

# Exam-style practice

## Mathematics

### A Level

### Paper 3: Statistics and Mechanics

Time: 2 hours

You must have: Mathematical Formulae and Statistical Tables, Calculator

#### SECTION A: STATISTICS

- 1 An electrical engineer makes components for computer systems. She claims that the components last longer than 500 hours on 52% of occasions.  
In a random sample of 40 of the components,  $X$  last longer than 500 hours.
  - a Find  $P(X \geq 22)$ . (1)
  - b Write down two conditions under which the normal approximation may be used as an approximation to the binomial distribution. (2)

A random sample of 250 components was taken and 120 lasted longer than 500 hours.

  - c Assuming the engineer's claim to be correct, use a normal approximation to find the probability that 120 or fewer components last longer than 500 hours. (3)
  - d Using your answer to part c, comment on the engineer's claim. (1)
  
- 2  $P(A) = 0.4$ ,  $P(B) = 0.55$  and  $P(C) = 0.26$ .  
Given that  $P(A \cap B) = 0.2$ , that events  $A$  and  $C$  are mutually exclusive and that events  $B$  and  $C$  are statistically independent,
  - a Draw a Venn diagram to illustrate events  $A$ ,  $B$  and  $C$ . (5)
  - b Show that events  $A$  and  $B$  are not statistically independent. (2)
  - c Find  $P(A|B')$ . (2)
  - d Find  $P(C|(A \cap B)')$ . (2)
  
- 3 The daily mean air temperature,  $t$  °C, is recorded in Perth for the month of October.
 

$t$	$12 \leq t < 15$	$15 \leq t < 18$	$18 \leq t < 20$	$20 \leq t < 22$	$22 \leq t < 26$
$f$	2	6	11	7	5

  - a State, with a reason, whether  $t$  is a discrete or continuous variable. (1)
  - b Use your calculator to find estimates for the mean and standard deviation of the temperatures. (2)
  - c Give two reasons why a histogram could be used to display this data. (2)
  - d Use linear interpolation to find the 10<sup>th</sup> to 90<sup>th</sup> interpercentile range. (3)

A meteorologist believes that there is a positive correlation between the daily mean air temperature and the number of hours of sunshine. She takes a random sample of 8 days from the data set above and finds that the product moment correlation coefficient is 0.612.

- e Stating your hypotheses clearly, test at the 5% level of significance, whether or not the product moment correlation coefficient for the population is greater than zero. (3)

- 4 An industrial chemical process produces an amount of a substance,  $q$  grams, dependent on the temperature,  $t$  °C applied. The table below shows the outcomes of five experiments.

$t$	10	20	32	41	57
$q$	160	700	2000	3300	6400

A chemist believes that the relationship between the variables can be modelled by an equation of the form  $q = kt^n$ , where  $k$  and  $n$  are constants to be determined. The data are coded using  $x = \log t$  and  $y = \log q$ . The product moment correlation coefficient between  $x$  and  $y$  is found to be 0.9998.

- a State with a reason whether this value supports the suggested model. (1)
- b Given that the equation of the regression line of  $y$  on  $x$  is  $y = 0.0761 + 2.1317x$ , find the value of  $k$  and the value of  $n$ . (3)
- c Explain, giving a reason, whether it would be sensible to use this model to predict the amount of substance produced when  $t = 85$  °C. (1)
- 5 The weights of cats in a particular town are normally distributed. A cat that weighs between 3.5 kg and 4.6 kg is said to be of 'standard' weight. Given that 2.5% of cats weigh less than 3.416 kg and 5% of cats weigh greater than 4.858 kg,
- a find the proportion of cats that are of standard weight. (6)
- 15 cats are chosen at random.
- b Find the probability that at least 10 of these cats are of standard weight. (2)
- In a second town, the weights of cats are also normally distributed with standard deviation 0.51 kg. A random sample of 12 cats was taken and the sample mean was 4.73 kg.
- c Test, at the 10% level of significance, whether or not the mean weight of all the cats in the town is different from 4.5 kg. State your hypotheses clearly. (4)
- 6 Jemima plays two games of tennis. The probability that she wins the first game is 0.62. If she wins the first game, the probability that she wins the second is 0.75. If she loses the first game, the probability that she wins the second is 0.45. Find the probability that she wins both games given that she wins the second game. (4)

## SECTION B: MECHANICS

- 7 At time  $t$  seconds, where  $t \geq 0$ , a particle  $P$  moves such that its velocity,  $\mathbf{v}$  m s<sup>-1</sup>, is given by

$$\mathbf{v} = (2 - 6t^2)\mathbf{i} - t\mathbf{j}$$

When  $t = 1$  the displacement of the particle from a fixed origin  $O$  is  $5\mathbf{i}$  m.

Find the distance of the particle from  $O$  when  $t = 3$  seconds, giving your answer to 3 significant figures. (7)

- 8 An arrow is fired from horizontal ground with an initial speed of  $100 \text{ m s}^{-1}$  at an angle of  $30^\circ$  above the horizontal.
- By modelling the arrow as a particle work out:
- the time taken for the arrow to hit the ground (3)
  - the maximum height of the arrow (3)
  - the speed of the arrow after 3 seconds. (6)

- 9 In this question  $\mathbf{i}$  and  $\mathbf{j}$  are the unit vectors due east and north respectively.
- A cyclist makes a journey between two points  $A$  and  $B$ . At time  $t = 0 \text{ s}$  the cyclist is moving due east at  $2 \text{ m s}^{-1}$ .

The cyclist is modelled as a particle.

Relative to  $A$ , the position vector of the cyclist at time  $t$  seconds is  $\mathbf{r}$  metres.

Given that the acceleration of the cyclist is constant and equal to  $0.2\mathbf{i} - 0.8\mathbf{j} \text{ m s}^{-2}$ , find:

- the position vector of the cyclist after 10 seconds (2)
- the distance of the cyclist from  $A$  after 10 seconds. (2)

After 10 seconds the cyclist stops accelerating and heads due east at a constant speed of  $5 \text{ m s}^{-1}$ .

- Find the value of  $t$  when the cyclist is directly south-east of  $A$ . (2)

After a further 30 s the cyclist reached point  $B$ .

- Work out the bearing of  $B$  from  $A$  to the nearest degree. (2)

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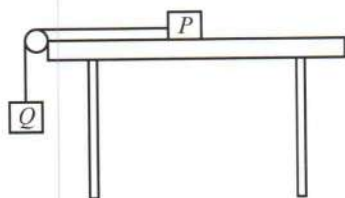


Figure 1

Two particles  $P$  and  $Q$ , of masses  $3 \text{ kg}$  and  $2 \text{ kg}$  respectively, are attached to the ends of a light inextensible string.  $P$  lies on a rough horizontal table. The string passes over a small smooth pulley fixed on the edge of the table.  $Q$  hangs freely below the pulley, as shown in Figure 1. The coefficient of friction between  $P$  and the table is  $\mu$ . The particles are released from rest with the string taut. Immediately after release,  $P$  accelerates at a rate of  $0.5 \text{ m s}^{-2}$ .

- Find the tension in the string immediately after the particles begin to move. (3)
- Show that  $\mu = 0.582$  (3 s.f) (3)

After two seconds the string breaks.

- Assuming that  $P$  remains on the table, calculate how long it takes  $P$  to come to rest. (6)
- State how you have used the information that the string is inextensible in your calculations. (1)



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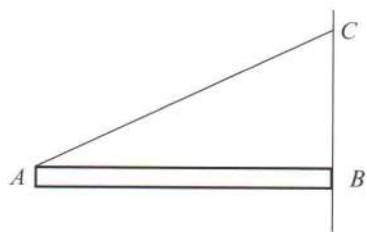


Figure 2

A uniform rod  $AB$  of length  $l$  m and mass  $m$  kg rests with one end touching a rough vertical wall at  $B$ .

The rod is kept horizontal by a light inextensible string  $AC$  where  $C$  lies on the wall directly above  $B$ .

The plane  $ABC$  is perpendicular to the wall and  $\angle BAC$  is  $\alpha$ , where  $\tan \alpha = \frac{5}{12}$ .

**a** Show that the tension in the string is  $\frac{13}{10}mg$  N. (4)

**b** Calculate the coefficient of friction between the rod and the wall. (6)

### Percentage points of the normal distribution

The values  $z$  in the table are those which a random variable  $Z \sim N(0, 1)$  exceeds with probability  $p$ ; that is,  $P(Z > z) = 1 - \Phi(z) = p$ .

$p$	$z$	$p$	$z$
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

### Critical values for correlation coefficients

This table concerns tests of the hypothesis that a population correlation coefficient  $\rho$  is 0. The values in the table are the minimum value which need to be reached by a sample correlation coefficient in order to be significant at the level shown, on a one-tailed test.

Product moment coefficient					
0.10	Level				Sample Level
	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10
0.4187	0.5214	0.6021	0.6851	0.7348	11
0.3981	0.4973	0.5760	0.6581	0.7079	12
0.3802	0.4762	0.5529	0.6339	0.6835	13
0.3646	0.4575	0.5324	0.6120	0.6614	14
0.3507	0.4409	0.5140	0.5923	0.6411	15
0.3383	0.4259	0.4973	0.5742	0.6226	16
0.3271	0.4124	0.4821	0.5577	0.6055	17
0.3170	0.4000	0.4683	0.5425	0.5897	18
0.3077	0.3887	0.4555	0.5285	0.5751	19
0.2992	0.3783	0.4438	0.5155	0.5614	20
0.2914	0.3687	0.4329	0.5034	0.5487	21
0.2841	0.3598	0.4227	0.4921	0.5368	22
0.2774	0.3515	0.4133	0.4815	0.5256	23
0.2711	0.3438	0.4044	0.4716	0.5151	24
0.2653	0.3365	0.3961	0.4622	0.5052	25
0.2598	0.3297	0.3882	0.4534	0.4958	26
0.2546	0.3233	0.3809	0.4451	0.4869	27
0.2497	0.3172	0.3739	0.4372	0.4785	28
0.2451	0.3115	0.3673	0.4297	0.4705	29
0.2407	0.3061	0.3610	0.4226	0.4629	30
0.2070	0.2638	0.3120	0.3665	0.4026	40
0.1843	0.2353	0.2787	0.3281	0.3610	50
0.1678	0.2144	0.2542	0.2997	0.3301	60
0.1550	0.1982	0.2352	0.2776	0.3060	70
0.1448	0.1852	0.2199	0.2597	0.2864	80
0.1364	0.1745	0.2072	0.2449	0.2702	90
0.1292	0.1654	0.1966	0.2324	0.2565	100

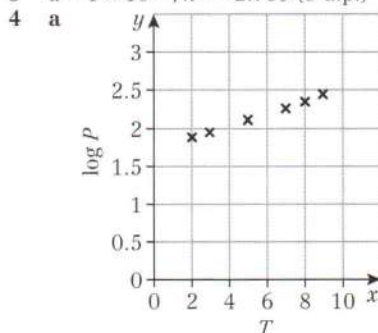
# Answers

## Prior knowledge 1

- 1 a  $A = \log 3$ ,  $B = \log 2$   
b Gradient =  $\log 2$ ,  $y$ -intercept =  $\log 3$
- 2 For each 1 cm increase in handspan, the height increases by approximately 11.3 cm.
- 3  $P(X \geq 32) = 0.0061 < 0.01$   
 $H_0$  can be rejected.

## Exercise 1A

- 1 a  $y = ax^n$  b  $a = 15.8$  (3 s.f.),  $n = 0.4$
- 2 a  $y = kb^x$  b  $k = 2.51$ ,  $b = 39.8$  (3 s.f.)
- 3  $a = 1 \times 10^{172}$ ,  $n = -2.739$  (3 d.p.)



$T$	2	3	5	7	8	9
$\log P$	1.86	1.93	2.10	2.25	2.33	2.41

- b Strong positive correlation
- c Yes – the variables show a linear relationship when  $\log P$  is plotted against  $T$ .
- d  $a = 50.1$  (3 s.f.),  $b = 1.2$
- e For every month that passes, the population of moles increases by 20%.
- 5 a  $t = a + bn$  would show a linear relationship. This graph is not a straight line.  
b  $a = 0.5$ ,  $k = 0.6$
- 6  $r = 0.389$  (3 s.f.)
- 7  $a = 1.0$ ,  $n = 1.8$
- 8 a  $a = 1.23$  (3 s.f.),  $b = 1.12$  (3 s.f.)  
b  $b$  is the rate of change of  $g$  per degree.  
c  $35^\circ\text{C}$  is outside the range of the data (extrapolation).

## Challenge

- a A graph of  $\log T$  against  $\log E$  shows a straight line.
- b  $\log E = 1.09 - 1.96(\log T)$ ,  $a = 12.3$  (3 s.f.),  $b = -1.96$  (3 s.f.)
- c  $\log 0$  is undefined.

## Exercise 1B

- 1 Answers close to  
a 0.9 b  $-0.7$  c  $-0.3$
- 2 a The type and strength of linear correlation between  $v$  and  $m$ .  
b 0.870
- 3 a  $-0.854$   
b There is negative correlation. The relatively older young people took less time to reach the required level.

4 a

Time, $t$	1	2	4	5	7
Atoms, $n$	231	41	17	7	2
$\log n$	2.36	1.61	1.23	0.845	0.301

- b  $-0.980$  (3 s.f.)
- c There is an almost perfect negative correlation with data in the form  $\log n$  against  $t$ , which suggests an exponential decay curve.
- d  $a = 307$  (3 s.f.),  $b = 0.479$  (3 s.f.)

5 a

Width, $w$	3	4	6	8	11
Mass, $m$	23	40	80	147	265
$\log w$	0.4771	0.6021	0.7782	0.9031	1.041
$\log m$	1.362	1.602	1.903	2.167	2.423

- b 0.9996
- c A graph of  $\log w$  against  $\log m$  is close to a straight line as the value of  $r$  is close to 1, therefore  $m = kw^n$  is a good model for this data.
- d  $n = 1.88$  (3 s.f.),  $k = 2.91$  (3 s.f.)
- 6 a  $-0.833$   
b  $-0.833$  is close to  $-1$  so the data values show a strong to moderate negative correlation. A linear regression model is suitable for these data.
- 7 a A 'trace or tr' of rain is an amount less than 0.05 mm.  
b  $-0.473$  (3 s.f.), treating 'tr' values as 0.  
c The data shows a weak negative correlation so a linear model may not be best, there may be other variables affecting the relationship or a different model might be a better fit.

## Challenge

$r$  for  $\log x$  and  $\log y$ : 1.00 (3 s.f.)

$r$  for  $x$  and  $\log y$ : 0.985 (3 s.f.)

Therefore the most suitable model would be in the form  $y = kx^n$

## Exercise 1C

- 1 a  $H_0: \rho = 0$ ,  $H_1: \rho \neq 0$ , critical value =  $\pm 0.3120$ . Reject  $H_0$ : there is reason to believe at the 5% level of significance that there is a correlation between the scores.  
b  $H_0: \rho = 0$ ,  $H_1: \rho \neq 0$ , critical value =  $\pm 0.3665$ . Do not reject  $H_0$ . There is no evidence of correlation between the two scores at the 2% level of significance.
- 2 a  $-0.960$  (3 s.f.)  
b  $H_0: \rho = 0$ ,  $H_1: \rho \neq 0$ , critical value =  $\pm 0.8745$ . Reject  $H_0$ : there is reason to believe at the 1% level of significance that there is a correlation between the scores.
- 3 a The type and strength of linear correlation between two variables.  
b 0.935 (3 s.f.)  
c  $H_0: \rho = 0$ ,  $H_1: \rho > 0$ , critical value = 0.4973. Reject  $H_0$ : there is reason to believe that students who do well in theoretical biology are likely to do well in practical biology.  
d There is a probability of less than 0.05 that the null hypothesis is true.
- 4 a 0.686 (3 s.f.)  
b  $H_0: \rho = 0$ ,  $H_1: \rho > 0$ , critical value = 0.6215. Reject  $H_0$ : there is reason to believe that there is a linear correlation between the English and Mathematics marks.
- 5  $H_0: \rho = 0$ ,  $H_1: \rho > 0$ ,  $r = 0.793$  (3 s.f.), critical value = 0.8822. Do not reject  $H_0$ . There is evidence that the company is incorrect to believe that profits increase with sales.





- 6  $H_0: \rho = 0$ ,  $H_1: \rho < 0$ , critical value =  $-0.4409$ . Do not reject  $H_0$ . There is evidence that the researcher is incorrect to believe that there is negative correlation between the amount of solvent and the rate of the reaction.
- 7 2.5%
- 8 8
- 9 a  $-0.846$  (3 s.f.)  
 b  $H_0: \rho = 0$ ,  $H_1: \rho < 0$ , critical value =  $-0.8822$ . Do not reject  $H_0$ . There is evidence that the employee is incorrect to believe that there is negative correlation between humidity and visibility.
- 10 a This is a two-tailed test, so the scientist would need to halve the significance level or double her  $p$ -value.  
 b  $\pm 0.4438$

### Mixed exercise 1

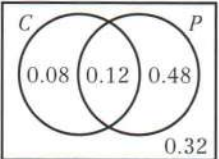
- 1 a 0.9998  
 b  $r$  is close to 1, so a graph of  $\log t$  against  $\log x$  shows a straight line, suggesting that the relationship is in the form  $t = ax^b$ .  
 c  $n = 1.38$ ,  $a = 0.617$  (3 s.f.)
- 2 a  $a = 0.232$  (3 s.f.),  $b = 1.08$  (3 s.f.)  
 b  $151^\circ\text{C}$  is outside the range of the data (extrapolation).
- 3 As a person's age increases, their score on the memory test decreases.
- 4 a Each cow should be given 7 units. The yield levels off at this point.  
 b 0.952 (3 s.f.)  
 c It would be less than 0.952. The yield of the last 3 cows is no greater than that of the 7th cow.
- 5 a  $-0.972$   
 b There is strong negative correlation. As  $c$  increases,  $f$  decreases.
- 6 a 0.340 (3 d.p.)  
 b  $H_0: \rho = 0$ ,  $H_1: \rho \neq 0$ , critical value =  $\pm 0.6319$ . Do not reject  $H_0$ . There is not enough evidence that there is a correlation between age and salary.
- 7 a 0.937 (3 s.f.)  
 b  $H_0: \rho = 0$ ,  $H_1: \rho \neq 0$ , critical value =  $\pm 0.6319$ . Reject  $H_0$ . There is evidence that there is a correlation between the age of a machine and its maintenance costs.
- 8  $H_0: \rho = 0$ ,  $H_1: \rho < 0$ , critical value =  $-0.5822$ . Reject  $H_0$ . There is evidence that the greater the altitude, the lower the temperature.
- 9  $H_0: \rho = 0$ ,  $H_1: \rho > 0$ , critical value 0.5822,  $0.972 > 0.5822$ . Reject  $H_0$ . There is evidence that age and weight are positively correlated.
- 10 a 0.940  
 b  $H_0: \rho = 0$ ,  $H_1: \rho > 0$ , critical value 0.7293. Reject  $H_0$ . There is evidence that sunshine hours and ice cream sales are positively correlated.
- 11  $r = 0.843$  (3 s.f.),  $H_0: \rho = 0$ ,  $H_1: \rho > 0$ , critical value 0.8054. Reject  $H_0$ . There is evidence that mean windspeed and daily maximum gust are positively correlated.
- 12  $r = -0.793$  (3 s.f.),  $H_0: \rho = 0$ ,  $H_1: \rho < 0$ , critical value  $-0.7545$ . Reject  $H_0$ . There is evidence that temperature and pressure are negatively correlated.

### Large data set

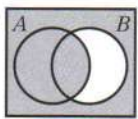
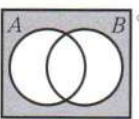
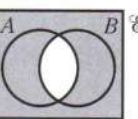
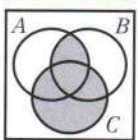
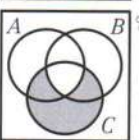
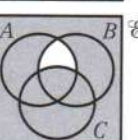
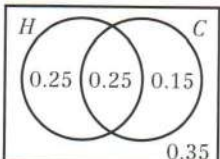
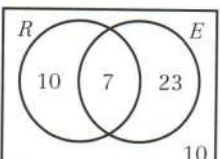
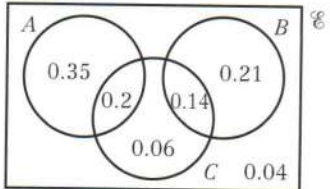
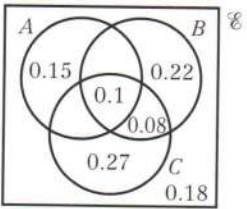
Student's own answers

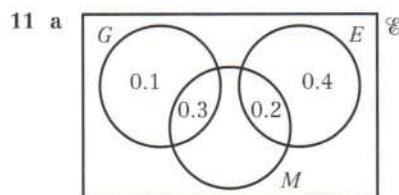
### Prior knowledge 2

- 1 a 0.75  
 b 0  
 c 0.25

- 2 a 0.12  
 b   
 c 0.32  
 3 a  $\frac{2}{7}$  b  $\frac{6}{7}$

### Exercise 2A

- 1 a  $A \cap B'$  b  $A' \cup B$   
 c  $(A \cap B) \cup (A' \cap B')$  d  $A \cap B \cap C$   
 e  $A \cup B \cup C$  f  $(A \cup B) \cap C'$
- 2 a  b  c 
- 3 a  b  c 
- 4 a  $\frac{1}{13}$  b  $\frac{1}{4}$  c  $\frac{1}{52}$   
 d  $\frac{4}{13}$  e  $\frac{3}{4}$  f  $\frac{3}{13}$
- 5 a 0.6 b 0.8 c 0.4 d 0.9  
 6 a 0.25 b 0.5 c 0.65 d 0.85
- 7 a   
 b i 0.65 ii 0.15 iii 0.85
- 8 a   
 b i 7 ii  $\frac{1}{5}$  iii  $\frac{43}{50}$
- 9 a   
 b i 0.1 ii 0.76 iii 1
- 10 a   
 b i 0.53 ii 0.18  
 c Not independent.  
 $P(A' \cap C) = 0.35$ ,  $P(A') \times P(C) = 0.75 \times 0.45 = 0.3375$



b i 0.6 ii 0.5

c Not independent.

$$P(G' \cap M) = 0.2, P(G') \times P(M) = 0.6 \times 0.5 = 0.3$$

12 a  $xy$  b  $x + y - xy$  c  $1 - y + xy$

### Challenge

a  $xyz$

b  $x + y + z + xyz - xy - yz - xz$

c  $z - yz + xyz$

### Exercise 2B

1 a  $\frac{29}{60}$  b  $\frac{18}{29}$  c  $\frac{18}{35}$  d  $\frac{14}{31}$

2 a

	Badminton	Squash	Total
Male	21	22	43
Female	15	17	32
Total	36	39	75

b i  $\frac{22}{39}$  ii  $\frac{15}{36}$  or  $\frac{5}{12}$  iii  $\frac{17}{32}$

3 a

	Girls	Boys	Total
Vanilla	13	2	15
Chocolate	12	10	22
Strawberry	20	23	43
Total	45	35	80

b i  $\frac{23}{43}$  ii  $\frac{13}{15}$  iii  $\frac{10}{35}$  or  $\frac{2}{7}$

4 a Blue spinner

Red spinner

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b i  $\frac{1}{4}$  ii  $\frac{1}{4}$  iii  $\frac{1}{4}$

5 a Dice 1

Dice 2

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b  $\frac{1}{6}$  c  $\frac{1}{4}$

d All outcomes are equally likely.

6 0.0769 (3 s.f.) or  $\frac{1}{13}$

7 a  $\frac{1}{3}$  b  $\frac{2}{3}$

c Assume that the coins are not biased.

8 a

	D	D'	Total
S	18	38	56
S'	59	5	64
Total	77	43	120

b i  $\frac{43}{120}$  ii  $\frac{5}{120}$  iii  $\frac{18}{77}$  iv  $\frac{38}{56}$

9 a

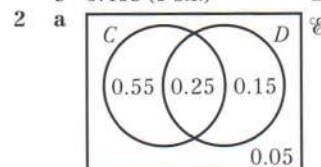
	Women	Men	Total
Stick	26	18	44
No stick	37	29	66
Total	63	47	110

b i  $\frac{44}{110}$  or  $\frac{2}{5}$  ii  $\frac{26}{63}$  iii  $\frac{18}{44}$  or  $\frac{9}{22}$

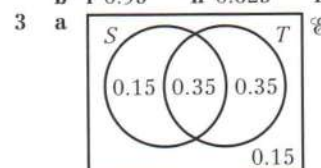
10 a  $\frac{6}{25}$  b  $\frac{13}{30}$  c  $\frac{29}{64}$  d  $\frac{31}{90}$

### Exercise 2C

1 a 0.7 b 0.3  
c 0.483 (3 s.f.) d 0.571 (3 s.f.)



b i 0.95 ii 0.625 iii 0.3125 iv 0.25



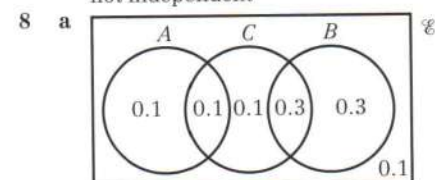
b i 0.35 ii 0.5 iii 0.7 iv 0.231 (3 s.f.)

4 a  $\frac{3}{8}$  b  $\frac{2}{5}$  c  $\frac{6}{11}$  d  $\frac{13}{19}$

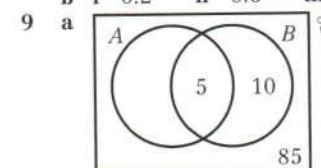
5 a  $\frac{9}{80}$  b  $\frac{9}{32}$  c  $\frac{1}{5}$  d  $\frac{12}{35}$

6 a 0.6 b 0.4  
c 0.299 (3 s.f.) d 0.329 (3 s.f.)

7 a  $\frac{9}{23}$  b  $\frac{3}{23}$   
c  $P(B|C) = 0.111... \neq P(B) = 0.345...$  So B and C are not independent



b i 0.2 ii 0.6 iii 0.5



b  $\frac{1}{3}$   
c No one who doesn't have the disease would be given a false negative result. However, only  $\frac{1}{3}$  of the people who have a positive result would have the disease.

10 a 0.7 b 0.7 c They are independent.

11  $x = 0.21, y = 0.49$

12 c  $\frac{7}{30}, d = \frac{4}{15}$

### Exercise 2D

1 a 0.3 b 0.6 c 0.8 d 0.9

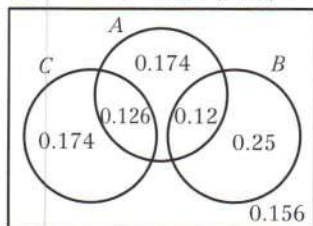
2 a 0.8  
b i 0.2 ii 0.615 (3 s.f.) iii 0.429 (3 s.f.)

c  $P(C \cap D) \neq P(C) \times P(D)$





- 3 a 0.9  
b i 0.8 ii 0.2 iii 0.5
- 4 a 0.15 b 0.45 c 0.55 d 0.25 e 0.3
- 5 0.1
- 6 a 0.5 b 0.3 c 0.3
- 7 a 0.3 b 0.35 c 0.4
- 8 a  $\frac{1}{12}$  b  $\frac{3}{20}$  c  $\frac{5}{14}$   
d  $\frac{7}{30}$  e  $\frac{18}{23}$
- 9 a 0.67 b 0.476 (3 s.f.) c 0.126  
d 0.294



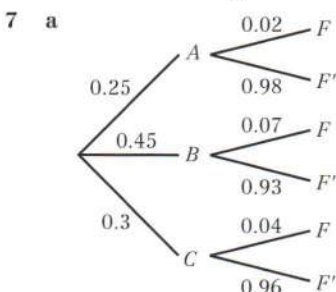
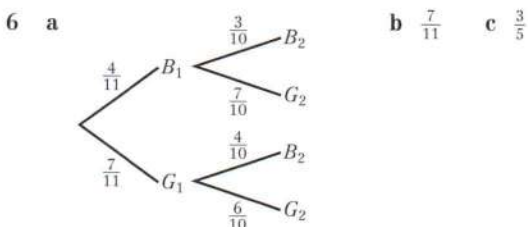
- 10 a 0.28 b 0.7  
c 0.333 (3 s.f.) d 0.467 (3 s.f.)
- 11 a 0.1 b 0.143 (3 s.f.)  
c  $P(A) \times P(B) = 0.3 \times 0.7 = 0.21$ ,  $P(A \cap B) = 0.1$ .  
This suggests that the events are not independent.  
If Anna is late, Bella is less likely to be late and vice versa.
- 12 a 0.5 b 0.333 (3 s.f.) c 0.833 (3 s.f.)  
d  $P(K|J) = 0.833 \dots \neq P(K) = 0.7$ . So  $J$  and  $K$  are not independent

### Challenge

- a  $\frac{1}{15}$  b  $\frac{5}{12}$  c  $\frac{4}{5}$

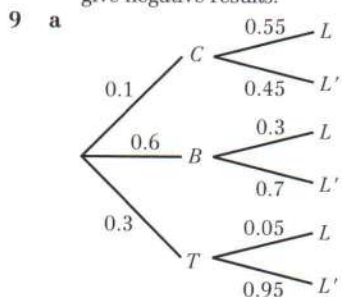
### Exercise 2E

- 1 a
- 
- 2 a  $\frac{5}{8}$  b  $\frac{5}{8}$  c  $\frac{1}{2}$  d  $\frac{1}{2}$  e  $\frac{1}{2}$
- 
- 3 a i 0.315 ii 0.195 iii 0.75
- 
- b 0.163 (3 s.f.) c 0.507 (3 s.f.) d 0.243 (3 s.f.)
- 4 0.36
- 5 a 0.25 b 0.333

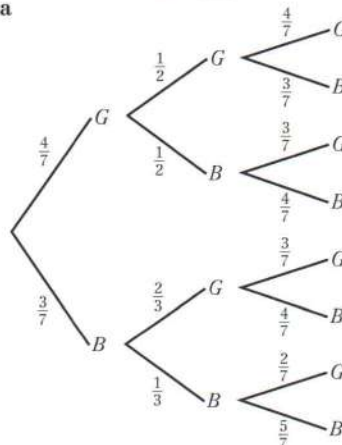


- b i 0.0315 ii 0.0485 c 0.103 (3 s.f.)
- 8 a
- 

- b 0.9448 c 0.00423
- d The probability that a positive result is a false positive (positive result for someone without the condition) =  $P(-|+) = 0.348$ . Over one third of positive results are false positives and 10% of people with the condition give negative results.



- b i 0.015 ii 0.25 c 0.78
- 10 a



- b  $\frac{3}{7}$   
 c Adding together the probabilities on the 4 branches of the tree diagram where the counter from box B is blue:  $\frac{12}{98} + \frac{16}{98} + \frac{24}{147} + \frac{15}{147} = \frac{27}{49}$   
 d Adding together the probabilities on the 2 branches of the tree diagram where events C and D both occur:  $\frac{12}{98} + \frac{15}{147} = \frac{11}{49}$   
 e  $\frac{37}{49}$  f  $\frac{8}{13}$   
 11 She has not taken into account the fact that the jelly bean is eaten after being selected. The correct answer is 0.5.

### Mixed exercise 2

- 1 a 0.55 b 0.45 c 0.5 d 0.429 (3 s.f.)  
 2 a   
 b i 0.6 ii 0.6 iii 0.222 (3 s.f.) iv 0.471 (3 s.f.)  
 3 a 0.433 (3 s.f.) b 0.6 c 0.72  
 d 0.25 e 0.577 (3 s.f.)  
 4 a

- b i  $\frac{12}{35}$  ii  $\frac{18}{35}$  d  $\frac{6}{65}$   
 c  $\frac{2}{5}$   
 5 a 0.74 b 0.757 (3 s.f.) c 0.703  
 6 a  $\frac{4}{15}$  b  $\frac{15}{41}$   
 c 0.117 (3 s.f.) d 0.146 (3 s.f.)  
 7 a 0.3 b 0.42  
 c

- d i 0.25 ii 0.28  
 8 a In some football matches, neither team scores.  
 b 0.12 c 0.179 (3 s.f.)

### Challenge

- a  $0.4 \leq p \leq 0.6$  b  $0.2 \leq q \leq 0.5$

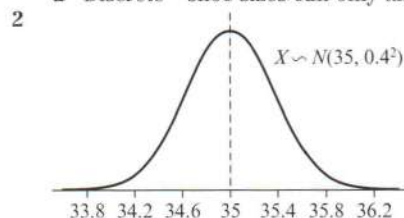
### Prior knowledge 3

- 1 a 0.540 (3 s.f.) b 0.390 (3 s.f.)  
 2 a 0.124 (3 s.f.) b 0.584 (3 s.f.) c 0.869 (3 s.f.)  
 3 a 0.211 (3 s.f.) b 0.599 (3 s.f.)

For Chapter 3, student answers may differ slightly from those shown here depending on whether tables or calculators were used.

### Exercise 3A

- 1 a Continuous – lengths can take any value  
 b Discrete – scores can only take certain values  
 c Continuous – masses can take any value  
 d Discrete – shoe sizes can only take certain values



- 3 The distribution is not symmetrical.  
 4 a 0.68 b 0.95  
 5 49  
 6 60 g  
 7  $\mu = 56.5, \sigma^2 = 4.5^2$   
 8 a 0.5 b 0.68 c 0.95  
 d Incorrect: although  $P(X > 100) > 0$ , it is very small since 100 is more than 3 standard deviations away from the mean, so the model as a whole is still reasonable.  
 9 a 36 b Between 2 and 3

### Exercise 3B

- 1 a 0.9332 b 0.9772 c 0.2119  
 2 a 0.0478 b 0.2525 c 0.2782  
 3 a i 19.12 (2 d.p.) ii 18.27 (2 d.p.) b 0.4985  
 c 0.5948  
 4 a i 70.60 (2 d.p.) ii 80.78 (2 d.p.) b 0.1714  
 c 0.0373  
 5 a i 81.01 (2 d.p.) ii 80.64 (2 d.p.)  
 b Sum is 1, combined probabilities include every possible value.  
 6 a 0.3176 b 0.6824  
 7 a 0.1814 b 0.4295  
 8 a i 0.1056 ii 0.1056 b 0.0012  
 9 a i 0.3605 ii 0.2375 b 0.3380  
 10 a 0.0766 b 0.1906 c 0.3296  
 11 a 0.0228 b 0.7345  
 12 a 0.4013 b 0.000199

### Exercise 3C

- 1 a 27.38 b 33.37 c 31.27 d 35.30  
 2 a 8.16 b 10.84 c 12.02 d 11.45  
 3 a i 19.1 ii 18.3  
 b 0.0915  
 4 a i 70.6 ii 80.8 b 0.075  
 5 a i 81.0 ii 80.6 b 0.0364  
 6 a 4.095 (3 d.p.) b 5.005 (3 d.p.)  
 c 4.5 is the mean, so 50% of badgers will have a mass less than 4.5.  
 7 a 73.52 (2 d.p.) b 8.09 (2 d.p.)  
 8 a 61.68 (2 d.p.) b 5.13 (2 d.p.)  
 c Tom is correct in this case; the normal distribution is symmetric about the mean, so 50% of bars will have mass less than the mean.  
 9 a Short: Up to 165 cm, Regular: Between 165 cm and 178 cm Long: Over 178 cm  
 b That the population follows the normal distribution over the whole range of values i.e. that there are no extreme outliers.

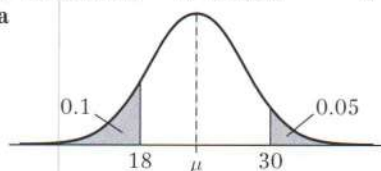


### Exercise 3D

- 1 a 0.9830 b 0.9131 c 0.2005 d 0.3520  
e 0.4893 f 0.0516 g 0.1823 h 0.8836
- 2 a 1.3298 b 1.8606 c 1.0364 d -1.6449  
e 1.0599 f 2.5491 g 1.2816 h 0.5244
- 3 a 0 b -0.16 c 0.2 d 0.74
- 4 a  $\Phi(0)$  b  $\Phi(0.5)$  c  $1 - \Phi(-0.25)$   
d  $\Phi(\frac{1}{12}) - \Phi(-\frac{7}{6})$
- 5 a 1.96 b 87.8 (3 s.f.)
- 6 a -1.0364 b 54.9 cm (3 s.f.)
- 7 a  $-1.2816 < z < 1.2816$  b 1103–1247 hours

### Exercise 3E

- 1 11.5
- 2 3.87
- 3 31.55
- 4 25
- 5  $\mu = 13.1, \sigma = 4.33$
- 6  $\mu = 28.3, \sigma = 2.59$
- 7  $\mu = 12, \sigma = 3.56$
- 8  $\mu = 35, \sigma = 14.8$
- 9 4.75
- 10  $\sigma = 1.99, a = 2.18$
- 11 a 203.37 mm b 0.1504 c 0.0516
- 12 a 0.1298 mm b 0.5591 c 0.0650
- 13 a



- b  $\mu = 23.26, \sigma = 4.101$  c 0.4459
- 14 a  $\mu = 16.79, \sigma = 0.9420$  b 1.27 (3 s.f.)

### Challenge

- a Let  $z$  be such that  $\Phi(z) = 0.75$ , then upper quartile  $= \mu + z\sigma$  and lower quartile  $= \mu - z\sigma$ , so  $q = (\mu + z\sigma) - (\mu - z\sigma) = 2z\sigma$ . Calculate that  $z = 0.67449$ , then  $q = 1.34898\sigma$  and thus  $\sigma = 0.741q$  (3 s.f.).
- b Since  $q = (\mu + z\sigma) - (\mu - z\sigma) = 2z\sigma$  (i.e. the  $\mu$ s cancel),  $q$  is not dependent on  $\mu$  and vice versa, and it is not possible to write  $\mu$  in terms of  $q$ .

### Exercise 3F

- 1 a i Yes,  $n$  is large ( $> 50$ ) and  $p$  is close to 0.5.  
ii  $X \sim N(72, 5.37^2)$   
b i No,  $n$  is not large enough ( $< 50$ ).  
c i Yes,  $n$  is large ( $> 50$ ) and  $p$  is close to 0.5.  
ii  $X \sim N(130, 7.90^2)$   
d i No,  $p$  is too far from 0.5.  
e i Yes,  $n$  is large ( $> 50$ ) and  $p$  is close to 0.5.  
ii  $X \sim N(192, 9.99^2)$   
f i Yes,  $n$  is large ( $> 50$ ) and  $p$  is close to 0.5.  
ii  $X \sim N(580, 15.6^2)$
- 2 a 0.1253 b 0.0946 c 0.6723
- 3 a 0.0097 b 0.5115 c 0.0559
- 4 a 0.6203 b 0.4540 c 0.0102
- 5 0.006
- 6 0.3767
- 7 a  $n$  large,  $p$  close to 0.5. b 0.1593  
c 0.5772 d 115
- 8 a 0.6277 b 0.8456
- 9 a 0.0784 (3 s.f.) b 0.31%

### Exercise 3G

- 1 a Not significant. Do not reject  $H_0$ .  
b Significant. Reject  $H_0$ .  
c Not significant. Do not reject  $H_0$ .  
d Significant. Reject  $H_0$ .  
e Not significant. Do not reject  $H_0$ .
- 2 a  $\bar{X} < 119.4$   
b  $\bar{X} > 13.2$   
c  $\bar{X} < 84.3$   
d  $\bar{X} > 0.877$  or  $\bar{X} < -0.877$   
e  $\bar{X} > -7.31$  or  $\bar{X} < -8.69$
- 3 Result is significant so reject  $H_0$ . There is evidence that the new formula is an improvement.
- 4 a  $\bar{X} > 103.29$   
b  $102.5 < 103.29$ , so there is not enough evidence to reject the null hypothesis
- 5 Insufficient evidence; do not reject  $H_0$ .
- 6 a 0.9256 b 0.9454  
c  $H_0: \mu = 5.7, H_1: \mu < 5.7$ . There is sufficient evidence to suggest mean diameter less than 5.7 mm.
- 7 a 136.48 g b 0.0129  
c  $H_0: \mu = 860, H_1: \mu \neq 860$ . Insufficient evidence to suggest mean mass is different to 860 g.
- 8  $H_0: \mu = 9.5, H_1: \mu > 9.5$ . Critical region is  $\bar{X} \geq 10.715$ .  $\bar{x} = 12.2 > 10.715$ , so reject  $H_0$  and conclude that the mean daily windspeed is greater than 9.5 knots.

### Mixed exercise 3

- 1 a 0.0401 b 0.3306 c 188 cm
- 2 a 12.8% b 51.1% c 0.9315
- 3 a 0.0668 b 0.0521 c 0.9315
- 4 a 3.65 b 0.136 c 32.5
- 5 a 8.60 ml b 0.122 c 109 ml
- 6 a  $\mu = 30, \sigma = 14.8$  b 38.0
- 7 Mean 10.2 cm, standard deviation 3.76 cm
- 8 a 0.3085  
b 0.3694  
c The first score was better, since fewer of the students got this score or more.
- 9 a 4.25 b 0.0499 c 0.8734
- 10 a 8.541 minutes b 0.1757
- 11 Mean 6.12 mm, standard deviation 0.398 mm
- 12 0.0778
- 13 a  $n$  is large and  $p$  is close to 0.5.  
b  $\mu = 40, \sigma^2 = 24$   
c 0.0262
- 14 a 0.0147  
b  $n$  is large and  $p$  is close to 0.5;  $\mu = 55.2, \sigma^2 = 29.808$   
c 0.20% (2 s.f.)
- 15 a  $n$  is large and  $p$  is close to 0.5.  
b 0.5232 c 166
- 16 0.6339
- 17 a 0.5914 b 0.0197  
c Assuming the claim is correct, there would be a less than 2% chance that 95 or fewer seedlings produce apples within 3 years. Therefore it is unlikely that the claim is correct.
- 18 a 0.5801 b 0.0594  
c Assuming that the claim is correct, there is a greater than 5% chance that 170 people out of 300 would be cured, therefore there is insufficient evidence to reject the herbalist's claim.
- 19  $\bar{X} > 7.66$  (3 s.f.)
- 20  $H_0: \mu = 125, H_1: \mu < 125, P(\bar{X} < 124.2) = 0.06066$   
Not significant so do not reject  $H_0$ . There is insufficient evidence to suggest that the mean contents of a bottle is lower than the manufacturer's claim.



- 21 a 0.0342 b 0.0692  
 c  $H_0: \mu = 170.2$ ,  $H_1: \mu > 170.2$ ,  $P(\bar{X} > 172.4) = 0.0692$   
 Not significant so do not reject  $H_0$ . Insufficient evidence of an increase in the mean breaking strength of climbing rope.
- 22 a  $Z = \pm 1.96 \rightarrow 1000 + 1.96\sigma = 1010$   
 $1.96\sigma = 10 \rightarrow \sigma^2 = 26.03$  (2 d.p.)  
 b  $H_0: \mu = 1010$ ,  $H_1: \mu \neq 1010$ ,  
 $P(\bar{X} > 1013.6375) = 0.0219$   
 Not significant so do not reject  $H_0$ . There is insufficient evidence of a deviation in mean from 1010. So we can assume condition i is being met.
- 23 a 0.8454  
 b  $H_0: \mu = 4.11$ ,  $H_1: \mu \neq 4.11$ ,  $P(\bar{X} > 4.3125) = 0.0129$   
 Significant so reject  $H_0$ . There is evidence that the mean length of eggs from this island is different from elsewhere.
- 24 a  $X \sim N(\mu, \frac{\sigma^2}{n})$  b Need  $n = 28$  or more

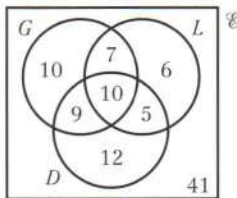
### Challenge

- a 0.2510 b  $\bar{X} \leq 104$ ,  $\bar{X} \geq 136$   
 c 102 is in the critical region, so at the 5% significance level there is evidence to reject the manager's claim. It is probable that less than 48% of people support the manager.

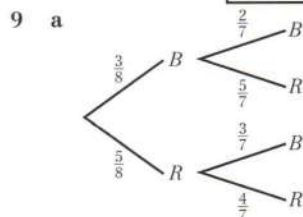
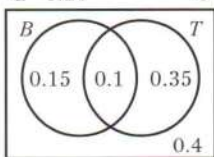
### Review exercise 1

Student answers may differ slightly from those shown here depending on whether tables or calculators were used.

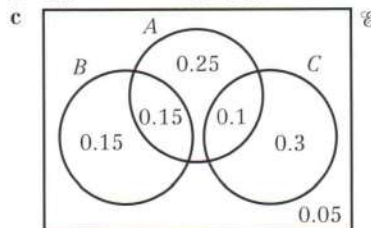
- 1 a 0.9992  
 b Very close to 1 suggesting a strong correlation between the two variables  
 c  $a = 0.1244$ ,  $n = 1.4437$
- 2 a  $a = 0.611$ ,  $b = 1.04$  b Not in range – extrapolation
- 3 a 0.93494...  
 b  $0.935 > 0.7155$  so reject  $H_0$ ; levels of serum and disease are positively correlated.
- 4  $r = -0.4063$ , critical value for  $n = 6$  is  $-0.6084$  so no evidence.
- 5 a  $H_0: \rho = 0$ ,  $H_1: \rho < 0$ ,  $r = -0.9313$ , critical value for  $n = 5$  is  $-0.8783$  so there is evidence. Reject  $H_0$ .  
 b Would lead to conclusion that there is no evidence (critical value =  $-0.9343$ ). Do not reject  $H_0$ .
- 6 a 0.338 (3 s.f.) b 0.46  
 c 0.743 (3 s.f.) d 0.218 (3 s.f.)
- 7 a



- c 0.41 d 0.21 e 0.667 (3 s.f.)  
 8 a 0.1 b



- b i  $\frac{21}{56}$  or  $\frac{3}{8}$  ii  $\frac{2}{7}$
- 10 a
- 
- b  $P(A) = 0.49$ ,  $P(B) = 0.28$   
 c  $\frac{17}{36}$  or 0.472 (3 s.f.)  
 d No:  $P(A) \times P(B) \neq P(A \cap B)$
- 11 a 0.5 b 0.35



- c
- d i 0.25 ii 0.4 iii  $\frac{2}{3}$
- 12 a 0.1975 b  $\frac{45}{79}$  or 0.570 (3 s.f.)  
 13 a 0.2743 b 12  
 14 a 0.0620 b 0.9545  
 c 0.00282 d This is a bad assumption.
- 15 a Mean 1.700 m, standard deviation 0.095 m  
 b 0.337
- 16 a 1.07 (3 s.f.) b 0.505 (3 s.f.) c 0.981 (3 s.f.)  
 17 a 0.2225 b 0.2607  
 c Accept 0.2566 ~ 0.2567 d 0.0946
- 18 a 0.06703 (5 d.p.) b 0.37% (2 s.f.)  
 19  $H_0: \mu = 18$ ,  $H_1: \mu < 18$ ,  $0.0264 < 0.05$  so reject  $H_0$  or in critical region. There is evidence that the (mean) time to complete the puzzles has reduced.
- 20 a 2.36 metres (3 s.f.) b 0.524 (3 s.f.)  
 c  $H_0: \mu = 1.9$ ,  $H_1: \mu \neq 1.9$ . Less than 1.769 m and greater than 2.031 m  
 d 2.09 m is in the critical region so there is evidence to suggest the mean length is not 1.9 m.
- 21  $H_0: \mu = 12$ ,  $H_1: \mu < 12$ . Given the mean =  $12^\circ\text{C}$ ,  $P(X < 11.1) < 0.05$ . Therefore there is evidence to suggest the mean is less than  $12^\circ\text{C}$ .

### Challenge

- 1 a  $0.4 \leq p \leq 0.7$  b  $0 \leq q \leq 0.25$   
 2 a CVs are 144.78 and 173.22 so CRs are  $\leq 144$  and  $\geq 174$   
 b 173 is not in the CR so there is evidence to support the politician's claim.

### Prior knowledge 4

- 1 13.1 cm, 12.0 cm  
 2 a 10.8 N b 21.6 N c 7.8 N (2 s.f.)

### Exercise 4A

- 1 a 6 Nm clockwise b 10.5 Nm clockwise  
 c 13 Nm anticlockwise d 0 Nm  
 2 a 10 Nm anticlockwise b 30.5 Nm anticlockwise  
 c 13.3 Nm clockwise d 33.8 Nm anticlockwise
- 3 a i 313.6 Nm clockwise ii 156.8 Nm anticlockwise  
 b Sign is a particle.  
 4 a 0 Nm b 0 Nm  
 c 36 Nm anticlockwise d 36 Nm anticlockwise  
 5 2.5 N



## Exercise 4B

- 1 a 5 Nm anticlockwise      b 13 Nm clockwise  
c 19 Nm anticlockwise      d 11 Nm anticlockwise  
e 4 Nm clockwise      f 7 Nm anticlockwise
- 2 a 16 Nm clockwise      b 1 Nm anticlockwise  
c 10 Nm clockwise      d 7 Nm clockwise  
e 0.5 Nm anticlockwise      f 9.59 Nm anticlockwise
- 3 6 m
- 4 1.6
- 5 528 448 Nm anticlockwise
- 6  $6000 \times x \sin \theta > 8000 \times \frac{1}{2} x \cos \theta$   
 $6000 \sin \theta > 4000 \cos \theta$   
 $\frac{\sin \theta}{\cos \theta} > \frac{4000}{6000}$   
 $\tan \theta > \frac{2}{3}$

## Exercise 4C

- 1 a 10 N, 10 N      b 15 N, 5 N  
c 12 N, 8 N      d 12.6 N, 7.4 N
- 2 a 7.5, 17.5      b 30, 35      c 245,  $2\frac{2}{3}$
- 3 0.5 m
- 4 59 N
- 5 31 cm from the broomhead
- 6 a 16.25 N, 13.75 N      b 3.2 m
- 7 a 784 N      b 0.625 m
- 8 a 122.5 N      b 1.17 m
- 9 a  $\frac{9}{2} T_C = 4W + 8 \times 30$   
 $\frac{9}{2} T_C = 4W + 240$   
 $9 T_C = 8W + 480$   
 $T_C = \frac{8}{9} W + \frac{160}{3}$   
b  $T_A = \frac{W}{9} - \frac{70}{3}$       c 750 N
- 10 14.1 N
- 11 a  $R_A = 60$  N      b 33.6 kg

## Challenge

3 kg, 5 kg, 1 kg, 2 kg, 4 kg (from left to right)

## Exercise 4D

- 1  $R_A = 2.4$  N,  $R_B = 3.6$  N
- 2 a 10 g N      b 3.5 m from A
- 3  $\frac{1}{3}$  m from A
- 4 a 29.4 N, 118 N      b 2.11 m
- 5 a 160 N      b 2.77 m
- 6 a 3 m  
b Centre of mass lies at the midpoint of the seesaw.  
c 2 m towards Sophia.
- 7  $R_C = 5R_D$   
 $R_C + R_D = 80 + W$   
 $R_D = \frac{80 + W}{6}$   
Taking moments about A:  $6R_C + 20R_D = 80 \times 10 + xW$   
 $50R_D = 800 + xW$   
 $25W - 3xW = 400$   
 $W = \frac{400}{25 - 3x}$
- 8 1.6 m

## Challenge

1.61 m

## Exercise 4E

- 1 5
- 2  $\frac{2}{3}$  m
- 3 2.05 m
- 4 a  $C = 15$  N,  $D = 5$  N      b  $2 \times 12 \neq 20 \times 0.5$   
c  $2.14 \leq x \leq 4.78$  m

- 5 2.5 m
- 6 a Taking moments about N:  
 $mg \times ON = \frac{3}{4} mg \times 2a$  so  $ON = \frac{3}{2} a$   
b  $\frac{23}{20} mg$  N
- 7 40 N

## Mixed exercise 4

- 1 a 105 N      b 140 N  
c 1.03 m to the right of D
- 2 a R(t) gives reaction at C = reaction at D =  $\frac{150 + W}{2}$   
M(C):  $(1 \times 150) + W(x - 1) = 1.5 \left( \frac{150 + W}{2} \right)$   
 $150 + Wx - W = 112.5 + 0.75W$   
 $37.5 = 1.75W - Wx \Rightarrow 150 = 7W - 4Wx$   
 $W = \frac{150}{7 - 4x}$   
b  $0 \leq x < \frac{7}{4}$
- 3 a 40 g      b 0.5 m  
c i The weight acts at the centre of the plank.  
ii The plank remains straight.  
iii The man's weight acts at a single point.
- 4 a  $2.5 \times 100 = 3.5W + 150(3.5 - x)$   
 $250 = 3.5W + 525 - 150x$   
 $150x = 3.5W + 275$   
 $300x = 7W + 550$   
b  $W = 790 - 300x$       c  $x = 2.53$ ,  $W = 30$
- 5 a 200 N      b 21 cm
- 6 a 36 kg      b 2.2 m
- 7 a 19.6 N      b 5
- 8  $\frac{2}{3}$  m
- 9 a 125 N      b 1.8 m
- 10 4.88 m
- 11 2.39 m
- 12 a  $\frac{10000}{x}$       b  $500 \text{ kg} \leq M \leq 2000 \text{ kg}$   
c This model has the crane only able to lift weights of 500 kg at full extension, not very practical.

## Challenge

- 1 3.28 m
- 2 a 69.1 N      b 163 N

## Prior knowledge 5

- 1 (i + 2j) ms<sup>-2</sup>
- 2 a 16.55      b 25.02°

## Exercise 5A

- 1 a i 11.3 N (3 s.f.)      ii 4.10 N (3 s.f.)  
iii (11.3i + 4.10j) N  
b i 0 N      ii -5 N      iii -5j N  
c i -5.14 N (3 s.f.)      ii 6.13 N (3 s.f.)  
iii (-5.14i + 6.13j) N  
d i -3.86 N (3 s.f.)      ii -4.60 N (3 s.f.)  
iii (-3.86i - 4.60j) N
- 2 a i -2 N      ii 6.93 N (3 s.f.)  
b i 8.13 N (3 s.f.)      ii 10.3 N (3 s.f.)  
c i ( $P \cos \alpha + Q - R \sin \beta$ ) N  
ii ( $P \sin \alpha - R \cos \beta$ ) N
- 3 a 39.3 N (3 s.f.) at an angle of 68.8° above the horizontal  
b 27.9 N (3 s.f.) at an angle of 16.2° above the horizontal  
c 3.01 N (3 s.f.) at an angle of 53.3° above the horizontal
- 4 a  $B = 30.4$  N,  $\theta = 4.72^\circ$   
b  $B = 28.5$  N,  $\theta = 29.8^\circ$   
c  $B = 13.9$  N,  $\theta = 7.52^\circ$

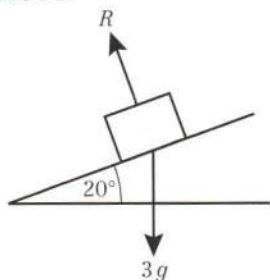
- 5 a  $\frac{\sqrt{3}}{5} \text{ m s}^{-2}$  b 48 N  
 6  $20\sqrt{2} \text{ N}$   
 7  $36.3 \text{ (3 s.f.)}$   
 8  $27\sin 60 + 20g = 80g$   
 $T = \frac{60g}{2\sin 60} = 20\sqrt{3}g$   
 9  $F_1 = 12\sqrt{3} \text{ N}, F_2 = 20 \text{ N}$

### Challenge

$$F_1 = 6\sqrt{2} - \sqrt{6} \text{ N}, F_2 = 2\sqrt{3} - 2$$

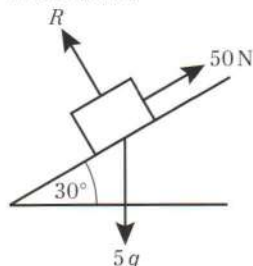
### Exercise 5B

1 a



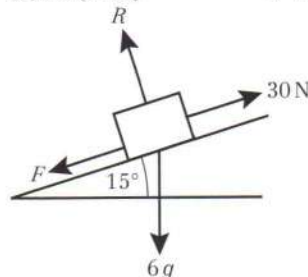
- b 27.6 N (3 s.f.) c  $3.35 \text{ ms}^{-2}$

2 a



- b 42.4 N (3 s.f.) c  $5.1 \text{ ms}^{-2}$   
 3 a 3.92 N (3 s.f.) b  $5.88 \text{ ms}^{-2}$  (3 s.f.)

4 a



- b 14.8 N (3 s.f.) b  $4.9 \text{ ms}^{-2}$

- 5 a 0.589 kg (3 s.f.)  
 6  $0.296 \text{ ms}^{-2}$  (3 s.f.)  
 7 15.0 N (3 s.f.)  
 8 R(↗):  $26\cos 45 - mg\sin \alpha - 12 = m \times 1$   
 $13\sqrt{2} - 12 = m + \frac{1}{2}mg$   
 $m = \frac{13\sqrt{2} - 12}{1 + \frac{g}{2}}$   
 $m = 1.08 \text{ kg}$  (3 s.f.)

### Challenge

- a  $mg\sin \theta = ma$  and  $mg\sin(\theta + 60) = 4ma$   
 $4\sin \theta = \sin(\theta + 60)$   
 $4\sin \theta = \sin \theta \cos 60 + \cos \theta \sin 60$   
 $4\sin \theta = \frac{1}{2}\sin \theta + \frac{\sqrt{3}}{2}\cos \theta$   
 $\frac{7}{2}\sin \theta = \frac{\sqrt{3}}{2}\cos \theta$   
 $\tan \theta = \frac{\sqrt{3}}{7}$   
 b  $13.9^\circ$

### Exercise 5C

- 1 a i 3 N ii  $F = 3 \text{ N}$  and body remains at rest  
 b i 7 N ii  $F = 7 \text{ N}$  and body remains at rest in limiting equilibrium  
 c i 7 N ii  $F = 7 \text{ N}$  and body accelerates  
 iii  $1 \text{ ms}^{-2}$   
 d i 6 N ii  $F = 6 \text{ N}$  and body remains at rest  
 e i 9 N  
 ii  $F = 9 \text{ N}$  and body remains at rest in limiting equilibrium  
 f i 9 N  
 ii  $F = 9 \text{ N}$  and body accelerates  
 iii  $0.6 \text{ ms}^{-2}$   
 g i 3 N ii  $F = 3 \text{ N}$  and body remains at rest  
 h i 5 N  
 ii  $F = 5 \text{ N}$  and body remains at rest in limiting equilibrium  
 i i 5 N ii  $F = 5 \text{ N}$  and body accelerates  
 iii  $0.2 \text{ ms}^{-2}$   
 j i 6 N ii  $F = 6 \text{ N}$  and body accelerates  
 iii  $1.22 \text{ ms}^{-2}$  (3 s.f.)  
 k i 5 N ii  $F = 5 \text{ N}$  and body accelerates  
 iii  $3.85 \text{ ms}^{-2}$  (3 s.f.)  
 l i  $12.7 \text{ N}$  (3 s.f.)  
 ii The body accelerates.  
 iii  $5.39 \text{ ms}^{-2}$  (3 s.f.)  
 2 a  $R = 88 \text{ N}, \mu = 0.083$  (2 s.f.)  
 b  $R = 80.679 \text{ N}, \mu = 0.062$  (2 s.f.)  
 c  $R = 118 \text{ N}, \mu = 0.13$  (2 s.f.)  
 3  $0.242$  (3 s.f.)  
 4  $0.778 \text{ N}$  (3 s.f.)  
 5  $56.1 \text{ N}$  (3 s.f.)  
 6  $16.5 \text{ N}$  (3 s.f.)  
 7 a Use  $v = u + at$  to find  $a = -\frac{2}{3} \text{ ms}^{-2}$   
 $R(\rightarrow): -\mu mg = -\frac{2}{3}m$   
 $\mu = \frac{2}{3g}$   
 b Let  $A$  be a constant resistive force of air resistance.  
 From part a,  $a = -\frac{2}{3}$   
 $R(\rightarrow)$  and using Newton's second law:  $-\mu mg - A = -\frac{2}{3}m$   
 So  $\mu mg = \frac{2}{3g} - \frac{A}{mg} < \frac{2}{3g}$

### Challenge

$$R(\swarrow): mg\sin \alpha - \mu mg\cos \alpha = ma$$

$$g\sin \alpha - \mu g\cos \alpha = a$$

### Mixed exercise 5

- 1 a 32.0 N (3 s.f.) b  $0.5 \text{ m s}^{-2}$   
 2  $F_1 = 27.8 \text{ N}, F_2 = 24.2 \text{ N}$  (3 s.f.)  
 3 a
- 
- b 13.9 N (3 s.f.)  
 c Res (↗):  $16 - 2g\sin 45 = 2a$   
 $a = \frac{16 - 2g\sin 45}{2} = 1.1 \text{ ms}^{-2}$  (2 s.f.)  
 4  $2.06 \text{ ms}^{-2}$  (3 s.f.)  
 5 R(→):  $F = 150\cos 45 + 100\cos 30$   
 $= \frac{150\sqrt{2}}{2} + \frac{100\sqrt{3}}{2}$   
 $= 25(3\sqrt{2} + 2\sqrt{3}) \text{ N}$   
 6  $\mu = \frac{5\sqrt{3}}{93}$





- 7 7.2 N  
 8  $3.41 \text{ ms}^{-2}$  (3 s.f.)  
 9 a 4400 N (2 s.f.)  
 b 0.59 (2 s.f.)  
 c i e.g. The force due to air resistance will not remain constant in the subsequent motion of the car.  
 ii e.g. Whilst skidding the car is unlikely to travel in a straight line.

**Challenge**

$$F_{\text{MAX}} = 0.2 \times 400g \cos 15^\circ = 760 \text{ N (2 s.f.)}$$

Component of weight that acts down the slipway:

$$400g \sin 15^\circ = 1000 \text{ N (2 s.f.)}$$

$1000 \text{ N} > 760 \text{ N}$  so boat will come to momentary rest then accelerate back down the slope.

The boat will take 6.9 seconds to reach the water.

**Prior knowledge 6**

- 1 a 11.5 m  
 b 3.1 s  
 2  $x = v \cos \theta$ ,  $y = v \sin \theta$   
 3 a i  $\cos \theta = \frac{12}{13}$  ii  $\tan \theta = \frac{5}{12}$   
 b i  $\sin \theta = \frac{8}{17}$  ii  $\cos \theta = \frac{15}{17}$

**Exercise 6A**

- 1 a 122.5 m b 100 m  
 2 a  $x = 36 \text{ m}$ ,  $y = 19.6 \text{ m}$  b 41 m  
 3  $u = 16.6 \text{ ms}^{-1}$   
 4  $77.5 \text{ ms}^{-1}$   
 5 0.59 s  
 6 1.9 m  
 7 a 0.51 s b 0.42 c 3.4 m

**Exercise 6B**

- 1 a  $u_x = 19.2 \text{ ms}^{-1}$ ,  $u_y = 16.1 \text{ ms}^{-1}$   
 b  $(19.2\mathbf{i} + 16.1\mathbf{j}) \text{ ms}^{-1}$   
 2 a  $u_x = 16.9 \text{ ms}^{-1}$ ,  $u_y = -6.2 \text{ ms}^{-1}$   
 b  $(16.9\mathbf{i} - 6.2\mathbf{j}) \text{ ms}^{-1}$   
 3 a  $u_x = 32.3 \text{ ms}^{-1}$ ,  $u_y = 13.5 \text{ ms}^{-1}$   
 b  $(32.3\mathbf{i} + 13.5\mathbf{j}) \text{ ms}^{-1}$   
 4 a  $u_x = 26.9 \text{ ms}^{-1}$ ,  $u_y = -7.8 \text{ ms}^{-1}$   
 b  $(26.9\mathbf{i} - 7.8\mathbf{j}) \text{ ms}^{-1}$   
 5  $10.8 \text{ ms}^{-1}$ ,  $56.3^\circ$   
 6  $6.4 \text{ ms}^{-1}$ ,  $51.3^\circ$  below the horizontal  
 7 a  $33.7^\circ$  b  $k = 3$  or  $k = -3$

**Exercise 6C**

- 1 3.1 s (2 s.f.)  
 2 8.5 m (2 s.f.)  
 3 a 45 m (2 s.f.) b 79 m  
 4 a 2.7 s (2 s.f.) b 790 m (2 s.f.)  
 5 a 10 m (2 s.f.) b 41 m (2 s.f.)  
 6 a 3.9 s (2 s.f.) b 56 m (2 s.f.)  
 7  $55^\circ$  (nearest degree)  
 8 a  $(36\mathbf{i} + 27.9\mathbf{j}) \text{ m}$  b  $13 \text{ ms}^{-1}$  (2 s.f.)  
 9 a  $22^\circ$  (2 s.f.) b  $97 \text{ m}$  (2 s.f.)  
 10 a 16 (2 s.f.) b  $1.6 \text{ s}$  (2 s.f.)  
 11 a 4.4 b 88 c  $50^\circ$  (2 s.f.)  
 12 a 1.1 s (2 s.f.) b 34 m (2 s.f.)  
 13 a  $\alpha = 40.6^\circ$  (nearest  $0.1^\circ$ )  
 $U = 44$  (2 s.f.)  
 14 a  $15.6 \text{ ms}^{-1}$  b 2.92 s c  $22.3 \text{ ms}^{-1}$   
 15 a  $k = 7.35$   
 b i  $13.6 \text{ ms}^{-1}$  ii  $72.9^\circ$  below the horizontal  
 16 a  $10.7 \text{ ms}^{-1}$  b e.g. weight of the ball; air resistance

**Challenge**

$$R(\rightarrow): s = 12t \text{ and } s = (20 \cos \alpha)t$$

$$\text{so } \cos \alpha = 0.6 \text{ and } \sin \alpha = 0.8$$

$$R(\uparrow): s = -4.9t^2 + 40 \text{ and } s = (20 \sin \alpha)t - 4.9t^2$$

$$\text{So } t = \frac{40}{20 \sin \alpha} = \frac{40}{16} = 2.5 \text{ seconds}$$

**Exercise 6D**

- 1  $R(\uparrow): v^2 = U^2 \sin^2 \alpha - 2gh$   
 At maximum height,  $v = 0$  so  $0 = U^2 \sin^2 \alpha - 2gh$   
 Rearrange to give  $h = \frac{U^2 \sin^2 \alpha}{2g}$   
 2 a  $R(\rightarrow): x = 21 \cos \alpha \times t$ , so  $t = \frac{x}{21 \cos \alpha}$   
 $R(\uparrow): y = 21 \sin \alpha \times \frac{x}{21 \cos \alpha} - \frac{1}{2}g\left(\frac{x}{21 \cos \alpha}\right)^2$   
 $y = x \tan \alpha - \frac{x^2}{90 \cos^2 \alpha}$   
 b  $\tan \alpha = 1.25$   
 3 a  $R(\uparrow): s = U \sin \alpha \times t - \frac{1}{2}gt^2$   
 When particle strikes plane,  $s = 0 = t(U \sin \alpha - \frac{1}{2}gt)$   
 So  $t = 0$  or  $t = \frac{2U \sin \alpha}{g}$   
 b  $R(\rightarrow): s = ut = U \cos \alpha \left(\frac{2U \sin \alpha}{g}\right) = \frac{U^2 \sin 2\alpha}{g}$   
 c Range  $s = \frac{U^2 \sin 2\alpha}{g}$  is greatest when  $\sin 2\alpha = 1$   
 Occurs when  $2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$   
 d  $12^\circ$  and  $78^\circ$   
 4 Using  $v = u + at$ , at max height  $t = \frac{v}{g}$   
 So time taken to return to the ground =  $\frac{v}{g}$   
 Using  $s = ut + \frac{1}{2}at^2$ , distance travelled by one part =  $2v\left(\frac{v}{g}\right) = \frac{2v^2}{g}$   
 So two parts of firework are  $\frac{2v^2}{g} + \frac{2v^2}{g} = \frac{4v^2}{g}$  apart.  
 5 a  $R(\rightarrow): x = U \cos \alpha \times t$ , so  $t = \frac{x}{U \cos \alpha}$   
 $R(\uparrow): y = U \sin \alpha \times t - \frac{1}{2}gt^2$   
 Substitute for  $t \Rightarrow y = U \sin \alpha \left(\frac{x}{U \cos \alpha}\right) - \frac{1}{2}g\left(\frac{x}{U \cos \alpha}\right)^2$   
 Use  $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$  and rearrange to give  
 $y = x \tan \alpha - \frac{gx^2}{2U^2 \cos^2 \alpha}$   
 b 13.7 m  
 6 a  $R(\rightarrow): x = U \cos \alpha \times t$ , so  $t = \frac{x}{U \cos \alpha}$   
 $R(\uparrow): y = U \sin \alpha \times t - \frac{1}{2}gt^2$   
 Substitute for  $t \Rightarrow y = U \sin \alpha \left(\frac{x}{U \cos \alpha}\right) - \frac{1}{2}g\left(\frac{x}{U \cos \alpha}\right)^2$   
 Use  $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$  and  $\frac{1}{\cos \alpha} = \sec \alpha$ , and rearrange to give  
 $y = x \tan \alpha - \frac{gx^2}{2U^2} \sec^2 \alpha$   
 Use  $\sec^2 \alpha = 1 + \tan^2 \alpha$ , and rearrange to give  
 $y = x \tan \alpha - \frac{gx^2}{2U^2} (1 + \tan^2 \alpha)$   
 b 93.8 m  
 c 4.4 s  
 7 a  $R(\rightarrow): x = 9 = U \cos \alpha \times t$ , so  $t = \frac{9}{U \cos \alpha}$   
 $R(\uparrow): y = U \sin \alpha \times t - \frac{1}{2}gt^2$   
 Substitute for  $t \Rightarrow y = U \sin \alpha \left(\frac{9}{U \cos \alpha}\right) - \frac{1}{2}g\left(\frac{9}{U \cos \alpha}\right)^2$   
 Use  $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$  and  $y = 0.9$ . Rearrange to give  
 $0.9 = 9 \tan \alpha - \frac{81g}{2U^2 \cos^2 \alpha}$   
 b  $10.3 \text{ ms}^{-1}$   
 8 a  $R(\rightarrow): x = kt$ , so  $t = \frac{x}{k}$   
 $R(\uparrow): y = 2kt - \frac{gt^2}{2}$   
 Substitute for  $t \Rightarrow y = 2x - \frac{gx^2}{2k^2}$   
 b i  $\frac{4k^2}{g} \text{ m}$  ii  $\frac{2k^2}{g} \text{ m}$

### Challenge

For the projectile:  $y = x \tan \alpha - \frac{gx^2}{2U^2 \cos^2 \alpha}$  so for  $\alpha = 45^\circ$

$$y = x - \frac{gx^2}{U^2}$$

For the slope:  $y = -x$

Projectile intersects the slope when  $-x = x - \frac{gx^2}{U^2} \Rightarrow x = \frac{2U^2}{g}$ ,

$$y = -\frac{2U^2}{g}$$

$$\text{Distance} = \sqrt{\left(\frac{2U^2}{g}\right)^2 + \left(\frac{2U^2}{g}\right)^2} = \sqrt{8\left(\frac{U^2}{g}\right)^2} = \frac{2\sqrt{2}U^2}{g}$$

### Mixed exercise 6

- 1 a 45 m b 6.1 s
- 2  $h = 35$  (2 s.f.) b 30 m (2 s.f.)
- 3 a 36 m b 36 ms<sup>-1</sup> (2 s.f.)
- 4 a 140 m (2 s.f.) b 36 ms<sup>-1</sup> (2 s.f.)
- 5 a  $R(t): s = U \sin \theta t - \frac{g}{2}t^2$   
When particle strikes plane,  $s = 0 = t(U \sin \theta - \frac{g}{2}t)$   
So  $t = 0$  or  $t = \frac{2U \sin \theta}{g}$   
 $R(\rightarrow): s = Ut = U \cos \theta \left(\frac{2U \sin \theta}{g}\right) = \frac{U^2 \sin 2\theta}{g}$   
b  $\frac{U^2}{g}$  c 20.9°, 69.1° (nearest 0.1°)
- 6 a 2.0 s (2 s.f.) b 3.1 s (2 s.f.) c 36 ms<sup>-1</sup> (2 s.f.)
- 7 a 2 m b 5.77° or 84.2°
- 8 a 0.65 s b 1.5 m c 23.8 ms<sup>-1</sup>
- 9 a Particle P:  $x = 18t$ , Particle Q:  $x = 30 \cos \alpha \times t$   
When particles collide:  $18t = 30 \cos \alpha \times t \Rightarrow \cos \alpha = \frac{3}{5}$   
b  $\frac{4}{3}$  s

### Challenge

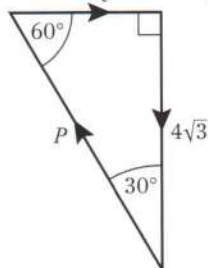
62 ms<sup>-1</sup>

### Prior knowledge 7

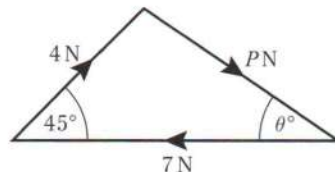
- 1 0.278 (3 s.f.)
- 2 12.25 N

### Exercise 7A

- 1 a i  $Q - 5 \cos 30^\circ = 0$  ii  $P - 5 \sin 30^\circ = 0$   
iii  $Q = 4.33$  N  $P = 2.5$  N
- b i  $P \cos \theta + 8 \sin 40^\circ - 7 \cos 35^\circ = 0$   
ii  $P \sin \theta + 7 \sin 35^\circ - 8 \cos 40^\circ = 0$   
iii  $\theta = 74.4^\circ$   $P = 2.19$  N
- c i  $9 - P \cos 30^\circ = 0$   
ii  $Q + P \sin 30^\circ - 8 = 0$   
iii  $Q = 2.80$  N  $P = 10.4$  N
- d i  $Q \cos 60^\circ + 6 \cos 45^\circ - P = 0$   
ii  $Q \sin 60^\circ - 6 \sin 45^\circ = 0$   
iii  $Q = 4.90$  N  $P = 6.69$  N
- e i  $6 \cos 45^\circ - 2 \cos 60^\circ - P \sin \theta = 0$   
ii  $6 \sin 45^\circ + 2 \sin 60^\circ - P \cos \theta - 4 = 0$   
iii  $\theta = 58.7^\circ$   $P = 3.80$  N
- f i  $9 \cos 40^\circ + 3 - P \cos \theta - 8 \sin 20^\circ = 0$   
ii  $P \sin \theta + 9 \sin 40^\circ - 8 \cos 20^\circ = 0$   
iii  $\theta = 13.6^\circ$   $P = 7.36$  N
- 2 a i  $Q$  ii  $Q = 4$  N,  $P = 8$  N

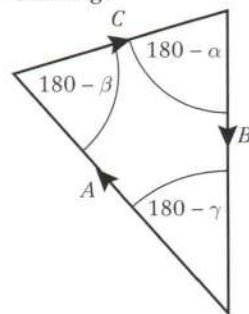


b i



- ii  $\theta = 34.1^\circ$ ,  $P = 5.04$  N
- 3 a  $P = 4.33$  N,  $Q = 2.5$  N b  $P = 7.07$  N,  $Q = 7.07$  N
- c  $P = 4.73$  N,  $Q = 4.20$  N d  $P = 3.00$  N,  $Q = 0.657$  N
- e  $P = 9.24$  N,  $Q = 4.62$  N

### Challenge



$$\frac{A}{\sin(180 - \alpha)} = \frac{B}{\sin(180 - \beta)} = \frac{C}{\sin(180 - \gamma)}$$

$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

### Exercise 7B

- 1 35 N (2 s.f.)
- 2 a 20 N b 1.77
- 3 a 33.7° b 14.4 N
- 4 30 N (right) and 43 N (left) (2 s.f.)
- 5 a 5.46 N b 0.76 kg
- c Assumption that there is no friction between the string and the bead.
- 6 a 1.46 N b 55 g
- 7 a 2.6 b 4.4 N
- 8 a  $F = 19.6$  m,  $R = 9.8$  m b  $F' = 17$  m (2 s.f.),  $R' = 0$
- 9 13.9 N 10 39.2 N
- 11 a 15.7 N (3 s.f.) b 37.2 N (3 s.f.)
- c Assumption that there is no friction between the string and the pulley.
- 12  $R = 0.40$  N (2 s.f.)

### Exercise 7C

- 1 0.446 2 0.123
- 3 a 1.5 N b Not limiting
- 4 a 40 kg
- b The assumption is that the crate and books may be modelled as a particle.
- 5 a 11.9 N b 6.40 N
- 6 0.601 (accept 0.6)
- 7 a 13.3 N b  $X = 9.54$  N
- 8 a 22.7 N
- b 9.97 N down the plane
- c  $\mu \geq 0.439$
- 9 a  $X = 44.8$  b  $R = 51.3$  N
- 10  $T = 102$  (3 s.f.) 11  $2.75 \leq T \leq 3.87$  N (3 s.f.)
- 12 0.758 13 67.2 N

### Exercise 7D

- 1 34.6 N, 50 N, 17.3 N, 0.35
- 2 a 22.8 N b 98 N, 22.8 N c 0.233
- d The weight of a uniform ladder passes through its midpoint.



- 3 a  $41.6^\circ$  b  $24.0^\circ$   
 c No friction at the wall.
- 4 a  $5\frac{1}{3}$  m  
 b i The ladder may not be uniform.  
 ii There would be friction between the ladder and the vertical wall.
- 5 a 1.99 N b 0.526  
 c There is no friction between the rail and the pole.
- 6 R( $\uparrow$ ):  $R = mg$ , R( $\rightarrow$ ):  $N = \mu mg$   
 Let the length of the ladder be  $2l$   
 Taking moments about A,  
 $mg l \cos \theta = 2\mu mg l \sin \theta$   
 Cancelling and rearranging gives  
 $2\mu \tan \theta = 1$
- 7 104 N, 64.5 N, 0.620
- 8 R( $\uparrow$ ):  $R + \mu_2 N = mg$ , R( $\rightarrow$ ):  $N = \mu_1 R$   
 Taking moments about the base of the ladder  
 $2N \tan \theta + 2\mu_2 N = mg$   
 $2 \tan \theta + 2\mu_2 = \frac{1}{\mu_1} + \mu_2$   
 $2 \tan \theta = \frac{1}{\mu_1} - \mu_2$   
 $\tan \theta = \frac{1 - \mu_1 \mu_2}{2\mu_1}$
- 9 a  $\frac{W\sqrt{3}}{6}$   
 b  $F = P = \frac{W\sqrt{3}}{6}$ ,  $R = W$ ,  $F \leq \mu R$ ,  $\mu \geq \frac{\sqrt{3}}{6}$   
 c  $\frac{W}{4}$
- 10 a 6.50 N, 10.8 N b  $\frac{1}{\sqrt{3}}$
- 11 a  $\frac{19}{2\sqrt{3}} W$  b  $\left(\frac{19}{2\sqrt{3}} - 2\right)W \leq P \leq \left(\frac{19}{2\sqrt{3}} + 2\right)W$   
 c It allows us to assume that the weight of the ladder acts through its midpoint.  
 d i The magnitude of the reaction at Y will get smaller.  
 ii The range of values for P will get smaller.
- 12 a 91.5 N (3 s.f.)  
 b 70.1 N (3 s.f.) parallel to and towards the rod
- 13 a M(A):  $a \cos 30^\circ \times mg = 2a \sin 30^\circ \times P \Rightarrow P = \frac{\sqrt{3}}{2} mg$  N  
 b  $\frac{\sqrt{7}}{2} mg$  N at an angle of  $49.1^\circ$  (3 s.f.) above the horizontal, away from the rod.

### Exercise 7E

- 1  $3.35 \text{ m s}^{-2}$  (3 s.f.)  
 2 a  $27.7 \text{ N}$  (3 s.f.) b  $2.12 \text{ m s}^{-2}$   
 3 a  $2.43 \text{ m s}^{-2}$  (3 s.f.) b  $4.93 \text{ m s}^{-1}$  (3 s.f.)  
 4 0.165 (3 s.f.) 5 0.20 (2 s.f.)  
 6 0.15 (2 s.f.)  
 7 a  $88.8 \text{ N}$  (3 s.f.) b 0.24 (2 s.f.)  
 8 a  $\frac{13g}{15}$  b  $23.5 \text{ m}$  (3 s.f.)  
 c  $2.35 \text{ s}$  (3 s.f.) d  $12.4 \text{ m s}^{-1}$  (3 s.f.)  
 9 0.180 (3 s.f.)  
 10 R( $\searrow$ ):  $R = mg \cos \alpha$ , R( $\nearrow$ ):  $\mu R - mg \sin \alpha = ma$   
 $\mu mg \cos \alpha - mg \sin \alpha = ma$   
 $\mu g \cos \alpha - g \sin \alpha = a$   
 11 a 0.155  
 b The particle will slide down the hill. The component of weight down the slope  $5g \sin 10 = 8.51 \text{ N}$  is greater than the friction  $0.15524 \dots \times 5g \cos 10 = 7.49 \text{ N}$ .

### Exercise 7F

- 1  $2.8 \text{ m s}^{-1}$   
 2 a  $1.12 \text{ m s}^{-2}$  b 4100 N

- c The resistances are unlikely to be constant as the resistance will increase as the speed increases.
- 3 a 21.9 N b 0.418 (3 s.f.) c 38 N (2 s.f.)  
 4 a 18 N (2 s.f.) b 2 c  $\frac{2}{7} \text{ s}$
- 5 a For particle on LHS  
 R( $\searrow$ ):  $R_1 = \sqrt{3}g$ , R( $\nearrow$ ):  $T = \frac{\sqrt{3}}{5}g + g$   
 For particle on RHS  
 R( $\nearrow$ ):  $R_2 = \frac{\sqrt{3}}{2}mg$ , R( $\searrow$ ):  $T = \frac{1}{2}mg - \frac{\sqrt{3}}{5}mg$   
 For maximum value of  $m$ , R( $\searrow$ ) is equal to R( $\nearrow$ ):  
 $\frac{1}{2}mg - \frac{\sqrt{3}}{5}mg = \frac{\sqrt{3}}{5}g + g \Rightarrow \frac{1}{2}m - \frac{\sqrt{3}}{5}m = \frac{\sqrt{3}}{5} + 1$   
 $\Rightarrow \left(\frac{1}{2} - \frac{\sqrt{3}}{5}\right)m = \frac{\sqrt{3}}{5} + 1 \Rightarrow m = \frac{10 + 2\sqrt{3}}{5 - 2\sqrt{3}}$
- b  $0.155 \text{ m s}^{-2}$  (3 s.f.)
- 6 a  $3 \text{ m s}^{-2}$  b 10.88 N  
 c R( $\rightarrow$ ):  $T - 1.5\mu g = 4.5$   
 $\mu = \frac{10.88 - 4.5}{1.5g} = \frac{319}{735} = 0.434$  (3 s.f.)  
 d The string doesn't stretch so the tension in the string is constant.

### Challenge

- a For particle on LHS: R( $\nearrow$ ):  $T = \frac{1}{2}m_1g$   
 For particle on RHS: R( $\searrow$ ):  $T = \frac{\sqrt{3}}{2}m_2g$   
 To prove, equate values of  $T$ .
- b If (attempted) motion is down slope on RHS  
 Consider particle on LHS:  $T = \frac{1}{2}m_1g + \frac{\sqrt{3}}{2}\mu m_1g$   
 Consider particle on RHS:  $T = \frac{\sqrt{3}}{2}m_2g - \frac{1}{2}\mu m_2g$   
 Equate values of  $T$  to find  $\frac{m_1}{m_2}$   
 If (attempted) motion is down slope on LHS  
 Consider particle on LHS:  $T = \frac{1}{2}m_1g - \frac{\sqrt{3}}{2}\mu m_1g$   
 Consider particle on RHS:  $T = \frac{1}{2}\mu m_2g + \frac{\sqrt{3}}{2}m_2g$   
 Equate values of  $T$  to find  $\frac{m_1}{m_2}$

### Mixed exercise 7

- 1 a  $32.3^\circ$  (3 s.f.) b  $16.3 \text{ N}$  (3 s.f.)  
 2 a  $18.0^\circ$  (3 s.f.) b  $43.3 \text{ N}$  (3 s.f.)  
 3  $T_1 = 1062 \text{ N}$ ,  $T_2 = 1013 \text{ N}$   
 4  $12 \text{ N}$  (2 s.f.)  
 5 a 12.25 N b  $46.6 \text{ N}$  (3 s.f.)  
 c  $F$  will be smaller as friction is acting up the slope.
- 6 a R( $\uparrow$ ):  $T \cos 20 = 2g + T \cos 70 \Rightarrow T = \frac{2g}{\cos 20 - \cos 70}$   
 $= 33 \text{ N}$  (2 s.f.)  
 b  $42 \text{ N}$  (2 s.f.)  
 7 a  $364 \text{ N}$  (3 s.f.) b Hill unlikely to be smooth.
- 8 R( $\rightarrow$ ):  $F = N$   
 Taking moments about A  
 $Fa \sin \theta + \frac{5}{2}mga \cos \theta = 5a N \sin \theta$   
 $\frac{5}{2}mg \cos \theta = 4F \sin \theta$   
 $\frac{5}{2}mg = F \tan \theta$
- 9  $\frac{7}{24}$
- 10 a  $\frac{8W}{9}$   
 b R( $\uparrow$ ):  $R + \mu N \geq W$ , R( $\rightarrow$ ):  $N = \frac{W}{3}$   
 $\frac{\mu W}{3} \geq W - \frac{8W}{9}$   
 c The ladder has negligible thickness/the ladder does not bend.



- 11 a Taking moments about point where ladder touches the ground  
 $R(\uparrow): R = W$ ,  $R(\rightarrow): N = 0.3R$   
 $1.5W = 1.2W$ . This cannot be true so the ladder cannot rest in this position.
- b  $R(\rightarrow): F = N$   
 Taking moments about point where ladder touches the ground  $1.5W = 4N$ ,  $F = N = \frac{3W}{8}$
- c  $\frac{W}{4g}$
- 12 18 N (2 s.f.)
- 13 a 94.3 N (3 s.f.)  
 b 80.6 N (3 s.f.), 54.2° (3 s.f.) to the horizontal
- 14 a 832 N (3 s.f.) b 620 N (3 s.f.)  
 c  $M(A): T \sin \theta = 40g$   
 $R(\uparrow): T \sin \theta = 40g + V \Rightarrow V = 0$ , so the force acts horizontally.
- 15 0.070 (2 s.f.)
- 16 6.35 ms<sup>-2</sup> (3 s.f.)
- 17  $R(\rightarrow): T - \mu m_1 g = m_1 a$ ,  $R(\uparrow): T = m_2 g - m_2 a$   
 $m_1 a + \mu m_1 g = m_2 g - m_2 a$   
 $g(m_2 - \mu m_1) = a(m_1 + m_2)$
- 18  $R(\nearrow): T - m_1 g \sin 30 = \frac{1}{2} m_1$ ,  
 $R(\searrow): m_2 g \cos 45 - T = \frac{1}{2} m_2$   
 and  $T = \frac{\sqrt{2}}{2} m_2 g - \frac{1}{2} m_2$ ,  $T = \frac{1}{2} m_1 + \frac{1}{2} m_1 g$   
 $\sqrt{2} m_2 g - m_2 = m_1 + m_1 g$   
 $m_2(\sqrt{2} g - 1) = m_1(1 + g)$

### Prior knowledge 8

- 1 a 19i - 43j b -10i + 11j c  $\frac{5}{13}i - \frac{12}{13}j$   
 2 a 23 ms<sup>-1</sup> b 52 m  
 3 a i 6e<sup>2x</sup> ii 6 cos 3x  
 b i  $\frac{4}{3}e^{3x+1}$  ii  $\frac{5}{2\pi} \sin 2\pi x$

### Exercise 8A

- 1 a 6i + 12j b -7i + 4j c -2i + 6j  
 d 10i - 13j e 2i - 3j f 4s
- 2  $\frac{\sqrt{85}}{2}$  ms<sup>-1</sup>, 319° 3 2.5 s
- 4 a  $\begin{pmatrix} 120 - 30t \\ -10 + 40t \end{pmatrix}$  b 4 s
- 5 2.03 ms<sup>-1</sup>
- 6 a 7ti + (400 + 7t)j, (500 - 3t)i + 15tj  
 b 350i + 750j
- 7 a  $\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$  ms<sup>-2</sup> b  $\begin{pmatrix} 15 \\ 2 \\ 10 \end{pmatrix}$  m
- 8 a  $-\frac{5}{2}i - \frac{7}{4}j$  ms<sup>-2</sup>  
 b  $(10 + 15t - \frac{5}{4}t^2)i + (-8 + 4t - \frac{7}{8}t^2)j$  m
- 9 a  $\begin{pmatrix} 60 \\ -15 \end{pmatrix}$  ms<sup>-1</sup> b 688 m
- 10  $\begin{pmatrix} 40 \\ -60 \end{pmatrix}$  m
- 11 a 12.1 ms<sup>-1</sup>, 6.08 ms<sup>-1</sup> b 18i - 3j  
 c 15i - 12j
- 12 a i - 7j ms<sup>-2</sup> b  $s = (-4t + 0.5t^2)i + (8t - 3.5t^2)j$   
 c 15:00 d -160j
- 13 a North-east of O when i and j components are equal  
 $2t^2 - 3 = 7 - 4t \Rightarrow 2t^2 + 4t - 10 = 0 \Rightarrow t^2 + 2t - 5 = 0$   
 b 1.70 m c 7.83 ms<sup>-1</sup>, 026.6° d 19.3 m

### Challenge

24 s

### Exercise 8B

- 1 a (36i + 27.9j) m b 13 ms<sup>-1</sup> (2 s.f.)
- 2 a  $r = (4t)i + (5t - 5t^2)j$  b 1.25 m
- 3 a Either answer with justification  
 e.g. The sea is likely to be horizontal and relatively flat, whereas the ball is subject to air resistance, so the assumption that sea is a horizontal plane is most reasonable.  
 Or e.g. Although the sea is horizontal it is unlikely to be flat because of waves, so the assumption that the ball is a particle is most reasonable.
- b  $v = (6.9i - 17j)$  ms<sup>-1</sup> (both values to 2 s.f.)  
 c 5.5 ms<sup>-2</sup> (2 s.f.)
- 4 a  $R(\uparrow): 0 = 4ut - \frac{g}{2}t^2 \Rightarrow t = \frac{8u}{g}$   
 $R(\rightarrow): 750 = 3ut = \frac{24u^2}{g} \Rightarrow u^2 = \frac{750g}{24} \Rightarrow u = 17.5$   
 b 250 m c 22° (nearest degree)
- 5 a 48 m b 120 m (2 s.f.)  
 c  $T = 2.5$  s,  $r = (20i - \frac{45}{8}j)$  m
- 6 a  $x = at \Rightarrow t = \frac{x}{a}$   
 $y = bt - 5t^2 \Rightarrow y = b(\frac{x}{a}) - 5(\frac{x}{a})^2 \Rightarrow y = \frac{bx}{a} - \frac{5x^2}{a^2}$   
 b i  $X = 1.6b$  ii  $Y = 0.05b^2$

### Exercise 8C

- 1 a  $v = t + \frac{\cos \pi t}{\pi} - \frac{1}{\pi}$  b  $s = \frac{t^2}{2} + \frac{\sin \pi t}{\pi^2} - \frac{t}{\pi}$
- 2 a  $v = -\frac{\cos 3\pi t}{3\pi} + \frac{2}{3\pi}$  b  $\frac{1}{\pi}$   
 c  $s = -\frac{\sin 3\pi t}{9\pi^2} + \frac{2t}{3\pi} + 1$
- 3 a  $v = -\frac{\sin 4\pi t}{4\pi}$  b  $\frac{1}{4\pi}$   
 c  $s = \frac{\cos 4\pi t}{16\pi^2} - \frac{1}{16\pi^2}$  d  $\frac{1}{8\pi^2}$  e 16
- 4 a 1.18 ms<sup>-1</sup> b -0.152 ms<sup>-2</sup>  
 c -0.759 N
- 5 a 0.5 ms<sup>-1</sup> b 0.1 ms<sup>-1</sup>
- 6 a 12.9 ms<sup>-1</sup> in the direction of s increasing  
 b 24 ms<sup>-1</sup> in the direction of s decreasing
- c 132 m d 20.8 m and 118.5 m
- 7 3.31 s
- 8 a  $k = 40$ ,  $T = 25$  b 4 ms<sup>-1</sup>  
 c  $v = \frac{20}{\sqrt{t}}$ , so for small  $t$ , the value of  $v$  is large  
 e.g.  $t = 0.01$ ,  $v = 200$  ms<sup>-1</sup>, so not realistic for small  $t$ .
- 9 a  $k = \frac{1}{2}$  b  $t = \pi, 3\pi$   
 c  $a = 4 \cos(\frac{t}{2})$ ,  $4a^2 = 64 \cos^2(\frac{t}{2})$   
 $v = 2 + 8 \sin(\frac{t}{2})$ ,  $(v - 2)^2 = 64 \sin^2(\frac{t}{2})$   
 $4a^2 = 64 - (v - 2)^2 \Rightarrow 64 \cos^2(\frac{t}{2}) = 64 - 64 \sin^2(\frac{t}{2})$   
 $\Rightarrow \cos^2(\frac{t}{2}) = 1 - \sin^2(\frac{t}{2})$   
 $\Rightarrow \cos^2 T + \sin^2 T = 1$   
 d 10 ms<sup>-1</sup>, 4 ms<sup>-2</sup>
- 10 a 24 ms<sup>-1</sup> b 54.4 m c 8.43 s d 177.58 m

### Exercise 8D

- 1 a (3i + 23j) ms<sup>-1</sup> b 18j ms<sup>-2</sup>
- 2 (0.024i + 0.006j) N
- 3 a 0.305 s b 6 ms<sup>-1</sup>  
 c i-component of velocity is negative, j-component of velocity = 0
- 4 a 20 ms<sup>-1</sup>  
 b  $a = 8i - 6j$ , no dependency on  $t$  therefore constant.  
 $|a| = 10$  ms<sup>-2</sup>
- 5 a  $6\sqrt{5}$  ms<sup>-1</sup> b  $t = 2$   
 c (-16i + 4j) m d 15.5 N (3 s.f.)



- 6 a  $k = -0.5, -8.5$   
 b  $10 \text{ ms}^{-2}$  for both values of  $k$   
 7 a  $52 \text{ ms}^{-1}$  b  $(12\mathbf{i} + \frac{15}{2}\mathbf{j}) \text{ ms}^{-2}$   
 8 a 4 b  $(-36\mathbf{i} + 8\mathbf{j}) \text{ ms}^{-2}$   
 9 a  $= 4\mathbf{i} + 2\mathbf{j}$ , no  $t$  dependency so constant,  $|\mathbf{a}| = 2\sqrt{5} \text{ ms}^{-2}$   
 10 a  $\sqrt{15}$  b  $(-4\sqrt{30}\mathbf{i} + 2\sqrt{15}\mathbf{j}) \text{ ms}^{-2}$

### Exercise 8E

- 1 a  $16\mathbf{i} \text{ ms}^{-1}$  b  $128\mathbf{i} - 140.8\mathbf{j} \text{ m}$   
 2 a 13 m b  $(4\mathbf{i} + 6\mathbf{j}) \text{ ms}^{-2}$   
 3 a  $\mathbf{v} = (t^2 - 4t + 2\pi - \frac{\pi^2}{4})\mathbf{i} - 6 \cos t \mathbf{j}$  b  $(2\pi^2 - 4\pi) \text{ ms}^{-1}$   
 4 a  $(\frac{5t^2}{2} - 3t + 2)\mathbf{i} + (8t - \frac{t^2}{2} - 5)\mathbf{j} \text{ ms}^{-1}$   
 b  $t = \frac{1}{2}$  c  $\frac{9\sqrt{2}}{8} \text{ ms}^{-1}$   
 5 a  $(t^2 + 6)\mathbf{i} + (2t - \frac{t^3}{3})\mathbf{j} \text{ m}$  b  $6\mathbf{j} \text{ m}$   
 6 a  $(-\mathbf{i} - \frac{1}{4}\mathbf{j}) \text{ ms}^{-1}$  b  $(94\mathbf{i} - 654\mathbf{j}) \text{ m}$   
 7 a  $((2t^4 - 3t^2 - 4)\mathbf{i} + (4t^2 - 3t - 7)\mathbf{j}) \text{ ms}^{-1}$   
 b  $t = \frac{7}{4}$   
 8 a  $\mathbf{r} = (2t^2 - 3t + 1)\mathbf{i} + (4t + 2)\mathbf{j} \text{ m}$   
 b i 3.4 ii  $\mathbf{r} = 36\mathbf{i} + 22\mathbf{j}$

### Challenge

$$\mathbf{r} = (\frac{3\pi}{2} + 1)\mathbf{i} + (\frac{5\pi^2}{8} + 1)\mathbf{j}$$

### Mixed exercise 8

- 1  $4.8\mathbf{i} - 6.4\mathbf{j}$   
 2 10.1 m  
 3 a  $2t\mathbf{i} + (-500 + 3t)\mathbf{j}$  b 721 m  
 4 a  $(1 + 2t)\mathbf{i} + (3 - t)\mathbf{j}$ ,  $(5 - t)\mathbf{i} + (-2 + 4t)\mathbf{j}$   
 b  $\mathbf{r}_{BA} = \mathbf{r}_B - \mathbf{r}_A = (5 - t)\mathbf{i} + (4t - 2)\mathbf{j} - ((1 + 2t)\mathbf{i} + (3 - t)\mathbf{j})$   
 $= (4 - 3t)\mathbf{i} + (5t - 5)\mathbf{j}$   
 c For A and B to collide  $\mathbf{r}_{AB} = 0$   
 Equating  $\mathbf{i} \rightarrow t = \frac{4}{3}$ , equating  $\mathbf{j} \rightarrow t = 1$ . Times are not the same therefore the ships do not collide.  
 d 5.39 km  
 5 a 48 m b 120 m (2 s.f.)  
 6 a  $p = 16$ ,  $q = 19.4$  b  $25.1 \text{ ms}^{-1}$   
 c  $\frac{97}{80}$  d 3.50 s (3 s.f.)  
 e e.g. weight of the ball, air resistance  
 7 a 76.6 m (3 s.f.) b 110 m (3 s.f.)  
 8 a  $k = -4$  b 4 m c 0.05  
 9 a  $0.3\sqrt{3} \text{ m}$  b  $t = 3$  c  $0.329 \text{ ms}^{-2}$  (3 s.f.)  
 10 a  $(\ln 2 - 2) \text{ ms}^{-2}$  in the direction of  $x$  increasing.  
 b  $\frac{8}{e} \text{ m}$   
 11 a  $V_P$  is  $(6t\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}$  and  $V_Q$  is  $(\mathbf{i} + 3t\mathbf{j}) \text{ ms}^{-1}$   
 b  $12.2 \text{ ms}^{-1}$  (3 s.f.)  
 c  $t = \frac{1}{3}$   
 d Equate  $\mathbf{i}$ -components and solve to get  $t = 1$ .  
 Equate  $\mathbf{j}$ -components and solve to get  $t = \frac{1}{3}$  or 1.  
 So  $t = 1$  and  $\mathbf{r} = (7\mathbf{i} + \frac{3}{2}\mathbf{j}) \text{ m}$   
 12 a Differentiate:  $\mathbf{v} = 6t\mathbf{i} - 8t\mathbf{j}$ ,  $\mathbf{a} = 6\mathbf{i} - 8\mathbf{j}$  so constant  
 b  $10 \text{ ms}^{-2}$   
 $143.1^\circ$  (nearest  $0.1^\circ$ )  
 13 a  $-6\mathbf{i} \text{ ms}^{-1}$   
 b Differentiate:  $\mathbf{v} = -6 \sin 3t\mathbf{i} - 6 \cos 3t\mathbf{j}$ ,  
 $\mathbf{a} = -18 \cos 3t\mathbf{i} + 18 \sin 3t\mathbf{j}$   
 $|\mathbf{a}| = \sqrt{18^2(\cos^2 t + \sin^2 t)} = 18 \text{ ms}^{-2}$  so constant  
 14 a Differentiate:  $\mathbf{a} = 4c\mathbf{i} + (14 - 2c)t\mathbf{j}$ ,  
 $\mathbf{F} = m\mathbf{a} = \frac{1}{2}(4c\mathbf{i} + (14 - 2c)t\mathbf{j})$   
 b 4,  $\frac{234}{29} \approx 8.07$   
 15 a  $((2t^4 - 3t^2 - 4)\mathbf{i} + (4t^2 - 3t - 7)\mathbf{j}) \text{ ms}^{-1}$   
 b  $t = \frac{7}{4}$   
 16  $10\sqrt{41} \text{ ms}^{-1}$   
 17 a  $\mathbf{v} = (t^2 + 3)\mathbf{i} + (3t + 13)\mathbf{j}$  b 5 s

### Challenge

- 1 a 20 m  
 b  $v = 0 \text{ ms}^{-1}$  when  $t = 1.64 \text{ s}$  (or  $-2.44 \text{ s}$ ) so only changes direction once  
 c  $2\sqrt{20}(\sqrt{20} + 1)^{\frac{1}{2}}$   
 2 a Differentiate:  $\mathbf{r} = (6\omega \cos \omega t)\mathbf{i} - (4\omega \sin \omega t)\mathbf{j}$   
 $|\dot{\mathbf{r}}|^2 = (36\omega^2 \cos^2 \omega t)\mathbf{i} + (16\omega^2 \sin^2 \omega t)\mathbf{j}$   
 use  $\sin^2 \omega t + \cos^2 \omega t = 1$  and  $2 \cos^2 \omega t = \cos 2\omega t + 1$   
 b  $v = \sqrt{26\omega^2 + 10\omega^2 \cos 2\omega t}$  max when  $\cos 2\omega t = 1$  and min when  $\cos 2\omega t = -1$   
 c  $70.2^\circ$  (1 d.p.)

### Review exercise 2

- 1 a 573 N (3 s.f.) b 427 N (3 s.f.)  
 c All his weight acts through a single point.  
 2 a Taking moments about C:  $2.5 mgl = 4R_B l$   
 So  $R_B = \frac{5}{8}mg$  (as required)  
 b Taking moments about B:  $1.5 mgl = 4R_C l$   
 So  $R_C = \frac{3}{8}mg$  (as required)  
 c i The weight of the plank acts through its midpoint.  
 ii By modelling a plank as a rod you can ignore its width.  
 3 a 125 N b 750 N  
 4 0.375 m  
 5 19.8 N (3 s.f.)  
 6  $R(\angle)$ :  $F \cos 30 - 2g \sin 45 = 4$   
 $\frac{\sqrt{3}}{2}F = 4 + \frac{2g\sqrt{2}}{2} \Rightarrow F = \frac{2}{\sqrt{3}}(4 + \sqrt{2}g) \text{ N}$  (as required)  
 7 a 144 767 N (to the nearest whole number)  
 b 0.10 (2 s.f.)  
 c 0.74 seconds (2 s.f.)  
 d Will not remain at rest.  $F_{\text{MAX}} = 15000 \text{ N}$  and component of weight down slope =  $26000 \text{ N}$  which is greater.  
 8 a 0.404 s (3 s.f.) b 0.808 m (3 s.f.)  
 9 a  $19.8 \text{ ms}^{-1}$  (3 s.f.)  
 b The ball as a projectile has negligible size and is subject to negligible air resistance.  
 Free fall acceleration remains constant during flight of ball.  
 10 a 2.7 s (2 s.f.) b 790 m (2 s.f.)  
 11 a  $u = 3.6$  (2 s.f.) b  $k = 86$  (2 s.f.)  
 c  $43^\circ$  (2 s.f.)  
 12 a  $(36\mathbf{i} + 27.9\mathbf{j}) \text{ m}$  b  $13 \text{ ms}^{-1}$  (2 s.f.)  
 13 a  $u_y = u \sin \alpha$ ,  $s_y = 0$ ,  $a = -g$   
 Using  $s = ut + \frac{1}{2}at^2$ :  $0 = ut \sin \alpha - \frac{1}{2}gt^2$   
 $\frac{1}{2}gt = u \sin \alpha$  so  $t = \frac{2u \sin \alpha}{g}$   
 b  $u_x = u \cos \alpha$ ,  $s_x = R$ ,  $a = 0$ ,  $t = \frac{2u \sin \alpha}{g}$   
 Using  $s = ut + \frac{1}{2}at^2$ :  
 $R = ut \cos \alpha = \frac{2u^2 \sin \alpha \cos \alpha}{g} = \frac{u^2 \sin 2\alpha}{g}$   
 c  $R$  is maximum when  $\sin 2\alpha$  is 1, which is when  $2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$   
 d  $12^\circ$  and  $78^\circ$  (nearest degree)  
 14 a  $R(t)$ :  $T \cos 30^\circ = g + T \cos 60^\circ$   
 $T = \frac{2g}{\sqrt{3} - 1}$  (as required)  
 b 36.6 N (3 s.f.)  
 c There are no frictional forces acting on the bead.  
 15 a Resolving perpendicular to the slope:  
 $R = 500g \cos \alpha$  and  $\cos \alpha = \frac{24}{25}$   
 So  $R = 480g$  (as required)  
 b 68g N

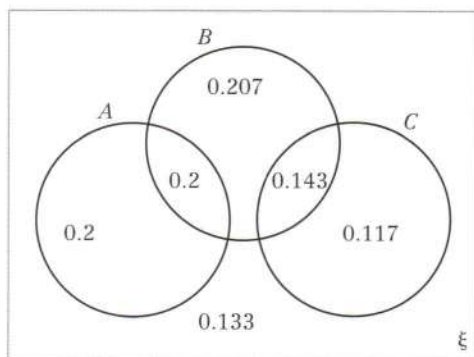
- 16 2.46 m (3 s.f.)  
 17  $\mu = \frac{1}{2 \tan \alpha}$   
 18 44.4 N (3 s.f.)  
 19 28.9 s (3 s.f.)  
 20 a  $\mathbf{r}_P = (400 + 300t)\mathbf{i} + (200 + 250t)\mathbf{j}$  and  
 $\mathbf{r}_Q = (500 + 600t)\mathbf{i} - (100 + 200t)\mathbf{j}$   
 b  $100\mathbf{i} - 300\mathbf{j}$   
 c 1390 km (3 s.f.)  
 21 a  $v = 3 + \frac{4k}{(2t-1)^2}$   
 $t = 0 \Rightarrow v = 3 + 4k = 10 \Rightarrow k = \frac{7}{4}$   
 b  $\frac{29}{6}\text{m}$   
 22 a  $((t^2 + 2)\mathbf{i} + t\mathbf{j})\text{ms}^{-1}$  b  $27.5\text{ms}^{-1}$  (3 s.f.)  
 c  $4.12\text{ms}^{-2}$  at an angle of  $14^\circ$  to the positive unit  $\mathbf{i}$  vector  
 23 a  $(24\mathbf{i} + 12\mathbf{j})\text{ms}^{-1}$  b  $\frac{d^2r}{dt^2} = 8\mathbf{i} + 4\mathbf{j}$   
 24 24 m (2 s.f.)  
 25 a  $\mathbf{v} = (t^2 - \frac{3}{4}t^4 + 3)\mathbf{i} - (4t^2 + 4t - 1)\mathbf{j}$   
 b  $0.207\text{s}$  (3 s.f.)  
 26 a  $\mathbf{v} = (-2t^2 + 8)\mathbf{i} - 2t\mathbf{j}$  b  $t = 2\text{s}$

### Challenge

- 1 Taking moments about  $B$ , in limiting equilibrium:  
 $0.4mgk = 0.4k \times 100 + 1.2Fk$   
 $12F = 4mg - 400$ , so  $F = \frac{1}{3}(mg - 100)\text{N}$   
 So in order for  $m$  to be lifted  $F > \frac{1}{3}(mg - 100)\text{N}$   
 2 Max distance: 12.3 m,  $t = 323.13$   
 3 R( $\rightarrow$ ):  $d \cos \theta = u \sin \theta \times t \Rightarrow t = \frac{d \cos \theta}{u \sin \theta}$   
 R( $\uparrow$ ):  $-d \sin \theta = u \cos \theta \times t - \frac{g}{2}t^2$   
 Substitute for  $t$ .  
 Rearrange to show that  $d = \frac{2u^2}{g} \tan \theta \sec \theta$

### Exam-style practice

- 1 a 0.4133  
 b  $n$  is large,  $p$  is close to 0.5  
 c 0.1146 (4 d.p.)  
 d The probability is not significant at the 5% level so there is no evidence that the claim is incorrect.  
 2 a



- b  $0.4 \times 0.55 \neq 0.2$  c  $\frac{4}{9}$ , 0.444 to 3 s.f.  
 d  $\frac{13}{40}$ , 0.325

- 3 a Continuous – it is a measured variable  
 b  $19.4^\circ\text{C}$  and  $2.81^\circ\text{C}$  (both to 3 s.f.)  
 c Continuous data, unequal class widths  
 d  $7.97^\circ\text{C}$   
 e  $H_0: \rho = 0$ ;  $H_1: \rho > 0$ . Critical value = 0.6215 so no evidence that the population correlation coefficient is greater than zero.  
 4 a Yes – the value of the PMCC is close to 1.  
 b  $k = 1.19$ ,  $n = 2.13$  (both to 3 s.f.)  
 c Value of  $t$  is a long way from the range of the experimental data so extrapolation – no it would not be sensible.  
 5 a 80.1% b 0.9406  
 c  $H_0: \mu = 4.5$ ,  $H_1: \mu \neq 4.5$  Probability = 0.059 so not significant – there is no evidence to suggest that the mean weight is not 4.5 kg.  
 6 0.731 (3 s.f.)  
 7 Position vector at  $t = 3$  is  $-43\mathbf{i} - 4\mathbf{j}$ .  
 distance = 43.2 m (3 s.f.)  
 8 a 10.2 s (3 s.f.) b 128 m (3 s.f.)  
 c  $89.0\text{ms}^{-1}$  (3 s.f.)  
 9 a  $30\mathbf{i} - 40\mathbf{j}$  b 50 m c 12 s d  $102^\circ$   
 10 a 18.6 N  
 b For  $P$ , R( $\leftarrow$ ):  $T - \mu 3g = 3a$  so  $18.6 - 29.4\mu = 1.5$  and  $\mu = 0.582$  (as required)  
 c 0.175 s  
 d Tension and acceleration are equal both sides of the pulley.  
 11 a Taking moments about  $B$   
 $\frac{1}{2}mgl = Tx$  where  $x = l \sin \alpha$  and  $\sin \alpha = \frac{5}{13}$   
 $\frac{1}{2}mgl = \frac{5}{13}lT$  so  $T = \frac{13}{10}mg$  (as required)  
 b  $\frac{5}{12}$  or 0.42 (2 s.f.)





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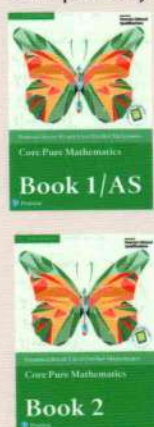


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ISBN 978-1-4469-4407-3



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