 = 0$ (ii) $2x^2 - 3x - 5 = 0$ (iii) $2x^2 - 3x - 10 = 0$ (iv) $2x^2 - 3x = 0$

Q5 a) Draw the graph of $y = x^2 + 3x - 7$ for $-6 \leq x \leq 3$.

b) By drawing a suitable straight line, use your graph to find the solutions to the quadratic equation $x^2 - 1 = 0$.



Q6 The graph of $y = x^3 - 2x^2 - 8x$ is shown on the right.
Find an estimate for the gradient of this curve when:

a) $x = -3$ b) $x = 2$

Q7 PROBLEM SOLVING
A circle is described by the equation $x^2 + y^2 = r^2$.
The line T is a tangent to the circle. Given that T meets the y-axis when $y = 5\sqrt{2}$ and the x-axis when $x = 10\sqrt{2}$, find the value of r, the radius of the circle.

Exam-Style Questions

Q1 A car is driven along a straight track to test its acceleration.
The distance in metres, x, that the car has travelled after t seconds is given by $x = 2t^2 + t$.

a) By calculating values of x at $t = 0, 1, 2, 3, 4$ and 5, draw a graph of x against t for $0 \leq t \leq 5$.

[2 marks]

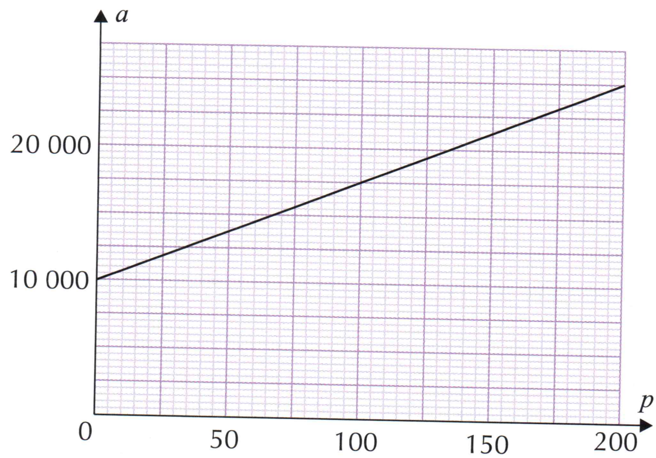
b) Use your graph to find the distance the car will have travelled after 3.5 seconds.

[1 mark]

c) Describe how the speed of the car changes with time.

[2 marks]

Q2 The graph below shows the relationship between the total amount in pounds (£), a, that a salesperson is paid in a year and the number of people, p, that they have signed up to a mobile phone contract in that year.



a) How much would the salesperson earn in a year in which they signed up 100 people?

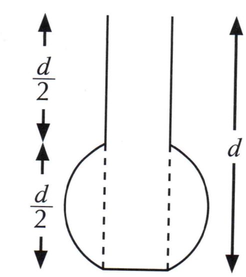
[1 mark]

b) Obtain a formula for a in terms of p.

[3 marks]

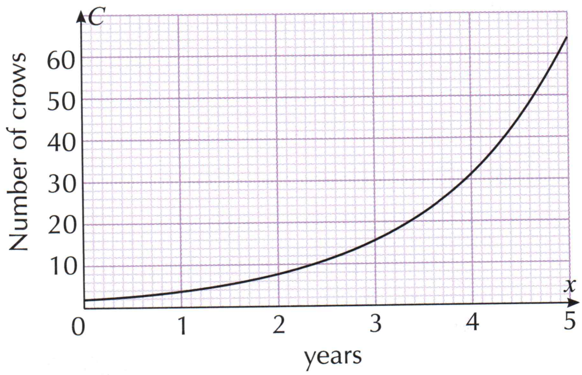
Q3 The diagram on the right shows the cross-section of a container used to store chemicals. It is made from a cylinder joined to part of a sphere. The container has a total depth of d, as shown.

A laboratory technician completely fills the container with water from a tap which is running at a constant rate.
On a set of axes, sketch a graph to show how the depth of water in the container changes with time.



[3 marks]

Q4 Five years ago, two crows were put on a deserted island. The number of crows on the island (C) is given by the equation $C = 2^{x+1}$, where x is the number of years since the crows were first put on the island. The graph of C is shown below.



a) Use the graph to find, to 1 decimal place, how many years it took for there to be 20 crows on the island.

[1 mark]

b) Find an estimate for the rate of change of the number of crows when they have been on the island for 4.5 years.

[2 marks]

The number of a particular tree species on the island, T, is given by $T = 5x + 10$.

c) By plotting an appropriate straight line, find how long it will take for the number of trees of this species to equal to the number of crows. Give your answer to 1 decimal place.

[2 marks]

Q5 Two tangents are drawn to the circle with equation $x^2 + y^2 = 25$.
The two tangents touch the circle at (4, 3) and (–4, 3).
Find the coordinates of the point where the two tangents intersect.

[5 marks]

Q6 Hamish has drawn the graph of $y = x^2 + 7x - 3$. He intends to use the graph by plotting straight lines onto it so that the x-coordinates of the intersections will be solutions to the equations he wants to solve. Find the equations of the straight lines he should plot in order to solve:

a) $x^2 + 7x - 6 = 0$

[1 mark]

b) $x^2 + 5x + 8 = 0$

[2 marks]