15.1 Straight-Line Graphs

This is a gentle introduction to straight-line graphs (or linear graphs) — it starts off with horizontal and vertical lines, before moving on to drawing other straight-line graphs from a given equation.

Horizontal and Vertical Lines

Learning Objectives — Spec Ref A9:

- Recognise horizontal and vertical lines from their equations.
- Be able to draw horizontal and vertical lines given their equations.

All **horizontal** lines have the equation $\boldsymbol{y = a}$ (where a is a number), since every point on the same horizontal line has the same y-coordinate (a). To **draw** the line $y = a$, draw a horizontal line that passes through a on the **y-axis**.

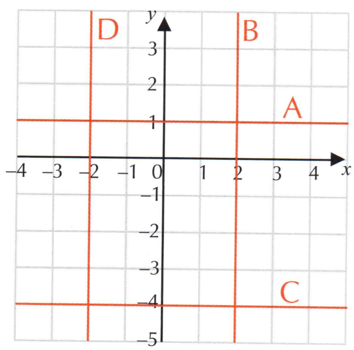
All **vertical** lines have the equation $\boldsymbol{x = b}$ (where b is a number), since every point on the same vertical line has the same x-coordinate (b). To **draw** the line $x = b$, draw a vertical line that passes through b on the **x-axis**.

The equation of the **x-axis** is $\boldsymbol{y = 0}$ and the equation of the **y-axis** is $\boldsymbol{x = 0}$.

Example 1

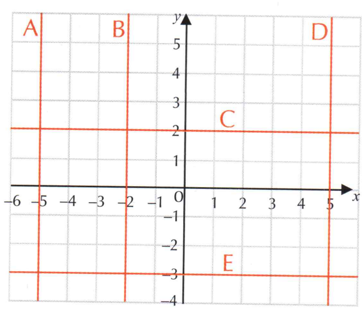
Write down the equations of the lines marked A-D.

1. Every point on the line marked A has y-coordinate 1. — A is the line $\boldsymbol{y = 1}$
2. Every point on the line marked B has x-coordinate 2. — B is the line $\boldsymbol{x = 2}$
3. Every point on the line marked C has y-coordinate –4. — C is the line $\boldsymbol{y = -4}$
4. Every point on the line marked D has x-coordinate –2. — D is the line $\boldsymbol{x = -2}$



Exercise 1

Q1 Write down the equations of each of the lines labelled A to E on the right.



Q2 Draw a set of coordinate axes and plot the graphs with the following equations.

a) $y = 3$ b) $y = -6$ c) $y = -1$

d) $x = 2$ e) $x = 4$ f) $x = -6$

Q3 Write down the equation of each of the following.

a) The line which is parallel to the x-axis, and which passes through the point (4, 8).

b) The line which is parallel to the y-axis, and which passes through the point (–2, –6).

c) The line which is parallel to the line $x = 4$, and which passes through the point (1, 1).

d) The line which is parallel to the line $y = -5$, and which passes through the point (0, 6).

Q4 Write down the coordinates of the points where the following pairs of lines intersect.

a) $x = 8$ and $y = -11$ b) $x = -5$ and $y = -13$ c) $x = -\frac{6}{11}$ and $y = -500$

Other Straight-Line Graphs

Learning Objective — Spec Ref A9:
Use the equation of a straight line to draw its graph.

The equation of a straight line which **isn't** horizontal or vertical contains **both x and y** — e.g. $y = 2x + 4$. If an equation **only** contains x and y terms (e.g. $y = 5x$), then the line passes through the **origin (0, 0)**.

There are a couple of different methods you can use to draw these straight-line graphs:

- Make a **table of values** — find the values of y for different values of x, plot the points and join with a straight line. You only have to plot two points to be able to sketch the graph — but it's often useful to plot more than two, in case one of the points you plot is incorrect.
- Find the **value of x** when $y = 0$ and the **value of y** when $x = 0$.
Plot these two points and join with a straight line. Both points should lie on the axes.
Extend your line to cover the range of x-values required (usually specified in the question).

Example 2

a) Complete the table to show the value of $y = 2x + 1$ for values of x from 0 to 5.

x	0	1	2	3	4	5
y						

Use the equation $y = 2x + 1$ to find the y-value corresponding to each value of x.

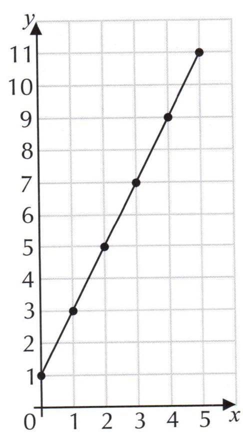
x	0	1	2	3	4	5
y	$2 \times 0 + 1 =$ **1**	$2 \times 1 + 1 =$ **3**	$2 \times 2 + 1 =$ **5**	$2 \times 3 + 1 =$ **7**	$2 \times 4 + 1 =$ **9**	$2 \times 5 + 1 =$ **11**

b) Plot the points from the table, and hence draw the graph of $y = 2x + 1$ for values of x from 0 to 5.

1. Use your table to find the coordinates to plot — just read off the x- and y-values from each column.

   The points to plot are (0, 1), (1, 3), (2, 5), (3, 7), (4, 9) and (5, 11).

2. Plot each point on the grid, then join them up with a straight line.



Example 3

Draw the graph of $y = 4 - 2x$ for $-1 \le x \le 3$.

1. Put $x = 0$ into the equation to find the value of y — this is where it crosses the y-axis.

   When $x = 0$, $y = 4 - 2(0) = \mathbf{4}$.

2. Put $y = 0$ into the equation to find the value of x — this is where it crosses the x-axis.

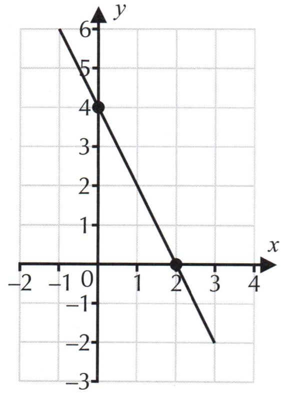
   When $y = 0$, $0 = 4 - 2x$

   $2x = 4$

   $x = \mathbf{2}$

   So the graph crosses the axes at **(0, 4)** and **(2, 0)**.

3. Mark the points (0, 4) and (2, 0) on your graph and draw a straight line passing through them. Make sure you extend it to cover the whole range of x-values asked for in the question.



Exercise 2

Q1 a) Copy and complete this table to show the value of $y = 2x$ and the coordinates of points on the line $y = 2x$ for values of x from –2 to 2.

x	–2	–1	0	1	2
y					
Coordinates					

b) Draw a set of axes with x-values from –5 to 5 and y-values from –10 to 10.
Plot the coordinates from your table.

c) Join up the points to draw the graph with equation $y = 2x$ for values of x from –2 to 2.

d) Use a ruler to extend your line to show the graph of $y = 2x$ for values of x from –5 to 5.

e) Use your graph to fill in the missing coordinates of these points on the line:

(i) (4, □) (ii) (–3, □) (iii) (□, –10)

Q2 a) Copy and complete the table to show the value of $y = 8 - x$ and the coordinates of points on the line $y = 8 - x$ for values of x from 0 to 4.

x	0	1	2	3	4
y					
Coordinates					

b) Draw a set of axes with x-values from –5 to 5 and y-values from 0 to 13.
Plot the coordinates from your table.

c) Join up the points to draw the graph of $y = 8 - x$ for values of x from 0 to 4.
Use a ruler to extend your line to show the graph of $y = 8 - x$ for values of x from –5 to 5.

Q3 For each of the following equations draw a graph for values of x from –5 to 5.

a) $y = -4x$ b) $y = \frac{x}{2}$ c) $y = 2x + 5$

d) $y = 4 - x$ e) $y = 8 - 3x$ f) $y = \frac{x}{4} + 1$

Q4 Draw a graph of the following equations for the given range of x-values.

a) $y = x + 7$ for $-7 \le x \le 0$ b) $y = -2x + 8$ for $0 \le x \le 5$

c) $y = 1.5x$ for $-5 \le x \le 5$ d) $y = 0.5x + 2$ for $-2 \le x \le 4$

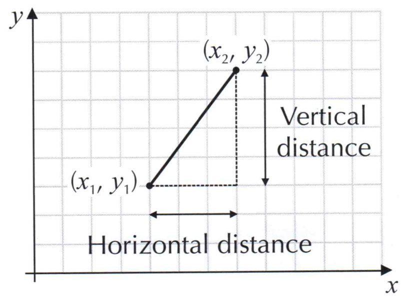
15.2 Gradients

The gradient of a straight line tells you how steep it is. The gradient comes in handy for lots of things (as you'll see later in this section), but first you need to know how to find it.

Learning Objective — Spec Ref A10:
Be able to find the gradient of a straight line.

To find the gradient of a line, divide the '**vertical distance**' (the change in the y-coordinates) between two points on the line by the '**horizontal distance**' (the change in the x-coordinates) between those points.

$$\text{Gradient} = \frac{\text{Vertical distance}}{\text{Horizontal distance}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$



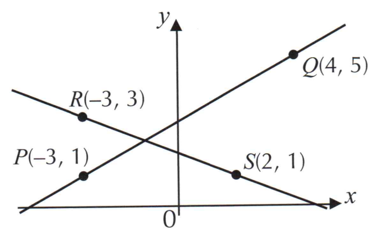
Make sure you subtract the x-coordinates and y-coordinates in the **same order** — i.e. if you do $y_2 - y_1$ on the numerator, you must do $x_2 - x_1$ on the denominator.

A line sloping **upwards** from left to right has a **positive gradient**.
A line sloping **downwards** from left to right has a **negative gradient**.

Regardless of the type of graph, the gradient always means '**y-axis units per x-axis units**'.

Example 1

Find the gradient of the line that passes through:
a) points $P(-3, 1)$ and $Q(4, 5)$ b) points $R(-3, 3)$ and $S(2, 1)$



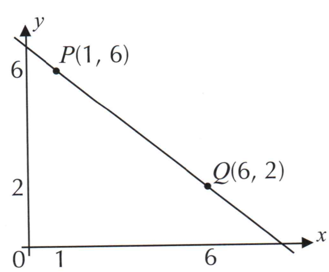
1. Call the coordinates of P (x_P, y_P), the coordinates of Q (x_Q, y_Q), the coordinates of R (x_R, y_R) and the coordinates of S (x_S, y_S).
2. Use the formula for the gradient.
3. The line through P and Q slopes upwards from left to right, so you should get a positive answer.
4. The line through R and S slopes downward from left to right, so you should get a negative answer.

a) Gradient of $PQ = \frac{y_Q - y_P}{x_Q - x_P} = \frac{5-1}{4-(-3)} = \mathbf{\frac{4}{7}}$

b) Gradient of $RS = \frac{y_R - y_S}{x_R - x_S} = \frac{3-1}{(-3)-2} = \mathbf{-\frac{2}{5}}$

Exercise 1

Q1 Points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are plotted on this graph.



a) Without doing any calculations, state whether the gradient of the line containing P and Q is positive or negative.

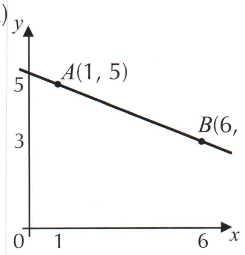
b) Calculate the vertical distance $y_2 - y_1$ between P and Q.

c) Calculate the horizontal distance $x_2 - x_1$ between P and Q.

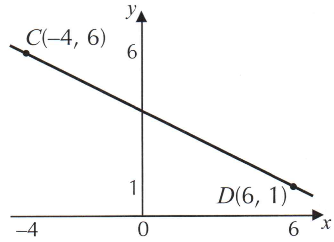
d) Find the gradient of the line containing P and Q.

Q2 Use the points shown to find the gradient of each of the following lines.

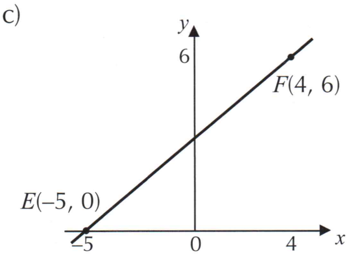
a)



b)

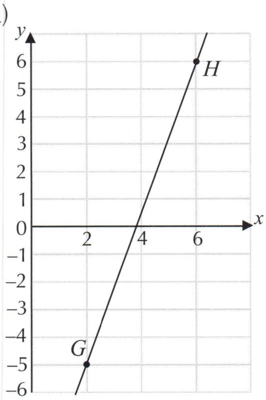


c)

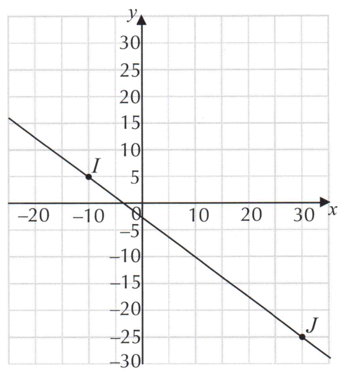


Q3 For each line shown below: (i) Use the axes to find the coordinates of each of the marked points.
(ii) Find the gradient of each of the lines.

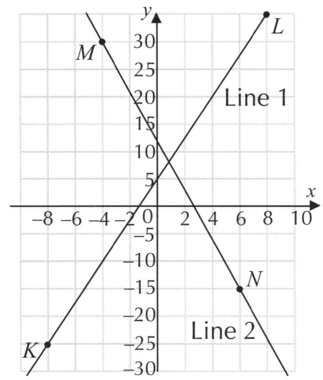
a)



b)



c)



Q4 a) Plot the points $U(-1, 2)$ and $V(2, 5)$ on a grid.

b) Find the gradient of the line joining points U and V.

Q5 a) Find the difference between the y-coordinates of the points $Y(2, 0)$ and $Z(-4, -3)$.

b) Find the difference between the x-coordinates of Y and Z.

c) Hence find the gradient of the line containing points Y and Z.

Q6 Find the gradients of the lines joining the following points.

a) $A(0, 4)$, $B(2, 10)$

b) $C(1, 3)$, $D(5, 11)$

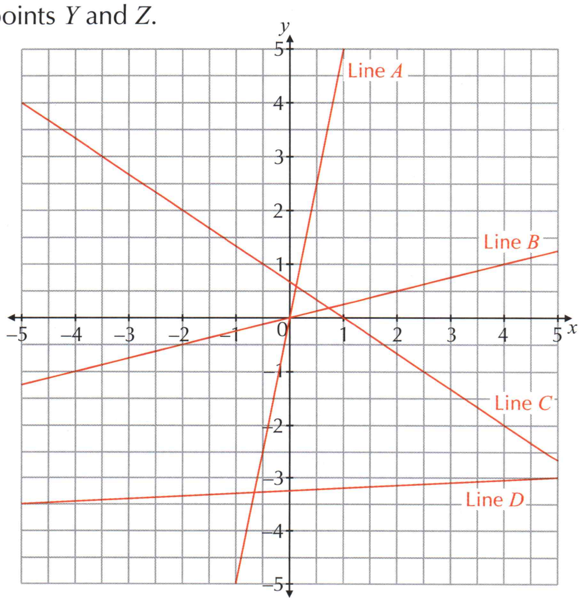
c) $E(-1, 3)$, $F(5, 7)$

d) $G(-3, -2)$, $H(1, -5)$

e) $I(5, -2)$, $J(1, 1)$

f) $K(-4, -3)$, $L(-8, -6)$

Q7 Find the gradients of lines A-D on the graph on the right.



15.3 Equations of Straight-Line Graphs

There are two key facts that help you find the equation of a straight line — the gradient and where it crosses the y-axis (the y-intercept). Armed with these two bits of information, there's nothing you can't do.

Gradients and y-intercepts

Learning Objective — Spec Ref A10:
Find the gradient and y-intercept of a line given its equation.

The equation of a straight line can be written in the form $\boldsymbol{y = mx + c}$.
E.g. for $y = 3x + 5$, $m = 3$ and $c = 5$.

When written in this form:

- m is the **gradient** of the line,
- c tells you the **y-intercept** — the point where the line crosses the y-axis.

If you're given an equation that isn't in $y = mx + c$ form, rearrange it into this format so that you can read off the values of m and c. For example:

Equation	$y = mx + c$ form	m	c
$y = 2 + 3x$	$y = 3x + 2$	3	2
$x - y = 0$	$y = x + 0$	1	0
$4x - 3 = 5y$	$y = \frac{4}{5}x - \frac{3}{5}$	$\frac{4}{5}$	$-\frac{3}{5}$

Make sure you don't mix up m and c when you get something like $y = 5 + 2x$. Remember, m is the number in front of the x and c is the number on its own. Watch out for **minus signs** too — both m and c can be **negative** (e.g. $y = -2x - 5$), so you have to include the minus sign when you state the gradient and y-intercept.

Example 1

Write down the gradient and the coordinates of the y-intercept of $y = 2x + 1$.

The equation is already in the form $y = mx + c$, so you just need to read the values for the gradient and y-intercept from the equation.

$y = (2)x + (1)$ — m = 2, c = 1

gradient = **2**
y-intercept = **(0, 1)**

Tip: The question asks for the coordinates of the y-intercept, so don't forget the x-coordinate (which is 0).

Example 2

Find the gradient and the coordinates of the y-intercept of $2x + 3y = 12$.

1. Rearrange the equation into the form $y = mx + c$.

$2x + 3y = 12$ $\quad (-\,2x)$

$3y = -2x + 12$ $\quad (\div\, 3)$

$y = \left(-\frac{2}{3}\right)x + (4)$ — $m = -\frac{2}{3}$, $c = 4$

2. Write down the values for the gradient and y-intercept. Notice that m is negative, so the line slopes downwards from left to right.

gradient = $\boldsymbol{-\frac{2}{3}}$
y-intercept = **(0, 4)**

Exercise 1

Q1 Write down the gradient and the coordinates of the y-intercept for each of the following graphs.

a) $y = 2x - 4$ b) $y = 5x - 11$ c) $y = -3x + 7$ d) $y = 4x$

e) $y = \frac{1}{2}x - 1$ f) $y = -x - \frac{1}{2}$ g) $y = 3 - x$ h) $y = 3$

Q2 Match the graphs to the correct equation from the box.

$y = x + 2$

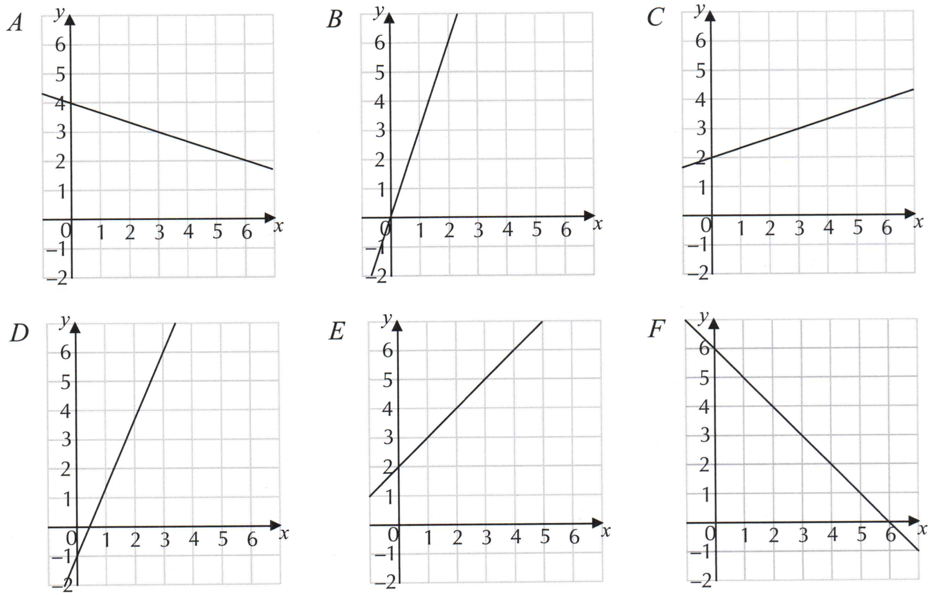
$y = \frac{7}{3}x - 1$

$y = -x + 6$

$y = 3x$

$y = -\frac{1}{3}x + 4$

$y = \frac{1}{3}x + 2$



Q3 Find the gradient and the coordinates of the y-intercept for each of the following graphs.

a) $3y = 9 - 3x$ b) $y - 5 = 7x$ c) $y + x = 8$ d) $x = 6 + 2y$

e) $3x + y = 1$ f) $3y - 6x = 15$ g) $4x = 5y - 5$ h) $8x - 2y = 14$

i) $5x + 4y = -3$ j) $4y - 6x + 8 = 0$ k) $6x - 3y + 1 = 0$ l) $\frac{1}{2} = -4x - 2y$

Finding the Equation of a Straight Line

Learning Objective — Spec Ref A9:
Find the equation of a straight line in the form $y = mx + c$.

Prior Knowledge Check: Be able to find the gradient of a line. See p.180.

You can find the equation of a line using its **gradient** and **one point** on the line.
First, substitute the values of the **gradient** (m) and the **coordinates** of the known point (x, y) into $y = mx + c$.
You'll be left with an equation where c is the only unknown, so **solve** this equation to find the value of c.
Finally, put your values of m and c into $y = mx + c$ to give the **equation of the line**.

If you only know **two points** on the line, you can calculate the gradient of the line using the method on p.180. Then follow the method above (using either of the two points) to find the **equation of the line**.

Example 3

Find the equation of the straight line that passes through the points $A(-3, -4)$ and $B(-1, 2)$.

1. Write down the equation for a straight line.
   $y = mx + c$ (m = gradient, c = y-intercept)

2. Find the gradient (m) of the line.
   gradient $(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{-1-(-3)} = \frac{6}{2} = 3$
   So the equation of the line must be $y = 3x + c$.

3. Substitute the value for the gradient and the x and y values for one of the points into $y = mx + c$, then solve to find c.
   At point B, $x = -1$ and $y = 2$.
   $2 = 3 \times (-1) + c$
   $2 = -3 + c \Rightarrow c = 5$

4. Finally, rewrite the equation using your values of m and c.
   So the equation of the line is $\mathbf{y = 3x + 5}$

Exercise 2

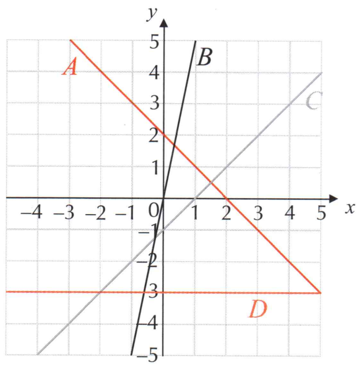
Q1 Find the equations of the following lines based on the information given.

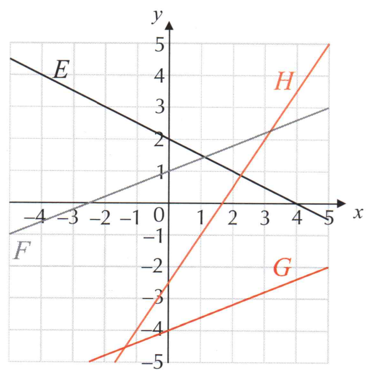
a) gradient = 8, passes through (0, 2)
b) gradient = –1, passes through (0, 7)
c) gradient = 3, passes through (1, 10)
d) gradient = $\frac{1}{2}$, passes through (4, –5)
e) gradient = –7, passes through (2, –4)
f) gradient = 5, passes through (–3, –7)

Q2 Find the equations of the lines passing through the following points.

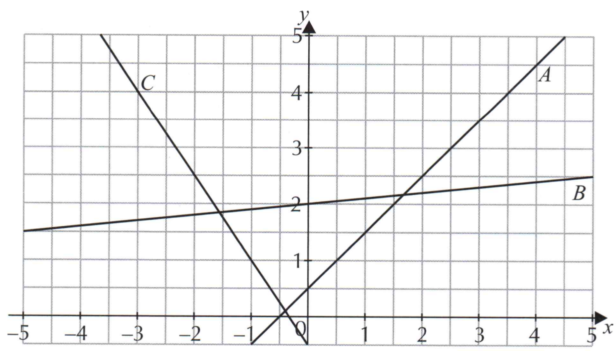
a) (3, 7) and (5, 11)
b) (5, 1) and (2, –5)
c) (4, 1) and (–3, –6)
d) (–2, 1) and (1, 7)
e) (2, 8) and (–1, –1)
f) (–3, 2) and (–2, 5)
g) (–7, 8) and (–1, 2)
h) (2, –1) and (4, –9)
i) (3, 4) and (–5, –8)

Q3 Find the equations of the lines A to H shown below. Write all your answers in the form $y = mx + c$.





Q4 Find the equations for the lines shown on the graph on the right.
Give your answers in the form $y = mx + c$.



15.4 Parallel and Perpendicular Lines

I told you gradients were going to come in handy — now you're going to use them to identify parallel and perpendicular lines. Remember, parallel lines are always the same distance apart, and never meet.

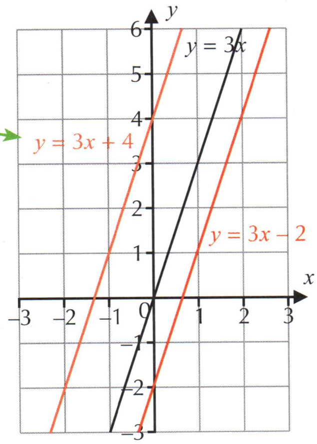
Parallel Lines

Learning Objectives — Spec Ref A9:
- Identify parallel lines from their equations.
- Be able to find the equation of a parallel line.

Prior Knowledge Check:
Know how to identify the gradient of a line and be able to find the equation of a line. See p.180-184.

Lines that are **parallel** have the **same gradient** — so their equations (in $y = mx + c$ form) all have the same value of $\boldsymbol{m}$.

For example, the lines $y = 3x$, $y = 3x - 2$ and $y = 3x + 4$ are all parallel.



To **check** if two lines are parallel, **rearrange** their equations so that they're both in $y = mx + c$ form, then **compare** the values of m.

If you have two parallel lines, A and B, and you know the **equation** of line A and **one point** on line B, you can find the equation of line B. First, find the **gradient** of line A (you can just read off this value if it's in $y = mx + c$ form). You know that line B has the **same gradient** as line A (as they're parallel), and you know the **coordinates** of one point on line B, so now you can use the method from p.183 to find the equation of line B.

Example 1

Which of the following lines is parallel to the line $2x + y = 5$?

A: $y = 3 - 2x$ B: $x + y = 5$ C: $y - 2x = 6$

1. Rearrange the equation into the form $y = mx + c$ to find its gradient.

$2x + y = 5$
$y = 5 - 2x$
$y = -2x + 5$, so the gradient $(m) = -2$.

2. Rearrange the other equations in the same way. Any that have $m = -2$ will be parallel to $2x + y = 5$.

A:	B:	C:
$y = 3 - 2x$	$x + y = 5$	$y - 2x = 6$
$y = -2x + 3$	$y = -x + 5$	$y = 2x + 6$
$m = -2$	$m = -1$	$m = 2$

So **line A** is parallel to $2x + y = 5$.

Example 2

Find the equation of line L, which passes through the point (5, 8) and is parallel to $y = 3x + 2$.

1. Find the gradient of the line $y = 3x + 2$.
2. The lines are parallel, so line L will have the same gradient.

Equation of a straight line: $y = mx + c$
The gradient of $y = 3x + 2$ is 3.
So the equation of line L must be $y = 3x + c$.

3. Substitute the values for x and y at the point (5, 8) into the equation. Solve to find c and hence the equation of line L.

At (5, 8), $x = 5$ and $y = 8$:
$8 = 3(5) + c$
$8 = 15 + c$
$c = -7$, so the equation of line L is $\boldsymbol{y = 3x - 7}$.

Exercise 1

Q1 Write down the equations of three lines that are parallel to: a) $y = 5x - 1$ b) $x + y = 7$

Q2 Work out which of the following are the equations of lines parallel to: a) $y = 2x - 1$ b) $2x - 3y = 0$

A: $y - 2x = 4$ B: $2y = 2x + 5$ C: $2x - y = 2$

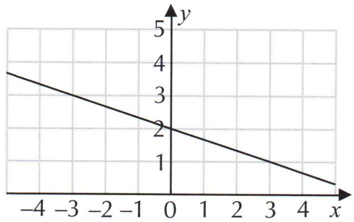
D: $2x + y + 7 = 0$ E: $3y + 2x = 2$ F: $6x - 9y = -2$

Q3 Which of the lines listed below are parallel to the line shown in the diagram?

A: $y + 3x = 2$ B: $3y = 7 - x$

C: $y = 4 - 3x$ D: $x - 3y = 8$

E: $y = 3 - \frac{1}{3}x$ F: $6y = -2x$



Q4 For each of the following, find the equation of the line which is parallel to the given line and passes through the given point. Give your answers in the form $y = mx + c$.

a) $y = 5x - 7$, (1, 8) b) $y = 2x$, (–1, 5) c) $y = \frac{1}{2}x + 3$, (6, –7)

d) $y = 8x - 1$, (–3, –5) e) $2y = 6x + 3$, (–3, 4) f) $y = 7 - 9x$, (1, –11)

g) $x + y = 4$, (8, 8) h) $2x + y = 12$, (–4, 0) i) $x + 3y + 1 = 0$, (–9, 9)

Perpendicular Lines

Learning Objectives — Spec Ref A9:

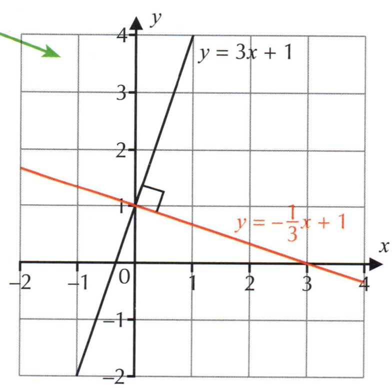
- Identify perpendicular lines from their equations.
- Be able to find the equation of a perpendicular line.

Prior Knowledge Check: Know how to identify the gradient of a line and be able to find the equation of a line. See p.180-184.

Lines that are **perpendicular** cross at a **right angle**.
If the gradient of a line is m, then the gradient of a line perpendicular to it is $-\frac{1}{m}$.

The product of the gradients of two perpendicular lines is **–1**: $m \times -\frac{1}{m} = -1$

For example, the lines $y = 3x + 1$ and $y = -\frac{1}{3}x + 1$ are perpendicular because $3 \times -\frac{1}{3} = -1$.



To **check** if two lines are perpendicular, **rearrange** their equations so that they're both in $y = mx + c$ form, then see if their values of m **multiply** to give **–1**.

If you have two perpendicular lines, P and Q, and you know the **equation** of line P and **one point** on line Q, you can find the equation of line Q. First, find the **gradient** of line P, then find the gradient of line Q (by doing –1 ÷ gradient of line P). You know the **coordinates** of one point on line Q and its gradient, so now use the method from p.183 to find the equation of line Q.

Example 3

Find the equation of line B, which is perpendicular to $y = 5x - 2$ and passes through the point (10, 4).

1. Use the gradient of $y = 5x - 2$ to find the gradient of line B.

The line $y = 5x - 2$ has gradient $m = 5$.

So the gradient of line B is $-\frac{1}{m} = -\frac{1}{5}$.

2. Substitute this gradient into the equation for a straight line along with the values for x and y at the point given.

The equation of line B is $y = -\frac{1}{5}x + c$.

At the point (10, 4), $x = 10$ and $y = 4$

$4 = -\frac{1}{5}(10) + c$

3. Solve this equation to find c and hence the equation of line B.

$4 = -2 + c$

$c = 6$

So the equation of line B is $\boldsymbol{y = -\frac{1}{5}x + 6}$.

Exercise 2

Q1 Find the gradient of a line which is perpendicular to a line with gradient:

a) 6 b) –3 c) $-\frac{1}{4}$ d) 12

e) –7 f) $\frac{2}{3}$ g) –2 h) 1.5

i) 0.3 j) –4.5 k) $-\frac{4}{3}$ l) $3\frac{1}{2}$

Q2 Write down the equation of any line which is perpendicular to:

a) $y = 2x + 3$ b) $y = -3x + 11$ c) $y = 5 - 6x$

d) $2y = 5x + 1$ e) $x + y = 2$ f) $5x - 10y = 4$

Q3 Match the following equations into pairs of perpendicular lines.

A: $y = 3x - 6$ B: $y = 2x - 3$ C: $8 - x = 3y$

D: $4x - 6y = 3$ E: $y + 3x = 2$ F: $2y - 3x = 6$

G: $x + 2y = 8$ H: $4y + 8x = 6$ I: $8y - 4x = 3$

J: $3y - 4 - x = 0$ K: $4x + 6y - 3 = 0$ L: $2y = 8 - 3x$

Q4 For each of the following, find the equation of the line which is perpendicular to the given line and passes through the given point. Give your answers in the form $y = mx + c$.

a) $y = -3x + 1$, (9, 8) b) $y = \frac{1}{2}x - 5$, (3, –4)

c) $y = \frac{1}{4}x - 7$, (1, –9) d) $y = \frac{4}{3}x + 15$, (12, –1)

e) $y = 8 - 2.5x$, (15, 2) f) $x + y = 8$, (3, 0)

g) $2y = 6x - 1$, (–6, 1) h) $3y + 8x = 1$, (8, 7)

i) $x + 2y = 6$, (1, 9) j) $x - 5y - 11 = 0$, (–2, 8)

15.5 Line Segments

A line segment is just part of a line between two end points — for example, line segment AB is the line joining points AB, rather than the line that passes through A and B and goes on forever in both directions.

Midpoint of a Line Segment

Learning Objective — Spec Ref G11:
Be able to find the midpoint of a line segment.

The **midpoint** of a line segment is **halfway** between the end points.

The ***x*-coordinate** of the midpoint is the **average** of the ***x*-coordinates** of the end points — so **add** the *x*-coordinates of the end points together and **divide by 2**.
The ***y*-coordinate** of the midpoint is the **average** of the ***y*-coordinates** of the end points — so **add** the *y*-coordinates of the end points together and **divide by 2**.

Example 1

Find the midpoint of the line segment *AB*, shown on the right.

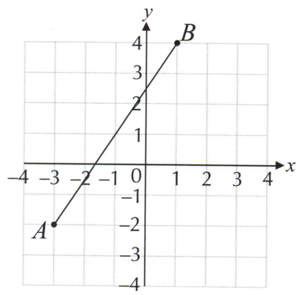
1. Write down the coordinates of the end points A and B.

   $A(-3, -2)$ and $B(1, 4)$

2. Find the average of the x-coordinates by adding them together and dividing by 2. Find the average of the y-coordinates in the same way.

   $\left(\frac{-3+1}{2}, \frac{-2+4}{2}\right) = (-1, 1)$

   The midpoint has coordinates **(–1, 1)**.



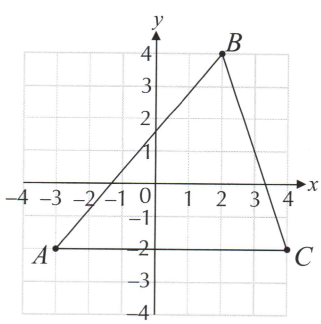
Exercise 1

Q1 Find the coordinates of the midpoint of the line segment AB, where A and B have coordinates:

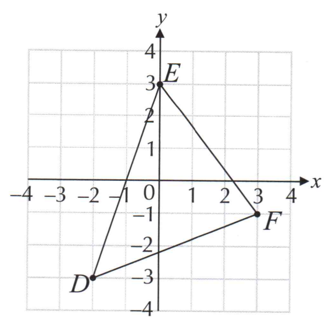
a) $A(8, 0)$, $B(4, 6)$
b) $A(-2, 3)$, $B(6, 5)$
c) $A(4, -7)$, $B(-2, 1)$
d) $A(-3, 0)$, $B(9, -2)$
e) $A(-6, -2)$, $B(-4, 6)$
f) $A(-1, 3)$, $B(-1, -7)$
g) $A(-\frac{1}{2}, 4)$, $B(\frac{1}{2}, -3)$
h) $A(2p, q)$, $B(6p, 7q)$
i) $A(8p, 2q)$, $B(2p, 14q)$

Q2 Find the midpoint of each side of the following triangles.

a)



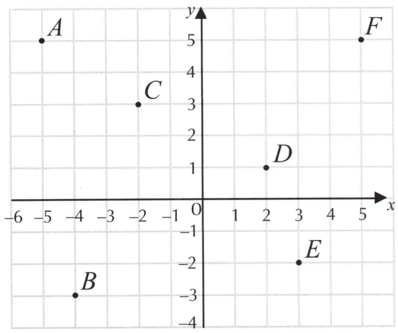
b)



Q3 Point A has the coordinates $A(1, 8)$. The midpoint, M, of the line segment AB has the coordinates $M(5, 3)$. Find the coordinates of B.

Q4 The coordinates of the endpoint, C, and midpoint, M, of the line segment CD are $C(6, -7)$ and $M(2, -1)$. Find the coordinates of point D.

Q5 Use the diagram on the right to find the midpoints of the following line segments.



a) AF b) AC c) DF

d) BE e) BF f) CE

Length of a Line Segment

Learning Objective — Spec Ref G11:
Be able to find the length of a line segment.

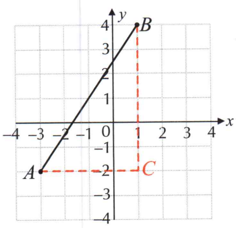
Prior Knowledge Check:
Be able to use Pythagoras' theorem. See p.318.

To find the **length** of a line segment (or the **distance** between two points), use **Pythagoras' theorem**:

1. Create a **right-angled triangle** using the line segment as the **hypotenuse**.
2. Work out the lengths of the two **shorter sides** — find the **difference** between the **x-coordinates** of the end points to find the length of the **horizontal** side, then find the **difference** between the **y-coordinates** of the end points to find the length of the **vertical** side.
3. Put these values into Pythagoras' theorem as a and b: $a^2 + b^2 = h^2$ and solve to find h.

Example 2

Calculate the length of the line segment AB. Give your answer to three significant figures.



1. Think of the line segment as the hypotenuse of a right-angled triangle ABC.
2. Calculate the length of AC and BC.
3. Calculate the length of AB using Pythagoras' theorem.

Length of $AC = 1 - (-3) = 4$
Length of $BC = 4 - (-2) = 6$
$AB = \sqrt{AC^2 + BC^2}$
$AB = \sqrt{4^2 + 6^2}$
$AB = \sqrt{52} = \mathbf{7.21}$ (to 3 s.f.)

Exercise 2

Q1 Find the length of the line segments with the following end point coordinates. Give your answers to 3 significant figures.

a) (5, 9) and (1, 6) b) (15, 3) and (11, 8) c) (5, 4) and (4, 1)

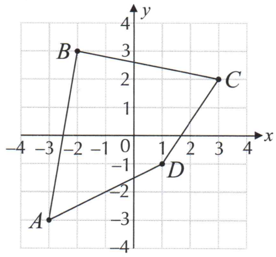
d) (3, 7) and (3, 14) e) (–1, 9) and (9, –3) f) (9, –4) and (–1, 12)

g) (1, –2) and (8, 2) h) (–3, 7) and (–2, –3) i) (–1, –1), (–5, 9)

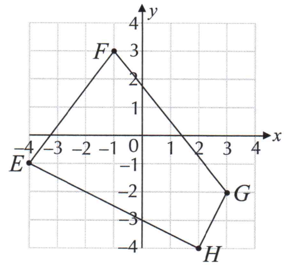
j) (2, 4) and (–1, –4) k) (0, –1) and (4, 8) l) (–2, –1) and (11, 8)

Q2 Find the length of each side of the shapes below. Give your answers to 3 significant figures.

a)



b)



Coordinates and Ratio

Learning Objective — Spec Ref G11:
Use ratios to find coordinates.

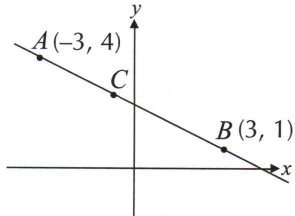
Prior Knowledge Check:
Work with ratios. See p.42-44.

Ratios can be used to express where a **point** is on a **line**. For example, if point R is a point on the line PQ such that $PR : RQ = 2 : 3$, then R is $\frac{2}{2+3} = \frac{2}{5}$ of the way from P to Q.

You can use ratios to find the **coordinates** of a point by treating the x- and y-coordinates **separately**.

Example 3

Points A, B and C lie on a straight line. Point A has coordinates (–3, 4) and point B has coordinates (3, 1). Point C lies on line segment AB such that $AC : CB = 1:2$. Find the coordinates of point C.



1. Find the difference between the x- and y-coordinates of A and B.
   x difference: $3 - (-3) = 6$
   y difference: $1 - 4 = -3$

2. Use the given ratio to see how far along AB point C is.
   C is $\frac{1}{1+2} = \frac{1}{3}$ of the way along AB, so x: $\frac{1}{3} \times 6 = 2$
   y: $\frac{1}{3} \times -3 = -1$

3. Add the results to the coordinates of A to get C.
   x-coordinate: $-3 + 2 = -1$
   y-coordinate: $4 + -1 = 3$
   Point C lies at **(–1, 3)**.

Exercise 3

Q1 Point C lies on the line segment AB. Find the coordinates of C given that:

a) $A(3, -3)$, $B(6, 6)$ $AC:CB = 1:2$
b) $A(-3, 5)$, $B(9, 1)$ $AC:CB = 3:1$
c) $A(0, 4)$, $B(10, -1)$ $AC:CB = 2:3$
d) $A(-20, 1)$, $B(8, -13)$ $AC:CB = 3:4$

Q2 Each set of three points below lies on a straight line. Use the points to find the specified ratios.

a) Find $AB:BC$ given $A(0, 0)$, $B(2, 2)$ and $C(6, 6)$.
b) Find $DE:EF$ given $D(1, 0)$, $E(-3, 4)$ and $F(-4, 5)$.
c) Find $GH:HI$ given $G(-1, -2)$, $H(5, 2)$ and $I(14, 8)$.

Q3 Point T lies on the line segment SU. Find the coordinates of U given that:

a) $S(6, 2)$, $T(12, -4)$ $ST:TU = 3:2$
b) $S(-2, -4)$, $T(18, 11)$ $ST:TU = 5:4$

Review Exercise

Q1 Draw the following lines.

a) $y = -2$ b) $x = 4$ c) $y = 4x - 7$

d) $y = -5x + 1$ e) $2x + y = 3$ f) $x = 2y - 5$

Q2 Find the gradient of the line joining the following points.

a) (2, 3) and (4, 7) b) (1, 4) and (3, 2) c) (3, 1) and (5, 4)

d) (–1, –2) and (2, –4) e) (4, –3) and (8, –1) f) (3, –4) and (–5, –2)

Q3 Match each of the equations with the graphs *A*-*E*.

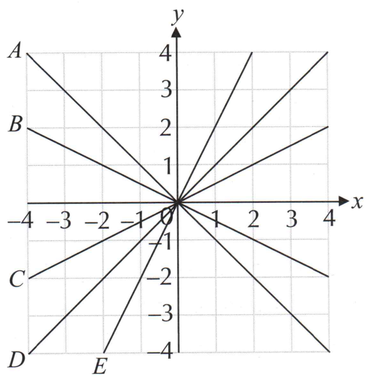
$y = 2x$

$y + x = 0$

$y - x = 0$

$y = -0.5x$

$2y = x$



Q4 a) The line through points $A(-1, 4)$ and $B(2, a)$ has a gradient of 5. Find a.

b) Points $C(2, 7)$ and $D(b, -2)$ lie on a line with a gradient of –3. Find b.

Q5 Find the gradient and the coordinates of the y-intercept of the following lines.

a) $y = 5x - 9$ b) $y = 11 - 2x$ c) $y = -8 + 3x$

d) $6 = 2x + 3y$ e) $x = 4y - 7$ f) $2y + 9 = 8x$

Q6 Match the equations below and the graphs shown on the right.

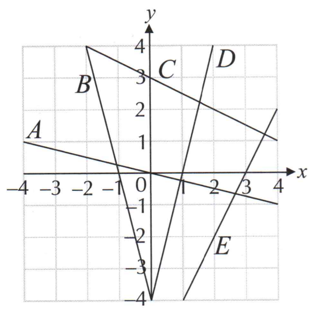
$x + 2y = 6$

$x = -4y$

$y = 2x - 6$

$y = 4(x - 1)$

$4x + y + 4 = 0$

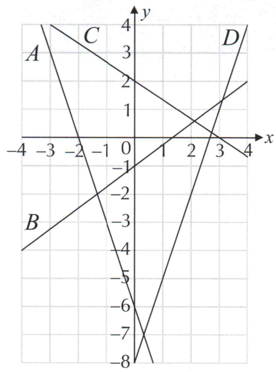


Q7 Find the equations of the lines through the following pairs of points.

a) (1, 2) and (0, 6) b) (8, 7) and (0, –9) c) (4, 5) and (6, 6)

d) (5, –8) and (–1, 10) e) (2, –9) and (5, –15) f) (2, 0) and (–6, –2)

Q8 Find the equations of the lines shown on the graph on the right.



Q9 Find the equation of the line parallel to the given line that passes through the given point.

a) $y = 10 - 7x$, $(1, 11)$ b) $y + 2 = 9x$, $(2, -16)$

c) $3x + 2y = 23$, $(8, 5)$ d) $x - 11 = 3y$, $(18, 1)$

Q10 Find the equation of the line perpendicular to the given line and passing through the given point.

a) $y = 4x + 1$, $(12, 9)$ b) $y = 5 - 2x$, $(14, 8)$ c) $y + 16 = 3x$, $(-6, -1)$

d) $x + 3y = 17$, $(-4, 2)$ e) $3x + 2y = 8$, $(9, -1)$ f) $x - 3 = 5y$, $(-2, 2)$

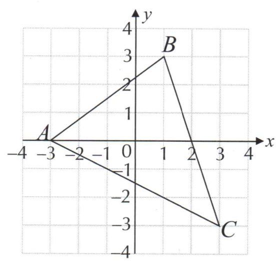
Q11 Decide whether each of the following lines are parallel to the line $y = \frac{1}{2}x + 8$, perpendicular to it, or neither.

A: $y = 3 - 2x$ B: $8y - 4x = 5$ C: $y - 2x = 4$ D: $3x - 6y = 1$

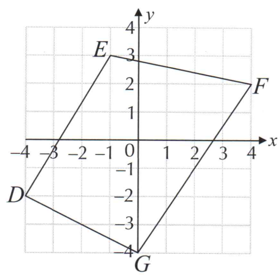
E: $4y + 2x = 8$ F: $y = 2(x - 4)$ G: $2y = x - 7$ H: $y + 2x = 8$

Q12 Find the gradient, length and midpoint of each side of the following shapes. Give your answers to 3 significant figures where appropriate.

a)

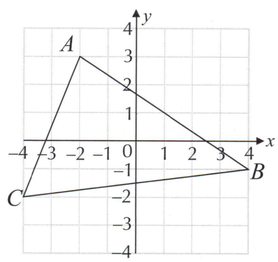


b)

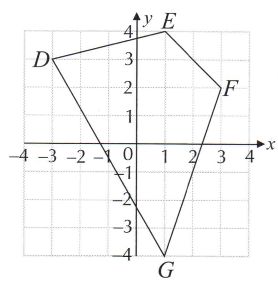


Q13 Find the total perimeter of the following shapes. Give your answers to 3 significant figures.

a)



b)



Q14 Jonah plots the points $A(-6, -2)$, $B(p, q)$ and $C(10, 6)$ on the line segment AC.

a) Given that $AB : BC = 1 : 3$, find the values of p and q.

b) Calculate the length of the line segment AB. Give your answer to one decimal place.

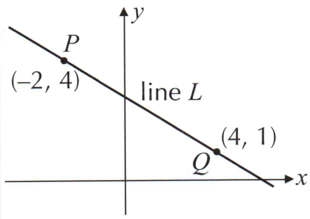
c) Jonah draws another point, D, on the line segment AC such that $AB : DC = 4 : 1$. What is the length of the line segment DC?

Exam-Style Questions

Q1 Line A has equation $5x + 2y - 8 = 0$.
Line B is parallel to line A but has a y-intercept which is triple that of line A.
Find the equation of line B.

[3 marks]

Q2 Line L passes through the points P and Q, as shown below.



a) Find the equation of line L.

[3 marks]

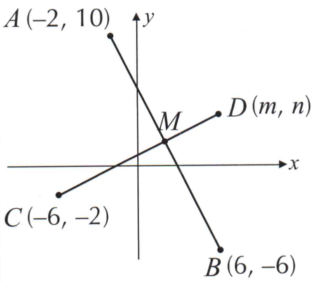
b) Lines L and M are perpendicular and intersect at the point (2, 2). Find the equation of line M.

[3 marks]

Q3 A straight line passes through the points (9, 110) and (–5, –100).
Does the point (33, 450) lie on the line? Justify your answer.

[4 marks]

Q4 The diagram below shows lines AB and CD.
Line CD intersects AB at the midpoint, M, of line AB.



a) Find the midpoint, M, of line AB.

[2 marks]

b) Given that $CM : MD = 2 : 1$, find the length of the line segment CD. Give your answer to one decimal place.

[3 marks]

Q5 A triangle has vertices $A(1, 5)$, $B(3, -1)$ and $C(6, 0)$.
Is triangle ABC right-angled? Justify your answer.

[4 marks]