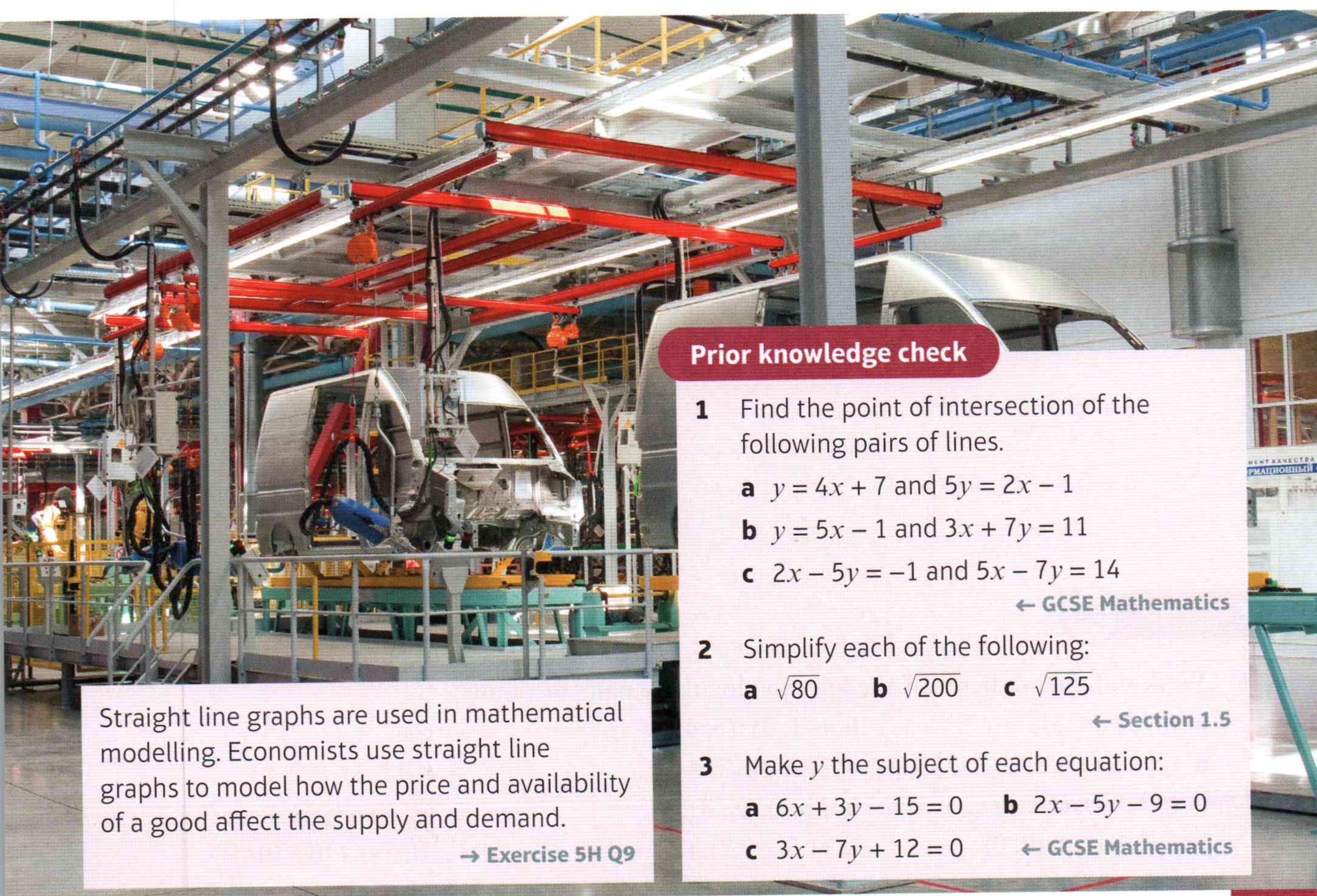
Straight line graphs

5

Objectives

After completing this unit you should be able to:

- Calculate the gradient of a line joining a pair of points → pages 90 – 91
- Understand the link between the equation of a line, and its gradient and intercept → pages 91 – 93
- Find the equation of a line given (i) the gradient and one point on the line or (ii) two points on the line → pages 93 – 95
- Find the point of intersection for a pair of straight lines → pages 95 – 96
- Know and use the rules for parallel and perpendicular gradients → pages 97 – 100
- Solve length and area problems on coordinate grids → pages 100 – 103
- Use straight line graphs to construct mathematical models → pages 103 – 108



Straight line graphs are used in mathematical modelling. Economists use straight line graphs to model how the price and availability of a good affect the supply and demand.

→ Exercise 5H Q9

Prior knowledge check

1 Find the point of intersection of the following pairs of lines.

a $y = 4x + 7$ and $5y = 2x - 1$

b $y = 5x - 1$ and $3x + 7y = 11$

c $2x - 5y = -1$ and $5x - 7y = 14$

← GCSE Mathematics

2 Simplify each of the following:

a $\sqrt{80}$ **b** $\sqrt{200}$ **c** $\sqrt{125}$

← Section 1.5

3 Make y the subject of each equation:

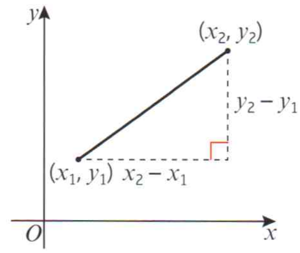
a $6x + 3y - 15 = 0$ **b** $2x - 5y - 9 = 0$

c $3x - 7y + 12 = 0$

← GCSE Mathematics

5.1 $y = mx + c$

You can find the gradient of a straight line joining two points by considering the vertical distance and the horizontal distance between the points.

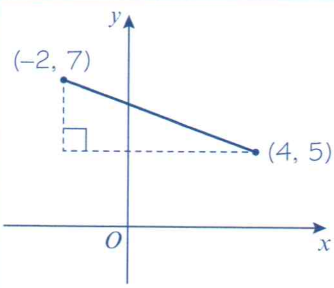


- **The gradient m of a line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$**

Online Explore the gradient formula using GeoGebra.

Example 1

Work out the gradient of the line joining (−2, 7) and (4, 5)



$m = \dfrac{5-7}{4-(-2)} = -\dfrac{2}{6} = -\dfrac{1}{3}$

Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Here $(x_1, y_1) = (-2, 7)$ and $(x_2, y_2) = (4, 5)$

Example 2

The line joining (2, −5) to (4, a) has gradient −1. Work out the value of a.

$\dfrac{a-(-5)}{4-2} = -1$

So $\dfrac{a+5}{2} = -1$

$a + 5 = -2$

$a = -7$

Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Here $m = -1$, $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (4, a)$.

Exercise 5A

1 Work out the gradients of the lines joining these pairs of points:

a (4, 2), (6, 3) **b** (−1, 3), (5, 4) **c** (−4, 5), (1, 2)

d (2, −3), (6, 5) **e** (−3, 4), (7, −6) **f** (−12, 3), (−2, 8)

g (−2, −4), (10, 2) **h** $(\frac{1}{2}, 2), (\frac{3}{4}, 4)$ **i** $(\frac{1}{4}, \frac{1}{2}), (\frac{1}{2}, \frac{2}{3})$

j (−2.4, 9.6), (0, 0) **k** (1.3, −2.2), (8.8, −4.7) **l** (0, 5a), (10a, 0)

m (3b, −2b), (7b, 2b) **n** $(p, p^2), (q, q^2)$

2 The line joining (3, −5) to (6, a) has a gradient 4. Work out the value of a.

3 The line joining (5, b) to (8, 3) has gradient −3. Work out the value of b.

4 The line joining (c, 4) to (7, 6) has gradient $\frac{3}{4}$. Work out the value of c.

5 The line joining (−1, 2d) to (1, 4) has gradient $-\frac{1}{4}$. Work out the value of d.

6 The line joining (−3, −2) to (2e, 5) has gradient 2. Work out the value of e.

7 The line joining (7, 2) to (f, 3f) has gradient 4. Work out the value of f.

8 The line joining (3, −4) to (−g, 2g) has gradient −3. Work out the value of g.

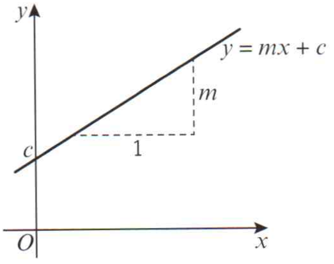
P 9 Show that the points A(2, 3), B(4, 4) and C(10,7) can be joined by a straight line.

Problem-solving Find the gradient of the line joining the points A and B and the line joining the points B and C.

E/P 10 Show that the points $A(-2a, 5a)$, $B(0, 4a)$ and points $C(6a, a)$ are collinear. **(3 marks)**

Notation Points are collinear if they all lie on the same straight line.

- **The equation of a straight line can be written in the form $y = mx + c$, where m is the gradient and c is the y-intercept.**
- **The equation of a straight line can also be written in the form $ax + by + c = 0$, where a, b and c are integers.**



Example 3

Write down the gradient and y-intercept of these lines:

a $y = -3x + 2$ **b** $4x - 3y + 5 = 0$

a Gradient = −3 and y-intercept = (0, 2).

Compare $y = -3x + 2$ with $y = mx + c$. From this, $m = -3$ and $c = 2$.

b $y = \frac{4}{3}x + \frac{5}{3}$

Gradient = $\frac{4}{3}$ and y-intercept = $(0, \frac{5}{3})$.

Rearrange the equation into the form $y = mx + c$. From this $m = \frac{4}{3}$ and $c = \frac{5}{3}$

Watch out Use fractions rather than decimals in coordinate geometry questions.

Example 4

Write these lines in the form $ax + by + c = 0$

a $y = 4x + 3$ **b** $y = -\frac{1}{2}x + 5$

a $4x - y + 3 = 0$

b $\frac{1}{2}x + y - 5 = 0$

$x + 2y - 10 = 0$

Rearrange the equation into the form $ax + by + c = 0$

Collect all the terms on one side of the equation.

Example 5

The line $y = 4x - 8$ meets the x-axis at the point P. Work out the coordinates of P.

$4x - 8 = 0$

$4x = 8$

$x = 2$

So P has coordinates (2, 0)

The line meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = 4x - 8$.

Rearrange the equation for x.

Always write down the coordinates of the point.

Exercise 5B

1 Work out the gradients of these lines:

a $y = -2x + 5$ **b** $y = -x + 7$ **c** $y = 4 + 3x$

d $y = \frac{1}{3}x - 2$ **e** $y = -\frac{2}{3}x$ **f** $y = \frac{5}{4}x + \frac{2}{3}$

g $2x - 4y + 5 = 0$ **h** $10x - 5y + 1 = 0$ **i** $-x + 2y - 4 = 0$

j $-3x + 6y + 7 = 0$ **k** $4x + 2y - 9 = 0$ **l** $9x + 6y + 2 = 0$

2 These lines cut the y-axis at $(0, c)$. Work out the value of c in each case.

a $y = -x + 4$ **b** $y = 2x - 5$ **c** $y = \frac{1}{2}x - \frac{2}{3}$

d $y = -3x$ **e** $y = \frac{6}{7}x + \frac{7}{5}$ **f** $y = 2 - 7x$

g $3x - 4y + 8 = 0$ **h** $4x - 5y - 10 = 0$ **i** $-2x + y - 9 = 0$

j $7x + 4y + 12 = 0$ **k** $7x - 2y + 3 = 0$ **l** $-5x + 4y + 2 = 0$

3 Write these lines in the form $ax + by + c = 0$.

a $y = 4x + 3$ **b** $y = 3x - 2$ **c** $y = -6x + 7$

d $y = \frac{4}{5}x - 6$ **e** $y = \frac{5}{3}x + 2$ **f** $y = \frac{7}{3}x$

g $y = 2x - \frac{4}{7}$ **h** $y = -3x + \frac{2}{9}$ **i** $y = -6x - \frac{2}{3}$

j $y = -\frac{1}{3}x + \frac{1}{2}$ **k** $y = \frac{2}{3}x + \frac{5}{6}$ **l** $y = \frac{3}{5}x + \frac{1}{2}$

4 The line $y = 6x - 18$ meets the x-axis at the point P. Work out the coordinates of P.

5 The line $3x + 2y = 0$ meets the x-axis at the point R. Work out the coordinates of R.

6 The line $5x - 4y + 20 = 0$ meets the y-axis at the point A and the x-axis at the point B. Work out the coordinates of A and B.

7 A line l passes through the points with coordinates (0, 5) and (6, 7).

a Find the gradient of the line.

b Find an equation of the line in the form $ax + by + c = 0$.

E **8** A line l cuts the x-axis at (5, 0) and the y-axis at (0, 2).

a Find the gradient of the line. **(1 mark)**

b Find an equation of the line in the form $ax + by + c = 0$. **(2 marks)**

P **9** Show that the line with equation $ax + by + c = 0$ has gradient $-\frac{a}{b}$ and cuts the y-axis at $-\frac{c}{b}$

Problem-solving

Try solving a similar problem with numbers first:

Find the gradient and y-intercept of the straight line with equation $3x + 7y + 2 = 0$.

E/P **10** The line l with gradient 3 and y-intercept (0, 5) has the equation $ax - 2y + c = 0$. Find the values of a and c. **(2 marks)**

E/P **11** The straight line l passes through (0, 6) and has gradient −2. It intersects the line with equation $5x - 8y - 15 = 0$ at point P. Find the coordinates of P. **(4 marks)**

E/P **12** The straight line l_1 with equation $y = 3x - 7$ intersects the straight line l_2 with equation $ax + 4y - 17 = 0$ at the point $P(-3, b)$.

a Find the value of b. **(1 mark)**

b Find the value of a. **(2 marks)**

Challenge

Show that the equation of a straight line through (0, a) and (b, 0) is $ax + by - ab = 0$.

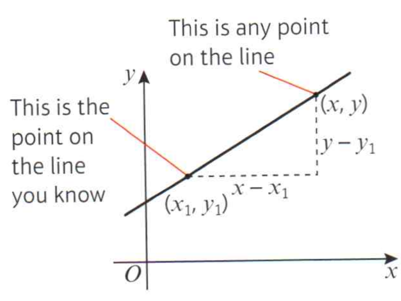
5.2 Equations of straight lines

You can define a straight line by giving:

- one point on the line and the gradient
- two different points on the line

You can find an equation of the line from either of these conditions.

- **The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as $y - y_1 = m(x - x_1)$.**



Example 6

Find the equation of the line with gradient 5 that passes through the point (3, 2).

$y - 2 = 5(x - 3)$

$y - 2 = 5x - 15$

$y = 5x - 13$

Online Explore lines of a given gradient passing through a given point using GeoGebra.

This is in the form $y - y_1 = m(x - x_1)$. Here $m = 5$ and $(x_1, y_1) = (3, 2)$.

Example 7

Find the equation of the line that passes through the points (5, 7) and (3, −1).

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-1)}{5 - 3} = \frac{8}{2} = 4$

So $y - y_1 = m(x - x_1)$

$y + 1 = 4(x - 3)$

$y + 1 = 4x - 12$

$y = 4x - 13$

First find the slope of the line.
Here $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (5, 7)$.
(x_1, y_1) and (x_2, y_2) have been chosen to make the denominator positive.

You know the gradient and a point on the line, so use $y - y_1 = m(x - x_1)$.
Use $m = 4$, $x_1 = 3$ and $y_1 = -1$.

Exercise 5C

1 Find the equation of the line with gradient m that passes through the point (x_1, y_1) when:

a $m = 2$ and $(x_1, y_1) = (2, 5)$
b $m = 3$ and $(x_1, y_1) = (-2, 1)$
c $m = -1$ and $(x_1, y_1) = (3, -6)$
d $m = -4$ and $(x_1, y_1) = (-2, -3)$
e $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 10)$
f $m = -\frac{2}{3}$ and $(x_1, y_1) = (-6, -1)$
g $m = 2$ and $(x_1, y_1) = (a, 2a)$
h $m = -\frac{1}{2}$ and $(x_1, y_1) = (-2b, 3b)$

2 Find the equations of the lines that pass through these pairs of points:

a (2, 4) and (3, 8)
b (0, 2) and (3, 5)
c (−2, 0) and (2, 8)
d (5, −3) and (7, 5)
e (3, −1) and (7, 3)
f (−4, −1) and (6, 4)
g (−1, −5) and (−3, 3)
h (−4, −1) and (−3, −9)
i $(\frac{1}{3}, \frac{2}{5})$ and $(\frac{2}{3}, \frac{4}{5})$
j $(-\frac{3}{4}, \frac{1}{7})$ and $(\frac{1}{4}, \frac{3}{7})$

Hint In each case find the gradient m then use $y - y_1 = m(x - x_1)$.

(E) 3 Find the equation of the line l which passes through the points $A(7, 2)$ and $B(9, -8)$. Give your answer in the form $ax + by + c = 0$. **(3 marks)**

4 The vertices of the triangle ABC have coordinates $A(3, 5)$, $B(-2, 0)$ and $C(4, -1)$. Find the equations of the sides of the triangle.

E/P **5** The straight line l passes through $(a, 4)$ and $(3a, 3)$. An equation of l is $x + 6y + c = 0$. Find the value of a and the value of c. **(3 marks)**

Problem-solving

It is often easier to find unknown values in the order they are given in the question. Find the value of a first then find the value of c.

E/P **6** The straight line l passes through $(7a, 5)$ and $(3a, 3)$. An equation of l is $x + by - 12 = 0$. Find the value of a and the value of b. **(3 marks)**

Challenge

Consider the line passing through points (x_1, y_1) and (x_2, y_2).

a Write down the formula for the gradient, m, of the line.

b Show that the general equation of the line can be written in the form $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

c Use the equation from part **b** to find the equation of the line passing through the points (−8, 4) and (−1, 7).

Example 8

The line $y = 3x - 9$ meets the x-axis at the point A. Find the equation of the line with gradient $\frac{2}{3}$ that passes through point A. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$0 = 3x - 9$ so $x = 3$. A is the point $(3, 0)$.

The line meets the x-axis when $y = 0$, so substitute $y = 0$ into $y = 3x - 9$.

$y - 0 = \frac{2}{3}(x - 3)$

$3y = 2x - 6$

$-2x + 3y + 6 = 0$

Use $y - y_1 = m(x - x_1)$. Here $m = \frac{2}{3}$ and $(x_1, y_1) = (3, 0)$.

Rearrange the equation into the form $ax + by + c = 0$.

Online Plot the solution on a graph using technology.

Example 9

The lines $y = 4x - 7$ and $2x + 3y - 21 = 0$ intersect at the point A. The point B has coordinates (−2, 8). Find the equation of the line that passes through the points A and B. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Online Check solutions to simultaneous equations using your calculator.

$2x + 3(4x - 7) - 21 = 0$

$2x + 12x - 21 - 21 = 0$

$14x = 42$

$x = 3$

$y = 4(3) - 7 = 5$ so A is the point $(3, 5)$.

Solve the equations simultaneously to find point A. Substitute $y = 4x - 7$ into $2x + 3y - 21 = 0$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 5}{-2 - 3} = \frac{3}{-5} = -\frac{3}{5}$

Find the slope of the line connecting A and B.

$y - 5 = -\frac{3}{5}(x - 3)$

$5y - 25 = -3x + 9$

$3x + 5y - 34 = 0$

Use $y - y_1 = m(x - x_1)$ with $m = -\frac{3}{5}$ and $(x_1, y_1) = (3, 5)$.

Exercise 5D

1 The line $y = 4x - 8$ meets the x-axis at the point A. Find the equation of the line with gradient 3 that passes through the point A.

2 The line $y = -2x + 8$ meets the y-axis at the point B. Find the equation of the line with gradient 2 that passes through the point B.

3 The line $y = \frac{1}{2}x + 6$ meets the x-axis at the point C. Find the equation of the line with gradient $\frac{2}{3}$ that passes through the point C. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(P) 4 The line $y = \frac{1}{4}x + 2$ meets the y-axis at the point B. The point C has coordinates $(-5, 3)$. Find the gradient of the line joining the points B and C.

(P) 5 The line that passes through the points $(2, -5)$ and $(-7, 4)$ meets the x-axis at the point P. Work out the coordinates of the point P.

Problem-solving

A sketch can help you check whether your answer looks right.

(P) 6 The line that passes through the points $(-3, -5)$ and $(4, 9)$ meets the y-axis at the point G. Work out the coordinates of the point G.

(P) 7 The line that passes through the points $(3, 2\frac{1}{2})$ and $(-1\frac{1}{2}, 4)$ meets the y-axis at the point J. Work out the coordinates of the point J.

(P) 8 The lines $y = x$ and $y = 2x - 5$ intersect at the point A. Find the equation of the line with gradient $\frac{2}{5}$ that passes through the point A.

(P) 9 The lines $y = 4x - 10$ and $y = x - 1$ intersect at the point T. Find the equation of the line with gradient $-\frac{2}{3}$ that passes through the point T. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(P) 10 The line p has gradient $\frac{2}{3}$ and passes through the point $(6, -12)$. The line q has gradient -1 and passes through the point $(5, 5)$. The line p meets the y-axis at A and the line q meets the x-axis at B. Work out the gradient of the line joining the points A and B.

(P) 11 The line $y = -2x + 6$ meets the x-axis at the point P. The line $y = \frac{3}{2}x - 4$ meets the y-axis at the point Q. Find the equation of the line joining the points P and Q.

(P) 12 The line $y = 3x - 5$ meets the x-axis at the point M. The line $y = -\frac{2}{3}x + \frac{2}{3}$ meets the y-axis at the point N. Find the equation of the line joining the points M and N. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(P) 13 The line $y = 2x - 10$ meets the x-axis at the point A. The line $y = -2x + 4$ meets the y-axis at the point B. Find the equation of the line joining the points A and B.

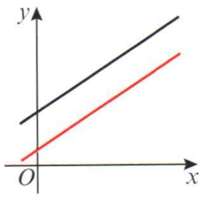
(P) 14 The line $y = 4x + 5$ meets the y-axis at the point C. The line $y = -3x - 15$ meets the x-axis at the point D. Find the equation of the line joining the points C and D. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(P) 15 The lines $y = x - 5$ and $y = 3x - 13$ intersect at the point S. The point T has coordinates $(-4, 2)$. Find the equation of the line that passes through the points S and T.

(P) 16 The lines $y = -2x + 1$ and $y = x + 7$ intersect at the point L. The point M has coordinates $(-3, 1)$. Find the equation of the line that passes through the points L and M.

5.3 Parallel and perpendicular lines

- **Parallel lines have the same gradient.**



Example 10

A line is parallel to the line $6x + 3y - 2 = 0$ and it passes through the point (0, 3).
Work out the equation of the line.

$6x + 3y - 2 = 0$
$3y - 2 = -6x$
$3y = -6x + 2$
$y = -2x + \frac{2}{3}$

Rearrange the equation into the form $y = mx + c$ to find m.

The gradient of this line is −2.

Compare $y = -2x + \frac{2}{3}$ with $y = mx + c$, so $m = -2$.
Parallel lines have the same gradient, so the gradient of the required line = −2.

The equation of the line is $y = -2x + 3$.

(0, 3) is the intercept on the y-axis, so $c = 3$.

Exercise 5E

1 Work out whether each pair of lines is parallel.

a $y = 5x - 2$
$15x - 3y + 9 = 0$

b $7x + 14y - 1 = 0$
$y = \frac{1}{2}x + 9$

c $4x - 3y - 8 = 0$
$3x - 4y - 8 = 0$

(P) **2** The line r passes through the points (1, 4) and (6, 8) and the line s passes through the points (5, −3) and (20, 9). Show that the lines r and s are parallel.

(P) **3** The coordinates of a quadrilateral $ABCD$ are $A(-6, 2)$, $B(4, 8)$, $C(6, 1)$ and $D(-9, -8)$. Show that the quadrilateral is a trapezium.

Hint A trapezium has exactly one pair of parallel sides.

4 A line is parallel to the line $y = 5x + 8$ and its y-intercept is (0, 3). Write down the equation of the line.

Hint The line will have gradient 5.

5 A line is parallel to the line $y = -\frac{2}{5}x + 1$ and its y-intercept is (0, −4). Work out the equation of the line. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

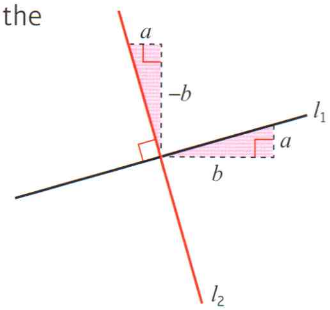
(P) **6** A line is parallel to the line $3x + 6y + 11 = 0$ and its intercept on the y-axis is (0, 7). Write down the equation of the line.

(P) **7** A line is parallel to the line $2x - 3y - 1 = 0$ and it passes through the point (0, 0). Write down the equation of the line.

8 Find an equation of the line that passes through the point (−2, 7) and is parallel to the line $y = 4x + 1$. Write your answer in the form $ax + by + c = 0$.

Perpendicular lines are at right angles to each other. If you know the gradient of one line, you can find the gradient of the other.

- **If a line has a gradient of m, a line perpendicular to it has a gradient of $-\frac{1}{m}$**
- **If two lines are perpendicular, the product of their gradients is −1.**



The shaded triangles are congruent.

Line l_1 has gradient $\frac{a}{b} = m$

Line l_2 has gradient $\frac{-b}{a} = -\frac{1}{m}$

Example 11

Work out whether these pairs of lines are parallel, perpendicular or neither:

a $3x - y - 2 = 0$
$x + 3y - 6 = 0$

b $y = \frac{1}{2}x$
$2x - y + 4 = 0$

a $3x - y - 2 = 0$
$3x - 2 = y$
So $y = 3x - 2$
The gradient of this line is 3.

Rearrange the equations into the form $y = mx + c$.

$x + 3y - 6 = 0$
$3y - 6 = -x$
$3y = -x + 6$
$y = -\frac{1}{3}x + 2$
The gradient of this line is $-\frac{1}{3}$

Compare $y = -\frac{1}{3}x + 2$ with $y = mx + c$, so $m = -\frac{1}{3}$

So the lines are perpendicular as $3 \times (-\frac{1}{3}) = -1$.

b $y = \frac{1}{2}x$
The gradient of this line is $\frac{1}{2}$

Compare $y = \frac{1}{2}x$ with $y = mx + c$, so $m = \frac{1}{2}$.

$2x - y + 4 = 0$
$2x + 4 = y$
So $y = 2x + 4$
The gradient of this line is 2.

Rearrange the equation into the form $y = mx + c$ to find m.

Compare $y = 2x + 4$ with $y = mx + c$, so $m = 2$.

The lines are not parallel as they have different gradients.
The lines are not perpendicular as $\frac{1}{2} \times 2 \neq -1$.

Online Explore this solution using technology.

Example 12

A line is perpendicular to the line $2y - x - 8 = 0$ and passes through the point (5, −7). Find the equation of the line.

Rearranging, $y = \frac{1}{2}x + 4$

Gradient of $y = \frac{1}{2}x + 4$ is $\frac{1}{2}$

So the gradient of the perpendicular line is −2.

$y - y_1 = m(x - x_1)$

$y + 7 = -2(x - 5)$

$y + 7 = -2x + 10$

$y = -2x + 3$

Problem-solving

You need to fill in the steps of this problem yourself:

- Rearrange the equation into the form $y = mx + c$ to find the gradient.
- Use $-\frac{1}{m}$ to find the gradient of a perpendicular line.
- Use $y - y_1 = m(x - x_1)$ to find the equation of the line.

Exercise 5F

1 Work out whether these pairs of lines are parallel, perpendicular or neither:

a $y = 4x + 2$
$y = -\frac{1}{4}x - 7$

b $y = \frac{2}{3}x - 1$
$y = \frac{2}{3}x - 11$

c $y = \frac{1}{5}x + 9$
$y = 5x + 9$

d $y = -3x + 2$
$y = \frac{1}{3}x - 7$

e $y = \frac{3}{5}x + 4$
$y = -\frac{5}{3}x - 1$

f $y = \frac{5}{7}x$
$y = \frac{5}{7}x - 3$

g $y = 5x - 3$
$5x - y + 4 = 0$

h $5x - y - 1 = 0$
$y = -\frac{1}{5}x$

i $y = -\frac{3}{2}x + 8$
$2x - 3y - 9 = 0$

j $4x - 5y + 1 = 0$
$8x - 10y - 2 = 0$

k $3x + 2y - 12 = 0$
$2x + 3y - 6 = 0$

l $5x - y + 2 = 0$
$2x + 10y - 4 = 0$

2 A line is perpendicular to the line $y = 6x - 9$ and passes through the point (0, 1). Find an equation of the line.

(P) **3** A line is perpendicular to the line $3x + 8y - 11 = 0$ and passes through the point (0, −8). Find an equation of the line.

4 Find an equation of the line that passes through the point (6, −2) and is perpendicular to the line $y = 3x + 5$.

5 Find an equation of the line that passes through the point (−2, 5) and is perpendicular to the line $y = 3x + 6$.

(P) **6** Find an equation of the line that passes through the point (3, 4) and is perpendicular to the line $4x - 6y + 7 = 0$.

7 Find an equation of the line that passes through the point (5, −5) and is perpendicular to the line $y = \frac{2}{3}x + 5$. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

8 Find an equation of the line that passes through the point (−2, −3) and is perpendicular to the line $y = -\frac{4}{7}x + 5$. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

P **9** The line l passes through the points (−3, 0) and (3, −2) and the line n passes through the points (1, 8) and (−1, 2). Show that the lines l and n are perpendicular.

Problem-solving

Don't do more work than you need to. You only need to find the gradients of both lines, not their equations.

P **10** The vertices of a quadrilateral $ABCD$ have coordinates $A(-1, 5)$, $B(7, 1)$, $C(5, -3)$ and $D(-3, 1)$. Show that the quadrilateral is a rectangle.

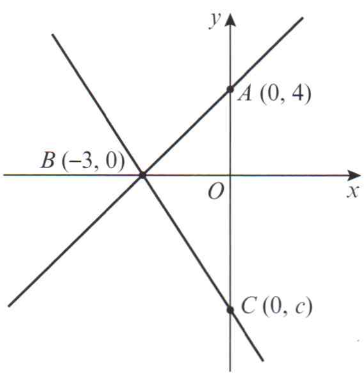
Hint The sides of a rectangle are perpendicular.

E/P **11** A line l_1 has equation $5x + 11y - 7 = 0$ and crosses the x-axis at A. The line l_2 is perpendicular to l_1 and passes through A.

a Find the coordinates of the point A. **(1 mark)**

b Find the equation of the line l_2. Write your answer in the form $ax + by + c = 0$. **(3 marks)**

E/P **12** The points A and C lie on the y-axis and the point B lies on the x-axis as shown in the diagram.



Problem-solving

Sketch graphs in coordinate geometry problems are not accurate, but you can use the graph to make sure that your answer makes sense. In this question c must be negative.

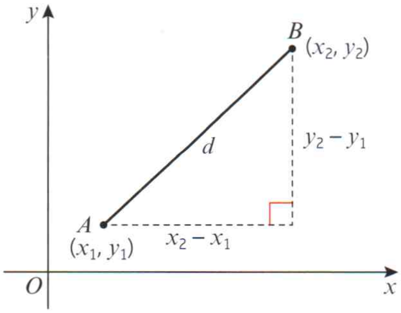
The line through points A and B is perpendicular to the line through points B and C. Find the value of c. **(6 marks)**

5.4 Length and area

You can find the distance between two points A and B by considering a right-angled triangle with hypotenuse AB.

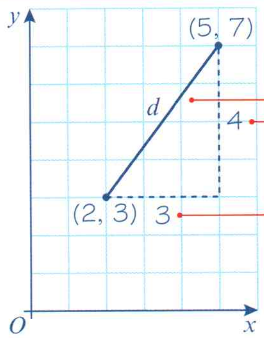
- **You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula**

  $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 13

Find the distance between (2, 3) and (5, 7).



Draw a sketch.

Let the distance between the points be d.

The difference in the y-coordinates is $7 - 3 = 4$.

The difference in the x-coordinates is $5 - 2 = 3$.

$d^2 = (5 - 2)^2 + (7 - 3)^2$

$d^2 = 3^2 + 4^2$

$d = \sqrt{(3^2 + 4^2)}$

$= \sqrt{25}$

$= 5$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (5, 7)$.

Example 14

Online Draw both lines and the triangle AOB on a graph using technology.

The straight line l_1 with equation $4x - y = 0$ and the straight line l_2 with equation $2x + 3y - 21 = 0$ intersect at point A.

a Work out the coordinates of A.

b Work out the area of triangle AOB where B is the point where l_2 meets the x-axis.

a Equation of l_1 is $y = 4x$ — Rewrite the equation of l_1 in the form $y = mx + c$.

$2x + 3y - 21 = 0$

$2x + 3(4x) - 21 = 0$ — Substitute $y = 4x$ into the equation for l_2 to find the point of intersection.

$14x - 21 = 0$

$14x = 21$

$x = \frac{3}{2}$ — Solve the equation to find the x-coordinate of point A.

$y = 4 \times \left(\frac{3}{2}\right) = 6$ — Substitute to find the y-coordinate of point A.

So point A has coordinates $\left(\frac{3}{2}, 6\right)$.

b The triangle AOB has a height of 6 units. — The height is the y-coordinate of point A.

$2x + 3y - 21 = 0$

$2x + 3(0) - 21 = 0$ — B is the point where the line l_2 intersects the x-axis. At B, the y-coordinate is zero.

$2x - 21 = 0$

$x = \frac{21}{2}$ — Solve the equation to find the x-coordinate of point B.

The triangle AOB has a base length of $\frac{21}{2}$ units.

Area $= \frac{1}{2} \times 6 \times \frac{21}{2} = \frac{63}{2}$ — Area $= \frac{1}{2} \times$ base $\times$ height. You don't need to give units for length and area problems on coordinate grids.

Exercise 5G

1 Find the distance between these pairs of points:

a $(0, 1), (6, 9)$ b $(4, -6), (9, 6)$ c $(3, 1), (-1, 4)$

d $(3, 5), (4, 7)$ e $(0, -4), (5, 5)$ f $(-2, -7), (5, 1)$

2 Consider the points $A(-3, 5)$, $B(-2, -2)$ and $C(3, -7)$. Determine whether the line joining the points A and B is congruent to the line joining the points B and C.

Hint Two line segments are congruent if they are the same length.

3 Consider the points $P(11, -8)$, $Q(4, -3)$ and $R(7, 5)$. Show that the line segment joining the points P and Q is not congruent to the line joining the points Q and R.

(P) 4 The distance between the points $(-1, 13)$ and $(x, 9)$ is $\sqrt{65}$. Find two possible values of x.

Problem-solving Use the distance formula to formulate a quadratic equation in x.

(P) 5 The distance between the points $(2, y)$ and $(5, 7)$ is $3\sqrt{10}$. Find two possible values of y.

(P) 6 a Show that the straight line l_1 with equation $y = 2x + 4$ is parallel to the straight line l_2 with equation $6x - 3y - 9 = 0$.

b Find the equation of the straight line l_3 that is perpendicular to l_1 and passes through the point $(3, 10)$.

c Find the point of intersection of the lines l_2 and l_3.

d Find the shortest distance between lines l_1 and l_2.

Problem-solving The shortest distance between two parallel lines is the perpendicular distance between them.

(E/P) 7 A point P lies on the line with equation $y = 4 - 3x$. The point P is a distance $\sqrt{34}$ from the origin. Find the two possible positions of point P. **(5 marks)**

(P) 8 The vertices of a triangle are $A(2, 7)$, $B(5, -6)$ and $C(8, -6)$.

a Show that the triangle is a scalene triangle.

b Find the area of the triangle ABC.

Notation Scalene triangles have three sides of different lengths.

Problem-solving Draw a sketch and label the points A, B and C. Find the length of the base and the height of the triangle.

9 The straight line l_1 has equation $y = 7x - 3$. The straight line l_2 has equation $4x + 3y - 41 = 0$. The lines intersect at the point A.

a Work out the coordinates of A.

The straight line l_2 crosses the x-axis at the point B.

b Work out the coordinates of B.

c Work out the area of triangle AOB.

10 The straight line l_1 has equation $4x - 5y - 10 = 0$ and intersects the x-axis at point A. The straight line l_2 has equation $4x - 2y + 20 = 0$ and intersects the x-axis at the point B.

a Work out the coordinates of A.

b Work out the coordinates of B.

The straight lines l_1 and l_2 intersect at the point C.

c Work out the coordinates of C.

d Work out the area of triangle ABC.

E **11** The points $R(5, -2)$ and $S(9, 0)$ lie on the straight line l_1 as shown.

a Work out an equation for straight line l_1. **(2 marks)**

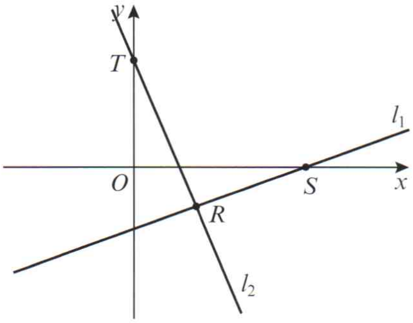
The straight line l_2 is perpendicular to l_1 and passes through the point R.

b Work out an equation for straight line l_2. **(2 marks)**

c Write down the coordinates of T. **(1 mark)**

d Work out the lengths of RS and TR leaving your answer in the form $k\sqrt{5}$. **(2 marks)**

e Work out the area of $\triangle RST$. **(2 marks)**



E/P **12** The straight line l_1 passes through the point $(-4, 14)$ and has gradient $-\frac{1}{4}$

a Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

b Write down the coordinates of A, the point where straight line l_1 crosses the y-axis. **(1 mark)**

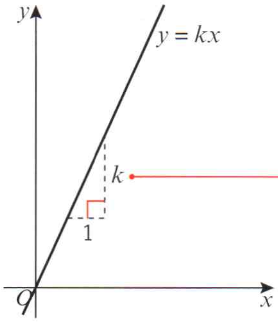
The straight line l_2 passes through the origin and has gradient 3. The lines l_1 and l_2 intersect at the point B.

c Calculate the coordinates of B. **(2 marks)**

d Calculate the exact area of $\triangle OAB$. **(2 marks)**

5.5 Modelling with straight lines

- **Two quantities are in direct proportion when they increase at the same rate. The graph of these quantities is a straight line through the origin.**

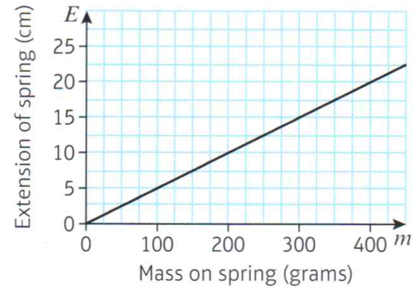


If x increases by 1 unit, y increases by k units

Notation These mean the same thing:
y is proportional to x
$y \propto x$
$y = kx$ for some real constant k.

Example 15

The graph shows the extension, E, of a spring when different masses, m, are attached to the end of the spring.



a Calculate the gradient, k, of the line.

b Write an equation linking E and m.

c Explain what the value of k represents in this situation.

a slope $= \frac{20-0}{400-0}$

$= \frac{20}{400} = \frac{1}{20}$

So $k = \frac{1}{20}$

Use any two points on the line to calculate the gradient. Here (0, 0) and (400, 20) are used.

Simplify the answer.

b $E = km$

$E = \frac{1}{20}m$

'$y = kx$' is the general form of a direct proportion equation. Here the variables are E and m.

c k represents the increase in extension in cms when the mass increases by 1 gram.

k is the gradient. When the m-value increases by 1, the E-value increases by k.

You can sometimes use a **linear model** to show the relationship between two variables, x and y. The graph of a linear model is a straight line, and the variables are related by an equation of the form $y = ax + b$.

A linear model can still be appropriate even if all the points do not lie directly on the line. In this case, the points should be **close** to the line. The further the points are from the line, the less appropriate a linear model is for the data.

- **A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problem in order to create a model.**

Example 16

A container was filled with water. A hole was made in the bottom of the container. The depth of water remaining was recorded at certain time intervals. The table shows the results.

Time, t seconds	0	10	30	60	100	120
Depth of water, d cm	19.1	17.8	15.2	11.3	6.1	3.5

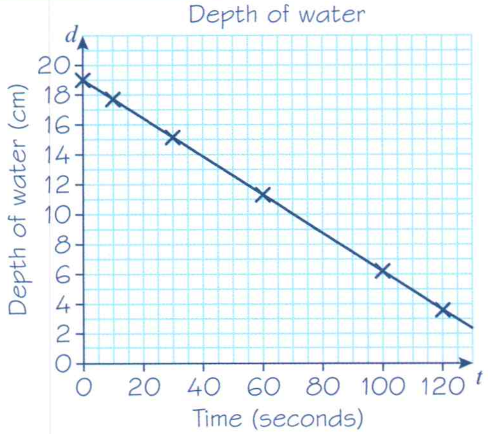
a Determine whether a linear model is appropriate by drawing a graph.

b Deduce an equation in the form $d = at + b$.

c Interpret the meaning of the coefficients a and b.

d Use the model to find the time when the container will be empty.

a



The points form a straight line, therefore a linear model is appropriate.

b $m = \frac{6.1 - 19.1}{100 - 0}$

$= -\frac{13}{100} = -0.13$

The d-intercept is 19.1. So $b = 19.1$

$d = at + b$

$d = -0.13t + 19.1$

c a is the change in depth of water in the container every second.

b is the depth of water in the container at the beginning of the experiment.

d $d = -0.13t + 19.1$

$0 = -0.13t + 19.1$

$0.13t = 19.1$

$t = 146.9$ seconds.

Problem-solving

You need to give your answer in the context of the question. Make sure you refer to the extension in the spring and the mass.

Pick any two points from the table. Here (0, 19.1) and (100, 6.1) are used.

The d-intercept is the d-value when $t = 0$.

State the linear equation using the variables in the question.

Substitute $a = -0.13$ and $b = 19.1$.

a represents the rate of change. Look at the problem and determine what is changing every second.

b is the value of d when $t = 0$. It represents the starting, or initial, value in the model.

State the linear equation using the variables in the question.

Substitute $d = 0$, as we want to know the time when the depth of water is zero.

Solve the equation to find t.

Example 17

In 1991 there were 18 500 people living in Bradley Stoke. Planners projected that the number of people living in Bradley Stoke would increase by 350 each year.

a Write a linear model for the population p of Bradley Stoke t years after 1991.

b Write down one reason why this might not be a realistic model.

a 1991 is the first year, so $t = 0$.

When $t = 0$, the population is 18 500.

18 500 is the p-intercept.

The population is expected to increase by 350 each year.

350 represents the gradient of the line.

$p = at + b$

$p = 350t + 18\,500$

b The number of people living in Bradley Stoke would probably not increase by exactly the same amount each year.

> The p-intercept is the population when $t = 0$.

> The gradient is the yearly change in population.

> State the linear equation using the variables in the question.

> Substitute $a = 350$ and $b = 18\,500$.

> **Problem-solving**
> Look at the question carefully. Which points did you accept without knowing them to be true? These are your **assumptions**.

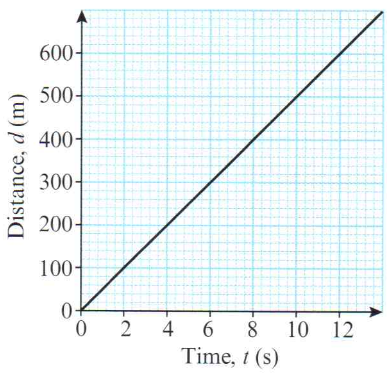
Exercise 5H

1 For each graph

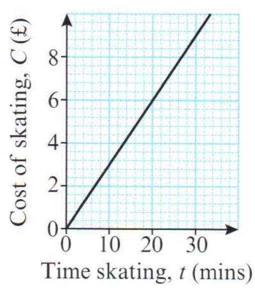
i calculate the gradient, k, of the line

ii write a direct proportion equation connecting the two variables.

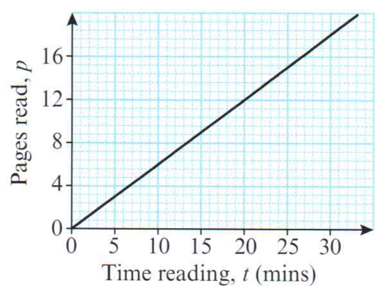
a



b



c



2 Draw a graph to determine whether a linear model would be appropriate for each set of data.

a

v	p
0	0
15	2
25	6
40	12
60	25
80	50

b

x	y
0	70
5	82.5
10	95
15	107.5
25	132.5
40	170

c

w	l
3.1	45
3.4	47
3.6	50
3.9	51
4.5	51
4.7	53

> **Hint** A linear model can be appropriate even if all the points do not lie exactly in a straight line. In these cases, the points should lie close to a straight line.

E/P **3** The cost of electricity, E, in pounds and the number of kilowatt hours, h, are shown in the table.

kilowatt hours, h	0	15	40	60	80	110
cost of electricity, E	45	46.8	49.8	52.2	54.6	58.2

a Draw a graph of the data. **(3 marks)**

b Explain how you know a linear model would be appropriate. **(1 mark)**

c Deduce an equation in the form $E = ah + b$. **(2 marks)**

d Interpret the meaning of the coefficients a and b. **(2 marks)**

e Use the model to find the cost of 65 kilowatt hours. **(1 mark)**

(P) **4** A racing car accelerates from rest to 90 m/s in 10 seconds. The table shows the total distance travelled by the racing car in each of the first 10 seconds.

time, *t* seconds	0	1	2	3	4	5	6	7	8	9	10
distance, *d* m	0	4.5	18	40.5	72	112.5	162	220.5	288	364.5	450

a Draw a graph of the data.

b Explain how you know a linear model would not be appropriate.

(E/P) **5** A website designer charges a flat fee and then a daily rate in order to design new websites for companies.

Company A's new website takes 6 days and they are charged £7100.
Company B's new website take 13 days and they are charged £9550.

Hint Let $(d_1, C_1) = (6, 7100)$ and $(d_2, C_2) = (13, 9550)$.

a Write an equation linking days, d and website cost, C in the form $C = ad + b$. **(3 marks)**

b Interpret the values of a and b. **(2 marks)**

c The web designer charges a third company £13 400. Calculate the number of days the designer spent working on the website. **(1 mark)**

(E/P) **6** The average August temperature in Exeter is 20 °C or 68 °F. The average January temperature in the same place is 9 °C or 48.2 °F.

a Write an equation linking Fahrenheit F and Celsius C in the form $F = aC + b$. **(3 marks)**

b Interpret the values of a and b. **(2 marks)**

c The highest temperature recorded in the UK was 101.3 °F. Calculate this temperature in Celsius. **(1 mark)**

d For what value is the temperature in Fahrenheit the same as the temperature in Celsius? **(3 marks)**

(P) **7** In 2004, in a city, there were 17 500 homes with internet connections. A service provider predicts that each year an additional 750 homes will get internet connections.

a Write a linear model for the number of homes n with internet connections t years after 2004.

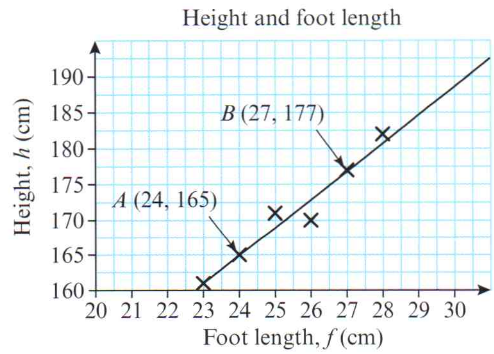
b Write down one assumption made by this model.

(E/P) **8** The scatter graph shows the height h and foot length f of 8 students. A line of best fit is drawn on the scatter graph.

a Explain why the data can be approximated to a linear model. **(1 mark)**

b Use points A and B on the scatter graph to write a linear equation in the form $h = af + b$. **(3 mark)**

c Calculate the expected height of a person with a foot length of 26.5 cm. **(1 mark)**



9 The price P of a good and the quantity Q of a good are linked.
The demand for a new pair of trainers can be modelled using the equation $P = -\frac{3}{4}Q + 35$.
The supply of the trainers can be modelled using the equation $P = \frac{2}{3}Q + 1$.

a Draw a sketch showing the demand and supply lines on the same pair of axes.

The equilibrium point is the point where the supply and demand lines meet.

b Find the values of P and Q at the equilibrium point.

Mixed exercise 5

E/P **1** The straight line passing through the point $P(2, 1)$ and the point $Q(k, 11)$ has gradient $-\frac{5}{12}$

a Find the equation of the line in terms of x and y only. **(2 marks)**

b Determine the value of k. **(2 marks)**

E/P **2** The points A and B have coordinates $(k, 1)$ and $(8, 2k - 1)$ respectively, where k is a constant.
Given that the gradient of AB is $\frac{1}{3}$

a show that $k = 2$ **(2 marks)**

b find an equation for the line through A and B. **(3 marks)**

E **3** The line L_1 has gradient $\frac{1}{7}$ and passes through the point $A(2, 2)$. The line L_2 has gradient -1 and passes through the point $B(4, 8)$. The lines L_1 and L_2 intersect at the point C.

a Find an equation for L_1 and an equation for L_2. **(4 marks)**

b Determine the coordinates of C. **(2 marks)**

E **4** **a** Find an equation of the line l which passes through the points $A(1, 0)$ and $B(5, 6)$. **(2 marks)**

The line m with equation $2x + 3y = 15$ meets l at the point C.

b Determine the coordinates of C. **(2 marks)**

E **5** The line L passes through the points $A(1, 3)$ and $B(-19, -19)$.
Find an equation of L in the form $ax + by + c = 0$. where a, b and c are integers. **(3 marks)**

E **6** The straight line l_1 passes through the points A and B with coordinats $(2, 2)$ and $(6, 0)$ respectively.

a Find an equation of l_1. **(3 marks)**

The straight line l_2 passes through the point C with coordinate $(-9, 0)$ and has gradient $\frac{1}{4}$.

b Find an equation of l_2. **(2 marks)**

E/P **7** The straight line l passes through $A(1, 3\sqrt{3})$ and $B(2 + \sqrt{3}, 3 + 4\sqrt{3})$.
Show that l meets the x-axis at the point $C(-2, 0)$. **(5 marks)**

E **8** The points A and B have coordinates $(-4, 6)$ and $(2, 8)$ respectively. A line p is drawn through B perpendicular to AB to meet the y-axis at the point C.

a Find an equation of the line p. **(3 marks)**

b Determine the coordinates of C. **(1 mark)**

E/P **9** The line l has equation $2x - y - 1 = 0$.

The line m passes through the point $A(0, 4)$ and is perpendicular to the line l.

a Find an equation of m. **(2 marks)**

b Show that the lines l and m intersect at the point $P(2, 3)$. **(2 marks)**

The line n passes through the point $B(3, 0)$ and is parallel to the line m.

c Find the coordinates of the point of intersection of the lines l and n. **(3 marks)**

E/P **10** The line l_1 passes through the points A and B with coordinates $(0, -2)$ and $(6, 7)$ respectively.

The line l_2 has equation $x + y = 8$ and cuts the y-axis at the point C.

The line l_1 and l_2 intersect at D.

Find the area of triangle ACD. **(6 marks)**

E **11** The points A and B have coordinates $(2, 16)$ and $(12, -4)$ respectively.

A straight line l_1 passes through A and B.

a Find an equation for l_1 in the form $ax + by = c$. **(2 marks)**

The line l_2 passes through the point C with coordinates $(-1, 1)$ and has gradient $\frac{1}{3}$

b Find an equation for l_2. **(2 marks)**

E/P **12** The points $A(-1, -2)$, $B(7, 2)$ and $C(k, 4)$, where k is a constant, are the vertices of $\triangle ABC$.

Angle ABC is a right angle.

a Find the gradient of AB. **(1 mark)**

b Calculate the value of k. **(2 marks)**

c Find an equation of the straight line passing through B and C. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers **(2 marks)**

d Calculate the area of $\triangle ABC$. **(2 marks)**

E/P **13** **a** Find an equation of the straight line passing through the points with coordinates $(-1, 5)$ and $(4, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

The line crosses the x-axis at the point A and the y-axis at the point B, and O is the origin.

b Find the area of $\triangle AOB$. **(3 marks)**

E **14** The straight line l_1 has equation $4y + x = 0$.

The straight line l_2 has equation $y = 2x - 3$.

a On the same axes, sketch the graphs of l_1 and l_2. Show clearly the coordinates of all points at which the graphs meet the coordinate axes. **(2 marks)**

The lines l_1 and l_2 intersect at the point A.

b Calculate, as exact fractions, the coordinates of A. **(2 marks)**

c Find an equation of the line through A which is perpendicular to l_1. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(2 marks)**

(E) **15** The points A and B have coordinates (4, 6) and (12, 2) respectively.
The straight line l_1 passes through A and B.

a Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

The straight line l_2 passes through the origin and has gradient $-\frac{2}{3}$

b Write down an equation for l_2. **(1 mark)**

The lines l_1 and l_2 intersect at the point C.

c Find the coordinates of C. **(2 marks)**

d Show that the lines OA and OC are perpendicular, where O is the origin. **(2 marks)**

e Work out the lengths of OA and OC. Write your answers in the form $k\sqrt{13}$. **(2 marks)**

f Hence calculate the area of $\triangle OAC$. **(2 marks)**

16 a Use the distance formula to find the distance between $(4a, a)$ and $(-3a, 2a)$.

Hence find the distance between the following pairs of points:

b (4, 1) and (−3, 2) **c** (12, 3) and (−9, 6) **d** (−20, −5) and (15, −10)

(E/P) **17** A is the point (−1, 5). Let (x, y) be any point on the line $y = 3x$.

a Write an equation in terms of x for the distance between (x, y) and $A(-1, 5)$. **(3 marks)**

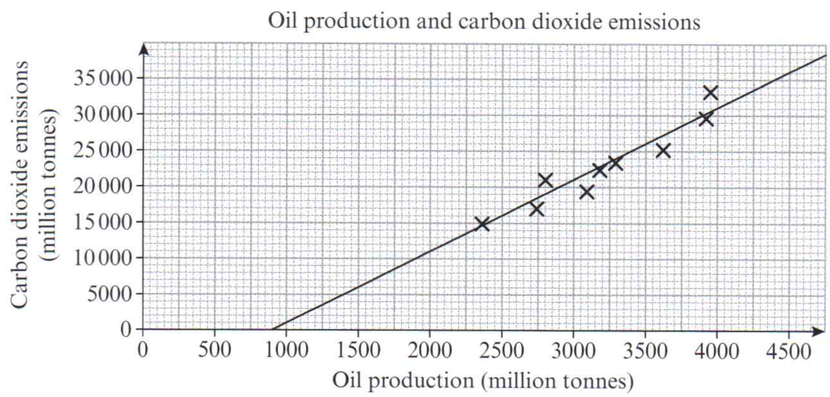
b Find the coordinates of the two points, B and C, on the line $y = 3x$ which are a distance of $\sqrt{74}$ from (−1, 5). **(3 marks)**

c Find the equation of the line l_1 that is perpendicular to $y = 3x$ and goes through the point (−1, 5). **(2 marks)**

d Find the coordinates of the point of intersection between l_1 and $y = 3x$. **(2 marks)**

e Find the area of triangle ABC. **(2 marks)**

(E/P) **18** The scatter graph shows the oil production P and carbon dioxide emissions C for various years since 1970. A line of best fit has been added to the scatter graph.



a Use two points on the line to calculate its gradient. **(1 mark)**

b Formulate a linear model linking oil production P and carbon dioxide emissions C, giving the relationship in the form $C = aP + b$. **(2 marks)**

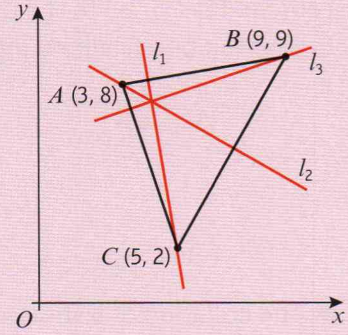
c Interpret the value of a in your model. **(1 mark)**

d With reference to your value of b, comment on the validity of the model for small values of P. **(1 mark)**

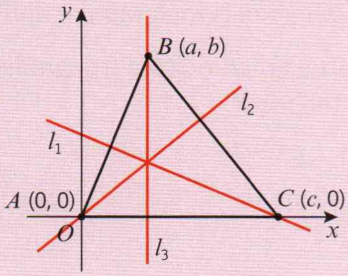
Challenge

1 Find the area of the triangle with vertices $A(-2, -2)$, $B(13, 8)$ and $C(-4, 14)$.

2 A triangle has vertices $A(3, 8)$, $B(9, 9)$ and $C(5, 2)$ as shown in the diagram.
The line l_1 is perpendicular to AB and passes through C.
The line l_2 is perpendicular to BC and passes through A.
The line l_3 is perpendicular to AC and passes through B.
Show that the lines l_1, l_2 and l_3 meet at a point and find the coordinates of that point.



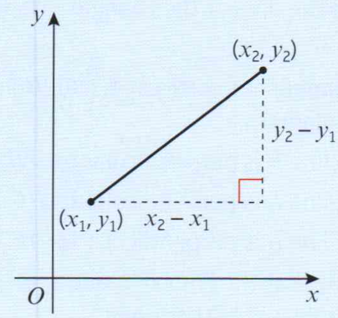
3 A triangle has vertices $A(0, 0)$, $B(a, b)$ and $C(c, 0)$ as shown in the diagram.
The line l_1 is perpendicular to AB and passes through C.
The line l_2 is perpendicular to BC and passes through A.
The line l_3 is perpendicular to AC and passes through B.
Find the coordinates of the point of intersection of l_1, l_2 and l_3.



Summary of key points

1 The gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



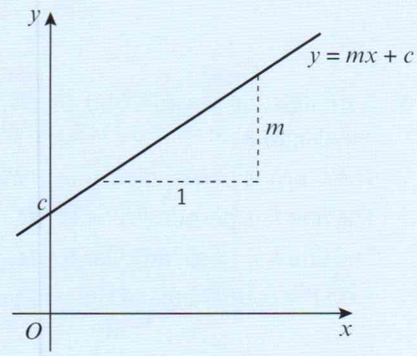
2
- The equation of a straight line can be written in the form

  $y = mx + c,$

  where m is the gradient and $(0, c)$ is the y-intercept.
- The equation of a straight line can also be written in the form

  $ax + by + c = 0,$

  where a, b and c are integers.



3 The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as $y - y_1 = m(x - x_1)$.

4 Parallel lines have the same gradient.

5 If a line has a gradient m, a line perpendicular to it has a gradient of $-\frac{1}{m}$

6 If two lines are perpendicular, the product of their gradients is −1.

7 You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

8 The point of intersection of two lines can be found using simultaneous equations.

9 Two quantities are in direct proportion when they increase at the same rate. The graph of these quantities is a straight line through the origin.

10 A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problems in order to create a model.