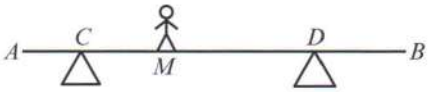
2 Review exercise

E 1



A uniform plank AB of length 5 m and weight 200 N, rests in a horizontal position on supports at C and D, where $AC = 0.5$ m and $BD = 0.75$ m. A builder of weight 800 N stands on the plank at M where $AM = 2$ m, as shown in the diagram. The builder is modelled as a particle and the plank is modelled as a rod. Calculate:

a the magnitude of the reaction at C **(3)**

b the magnitude of the reaction at D. **(3)**

c State how you have used the modelling assumption that the builder is a particle. **(1)**

← Section 4.3

E/P 2

A uniform plank AC of length $5l$ m and mass m kg, rests in a horizontal position on supports at B and C, where $AB = l$ m and $BC = 4l$ m. The plank is modelled as a rod. Show that:

a the magnitude of the reaction at B is $\frac{5}{8}mg$ **(3)**

b the magnitude of the reaction at C is $\frac{3}{8}mg$. **(3)**

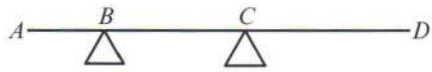
c State how you have used the modelling assumption that the plank is:

i uniform **(1)**

ii a rod. **(1)**

← Section 4.3

E 3



A uniform rod AD of length 10 m and weight 500 N, rests in a horizontal position on supports at B and C, where $AB = 2$ m and $BC = 4$ m.

a Calculate the largest weight that can be placed at D before the rod starts to tip. **(3)**

b Calculate the largest weight that can be placed at A before the rod starts to tip. **(3)**

← Sections 4.3, 4.5

4

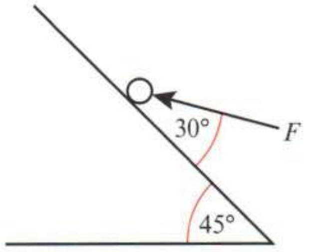
A lever consists of a uniform steel rod AB of weight 200 N and length 3 m, which rests on a pivot at C. A 2000 N weight is placed at B, and is supported by a force of 200 N applied vertically downwards at A. Given that the lever is in equilibrium, calculate the length CB.

← Sections 4.3, 4.5

5 A particle of mass 3 kg is moving up a rough slope that is inclined at an angle α to the horizontal where $\tan \alpha = \frac{5}{12}$. A force of magnitude P N acts horizontally on the particle towards the plane. Given that the coefficient of friction between the particle and the slope is 0.2 and that the particle is moving at a constant velocity, calculate the value of P.

← Sections 5.2, 5.3

(P) **6** A particle of mass 2 kg sits on a smooth slope that is inclined at 45° to the horizontal. A force of F N acts at an angle of 30° to the plane on the particle causing it to accelerate up the hill at $2\,\text{m}\,\text{s}^{-2}$.



Show that $F = \frac{2}{\sqrt{3}}(4 + \sqrt{2}g)$ N.

← Sections 5.2, 5.3

(E/P) **7** A shipping container of mass 15000 kg is being pulled by a winch up a rough slope that is inclined at 10° to the horizontal. The winch line imparts a constant force of 42000 N, which acts parallel to and up the slope, causing the shipping container to accelerate at a constant rate of $0.1\,\text{m}\,\text{s}^{-2}$. Calculate:

a the reaction between the shipping container and the slope **(2)**

b the coefficient of friction, μ, between the shipping container and the slope. **(3)**

When the shipping container is travelling at $2\,\text{m}\,\text{s}^{-1}$ the engine is turned off.

c Find the time taken for the shipping container to come to rest. **(3)**

d Determine whether the shipping container will remain at rest, justifying your answer carefully. **(2)**

← Sections 5.2, 5.3

(E) **8** A ball is projected horizontally from a tabletop at a height of 0.8 m above level ground. Given that the initial velocity of the ball is $2\,\text{m}\,\text{s}^{-1}$, find:

a the time taken for the ball to reach the ground **(3)**

b the horizontal distance between the table edge and the point where the ball lands. **(2)**

← Section 6.1

(E/P) **9** A football is kicked horizontally off a platform of height 20 m and lands a horizontal distance of 40.0 m from the edge of the platform.

a Find the initial horizontal velocity of the football. **(5)**

b State two assumptions you have made in your calculations, and comment on the validity of each assumption. **(2)**

← Section 6.1

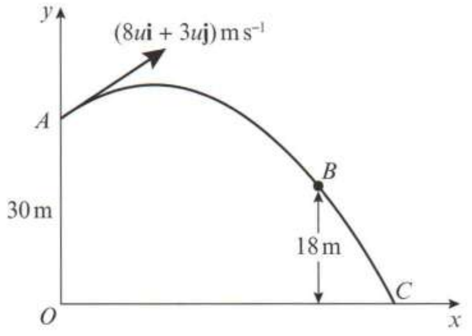
(E) **10** A projectile is launched from a point on horizontal ground with speed $150\,\text{m}\,\text{s}^{-1}$ at an angle of 10° above the horizontal. Find:

a the time the projectile takes to reach its highest point above the ground **(4)**

b the range of the projectile. **(4)**

← Sections 6.2, 6.3

(E/P) **11**



In this question, the unit vectors **i** and **j** are in a vertical plane, **i** being horizontal and **j** being vertical.

A particle P is projected from a point A with position vector $30\mathbf{j}$ m with respect to a fixed origin O. The velocity of projection is $(8u\mathbf{i} + 3u\mathbf{j})\,\text{m}\,\text{s}^{-1}$. The particle moves freely under gravity, passing through a point B, which has position vector $(k\mathbf{i} + 18\mathbf{j})$ m, where k is a constant, before reaching the point C on the x-axis, as shown in the figure above. The particle takes 3 s to move from A to B.

Find:

a the value of u **(4)**

b the value of k **(2)**

c the angle the velocity of P makes with the x-axis as it reaches C. **(6)**

← Sections 6.2, 6.3

(E) **12** A particle P is projected from the origin with velocity $(12\mathbf{i} + 24\mathbf{j})\,\text{m s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively. The particle moves freely under gravity. Find:

a the position vector of P after 3 s **(4)**

b the speed of P after 3 s. **(4)**

← Sections 6.2, 6.3

(P) **13** A projectile is launched from a point on a horizontal plane with initial speed $u\,\text{m s}^{-1}$ at an angle of elevation α. The particle moves freely under gravity until it strikes the plane. The range of the projectile is R m.

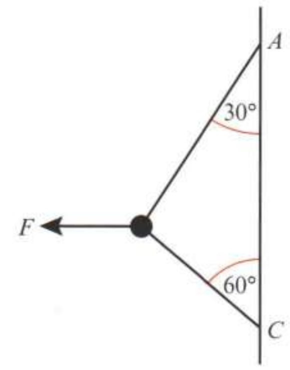
a Show that the time of flight of the particle is $\frac{2u\sin\alpha}{g}$ seconds.

b Show that $R = \frac{u^2\sin 2\alpha}{g}$.

c Deduce that, for a fixed u, the greatest possible range is when $\alpha = 45°$.

d Given that $R = \frac{2u^2}{5g}$, find the two possible values of the angle of elevation at which the projectile could have been launched. ← Section 6.4

(E/P) **14** A smooth bead B of mass 1 kg is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C where A is vertically above C. The bead is held in equilibrium by a horizontal force F. AB and BC make angles of 30° and 60° respectively with the vertical, as shown in diagram.



a Show that the tension in the string is $\frac{2g}{\sqrt{3}-1}$ N. **(3)**

b Calculate the magnitude of F. **(3)**

c State how you have used the fact that the bead is smooth in your calculations. **(1)**

← Section 7.2

(E) **15** A crate of mass 500 kg sits on a hill which is inclined at an angle α to the horizontal where $\tan\alpha = \frac{7}{24}$. The coefficient of friction between the hill and the crate is 0.15 , and the crate is held at rest by a force of magnitude F N which acts parallel to and up the line of greatest slope of the hill.

By modelling the crate as a particle,

a show that the normal reaction of the hill on the crate is $480g$ N **(3)**

b work out the minimum value of F. **(3)**

← Section 7.3

(E/P) **16** A ladder PQ of mass 25 kg and length 6 metres, rests with its base, P, on rough horizontal ground and its top, Q, leaning against a smooth vertical wall. The coefficient of friction between the ladder and the ground is 0.25. The ladder lies in a vertical plane perpendicular to the wall and the ground, and is inclined at an angle 60° to the horizontal.

A builder of mass 75 kg climbs up the ladder. Modelling the builder as a particle and the ladder as a uniform rod, find the maximum distance up the ladder the builder can climb before the ladder begins to slip. **(10)**

← Section 7.4

P **17** A uniform ladder PQ of mass m kg and length l metres, rests with one end P on rough horizontal ground and the other end Q against a smooth vertical wall. The coefficient of friction between the ladder and the ground is μ. The ladder lies in a vertical plane perpendicular to the wall and the ground, and is inclined at an angle α to the horizontal. Given that the ladder is on the point of slipping, find an expression for μ in terms of α. **(10)**

← Section 7.4

P **18** A non-uniform ladder AB of weight 240 N and length 6 m rests with its end A on smooth horizontal ground and its end B against a rough vertical wall. The coefficient of friction between the ladder and the wall is 0.3.The centre of mass of the ladder is 2 m from A. The ladder lies in a vertical plane perpendicular to the wall and the ground, and is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{2}$. The ladder can be prevented from sliding down the wall by applying a horizontal force of magnitude P N to the bottom of the ladder. By modelling the ladder as a non-uniform rod determine the minimum value of P. **(10)**

← Section 7.4

P **19** A sled of mass 5 kg is released from rest on a hill that is angled at α to the horizontal where $\tan\alpha = \frac{1}{5}$. The coefficient of friction between the sled and the hill is 0.15. By modelling the sled as a particle work out how long it takes the sled to travel 200 m. **(6)**

← Section 7.5

E/P **20** At 10 am two aeroplanes P and Q have position vectors $\mathbf{r}_P = (400\mathbf{i} + 200\mathbf{j})$ km and $\mathbf{r}_Q = (500\mathbf{i} - 100\mathbf{j})$ km relative to a fixed origin O. Their velocities are $\mathbf{v}_P = (300\mathbf{i} + 250\mathbf{j})\text{ km h}^{-1}$ and $\mathbf{v}_Q = (600\mathbf{i} - 200\mathbf{j})\text{ km h}^{-1}$.

a Write down expressions for the position vectors of P and Q after a time t hours. **(4)**

b Find the displacement vector of Q relative to P at 10 am. **(2)**

c Work out the distance between P and Q at noon. **(4)**

← Section 8.1

E/P **21** A particle P of mass 2 kg moves in a straight line under the action of a variable force F N. At time t ($t \geqslant 0$), the displacement x m of P from a fixed point O is given by $x = 3t - \dfrac{2k}{2t-1}$, where k is a constant. When $t = 0$, the velocity of P is 10 m s^{-1}.

a Show that $k = \frac{7}{4}$ **(4)**

b Find the distance of P from O when $t = 2$ s. **(2)**

← Sections 8.3, 8.4

E **22** A particle P moves in a plane such that at time t seconds, where $t \geqslant 0$, it has position vector

$$\mathbf{r} = \left(\left(\tfrac{1}{3}t^3 + 2t\right)\mathbf{i} + \left(\tfrac{1}{2}t^2 - 1\right)\mathbf{j}\right)\text{ m}$$

Find:

a the velocity vector of P at time t seconds **(2)**

b the speed of P when $t = 5$ s **(3)**

c the magnitude and direction of the acceleration of P when $t = 2$ s. **(4)**

← Sections 8.3, 8.4

E **23** A particle is acted upon by a variable force F. At time t seconds the displacement of the particle in metres relative to a fixed origin O is given by

$$\mathbf{r} = (4t^2 + 1)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

a Find the velocity of the particle when $t = 3$ s. **(3)**

b Show that the acceleration of the particle is constant. **(2)**

← Sections 8.3, 8.4

E/P **24** A particle P moves so that its velocity $\mathbf{v}$ m s^{-1} at time t seconds, where $t \geqslant 0$, is given by $\mathbf{v} = -2t\mathbf{i} + 3\sqrt{t}\mathbf{j}$. When $t = 0$, the displacement of P relative to a fixed origin is $2\mathbf{j}$ m.

Find the distance of P from O when $t = 4$ s. **(7)**

← Sections 8.3, 8.5

E **25** A particle moves in a plane with acceleration $\mathbf{a}$ m s^{-2} where

$$\mathbf{a} = t(2 - 3t^2)\mathbf{i} - 4(2t + 1)\mathbf{j},\ t \geqslant 0$$

When $t = 0$, the velocity of P is $(3\mathbf{i} + \mathbf{j})$ m s^{-1}. Find:

a the velocity of P after t s **(4)**

b the time at which P is moving in the direction of $\mathbf{i}$. **(2)**

← Sections 8.3, 8.5

E **26** In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively.

A wind surfer is surfing on a lake. The acceleration of the wind surfer at time t s is given by $\mathbf{a} = (-4t\mathbf{i} - 2\mathbf{j})$ m s^{-2}. At time $t = 0$ s the windsurfer is moving directly east at a speed of 8 m s^{-1}.

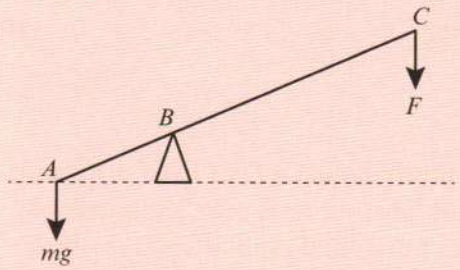
a Find $\mathbf{v}$ in terms of t. **(4)**

b Find the value of t when the windsurfer is moving in a southerly direction. **(3)**

← Sections 8.3, 8.5

Challenge

1



A lever consists of a uniform steel rod AC of weight 100 N and length $2k$ m, which rests on a pivot at B that has a height of $0.3k$ m. $AB = 0.5k$ m. A mass m kg is attached to the lever at A. The mass is lifted by means of a force of magnitude F N that is applied vertically downwards at C. Show that $F > \frac{1}{3}(mg - 100)$.

← Section 4.3, 4.5

2 A particle P travels in a straight line such that its velocity, v m s^{-1} at time t seconds, is given by

$$v = 3\sin kt + \cos kt,\ t \geqslant 0$$

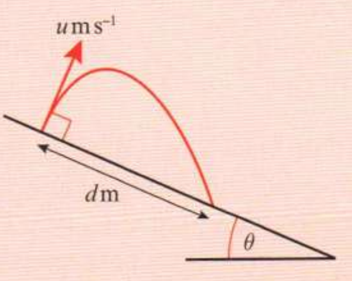
where k is a constant and angles are measured in degrees. At time $t = 0$, the particle is at a fixed origin, O, and has acceleration 1.5 m s^{-2}.

Work out the maximum distance of the particle from the origin in its subsequent motion, and the first time at which this occurs.

← Section 8.1

3 A straight hill slopes upwards at an angle of θ to the horizontal, where $0 \leqslant \theta < 90°$. A projectile is launched perpendicular to the plane of the hill, with an initial velocity of u m s^{-1}, and lands a distance d m down the hill.

Show that $d = \frac{2u^2}{g}\tan\theta\sec\theta$.



← Section 6.4