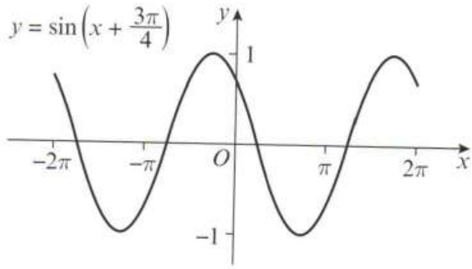
Review exercise 2

E **1** The diagram shows the curve with equation $y = \sin\left(x + \frac{3\pi}{4}\right)$, $-2\pi \leqslant x \leqslant 2\pi$.



Calculate the coordinates of the points at which the curve meets the coordinate axes. **(3)**

← Section 5.1

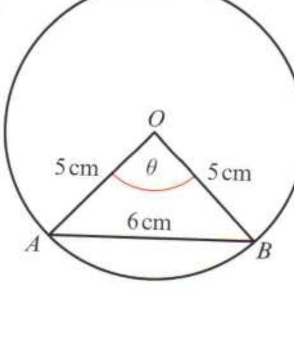
2 **a** Sketch, for $0 \leqslant x \leqslant 2\pi$, the graph of $y = \cos\left(x - \frac{\pi}{3}\right)$ **(2)**

b Write down the exact coordinates of the points where the graph meets the coordinate axes. **(3)**

c Solve, for $0 \leqslant x \leqslant 2\pi$, the equation $\cos\left(x - \frac{\pi}{3}\right) = -0.27$, giving your answers in radians to 2 decimal places. **(5)**

← Section 5.1

3 In the diagram, A and B are points on the circumference of a circle centre O and radius 5 cm.
$\angle AOB = \theta$ radians
$AB = 6$ cm

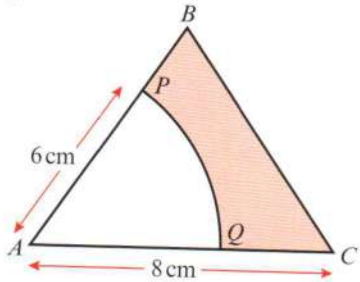


a Find the value of θ. **(2)**

b Calculate the length of the minor arc AB to 3 s.f. **(2)**

← Section 5.2

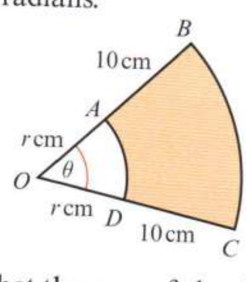
E/P **4** In the diagram, ABC is an equilateral triangle with side 8 cm.
PQ is an arc of a circle centre, A, radius 6 cm.
Find the perimeter of the shaded region in the diagram.



(5)

← Section 5.2

E/P **5** In the diagram, AD and BC are arcs of circles with centre O, such that $OA = OD = r$ cm, $AB = DC = 10$ cm and $\angle BOC = \theta$ radians.

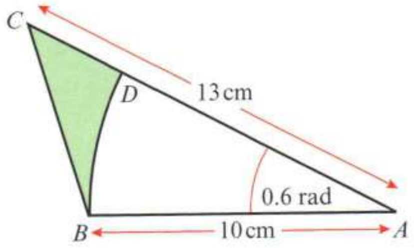


a Given that the area of the shaded region is 40 cm², show that $r = \frac{4}{\theta} - 5$. **(4)**

b Given also that $r = 6\theta$, calculate the perimeter of the shaded region. **(6)**

← Sections 5.2, 5.3

E/P **6** In the diagram,
$AB = 10$ cm, $AC = 13$ cm.
$\angle CAB = 0.6$ radians.
BD is an arc of a circle centre A and radius 10 cm.

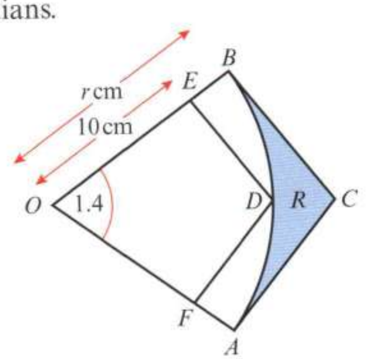


a Calculate the length of the arc BD. **(2)**

b Calculate the shaded area in the diagram to 1 d.p. **(3)**

← Sections 5.2, 5.3

E/P **7** The diagram shows the sector OAB of a circle with centre O, radius r cm and angle 1.4 radians.



The lines AC and BC are tangent to the circle with centre O. OEB and OFA are straight lines. The line ED is parallel to BC and the line FD is parallel to AC.

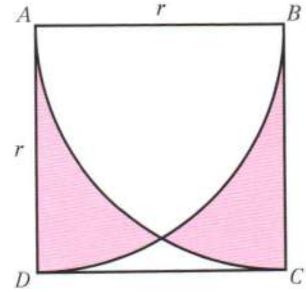
a Find the area of sector OAB, giving your answer to 1 decimal place. **(4)**

The region R is bounded by the arc AB and the lines AC and CB.

b Find the perimeter of R, giving your answer to 1 decimal place. **(6)**

← Sections 5.2, 5.3

E/P **8** The diagram shows a square, $ABCD$, with side length r, and 2 arcs of circles with centres A and B.



Show that the area of the shaded region is $\frac{r^2}{6}(3\sqrt{3} - \pi)$. **(5)**

← Sections 5.2, 5.3

E/P **9** **a** Show that the equation $3\sin^2 x + 7\cos x + 3 = 0$ can be written as $3\cos^2 x - 7\cos x - 6 = 0$. **(2)**

b Hence solve, for $0 \leqslant x < 2\pi$, $3\sin^2 x + 7\cos x + 3 = 0$, giving your answers to 2 decimal places. **(3)**

← Section 5.4

E **10** **a** Show that, when θ is small, $\sin 4\theta - \cos 4\theta + \tan 3\theta \approx 8\theta^2 + 7\theta - 1$ **(3)**

b Hence state the approximate value of $\sin 4\theta - \cos 4\theta + \tan 3\theta$ for small values of θ. **(1)**

← Section 5.5

E/P **11** **a** Sketch, in the interval $-2\pi \leqslant x \leqslant 2\pi$, the graph of $y = 4 - 2\operatorname{cosec} x$. Mark any asymptotes on your graph. **(3)**

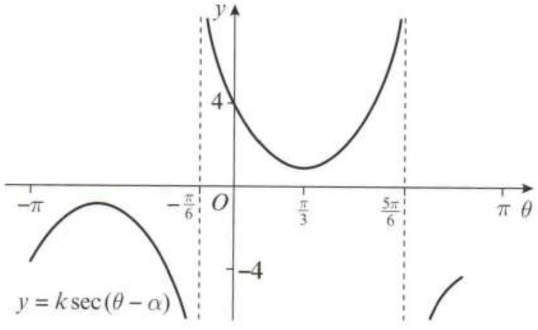
b Hence deduce the range of values of k for which the equation $4 - 2\operatorname{cosec} x = k$ has no solutions. **(2)**

← Sections 6.1, 6.2

P **12** The diagram shows the graph of

$$y = k\sec(\theta - \alpha)$$

The curve crosses the y-axis at the point $(0, 4)$, and the θ-coordinate of its minimum point is $\frac{\pi}{3}$



a State, as a multiple of π, the value of α. **(1)**

b Find the value of k. **(2)**

c Find the exact values of θ at the points where the graph crosses the line $y = -2\sqrt{2}$. **(3)**

← Section 6.2

P **13** **a** Show that

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} \equiv 2\sec x$$

(4)

b Hence solve, in the interval $0 \leqslant x \leqslant 4\pi$, $\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = -2\sqrt{2}$ **(4)**

← Section 6.3

P **14** **a** Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta, \theta \neq 90n°$$

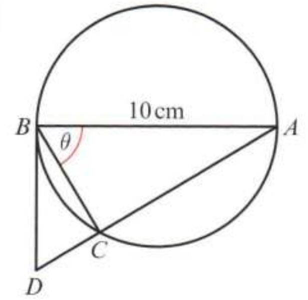
(3)

b Sketch the graph of $y = 2\operatorname{cosec} 2\theta$ for $0° < \theta < 360°$. **(3)**

c Solve, for $0° < \theta < 360°$, the equation $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$, giving your answer to 1 decimal place. **(4)**

← Section 6.3

E/P **15**



In the diagram, $AB = 10$ cm is the diameter of the circle and BD is the tangent to the circle at B. The chord AC is extended to meet this tangent at D and $\angle ABC = \theta$.

a Show that $BD = 10\cot\theta$. **(4)**

b Given that $BD = \frac{10}{\sqrt{3}}$ cm, calculate the exact length of DC. **(3)**

← Section 6.4

E/P **16** **a** Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta = \sec^2\theta$. **(2)**

b Solve, for $0° \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1$$

giving your answers to 1 decimal place. **(6)**

← Section 6.3

P **17** Given that $a = \operatorname{cosec} x$ and $b = 2\sin x$.

a express a in terms of b

b find the value of $\frac{4 - b^2}{a^2 - 1}$ in terms of b.

← Section 6.4

E/P **18** Given that

$$y = \arcsin x, -1 \leqslant x \leqslant 1, -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$$

a express $\arccos x$ in terms of y. **(2)**

b Hence find, in terms of π, the value of $\arcsin x + \arccos x$. **(1)**

← Section 6.5

E **19** **a** Prove that for $x \geqslant 1$,

$$\arccos\frac{1}{x} = \arcsin\frac{\sqrt{x^2 - 1}}{x}$$ **(4)**

b Explain why this identity is not true for $0 \leqslant x < 1$. **(2)**

← Section 6.5

E **20** **a** Sketch the graph of $y = 2\arccos x - \frac{\pi}{2}$, showing clearly the exact endpoints of the curve. **(4)**

b Find the exact coordinates of the point where the curve crosses the x-axis. **(3)**

← Section 6.5

E **21** Given that $\tan\left(x + \frac{\pi}{6}\right) = \frac{1}{6}$, show that

$$\tan x = \frac{72 - 111\sqrt{3}}{321}$$ **(5)**

← Section 7.1

E/P **22** Given that $\sin(x + 30°) = 2\sin(x - 60°)$

a show that $\tan x = 8 + 5\sqrt{3}$. **(4)**

b Hence express $\tan(x + 60°)$ in the form $a + b\sqrt{3}$. **(3)**

← Section 7.1

E/P **23** **a** Use $\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$, or otherwise, to show that

$$\sin 165° = \frac{\sqrt{6} - \sqrt{2}}{4}$$ **(4)**

b Hence, or otherwise, show that $\operatorname{cosec} 165° = \sqrt{a} + \sqrt{b}$, where a and b are constants to be found. **(3)**

← Sections 7.1, 7.2

E/P **24** Given that $\cos A = \frac{3}{4}$ where $270° < A < 360°$,

a find the exact value of $\sin 2A$ **(3)**

b show that $\tan 2A = -3\sqrt{7}$. **(3)**

← Section 7.3

E/P **25** Solve, in the interval $-180° \leqslant x \leqslant 180°$, the equations

a $\cos 2x + \sin x = 1$ **(3)**

b $\sin x(\cos x + \operatorname{cosec} x) = 2\cos^2 x$ **(3)**

giving your answers to 1 decimal place.

← Section 7.4

E **26** $f(x) = 3\sin x + 2\cos x$
Given $f(x) = R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

a find the value of R and the value of α. **(4)**

b Hence find the greatest value of $(3\sin x + 2\cos x)^4$ **(2)**

c Hence, or otherwise, solve for $0 \leqslant \theta < 2\pi$, $f(x) = 1$, rounding your answers to 3 decimal places. **(3)**

← Section 7.5

E **27** **a** Prove that

$$\cot\theta - \tan\theta \equiv 2\cot 2\theta,\ \theta \neq \frac{n\pi}{2}$$ **(3)**

b Solve, for $-\pi < \theta < \pi$, the equation

$$\cot\theta - \tan\theta = 5,$$

giving your answers to 3 significant figures. **(3)**

← Sections 6.3, 7.6

E **28** **a** By writing $\cos 3\theta$ as $\cos(2\theta + \theta)$, show that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$ **(4)**

b Given that $\cos\theta = \frac{\sqrt{2}}{3}$, find the exact value of $\sec 3\theta$. Give your answer in the form $k\sqrt{2}$ where k is a rational constant to be found. **(2)**

← Sections 6.3, 7.1

E **29** Show that $\sin^4\theta \equiv \frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$. You must show each stage of your working. **(6)**

← Section 7.6

E/P **30** **a** Express $6\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $r < 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 2 decimal places. **(4)**

b **i** Find the maximum value of $6\sin\theta + 2\cos\theta$ **(2)**

ii Find the value of θ, for $0 < \theta < \pi$, at which the maximum occurs, giving the value to 2 d.p. **(1)**

The temperature, in T °C, on a particular day is modelled by the equation

$$T = 9 + 6\sin\left(\frac{\pi t}{12}\right) + 2\cos\left(\frac{\pi t}{12}\right),$$

$0 \leqslant t \leqslant 24$ where t is the number of hours after 9 a.m.

c Calculate the minimum value of T predicted by this model, and the value of t, to 2 decimal places, when this minimum occurs. **(3)**

d Calculate, to the nearest minute, the times in the first day when the temperature is predicted by this model, to be exactly 14°C. **(4)**

← Section 7.5, 7.7

E **31** A curve C has parametric equations

$$x = 1 - \frac{4}{t}, y = t^2 - 3t + 1, t \in \mathrm{R}, t \neq 0$$

a Determine the ranges of x and y in the given domain of t. **(3)**

b Show that the Cartesian equation of C can be written in the form $y = \frac{ax^2 + bx + c}{(1 - x)^2}$, where a, b and c are integers to be found. **(3)**

← Section 8.1

E **32** A curve has parametric equations

$$x = \ln(t + 2), y = \frac{3t}{t + 3}, t > 4$$

a Find a Cartesian equation of this curve in the form $y = \mathrm{f}(x)$, $x > \mathrm{k}$, where k is an exact constant to be found. **(4)**

b Write down the range of $\mathrm{f}(x)$ in the form $a < y < b$, where a and b are constants to be found. **(2)**

← Section 8.1

E **33** A curve C has parametric equations

$$x = \frac{1}{1 + t}, y = \frac{1}{1 - t}, -1 < t < 1$$

Show that a Cartesian equation of C is

$y = \frac{x}{2x - 1}$ **(4)**

← Section 8.1

E/P **34** A curve C has parametric equations

$$x = 2\cos t, y = \cos 3t, 0 \leqslant t \leqslant \frac{\pi}{2}$$

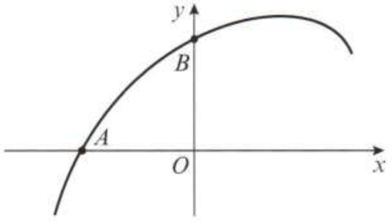
a Find a Cartesian equation of the curve in the form $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a cubic function. **(5)**

b State the domain and range of $\mathrm{f}(x)$ for the given domain of t. **(2)**

← Section 8.2

E/P **35** The curve shown in the figure has parametric equations

$$x = \sin t, y = \sin\left(t + \frac{\pi}{6}\right), -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$



a Show that a Cartesian equation of the curve is

$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, -1 \leqslant x \leqslant 1$ **(4)**

b Find the coordinates of the points A and B, where the curve intercepts the x- and y-axes. **(3)**

← Section 8.2

E **36** The curve C has parametric equations

$$x = 3\cos t, y = \cos 2t, 0 \leqslant t \leqslant \pi$$

a Find a Cartesian equation of C. **(4)**

b Sketch the curve C on the appropriate domain, labelling the points where the curve intercepts the x- and y-axes. **(3)**

← Section 8.2, 8.3

E/P **37** The curve C has parametric equations

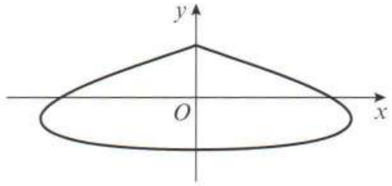
$$x = 4t, y = 8t(2t - 1), t \in \mathbb{R}.$$

Given that the line with equation $y = 3x + c$, where c is a constant, does not intersect C, find the range of possible values of c. **(5)**

← Section 8.4

E **38** A curve has parametric equations

$x = 3\sin 2t, y = 2\cos t + 1, \frac{\pi}{2} \leqslant t \leqslant \frac{3\pi}{2}$



a Find the coordinates of the points where the curve intersects the x-axis. **(4)**

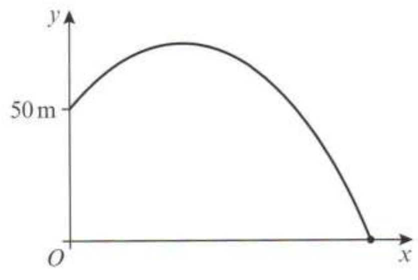
b Show that the curve crosses the line $x = 1.5$ when $t = \frac{13\pi}{12}$ and $t = \frac{17\pi}{12}$ **(3)**

← Section 8.4

E/P **39** A golf ball is hit from an elevation of 50 m, with an initial speed of $50\,\text{m s}^{-1}$ at an angle of 30° above the horizontal. Its position after t seconds can be described using the following parametric equations:

$x = (25\sqrt{3})t, y = 25t - 4.9t^2 + 50, 0 \leqslant t \leqslant k$

where x is the horizontal distance in metres, y is the vertical distance in metres from the ground and k is a constant.



Given that the model is valid from the time the golf ball is hit until the time it hits the ground,

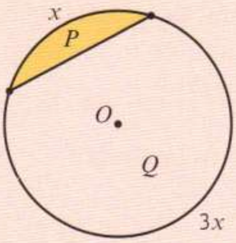
a find the value of k to 2 decimal places. **(3)**

b Find a Cartesian equation for the path of the golf ball in the form $y = f(x)$, and determine the domain of $f(x)$. Give the domain to 1 d.p. **(5)**

← Section 8.5

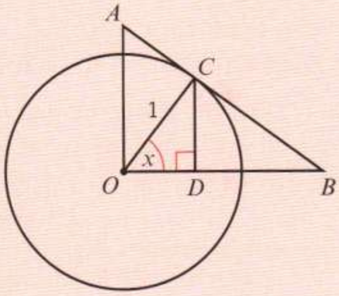
Challenge

1 A chord of a circle, centre O and radius r, divides the circumference in the ratio 1 : 3, as shown in the diagram. Find the ratio of the area of region P to the area of region Q.



← Section 5.3

2 The diagram shows a circle, centre O. The radius of the circle, OC, is 1, and $\angle CDO = 90°$.



Given that $\angle COD = x$, express the following lengths as single trigonometric functions of x.

a CD **b** OD **c** OA
d AC **e** CB **f** OB

← Section 6.1

3 The curve C has parametric equations

$x = 4\sin t + 3, y = 4\cos t - 1, -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{4}$

a By finding a Cartesian equation of C in the form $(x - a)^2 + (y - b)^2 = c$, or otherwise, sketch C, labelling the endpoints of the curve with their exact coordinates.

b Find the length of C, giving your answer in terms of π. ← Section 8.3