Section 1 Arithmetic, Multiples and Factors

1.1 Calculations

Here's a nice gentle start to the book — non-calculator arithmetic (so put your calculator away). All you have to do here is add, subtract, multiply and divide — but you need to know the order in which to do them if you have to do multiple operations, and how to handle negative numbers and decimals.

Order of Operations

Learning Objective — Spec Ref N3:
Know the correct order in which to apply operations, including dealing with brackets.

BODMAS tells you the correct order to carry out mathematical operations:

Brackets, **O**ther, **D**ivision, **M**ultiplication, **A**ddition, **S**ubtraction

If there are two or more **consecutive divisions and/or multiplications** (e.g. $3 \times 6 \div 9 \times 5$), do them in order **from left to right**. The same goes for **addition and subtraction**.

Example 1

Work out: **a)** $20 - 3 \times 4$ **b)** $30 \div (15 - 12)$

a) There are no brackets, 'other' operations or divisions in this calculation, so do the multiplication first then do the subtraction.

$$20 - 3 \times 4 = 20 - 12 = \mathbf{8}$$

b) Do the bit in brackets first then do the division.

$$30 \div (15 - 12) = 30 \div 3 = \mathbf{10}$$

Tip: Just ignore any bits of BODMAS that you don't need.

Exercise 1

Q1 Work out the following.

a) $5 + 1 \times 3$ b) $11 - 2 \times 5$ c) $18 - 10 \div 5$

d) $24 \div 4 + 2$ e) $35 \div 5 + 2$ f) $36 - 12 \div 4$

Q2 Work out the following.

a) $2 \times (4 + 10)$ b) $(7 - 2) \times 3$ c) $4 + (48 \div 8)$

d) $56 \div (2 \times 4)$ e) $(3 + 2) \times (9 - 4)$ f) $(8 - 7) \times (6 + 5)$

Q3 Work out the following.

a) $2 \times (8 + 4) - 7$ b) $100 \div (8 + 3 \times 4)$ c) $7 + (10 - 9 \div 3)$

d) $20 - (5 \times 3 + 2)$ e) $48 \div 3 - 7 \times 2$ f) $36 - (7 + 4 \times 4)$

Q4 Work out the following. a) $4 - 5 + 2 - 1$ b) $5 \times 4 \div 10 \times 6$

Example 2

Work out: $\dfrac{18-2\times3}{8\div2}$

1. Think of the top and bottom of the fraction as having 'invisible brackets' around them.
2. To evaluate each set of brackets, apply BODMAS to the expression inside.

$$\frac{18-2\times3}{8\div2}=\frac{(18-2\times3)}{(8\div2)}$$
$$=\frac{(18-6)}{4}$$
$$=\frac{12}{4}=\mathbf{3}$$

Exercise 2

Q1 Work out the following.

a) $\dfrac{16}{4\times(5-3)}$ b) $\dfrac{8+2}{15\div3}$ c) $\dfrac{4\times(7+5)}{6+3\times2}$ d) $\dfrac{6+(11-8)}{7-5}$

Q2 Work out the following.

a) $\dfrac{12\div(9-5)}{25\div5}$ b) $\dfrac{8\times2\div4}{5-6+7}$ c) $\dfrac{3\times3}{21\div(12-5)}$ d) $\dfrac{36\div(11-2)}{8-8\div2}$

Negative Numbers

Learning Objective — Spec Ref N2:
Add, subtract, multiply and divide negative numbers.

Adding and Subtracting Negative Numbers

When adding and subtracting negative numbers, there are two rules you need to know:

1. Adding a negative number is the same as subtracting a positive number.
   So '+' next to '–' means **subtract**, i.e. $a + (-b) = a - b$.
2. Subtracting a negative number is the same as adding a positive number.
   So '–' next to '–' means **add**, i.e. $a - (-b) = a + b$.

Example 3

Work out: **a) 1 – (–4)** **b) –5 + (–2)** **c) –3 – (–7)**

a) A '–' next to a '–' means add.

$1 - (-4) = 1 + 4$
$= \mathbf{5}$

b) A '+' next to a '–' means subtract.

$-5 + (-2) = -5 - 2$
$= \mathbf{-7}$

c) A '–' next to a '–' means add.

$-3 - (-7) = -3 + 7$
$= \mathbf{4}$

Exercise 3

Q1 Work out the following.

a) $-4 + 3$ b) $-1 - 4$ c) $-12 + 15$

d) $6 - 17$ e) $4 - (-2)$ f) $-6 - (-2)$

Q2 Work out the following.

a) $-5 + (-5)$ b) $-5 - (-5)$ c) $-23 - (-35)$

d) $48 + (-22)$ e) $-27 + (-33)$ f) $61 - (-29)$

Multiplying and Dividing Negative Numbers

1. When you multiply or divide two numbers which have the **same** sign, the answer is **positive** — for example, $(-6) \times (-7) = 42$ and $(-40) \div (-8) = 5$.
2. When you multiply or divide two numbers which have **opposite** signs, the answer is **negative** — for example, $(-4) \times 9 = -36$ and $35 \div (-5) = -7$.

Example 4

Work out: **a) $24 \div (-6)$** **b) $(-5) \times (-8)$** **c) $[(-27) \div 3] \times (-4)$**

a) The signs are different so the answer will be negative. $24 \div (-6) = \mathbf{-4}$

b) The signs are the same so the answer will be positive. $(-5) \times (-8) = \mathbf{40}$

c) You'll have to use BODMAS here:
1) Do the bit in brackets first (the signs are different so the result will be negative).
2) Then do the multiplication — because you ended up with a negative number inside the brackets, the signs are the same so the answer will be positive.

$[(-27) \div 3] \times (-4) = (-9) \times (-4)$
$= \mathbf{36}$

Exercise 4

Q1 Work out the following.

a) $(-15) \div (-3)$ b) $12 \div (-4)$ c) $(-72) \div (-6)$

d) $56 \div (-8)$ e) $(-16) \times (-3)$ f) $(-81) \div (-9)$

g) $(-13) \times (-3)$ h) $7 \times (-6)$ i) $(-34) \times 2$

Q2 Work out the following.

a) $[(-3) \times 7] \div (-21)$ b) $[(-24) \div 8] \div 3$ c) $[55 \div (-11)] \times (-9)$

d) $[(-63) \div (-9)] \times (-7)$ e) $[35 \div (-7)] \times (-8)$ f) $[(-12) \times 3] \times (-2)$

Q3 Copy the following calculations and fill in the blanks.

a) $(-3) \times \square = -6$ b) $(-14) \div \square = -2$ c) $\square \times 4 = -16$

d) $(-8) \times \square = -24$ e) $\square \times (-3) = 36$ f) $\square \div 11 = -7$

Decimals

Learning Objective — Spec Ref N2:
Add, subtract, multiply and divide decimals.

Adding and Subtracting Decimals

To add and subtract **decimals**, arrange them in columns like you would for normal numbers — just make sure you line up the **decimal points**. You might have to **add in 0s** to fill in any gaps.

Example 5

Work out: **a) 4.53 + 1.6** **b) 8.5 – 3.07**

1. Set out the sum by lining up the decimal points.
2. Fill in any gaps with 0s.
3. Add or subtract the digits one column at a time, right to left. Carry or borrow digits as necessary.

a)
$$\begin{array}{r} 4.53 \\ +\ 1.60 \\ \hline 6_{1}.13 \end{array}$$

b)
$$\begin{array}{r} 8.{}^{4}\cancel{5}{}^{1}0 \\ -\ 3.07 \\ \hline 5.43 \end{array}$$

Exercise 5

Q1 Work out the following.

a) 12.74 + 7 b) 0.8 – 0.03 c) 10.83 + 7.4 d) 91.7 + 0.492

e) 6.474 + 0.92 f) 16.3 – 5.16 g) 9.241 – 2.8 h) 23 – 18.591

Q2 Copy the following calculations and fill in the blanks.

a)
$$\begin{array}{r} \square.6\square \\ +\ 0.\square 0 \\ \hline 8.21 \end{array}$$

b)
$$\begin{array}{r} 5.\square 3 \\ -\ 2.1\square \\ \hline \square.31 \end{array}$$

Q3 Sunita buys a hat for £18.50 and a bag for £31.99. How much does she spend altogether?

Q4 Jay's meal costs £66.49. He uses a £15.25 off voucher. How much does he have left to pay?

Multiplying Decimals

To **multiply** decimals, multiply each decimal by a **power of 10** to get a whole number multiplication. Do this multiplication in the normal way. Finally, **divide** your answer by the product of the powers of 10 to get back to the original multiplication.

Example 6

Work out 0.32 × 0.6

1. Multiply each decimal by a power of 10 to get a whole number multiplication.
2. Multiply the whole numbers.
3. Divide by the product of the powers of 10 you multiplied by in Step 1.

0.32×0.6

$\times 100$ ↘ 32×6 ↙ $\times 10$

$$\begin{array}{r} 32 \\ \times\ 6 \\ \hline 192 \\ {\scriptstyle 1} \end{array}$$

So $0.32 \times 0.6 = 192 \div 1000$
$= \mathbf{0.192}$

Exercise 6

Q1 $132 \times 238 = 31\,416$. Use this information to work out the following.

a) 13.2×238 b) 1.32×23.8 c) 1.32×0.238 d) 0.132×0.238

Q2 Work out the following.

a) 16.7×8 b) 31.2×6 c) 3.1×40 d) 0.7×600

e) 0.05×0.04 f) 0.08×0.5 g) 2.1×0.6 h) 1.6×0.04

i) 5.2×0.09 j) 3.9×8.3 k) 0.16×3.3 l) 0.64×0.42

Q3 1 litre is equal to 1.76 pints. What is 5 litres in pints?

Q4 Petrol costs £1.35 per litre. A car uses 9.2 litres during a journey. How much does this cost?

Dividing Decimals

1. To divide a decimal by a **whole number**, you can just treat it as a normal division, but set it out so that the decimal points in the question and answer are **lined up**.
2. To divide **by a decimal**, multiply both numbers by the **same power of 10** to turn the calculation into a division by a whole number. Then do the division, making sure you line up the decimal points in your calculation. You **don't** need to divide by the power of 10 at the end — you multiplied both numbers, so the answer wasn't affected.

Example 7

Work out $0.516 \div 0.8$

1. Multiply both numbers by 10, so you're dividing by a whole number.

$0.516 \div 0.8 = 5.16 \div 8$

2. Line up the decimal points to set out the calculation, then divide.

$$\begin{array}{r} 0.645 \\ 8\overline{)5.^{5}1^{3}6^{4}0} \end{array}$$

So $0.516 \div 0.8 = \mathbf{0.645}$

Tip: This example uses short division, but you can use long division here if you want.

Exercise 7

Q1 Work out the following.

a) $8.52 \div 4$ b) $2.14 \div 4$ c) $8.62 \div 5$

d) $17.1 \div 6$ e) $0.081 \div 9$ f) $12.06 \div 8$

Q2 Work out the following.

a) $1.56 \div 0.2$ b) $0.624 \div 0.3$ c) $0.275 \div 0.5$ d) $16.42 \div 0.02$

e) $0.257 \div 0.05$ f) $7.665 \div 0.03$ g) $0.039 \div 0.06$ h) $50.4 \div 0.07$

i) $0.71 \div 0.002$ j) $108 \div 0.4$ k) $20.16 \div 0.007$ l) $1.44 \div 1.2$

Q3 A 2.72 m ribbon is cut into equal pieces of length 0.08 m. How many pieces will there be?

Q4 It costs £6.93 to buy 3.5 kg of pears. How much do pears cost per kg?

1.2 Multiples and Factors

To tackle multiples and factors, you need to know your times tables — that's all there is to it.

Multiples

Learning Objective — Spec Ref N4:
Identify and find multiples and common multiples.

The **multiples** of a number are just the numbers in its **times table**. E.g. the multiples of 4 are 4, 8, 12, 16, ...

A **common multiple** of two (or more) numbers is a multiple of both (or all) of those numbers.

Example 1

a) List the multiples of 5 between 23 and 43.

These are the numbers between 23 and 43 that are in the 5 times table. **25, 30, 35, 40**

b) Which of the numbers in the box on the right are common multiples of 2 and 7?

24	7	28	42	35

The multiples of 2 in the box are 24, 28 and 42.
The multiples of 7 in the box are 7, 28, 35 and 42.
So 28 and 42 are multiples of both numbers. **28, 42**

Exercise 1

Q1 List the first five multiples of: a) 9 b) 13 c) 16

Q2 a) List the multiples of 12 between 20 and 100.
b) List the multiples of 14 between 25 and 90.

Q3 Write down the numbers from the box that are:
a) multiples of 10
b) multiples of 15
c) common multiples of 10 and 15

5	10	15	20	25	30	35
40	45	50	55	60	65	70
75	80	85	90	95	100	105

Q4 a) List the multiples of 3 between 19 and 35. b) List the multiples of 4 between 19 and 35.
c) List the common multiples of 3 and 4 between 19 and 35.

Q5 List all the common multiples of 5 and 6 between 1 and 40.

Q6 List all the common multiples of 6, 8 and 10 between 1 and 100.

Q7 List all the common multiples of 9, 12 and 15 between 1 and 100.

Q8 List the first five common multiples of 3, 6 and 9.

Factors

Learning Objective — Spec Ref N4:
Identify and find factors and common factors.

The **factors** of a number are the numbers that divide into it exactly.
E.g. the factors of 8 are 1, 2, 4 and 8 — dividing 8 by any of these numbers gives a whole number.

A **common factor** of two numbers is a factor of both of those numbers.
E.g. the factors of 12 are 1, 2, 3, 4, 6 and 12, so the common factors of 8 and 12 are 1, 2 and 4.

Example 2

a) Write down all the factors of: (i) 18 (ii) 30

1. Check if 1, 2, 3, etc. divide into the number.
2. Stop when a factor is repeated — factors come in pairs, so once you get to one that's already been found, you'll have all the factors of the number.

(i) $1 \times 18 = 18$ — so 1 and 18 are factors
$2 \times 9 = 18$ — so 2 and 9 are factors
$3 \times 6 = 18$ — so 3 and 6 are factors
$4 \times \text{—} = 18$ — so 4 is not a factor
$5 \times \text{—} = 18$ — so 5 is not a factor
$6 \times 3 = 18$ — 6 and 3 are repeated so stop
So the factors of 18 are **1, 2, 3, 6, 9 and 18.**

Tip: Any two factors that multiply to give the number are called a factor pair — e.g. 2 and 9 are a factor pair of 18.

(ii) $1 \times 30 = 30$ — so 1 and 30 are factors
$2 \times 15 = 30$ — so 2 and 15 are factors
$3 \times 10 = 30$ — so 3 and 10 are factors
$4 \times \text{—} = 30$ — so 4 is not a factor
$5 \times 6 = 30$ — so 5 and 6 are factors
$6 \times 5 = 30$ — 6 and 5 are repeated so stop
So the factors of 30 are **1, 2, 3, 5, 6, 10, 15 and 30.**

b) Write down the common factors of 18 and 30.
These are the numbers which appear in both lists from part a). **1, 2, 3 and 6**

Exercise 2

Q1 List all the factors of each of the following numbers.
a) 13 b) 25 c) 24 d) 35
e) 32 f) 40 g) 50 h) 49

Q2 A baker has 12 identical cakes. In how many different ways can he divide them up into equal packets? List all the possibilities.

Q3 In how many different ways can 100 people be arranged into groups of equal size? List all the possibilities.

Q4 a) List all the factors of: (i) 15 (ii) 21
b) Hence list the common factors of 15 and 21.

Q5 List the common factors of each of the following pairs of numbers.
a) 15, 20 b) 50, 90 c) 24, 32
d) 64, 80 e) 45, 81 f) 96, 108

Q6 List the common factors of each of the following sets of numbers.
a) 30, 45, 50 b) 8, 12, 20 c) 9, 27, 36 d) 24, 48, 96

1.3 Prime Numbers and Prime Factors

There's a key definition coming up — prime numbers. Once you know what they are (and how to find them), you can move on to prime factors — which will come in very handy later in this section.

Prime Numbers

Learning Objective — Spec Ref N4:
Identify and find prime numbers.

A **prime number** is a number that has no other factors except **itself** and **1**.

Here are a few things to note about prime numbers:

- 1 is **not** classed as a prime number — this is a common mistake.
- 2 is the only **even** prime number.
- Prime numbers end in **1**, **3**, **7** or **9** (2 and 5 are the only exceptions to this rule).
  But **not all** numbers ending in 1, 3, 7 or 9 are prime (e.g. $27 = 3 \times 9$, so it isn't prime).

Example 1

Which of the numbers in the box on the right are prime? 15 16 17 18 19 20

1. Ignore any even numbers, and any ending in 5.
2. Look for factors of each of the remaining numbers. If there aren't any, it's prime.

16, 18 and 20 are even, so can't be prime.
15 ends in 5, so can't be prime.
17 has no factors other than 1 and 17.
19 has no factors other than 1 and 19.
So the prime numbers are **17** and **19**.

Exercise 1

Q1 Consider the following list of numbers: 11, 13, 15, 17, 19
a) Which number in the list is not prime?
b) Find two factors greater than 1 that can be multiplied together to give this number.

Q2 a) Which three numbers in the box on the right are not prime? 31 33 35 37 39
b) Find a factor pair (where each factor is greater than 1) of each of your answers to (a).

Q3 Write down the prime numbers in this list: 5, 15, 22, 34, 47, 51, 59

Q4 a) Write down all the prime numbers less than 10.
b) Find all the prime numbers between 20 and 50.

Q5 a) For each of the following, find a factor greater than 1 but less than the number itself.
(i) 4 (ii) 14 (iii) 34 (iv) 74
b) Explain why any number with last digit 4 cannot be prime.

Writing a Number as a Product of Prime Factors

Learning Objectives — Spec Ref N4:

- Understand the unique factorisation theorem.
- Find the prime factorisation of a number.
- Write a prime factorisation using product notation.

Prior Knowledge Check:
Be able to find factors, recognise prime numbers and use index notation. See p.8-9 and p.92.

Any integer greater than 1 can be broken down into a string of **prime numbers** all multiplied together — this is known as the **prime factorisation** of the number. For example, the prime factorisation of 112 is $2 \times 2 \times 2 \times 2 \times 7$. The prime factorisation of every number is **unique** — each number only has **one** prime factorisation, and no two numbers can have the **same one**.

If the prime factorisation has **repeated factors**, you can write it using **index notation** (i.e. as a product of powers). So the prime factorisation of 112 can be written as $2^4 \times 7$.

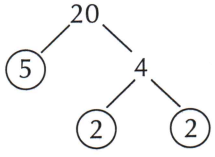
To find a prime factorisation, you can use a **factor tree**. A factor tree breaks a number into factors, then breaks these factors into smaller factors, and keeps going until all of the factors are prime.

Example 2

Write 20 as a product of prime factors. Give your answer in index form.

Make a factor tree.

1. Find any factor pair of 20. Circle any factors that are prime.
2. Repeat step 1 for any factors which aren't prime.
3. Stop when all the factor tree's branches end in a prime.
4. Give any repeated factors as a power, e.g. $2 \times 2 = 2^2$.



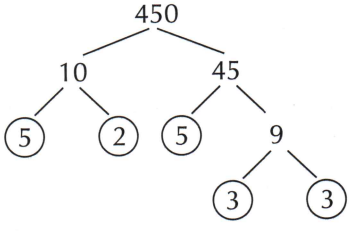
$20 = 2 \times 2 \times 5 = \mathbf{2^2 \times 5}$

Example 3

Write 450 as a product of prime factors. Give your answer in index form.

Make a factor tree.

1. Find any factor pair of 450. Neither of these are prime, so carry on to step 2.
2. Repeat step 1 for any factors which aren't prime. Circle any factors that are prime.
3. Stop when all the factor tree's branches end in a prime.
4. Give any repeated factors as a power.



$450 = 2 \times 3 \times 3 \times 5 \times 5 = \mathbf{2 \times 3^2 \times 5^2}$

Tip: You could have started with any two factors of 450 on the first branches — e.g. 9 and 50. The rest of the tree would look a bit different, but the final prime factorisation would be exactly the same.

Exercise 2

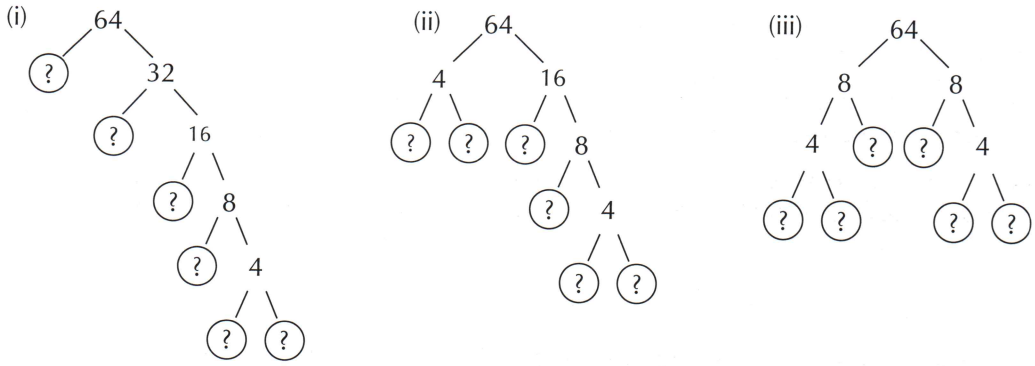
Give your answers to these questions in index form where appropriate.

Q1 Write each of the following as the product of two prime factors.

a) 14 b) 55 c) 15 d) 21

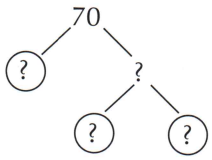
e) 35 f) 39 g) 77 h) 121

Q2 a) Copy and complete the three factor trees below.



b) Use each of your factor trees to write down the prime factors of 64. What do you notice?

Q3 Copy and complete the factor tree on the right and use it to find the prime factorisation of 70.



Q4 Write each of the following as the product of prime factors.

a) 30 b) 42 c) 66 d) 46

e) 78 f) 190 g) 210 h) 138

Q5 Write each of the following as the product of prime factors.

a) 44 b) 48 c) 72 d) 90

e) 50 f) 28 g) 98 h) 150

i) 132 j) 168 k) 325 l) 1000

Q6 Square numbers have all their prime factors raised to even powers.
For example, $36 = 2^2 \times 3^2$ and $64 = 2^6$.

a) Write 75 as a product of prime factors.

b) What is the smallest integer you could multiply 75 by to form a square number? Explain your answer.

Q7 By first writing each of the following as a product of prime factors, find the smallest integer that you could multiply each number by to give a square number.

a) 250 b) 416 c) 756 d) 1215

1.4. LCM and HCF

Now that you've got your head around multiples and factors, it's time to start looking at the 'lowest common multiple' and 'highest common factor' — or LCM and HCF for short.

LCM — 'Lowest Common Multiple'

Learning Objectives — Spec Ref N4:

- Understand the term 'lowest common multiple'.
- Be able to find the lowest common multiple of a set of numbers.

Prior Knowledge Check: Be able to work with multiples. See p.7.

If you're given two (or more) numbers, then you can find their **lowest common multiple** (LCM). As the name suggests, the LCM is basically the smallest number that is a multiple of both (or all) of the numbers. In other words:

The LCM is the **smallest** number that will **divide** by **all** the numbers in a list.

If you're given a set of numbers, you can find their LCM by listing multiples of each number, then identifying the first one that appears in every list.

Tip: The LCM is the smallest number in the times tables of all the numbers in the list.

Example 1

Find the LCM of 4, 6 and 8.

1. List multiples of 4, 6 and 8.

   Multiples of 4: 4, 8, 12, 16, 20, (24), 28...

   Multiples of 6: 6, 12, 18, (24) 30...

   Multiples of 8: 8, 16, (24), 32...

2. The LCM is the smallest number that appears in all three lists.

   So the LCM of 4, 6 and 8 is **24**.

Exercise 1

Q1 Find the LCM of each of the following pairs of numbers.

a) 3 and 5 b) 6 and 8 c) 2 and 10 d) 6 and 7

Q2 Find the LCM of each of the following sets of numbers.

a) 3, 6, 8 b) 2, 5, 6 c) 4, 9, 12 d) 5, 7, 10

Q3 Mike visits Oscar every 4 days, while Narinda visits Oscar every 5 days. If they both visited today, how many days will it be before they visit on the same day again? PROBLEM SOLVING

Q4 A garden centre has between 95 and 205 potted plants. They can be arranged exactly in rows of 25 and exactly in rows of 30. How many plants are there? PROBLEM SOLVING

Q5 There are between 240 and 300 decorated plates hanging on a wall, and the number of plates divides exactly by both 40 and 70. How many plates are there?

HCF — 'Highest Common Factor'

Learning Objectives — Spec Ref N4:

- Understand the term 'highest common factor'.
- Be able to find the highest common factor of a set of numbers.

Prior Knowledge Check: Be able to find the factors of a number. See p.8.

You may also be asked to find the **highest common factor** (HCF) of a list of numbers.
The HCF is just the largest value that is a factor of all the numbers in the list. In other words:

HCF is the **largest** number that will **divide into all** the numbers in your list.

To find the HCF of a set of numbers, list the factors of each number, then pick the biggest one that's in every list.

Example 2

Find the HCF of 12 and 15.

1. Find the factors of 12 and the factors of 15.
2. Circle the common factors (the ones that appear in both lists).
3. The HCF is the biggest number that appears in both lists.

Factors of 12: (1), (2), (3), 4, 6, 12
Factors of 15: (1), (3), 5, 15

So the HCF of 12 and 15 is **3**.

Exercise 2

Q1 a) Find the common factors of 12 and 20.

b) Hence find the highest common factor (HCF) of 12 and 20.

Q2 Find the HCF of each of the following pairs of numbers.

a) 24 and 32 b) 36 and 60 c) 14 and 15 d) 12 and 36

Q3 Find the HCF of each of the following sets of numbers.

a) 6, 8, 16 b) 12, 15, 18 c) 18, 36, 72 d) 36, 48, 60

Q4 An artist is using blue and pink ribbon in her artwork. She has 63 cm of blue ribbon and 91 cm of pink ribbon. She wants to cut the ribbon into pieces so each piece is of equal length and as long as possible.

PROBLEM SOLVING

a) How long should she make each piece?

b) How many pieces of each colour will she have in the end?

Q5 Kim is dividing counters into equal piles. She has 135 tangerine counters and 165 gold counters. Each pile must contain only one of the colours. What is the least number of piles she can make in total?

PROBLEM SOLVING

LCM and HCF using prime factors

Learning Objective — Spec Ref N4:
Be able to find the lowest common multiple and highest common factor of a set of numbers using prime factorisation.

Prior Knowledge Check:
Be able to find the prime factorisation of a number. See p.10.

You've just seen simple methods for finding the LCM and HCF by listing multiples and factors and looking for the smallest or largest. Now it's time to see a more sophisticated method that uses **prime factorisation**.

LCM using prime factors

To find the LCM of a set of numbers using prime factors:

1. Write each number as a product of its prime factors.
2. For each prime factor, find the highest power of it that appears in **any** of the lists.
3. Multiply these numbers to find the lowest common multiple.

This method is especially useful when the numbers you're given are quite large — then you're not stuck trying to write down huge lists of multiples.

Example 3

Use prime factors to find the LCM of 84 and 98.

1. Write 84 and 98 as products of prime factors.
2. Find the highest power of each prime factor that appears in either list. In this case, the highest power of 2 is 2^2, the highest power of 3 is just 3, and the highest power of 7 is 7^2.
3. The LCM is the product of these numbers.

$84 = (2^2) \times (3) \times 7$

$98 = 2 \times (7^2)$

So the LCM is:
$2^2 \times 3 \times 7^2 = \mathbf{588}$

Tip: Remember to use index form (powers) to write any factors which appear more than once.

The method works because a multiple of a number must contain **at least** its prime factorisation (e.g. any multiple of 12 must be 'something' $\times 2^2 \times 3$). Therefore a common multiple of several numbers must contain the prime factorisations of **all** the numbers — you have to take the highest power of each factor to ensure this. You're not multiplying by anything else, so this is the LCM.

HCF using prime factors

A very similar method is used to find the HCF of a set of numbers using prime factors:

1. Write each number as a product of its prime factors.
2. For each prime factor, find the highest power of it that appears in **all** of the lists.
3. Multiply these numbers to find the highest common factor.

The only difference to the method for the LCM is that you only want factors that appear in **all** the prime factorisations — if one prime factor only appears in one, it won't be part of the HCF.

Example 4

Use prime factors to find the HCF of 60 and 72.

1. Write 60 and 72 as products of prime factors.
   $60 = 2^2 \times 3 \times 5$
   $72 = 2^3 \times 3^2$
2. Find the highest power of each prime factor that appears in both lists.
   Both lists contain $2^2 \times 3$
3. The HCF is the product of these numbers.
   So the HCF is: $2^2 \times 3 = \mathbf{12}$

To understand how this method works, note that a factor of a number contains **some combination** of its prime factors. E.g. 2, 2^2 and 3×2 are factors of 12 because $12 = 2^2 \times 3$.

So a common factor of several numbers will be made up of some prime factors that the numbers have in common. And the **highest** common factor will contain **every** prime factor (including repeats) that the numbers have in common. Taking the highest power ensures that you catch all of them.

Exercise 3

Q1 a) Write 120 and 155 as products of their prime factors.
b) Hence find the HCF of 120 and 155.

Q2 a) Write 76 and 88 as products of their prime factors.
b) Hence find the LCM of 76 and 88.

Q3 Use prime factors to find (i) the HCF and (ii) the LCM of each of the following pairs of numbers.
a) 60 and 75 b) 54 and 96 c) 108 and 144
d) 200 and 240 e) 168 and 196 f) 150 and 180

Q4 Use prime factors to find the highest number that will divide into both 93 and 155.

Q5 Use prime factors to find the lowest number that divides exactly by both 316 and 408.

Q6 One day, Arran divides his action figures into equal groups of 26.
The next day, he divides them up into equal groups of 12.
Use prime factors to find the lowest possible number of action figures he owns.

Q7 Jess goes swimming every 21 days. Seamus goes swimming every 35 days.
They both went swimming today. Use prime factors to find the number of days it will be until they both go swimming on the same day again.

Q8 a) Write 30, 140 and 210 as products of their prime factors.
b) Hence find the HCF of 30, 140 and 210.

Q9 a) Write 121, 280 and 550 as products of their prime factors.
b) Hence find the LCM of 121, 280 and 550.

Q10 Use prime factors to find (i) the HCF and (ii) the LCM of each of the following sets of numbers.
a) 65, 143 and 231 b) 175, 245 and 1225 c) 104, 338 and 1078

Q11 a) Use prime factors to find a pair of numbers that have HCF = 12 and LCM = 120.
b) Use prime factors to find a pair of numbers that have HCF = 20 and LCM = 300.

Review Exercise

Q1 Work out the following. a) $5 \times 6 - 8 \div 2$ b) $18 \div (9 - 12 \div 4)$ 

Q2 At midday the temperature was 6 °C. By midnight, the temperature had decreased by 7 °C. What was the temperature at midnight? 

Q3 Work out: $-4.2 - (1.5 \times -0.3)$ 

Q4 Milo spends £71.42 at the supermarket. His receipt says that he has saved £11.79 on special offers. How much would he have spent if there had been no special offers? 

Q5 Asha bought 2 CDs each costing £11.95 and 3 CDs each costing £6.59. She paid with a £50 note. How much change did she receive? 

Q6 It costs £31.85 to buy 7 identical DVDs. How much would it cost to buy 3 DVDs? 

Q7 a) List all the common multiples of 12 and 16 between 1 and 100.
b) List all the common multiples of 6, 15 and 20 between 1 and 100.

Q8 A butcher has 54 identical sausages. In how many different ways can she divide them up into equal packets? List the possibilities.

Q9 List the common factors of each of the following pairs of numbers.
a) 12, 15 b) 25, 50 c) 36, 48 d) 64, 80

Q10 Without doing any calculations, explain how you can tell that none of the numbers in this box are prime.

20	30	40	50
70	90	110	130

Q11 Write each of the following as the product of prime factors. 
a) 6 b) 40 c) 24 d) 110
e) 255 f) 60 g) 360 h) 225

Q12 Jill divides a pile of sweets into 5 equal piles. Kay then divides the same sweets into 7 equal piles. What is the smallest number of sweets there could be? 

Q13 Find the HCF of each of the following pairs of numbers.
a) 8 and 12 b) 18 and 24 c) 35 and 42 d) 56 and 63

Q14 Use prime factors to find (i) the HCF and (ii) the LCM of each of the following sets of numbers.
a) 36 and 48 b) 210 and 308 c) 126, 150 and 1029

Exam-Style Questions

Q1 $539 \times 28 = 15\,092$ 

Use the above result to work out the value of:

a) 539×14

[1 mark]

b) 5390×0.28

[1 mark]

c) $1\,509\,200 \div 53.9$

[2 marks]

Q2 Work out $3.774 \div 0.4$ 

[3 marks]

Q3 A single pack of salt and vinegar crisps costs 70p.
A single pack of cheese and onion crisps costs 65p. 
A multipack of 3 salt and vinegar and 3 cheese and onion costs £3.19.
How much would you save buying a multipack instead of the equivalent amount in individual packs?

[3 marks]

Q4 The lowest common multiple of two numbers is 60.
The highest common factor of the same two numbers is 4.
Neither of the numbers is 4 or 60. What are the numbers? 

[3 marks]

Q5 p and q are prime numbers. $50 < p < 60$ and $60 < q < 70$.
If $pq = 3599$, find the values of p and q. 

[3 marks]

Q6 When written as a product of prime factors in index form, $60 = 2^2 \times 3 \times 5$. 

a) Write 135 as a product of prime factors in index form.

[2 marks]

b) Use the prime factor forms of 60 and 135 to show that $\sqrt{60 \times 135} = 90$.

[2 marks]