13.1 Solving Inequalities

Inequality symbols can be used to compare numbers (e.g. $3 < 5$ and $10 > 2$) or to compare algebraic expressions (e.g. $5x + 4 < 2x - 1$). They can also be used to describe a range of values (e.g. $x < 4$ or $-3 > y$).

Simple Inequalities

Learning Objectives — Spec Ref N1/A22:
- Solve simple linear inequalities.
- Represent inequalities on a number line and using set notation.

Prior Knowledge Check: Be able to solve linear equations (see p.113-115) and use set notation (see p.236-237).

Inequalities are written using the following symbols:

> greater than | < less than | ≥ greater than or equal to | ≤ less than or equal to

Greater than (>) and less than (<) are called **strict inequalities**.

The rules for **solving inequalities** are very similar to the rules for **solving equations**. The only extra rule you need to know is if you're **multiplying** or **dividing** by a **negative number** then the inequality sign **'flips over'**. For example, to solve $-2x > 6$ you need to divide both sides by –2 and 'flip' the inequality sign to give $x < -3$.

The **solution** to an inequality will usually be an inequality with x on one side and a number on the other. You can also write the solution using **set notation** (see p.236-237) — e.g. if the solution is $x < -3$, you can write it as $\{x : x < -3\}$.

Example 1

a) **Solve the inequality $2x + 4 < 8$.**

Just treat the inequality like an equation. Subtract 4 from both sides then divide by 2.

$2x + 4 < 8$

$2x < 4$

$\mathbf{x < 2}$

b) **Solve $-\frac{x}{3} \geq -5$, giving your answer using set notation.**

1. You need to multiply by –3 to leave x on its own. Remember to 'flip' the inequality sign as you're multiplying by a negative number.
2. Don't forget to write your answer using set notation.

$-\frac{x}{3} \geq -5$

$x \leq -5 \times -3$

$x \leq 15$

$\mathbf{\{x : x \leq 15\}}$

Tip: To see why the sign 'flips', rearrange the inequality to give $5 \geq \frac{x}{3}$, then multiply by 3 to leave $15 \geq x$ (which is the same as $x \leq 15$).

Exercise 1

Q1 Solve the following inequalities.

a) $3x \geq 36$ b) $-96 \leq -12x$ c) $-\frac{x}{3} < -28$ d) $11 \leq -\frac{x}{7}$

e) $4x + 11 < 23$ f) $5x + 3 \leq 43$ g) $-3x - 7 \geq -1$ h) $65 < 7x - 12$

Q2 Solve these inequalities, giving your answers using set notation.

a) $4x < 16$ b) $-33 \leq 11x$ c) $\frac{x}{6} > -3$ d) $-\frac{x}{5} > -1$

e) $2x + 15 < 21$ f) $-5x - 8 \geq 12$ g) $\frac{x}{3} - 13 \geq -1$ h) $44 < 8x + 16$

Q3 Solve the following inequalities.

a) $\frac{x+2}{3} < 1$ b) $\frac{x+4}{5} \geq 2$ c) $\frac{x-8}{2} > 7$ d) $\frac{x-8}{4} \leq 0.5$

e) $\frac{x}{4} - 2.5 \geq 1$ f) $1 < \frac{x+5.5}{2}$ g) $-1 > \frac{x-3.1}{8}$ h) $-\frac{x}{3} + 1.3 \leq 3.3$

Q4 Solve the following inequalities, giving your answers using set notation.

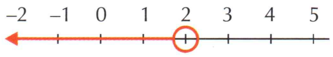
a) $4x + 2 < 2x - 2$ b) $3x + 5 \leq 4 + x$ c) $3x - 3 \geq -1 + x$ d) $6 - x < 7x - 2$

e) $\frac{x}{2} - 5 \geq 3 - \frac{x}{2}$ f) $1 - 2x < \frac{x+3}{2}$ g) $2x + 4 > \frac{2x-3}{8}$ h) $\frac{x}{4} + \frac{3}{2} \leq \frac{1}{4} - x$

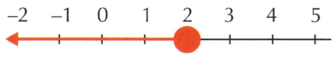
You can also represent the solution to a simple inequality on a **number line**, using a **circle** to show the **boundary value** and an **arrow** to show which values are part of the solution.

Use an **open circle** (○) to represent **strict inequalities** (i.e < or >) and a **solid circle** (●) to represent **non-strict inequalities** (i.e ≤ or ≥). For example:

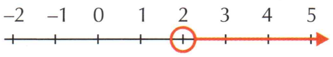
$x < 2$



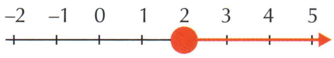
$x \leq 2$



$x > 2$



$x \geq 2$



Example 2

Solve the following inequalities. Show the solutions on a number line.

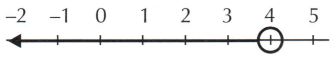
a) $x + 5 < 9$

1. Solve the inequality in the usual way — subtract 5 from both sides.
2. It's a strict inequality so use an open circle to show that the number is not included.

$x + 5 < 9$

$x < 9 - 5$

$\mathbf{x < 4}$



b) $1 + 2x \leq 4x - 5$

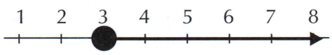
1. Solve the inequality in the usual way.
2. It's a non-strict inequality so use a solid circle to show that the number is included.

$1 + 2x \leq 4x - 5$

$1 + 5 \leq 4x - 2x$

$6 \leq 2x$

$\mathbf{3 \leq x}$ (or $\mathbf{x \geq 3}$)



Exercise 2

For Questions 1-3, solve each inequality and show the solution on a number line.

Q1 a) $x + 9 > 14$ b) $x + 3 \leq 12$ c) $x - 2 \geq 14$ d) $x - 7 < 19$

e) $18 < x + 2$ f) $12 \leq x - 4$ g) $1 > x - 17$ h) $31 \geq x + 30$

Q2 a) $3x \geq 9$ b) $5x < 25$ c) $4x < -16$ d) $9x > -72$

e) $\frac{x}{2} \geq 3$ f) $2 > \frac{x}{5}$ g) $\frac{x}{3} < 8$ h) $\frac{x}{7} \leq 5$

Q3 a) $4x + 3 > x + 15$ b) $x + 2 \leq -3x + 14$ c) $5x + 20 > 7x - 25$

Compound Inequalities

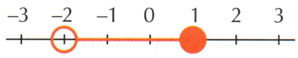
Learning Objectives — Spec Ref N1/A22:

- Solve compound linear inequalities.
- Represent compound inequalities on a number line and using set notation.

A **compound inequality** combines multiple inequalities into one. For example, $3 < x \leq 9$ means that $x > 3$ **and** $x \leq 9$ — so if x is an integer, the solutions are 4, 5, 6, 7, 8 and 9.

To solve a compound inequality, you can just **split it up** into two simple inequalities and solve each one separately. Then **combine** your solutions back into one inequality at the end.

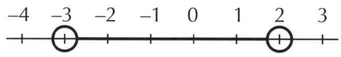
Just like for simple inequalities, you can give a solution using **set notation** or on a **number line** — this time the solution will be shown by two circles with a line between them. For example, if the solution is $-2 < x \leq 1$, then in set notation you write $\{x : -2 < x \leq 1\}$ and on a number line it would be:



Example 3

Solve the inequality $-4 < 2x + 2 < 6$. Show your solution on a number line.

1. Write down two separate inequalities. $-4 < 2x + 2$ and $2x + 2 < 6$
2. Solve the inequalities separately.
   ① $-4 < 2x + 2$ ② $2x + 2 < 6$
   $-6 < 2x$ $2x < 4$
   $-3 < x$ $x < 2$
3. Combine the inequalities into one and draw the number line — use open circles as both inequalities are strict. So $\mathbf{-3 < x < 2}$



Tip: You can solve without splitting it up — just do the same thing to each part of the inequality. For example:
$-4 < 2x + 2 < 6$
$-6 < 2x < 4$
$-3 < x < 2$

Exercise 3

Q1 Solve the following inequalities. Show each solution on a number line.

a) $7 < x + 3 \leq 15$ b) $2 \leq x - 4 \leq 12$ c) $-1 \leq x + 5 \leq 4$ d) $21 \leq x - 16 \leq 44$

Q2 Solve the following inequalities, giving your answers using set notation.

a) $16 < 4x < 28$ b) $32 < 2x \leq 42$ c) $27 < 4.5x \leq 72$ d) $-22.5 \leq 7.5x < 30$

Q3 Solve the following inequalities:

a) $17 < 6x + 5 < 29$ b) $8 < 3x - 4 \leq 26$ c) $-42 < 7x + 7 \leq 91$

d) $-2 < 2x + 3 < 5$ e) $5.1 \leq -x + 2.5 < 9.7$ f) $-5.6 < -x - 6.8 < 12.9$

Q4 Find the integer solutions that satisfy both of the inequalities.

a) $24 > 8x$ and $9x \geq -18$ b) $-3 > 2x + 5$ and $-4x \leq 32$ c) $9 - x > 8x + 9$ and $2x + 9 > 3$

13.2 Quadratic Inequalities

An inequality where the highest power of x is x^2 is called a quadratic inequality.

Learning Objective — Spec Ref A22:
Solve quadratic inequalities, representing the solution on a number line and using set notation.

Prior Knowledge Check:
Be able to solve quadratic equations (see Section 11) and sketch quadratic graphs (see p.194-197).

In order to solve a **quadratic inequality** you need to sketch a **quadratic graph**. To do this, first solve the equivalent **quadratic equation**. **Replace** the inequality sign with an **equals sign**, then solve the quadratic equation using a method from Section 11 — the solutions are the **boundary values** of the inequality. Next, rearrange the inequality so you have $ax^2 + bx + c$ on one side and **0** on the other. **Sketch the graph** of $y = ax^2 + bx + c$ — the shape of the graph helps you to determine where the quadratic inequality is **satisfied**.

- If the inequality is satisfied **between** the two boundary values then your solution will be a **compound inequality**, e.g. $-5 < x < 5$ or $-1 \leq x \leq 3$.
- If the inequality is satisfied **outside** the two boundary values then your solution will be a **pair** of linear inequalities, e.g. $x \leq -1$ or $x \geq 3$.

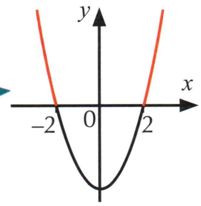
Tip: The solutions of the quadratic equation are needed because they tell you where the graph crosses the x-axis — i.e. where it's positive and where it's negative.

Just like for linear inequalities, you can give your solution using **set notation** or on a **number line**.

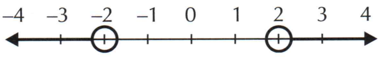
Example 1

Solve the inequality $x^2 > 4$. Show your solution on a number line.

1. Rewrite the inequality with an equals sign. $x^2 = 4$
2. Take the square root of both sides to solve the equation. $x = \pm\sqrt{4}$, $x = -2$ or $x = 2$
3. $x^2 > 4 \Rightarrow x^2 - 4 > 0$ so sketch the graph of $y = x^2 - 4$. It's u-shaped and crosses the x-axis at $x = -2$ and $x = 2$.



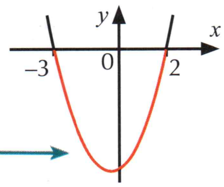
4. Now, $x^2 - 4$ is greater than zero when the graph is above the x-axis (the orange part of the graph). So $\boldsymbol{x < -2}$ or $\boldsymbol{x > 2}$
5. Show both simple inequalities on the same number line — use open circles as you have strict inequalities.



Example 2

Find the integer solutions to the inequality $x + x^2 \leq 6$. Give your answer using set notation.

1. Rewrite the inequality with an equals sign. $x + x^2 = 6$
2. Rearrange the equation and solve it by factorisation. $x^2 + x - 6 = 0$, $(x + 3)(x - 2) = 0$, $x = -3$ or $x = 2$
3. $x + x^2 \leq 6 \Rightarrow x^2 + x - 6 \leq 0$ so sketch the graph of $y = x^2 + x - 6$. It is u-shaped and crosses the x-axis at $x = 2$ and $x = -3$.



4. $x^2 + x - 6$ is less than or equal to 0 below the x-axis (the orange part of the graph) so the solution is a compound inequality. So $-3 \leq x \leq 2$
5. Write the integer solutions to this inequality using set notation. The set of solutions is **{−3, −2, −1, 0, 1, 2}**.

Tip: It's not a strict inequality so remember to include the boundary values in the solution.

Exercise 1

Q1 Consider the inequality $x^2 > 16$.

a) Rearrange the inequality into the form $f(x) > 0$, where $f(x)$ is a quadratic expression.

b) (i) Factorise $f(x)$.

(ii) Write down the x-coordinates of the points where the graph of $y = f(x)$ crosses the x-axis.

c) Hence solve the inequality $x^2 > 16$.

Q2 Solve each quadratic inequality and show the solution on a number line.

a) $x^2 < 4$ b) $x^2 \leq 9$ c) $x^2 > 25$ d) $x^2 \geq 36$

e) $x^2 < 1$ f) $x^2 \leq 49$ g) $x^2 > 64$ h) $x^2 < 100$

Q3 Solve each quadratic inequality, giving your solution using set notation.

a) $x^2 < \frac{1}{4}$ b) $x^2 \geq \frac{1}{25}$ c) $x^2 \leq \frac{1}{121}$ d) $x^2 > \frac{1}{36}$

e) $x^2 < \frac{4}{9}$ f) $x^2 \geq \frac{25}{49}$ g) $x^2 \leq \frac{9}{16}$ h) $x^2 < \frac{16}{169}$

Q4 Find the integer solutions to these inequalities. Give your answers using set notation.

a) $2x^2 < 18$ b) $3x^2 \leq 75$ c) $5x^2 < 80$ d) $2x^2 \leq 72$

Q5 Consider the inequality $4x \leq 12 - x^2$.

a) Rearrange the inequality into the form $g(x) \leq 0$, where $g(x)$ is a quadratic expression.

b) (i) Factorise $g(x)$.

(ii) Write down the x-coordinates of the points where the graph of $y = g(x)$ crosses the x-axis.

c) Hence solve the inequality $4x \leq 12 - x^2$.

Q6 Solve each of these inequalities.

a) $x^2 + x - 2 < 0$ b) $x^2 - x - 2 \leq 0$ c) $x^2 - 8x + 15 > 0$

d) $x^2 + 6x + 5 \leq 0$ e) $x^2 - x - 12 \geq 0$ f) $x^2 - 6x - 7 < 0$

g) $x^2 - 7x + 12 \leq 0$ h) $x^2 + 10x + 24 \geq 0$ i) $x^2 - 6x - 16 < 0$

j) $x^2 + 2x - 15 < 0$ k) $x^2 - 10x - 11 \leq 0$ l) $x^2 + 11x + 18 < 0$

Q7 Solve each quadratic inequality and show the solution on a number line.

a) $x^2 - 2x > 48$ b) $x^2 - 3x \leq 10$ c) $x^2 + 20 < 9x$ d) $x^2 + 18 \leq 9x$

e) $5x \geq 36 - x^2$ f) $x^2 < 9x + 22$ g) $x^2 < 6x + 27$ h) $32 < 12x - x^2$

Q8 Solve each quadratic inequality, giving your solution using set notation.

a) $x^2 - 4x > 0$ b) $x^2 + 3x \leq 0$ c) $x^2 - 5x < 0$ d) $x^2 + 8x \geq 0$

e) $x^2 > 12x$ f) $x^2 \leq 2x$ g) $x^2 \geq 9x$ h) $3x \leq -x^2$

13.3 Graphing Inequalities

Another way that you can represent inequalities is by using a graph. Graphs can be used to show the solutions to inequalities in one variable (e.g. $x \geq -2$) and in two variables (e.g. $y < 2x + 5$).

Graphing Inequalities

Learning Objective — Spec Ref A22:
Be able to show inequalities on a graph.

Prior Knowledge Check:
Be able to draw a straight-line graph.
See Section 15.

A linear inequality is shown on a graph using a **straight line** — the set of solutions satisfying the inequality will be the **entire region** on **one side** of the line.

To draw the straight line, replace the inequality sign with an **equals sign** and use one of the methods from Section 15 to draw the line. If you have a **strict inequality** (e.g. < or >) then you should draw a **dashed line** to show that the values on the line are **not included**. If you have a **non-strict inequality** (e.g. ≤ or ≥) then draw a **solid line** to show that the values on the line are **included**.

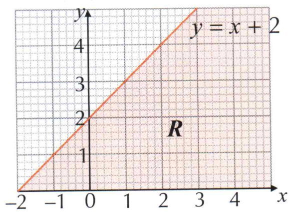
Once you've drawn the straight line, you need to decide which side of the line **satisfies** the inequality, then **shade** and **label** it. Choose any point on either side of the line. If that point satisfies the inequality (i.e. makes it true), it lies on the required side. If it doesn't, it's the other side you want.

Tip: You can also shade the region that you don't want and then label the region you do want.

Example 1

On a graph, use shading to show the region *R* that satisfies the inequality $y \leq x + 2$.

1. Draw the line $y = x + 2$ — it's a straight line with a gradient of 1 that crosses the y-axis at (0, 2). It's not a strict inequality so draw a solid line to show that the line is included.
2. Choose a point that's not on the line and see if its coordinates satisfy the inequality. E.g. (0, 0) satisfies the inequality because $0 \leq 0 + 2$ is true. Shade and label the region which includes (0, 0) — below the line.



Exercise 1

In Questions 1-2, use shading and an (x, y)-coordinate grid to show the region R that satisfies the inequality.

Q1 a) $x \leq 2$ b) $x < -1$ c) $y < 2$ d) $y \leq -1$

Q2 a) $y < x + 1$ b) $y > x - 3$ c) $y \geq x + 3$ d) $y \leq 5 - x$
e) $y \leq 2x$ f) $y > 2x + 3$ g) $y < 3x - 1$ h) $y > 2x - 5$
i) $y < \frac{1}{2}x + 2$ j) $y \leq \frac{1}{2}x - 3$ k) $y < \frac{x-1}{2}$ l) $y > \frac{3x+5}{2}$

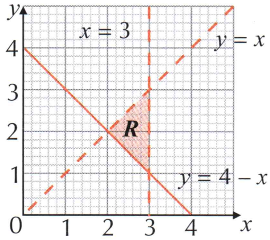
Q3 a) Rearrange the inequality $2x > 6 - y$ into the form '$y > \ldots$'
b) Hence draw a graph and show, by shading, the region R that satisfies the inequality $2x > 6 - y$.

Q4 Draw a graph and use shading to show the region R that satisfies each inequality.
a) $x + y \leq 5$ b) $x + y > 0$ c) $y - x > 4$ d) $x - y \leq 3$
e) $2x - y \geq 8$ f) $x + 2y \geq 4$ g) $2x + 3y < 6$ h) $3x + 2y \geq 6$

Example 2

On a graph, use shading to show the region R that satisfies the inequalities $y < x$, $x < 3$ and $y \geq 4 - x$.

1. Draw each of the lines $y = x$, $x = 3$ and $y = 4 - x$.
2. $y = x$ should be a dotted line as $y < x$. Point (1, 0) satisfies the inequality ($0 < 1$) so you want the region below $y = x$.
3. $x = 3$ should be a dotted line as $x < 3$. Point (0, 0) satisfies the inequality ($0 < 3$) so you want the region to the left of $x = 3$.
4. $y = 4 - x$ should be a solid line as $y \geq 4 - x$. Point (0, 0) doesn't satisfy the inequality ($0 \geq 4 - 0$) so you want the region above $y = 4 - x$.
5. The shaded region R satisfies all three inequalities.



Exercise 2

In Questions 1-6, draw a graph and use shading to show the region R that satisfies the inequalities.

Q1 a) $x > 1$ and $y \leq 2$ b) $x \leq 4$ and $y \leq 3$ c) $x \geq 0$ and $y > 1$

Q2 a) $1 < x < 4$ b) $-2 < x \leq 1$ c) $-4 < y < 2$

Q3 a) $-2 < x < 0$ and $y > 1$ b) $2 < x \leq 6$ and $y \leq 5$ c) $x \geq 0$ and $-6 < y < -2$
d) $x < -1$ and $-1 \leq y \leq 4$ e) $1 \leq x < 4$ and $2 < y < 5$ f) $-2 < x \leq 3$ and $2 \leq y \leq 6$

Q4 a) $y > 1$ and $y + x < 4$ b) $x < 2$ and $x + y \geq 1$ c) $y > -1$ and $y \leq x - 2$
d) $x > -4$ and $y \geq x + 3$ e) $y \geq 0$ and $x < 6 - y$ f) $x < 3$ and $3 \leq x - y$

Q5 a) $x > -2$, $y < 5$ and $y > x + 4$ b) $x \geq -1$, $y < 4$ and $y < x + 3$ c) $x \leq 6$, $y > -1$ and $6 < 2x + 2y$
d) $y > x - 2$ and $x + y \leq 4$ e) $x + y \geq -4$ and $y \leq x + 5$ f) $y \geq x - 3$ and $4 < x + y$

Q6 a) $x > -5$, $y \geq x - 3$ and $x + y < 7$ b) $x < 6$, $x + y > -5$ and $y \leq 2x + 1$
c) $y < 2x - 4$, $y > x - 4$ and $x + y \leq 8$ d) $y < 3x - 4$, $4y \geq x - 12$ and $x + y \leq 6$

Finding Inequalities from Graphs

Learning Objective — Spec Ref A22:
Be able to describe a region of a graph using inequalities.

Prior Knowledge Check:
Recognise a straight-line equation from its graph. See Section 15.

To find the inequalities that define a region on a graph, first **find the equations** of the boundary lines using one of the methods covered in Section 15. Then decide which **inequality sign** to use.

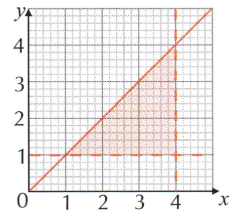
- Look at the type of boundary line. If it's **dashed**, the inequality sign will be **< or >**. If it's a **solid** line, it'll be either **≤ or ≥**.
- Choose a **point** in the region you're trying to define and put the **coordinates** into the equation for each boundary line. Use the result to decide which way round the **inequality sign** should be.

E.g. if you have the equation $y = 4x + 1$ and the point (0, 0) is in the region you want, you get $0 = 1$ — so you should use $<$ or $\leq$ (depending on whether it is a **dotted** or **dashed line**) to make the statement true.

Example 3

Find the inequalities which are represented by the shaded region shown.

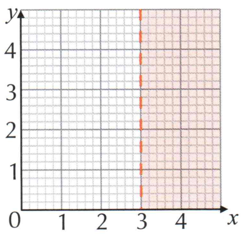
1. The horizontal line is $y = 1$. The point (3, 2) is in the shaded region, and $2 > 1$. The line is dashed, so this represents the inequality $y > 1$.
2. The vertical line is $x = 4$. The point (3, 2) is in the shaded region, and $3 < 4$. The line is dashed, so this represents the inequality $x < 4$.
3. The diagonal line is $y = x$. The point (3, 2) is in the region and $2 < 3$ (i.e. $y < x$). The line is solid, so this represents the inequality $y \leq x$.

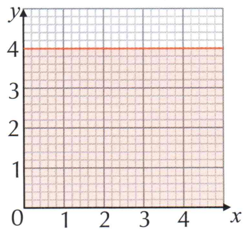


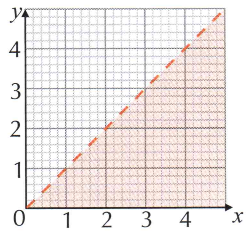
$\boldsymbol{y > 1, x < 4}$ and $\boldsymbol{y \leq x}$

Exercise 3

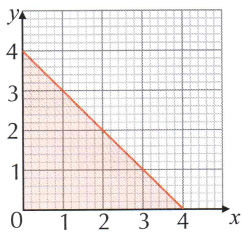
Q1 Write down the inequality represented by the shaded region in each of the following graphs.

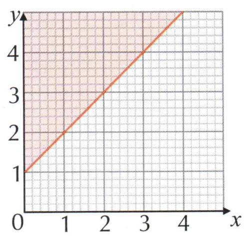
a) 

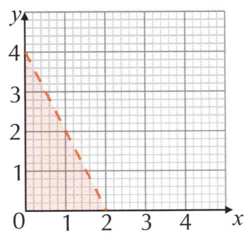
b) 

c) 

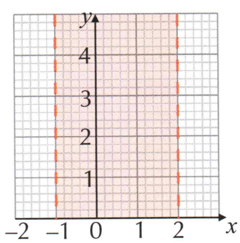
Q2 By first finding the equation of the line shown, write down the inequality represented by the shaded region in each of the following graphs.

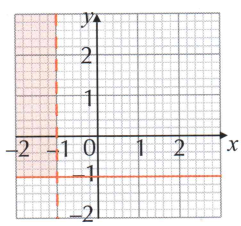
a) 

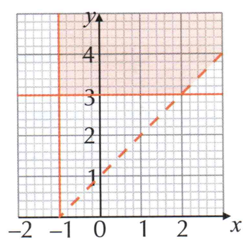
b) 

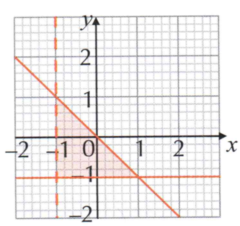
c) 

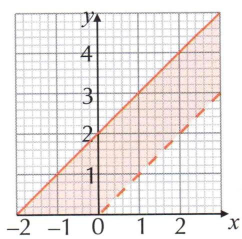
Q3 Find the inequalities which are represented by the shaded region in each of the graphs.

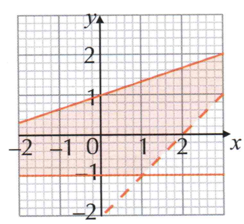
a) 

b) 

c) 

d) 

e) 

f) 

Review Exercise

Q1 Solve the following inequalities. Give your answers:

(i) using set notation. (ii) on a number line.

a) $-1 > \frac{x}{8}$ b) $\frac{x}{6} \leq 0.5$ c) $\frac{x}{1.1} \geq 10$ d) $\frac{x}{0.2} < -3.2$

e) $2x + 16 \geq -8$ f) $-5 < -9x - 14$ g) $8x - 3.4 < 12.6$ h) $-4x + 2.6 \leq 26.6$

Q2 Find the integer solutions to the following compound inequalities. Give your answers using set notation.

a) $-7 < x - 6 < 4$ b) $-4 \leq 2x + 2 \leq 10$ c) $-3 < 6x + 3 < 27$

Q3 Solve each of these quadratic inequalities.

a) $x^2 < 81$ b) $x^2 \geq 81$ c) $3x^2 > 48$

d) $x^2 + 7x + 12 > 0$ e) $x^2 - 4x < 5$ f) $-x^2 > 8x - 20$

Q4 Draw a graph and show, by shading, the region R that satisfies each inequality.

a) $x \geq -5$ b) $y > -4$ c) $y \geq 2x + 3$

d) $y < -x - 2$ e) $2x + 5y > 10$ f) $5 < -10x - y$

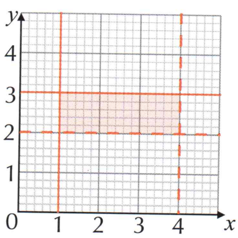
g) $x - 2y - 6 \leq 0$ h) $-4 \leq x < -1$ i) $3 < y \leq 6$

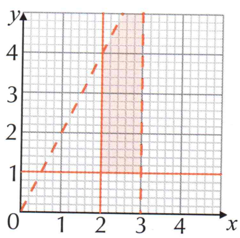
Q5 Draw a graph and show, by shading, the region R that satisfies the inequalities.

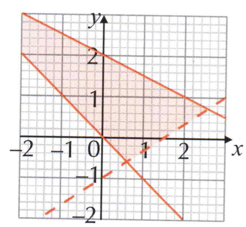
a) $-5 \leq y \leq -1$ and $y < -x$ b) $-5 \leq x < -2$ and $x + 1 < y$

c) $y < 6$, $2y > x - 4$ and $5 < x + y$ d) $y \geq -4$, $y < 2x - 2$ and $x + 2y \leq 8$

Q6 Find the inequalities which are represented by the shaded region in each of the graphs.

a) 

b) 

c) 

Q7 A delivery company is buying x motorbikes and y vans. Motorbikes cost £8000 and vans cost £16 000. They have £80 000 to spend on buying vehicles, and must buy at least 7 vehicles, including at least 1 van.

a) (i) Write down 4 inequalities which the company must satisfy.

(ii) Represent these inequalities graphically.

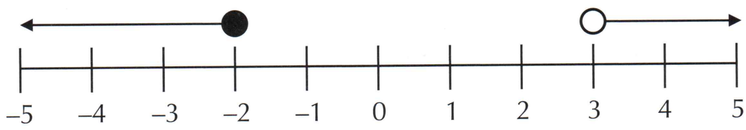
b) List the different possible combinations of motorbikes and vans that are available to the company.

Exam-Style Questions

Q1 Solve $4(x + 3) > 2(x - 3)$

[2 marks]

Q2 Elisa has drawn the following diagram to show the inequality $-2 < x \leq 3$.



Identify the problems with the inequality Elisa has drawn.

[2 marks]

Q3 Solve $6x^2 + 9x - 15 \leq 0$

[3 marks]

Q4 A charity sells a range of wristbands which all cost the same amount.
Eve buys four wristbands and also makes a £2 donation to the charity.
Faisal buys six wristbands and makes a £1 donation.
Faisal spends less than £13 in total and he spends more than Eve.
If x is the cost of the wristband in £, use inequalities to work out the range of possible values for x, giving your answer in the form $a < x < b$.

PROBLEM SOLVING

[4 marks]

Q5 A decorator is buying paint from his supplier — undercoat, which costs £12 per tin, and matt emulsion, which costs £24 per tin. He needs to buy at least 5 tins of undercoat and at least as many tins of matt emulsion as undercoat. He has £216 to spend in total. Let x be the number of tins of undercoat he buys and y be the number of tins of matt emulsion he buys.

a) Write down 3 inequalities that the decorator must satisfy when buying his paint. Simplify the inequalities if possible.

PROBLEM SOLVING

[3 marks]

b) Represent these inequalities graphically.

[3 marks]

c) Use your graph to list all the possible combinations of tins of paint he can buy.

[2 marks]