5 Forces and friction

Objectives

After completing this chapter you should be able to:

- Resolve forces into components → pages 91–96
- Use the triangle law to find a resultant force → pages 93–96
- Solve problems involving smooth or rough inclined planes → pages 96–99
- Understand friction and the coefficient of friction. → pages 100–103
- Use $F \leqslant \mu R$ → pages 100–103

Prior knowledge check

1 A particle of mass 5 kg is acted on by two forces:

$\mathbf{F}_1 = (8\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (-3\mathbf{i} + 8\mathbf{j})$ N.

Find the acceleration of the particle in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-2}.

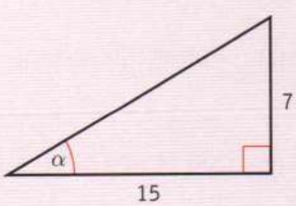
← Year 1, Chapter 10

2 In the diagram below, calculate

a the length of the hypotenuse

b the size of α.

Give your answers correct to 2 d.p.



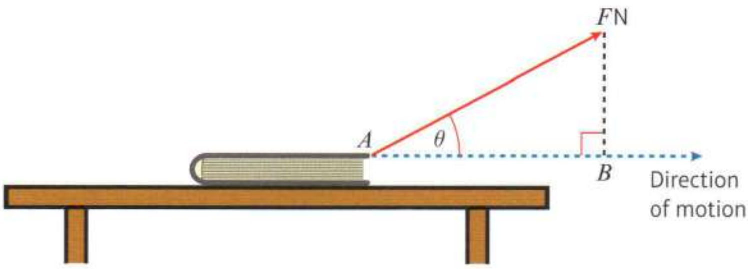
← GCSE Mathematics

A car's braking distance is determined by its speed and the frictional force between the car's wheels and the road. In wet or icy conditions, friction is reduced so the braking distance is increased. → Mixed exercise Q9

5.1 Resolving forces

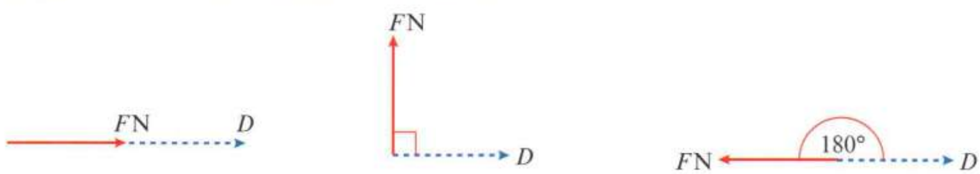
- **If a force is applied at an angle to the direction of motion you can resolve it to find the component of the force that acts in the direction of motion.**

This book is being dragged along the table by means of a force of magnitude F. The book is moving horizontally, and the angle between the force and the direction of motion is θ.



The effect of the force in the direction of motion is the length of the line AB. This is called the **component of the force in the direction of motion**. Using the rule for a right-angled triangle $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, you can see that the length of AB is $F \times \cos\theta$. Finding this value is called **resolving** the force in the direction of motion.

- **The component of a force of magnitude F in a certain direction is $F\cos\theta$, where θ is the size of the angle between the force and the direction.**



If F acts in the direction D, then the component of F in that direction is $F\cos 0° = F \times 1 = F$

If F acts at the right angles to D, then the component of F in that direction is $F\cos 90° = F \times 0 = 0$

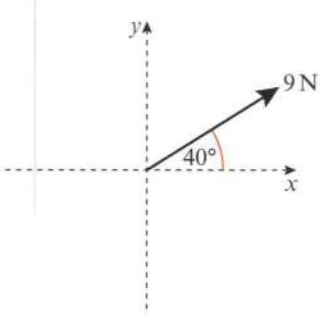
If F acts in the opposite direction to D, then the component of F in that direction is $F\cos 180° = F \times -1 = -F$

Example 1

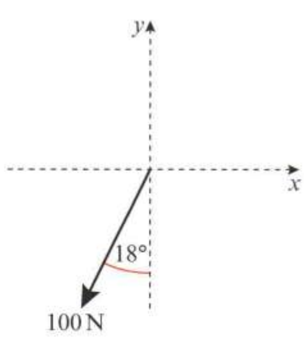
Find the component of each force in **i** the x-direction **ii** the y-direction

iii Hence write each force in the form $p\mathbf{i} + q\mathbf{j}$ where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the x and y directions respectively.

a



b



a i $\theta = 40°$

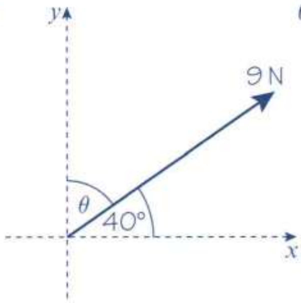
Component in x-direction $= F\cos\theta$

$= 9 \times \cos 40°$

$= 6.89\text{ N (3 s.f.)}$

> Give your answers correct to three significant figures.

ii



$\theta = 90° - 40°$

$= 50°$

> Make sure you find the angle between the force and the direction you are resolving in.

Component in y-direction $= F\cos\theta$

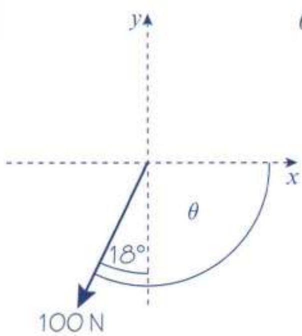
$= 9 \times \cos 50°$

$= 5.79\text{ N (3 s.f.)}$

> You could also use $F\sin 40°$ as $\sin 40° = \cos(90° - 40°) = \cos 50°$

iii $(6.89\mathbf{i} + 5.79\mathbf{j})$ N

b i



$\theta = 90° + 18°$

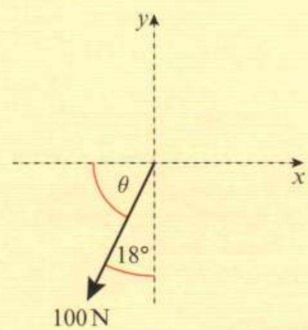
$= 108°$

Component in x-direction $= F\cos\theta$

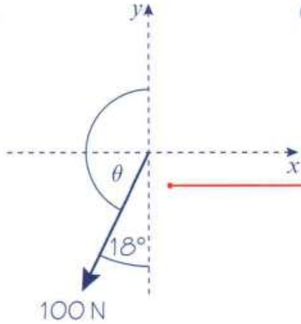
$= 100 \times \cos 108°$

$= -30.9\text{ N (3 s.f.)}$

> You get a negative answer because you are resolving in the positive x-direction. You could also resolve in the negative x-direction using $\theta = 90° - 18° = 72°$, then change the sign of your answer from positive to negative:



ii



$\theta = 180° - 18°$

$= 162°$

> You could use $\theta = 18°$ then change the sign of your answer from positive to negative: $-100\cos 18° = -95.1\text{ N (3 s.f.)}$.

> You can measure θ in either the clockwise or the anticlockwise direction since $\cos\theta = \cos(360° - \theta)$.

Component in y-direction $= F\cos\theta$

$= 100 \times \cos 162°$

$= -95.1\text{ N (3 s.f.)}$

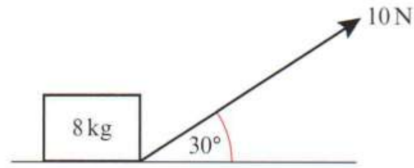
iii $(-30.9\mathbf{i} - 95.1\mathbf{j})$ N

Example 2

A box of mass 8 kg lies on a smooth horizontal floor.
A force of 10 N is applied at an angle of 30° causing the box to accelerate horizontally along the floor.

a Work out the acceleration of the box.

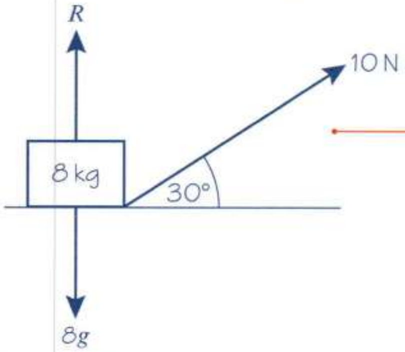
b Calculate the normal reaction between the box and the floor.



a R(→), $10\cos 30° = 8a$
$8a = 5\sqrt{3}$
$a = \frac{5\sqrt{3}}{8}\ \text{m s}^{-2}$

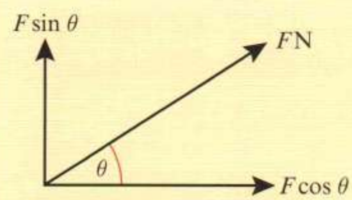
Resolve the force horizontally and write an equation of motion for the box. ← Year 1, Chapter 10

b



Add the weight of the box and the normal reaction to the force diagram.

R(↑), $R + 10\sin 30° = 8g$
$R = 78.4 - 5$
$= 73\ \text{N}$ (2 s.f.)

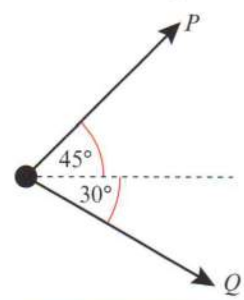


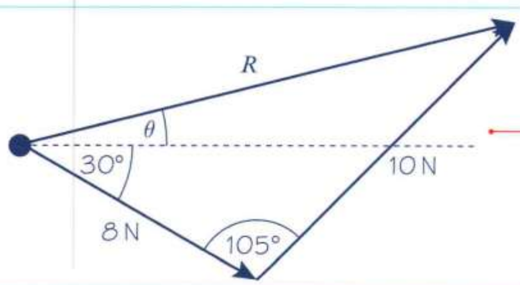
The component in the y-direction is $F\cos(90° - \theta) = F\sin\theta$

You can use the **triangle law** of vector addition to find the resultant of two forces acting at an angle without resolving them into components.

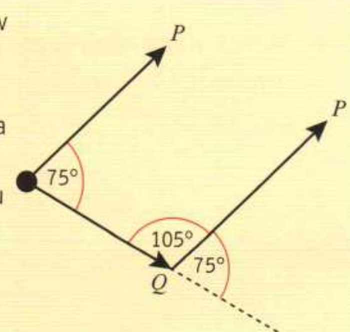
Example 3

Two forces P and Q act on a particle as shown. P has a magnitude of 10 N and Q has a magnitude of 8 N. Work out the magnitude and direction of the resultant force.





Use the triangle law for vector addition. The resultant force is the third side of a triangle formed by forces P and Q. You might need to use geometry to work out missing angles in the triangle:



$R^2 = 8^2 + 10^2 - 2 \times 8 \times 10\cos 105°$
$= 164 - 160\cos 105° = 205.411...$
$R = 14.332... = 14.3\text{ N (3 s.f.)}$

$\frac{\sin(\theta + 30°)}{10} = \frac{\sin 105°}{14.332...}$

$\sin(\theta + 30°) = \frac{10\sin 105°}{14.332...} = 0.673...$

$\theta + 30° = 42.373...$
$\theta = 12.4°\text{ (3 s.f.)}$

The resultant force R has a magnitude of 14.3 N and acts at an angle of 12.4° above the horizontal.

Use the cosine rule to calculate the magnitude of R.

Use the sine rule to work out θ.

Remember to use unrounded values in your calculations then round your final answer.

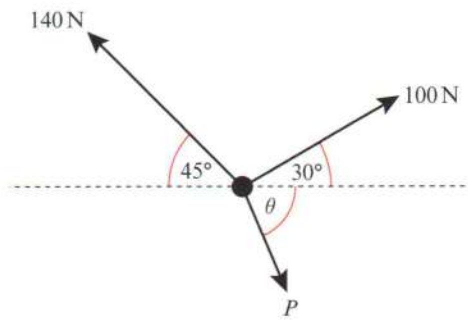
Use your diagram to check that your answer makes sense.

Online Explore the resultant of two forces using GeoGebra.

Example 4

Three forces act upon a particle as shown.

Given that the particle is in equilibrium, calculate the magnitude of P.



R(→), $100\cos 30° + P\cos\theta = 140\cos 45°$
$P\cos\theta = 12.392...$ (1)

R(↑), $100\sin 30° + 140\sin 45° = P\sin\theta$
$P\sin\theta = 148.994...$ (2)

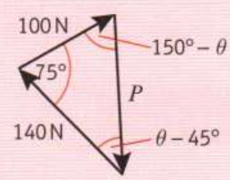
$\frac{P\sin\theta}{P\cos\theta} = \frac{148.994...}{12.392...}$

$\tan\theta = 12.023...$
$\theta = 85.245...°$
$P\cos 85.2454...° = 12.392...$
$P = 150\text{ N (3 s.f.)}$

Resolve horizontally and vertically. You can solve these two equations simultaneously by dividing to eliminate P.

Problem-solving

You could also solve this problem by drawing a **triangle of forces**. The particle is in equilibrium, so the three forces will form a closed triangle:

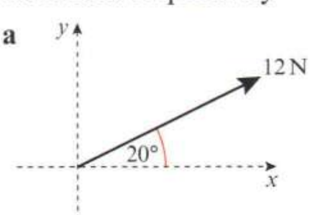


Exercise 5A

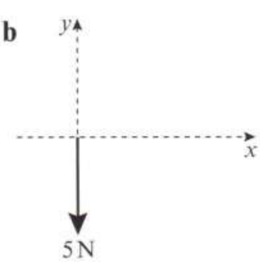
1 Find the component of each force in **i** the x-direction **ii** the y-direction

iii Hence write each force in the form $p\mathbf{i} + q\mathbf{j}$ where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the x and y directions respectively.

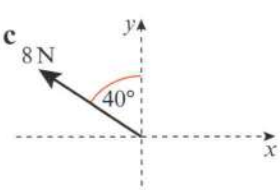
a



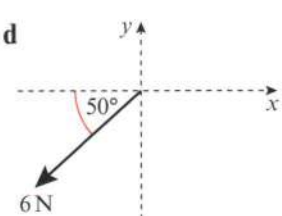
b



c

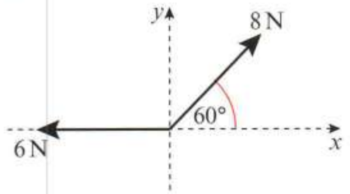


d

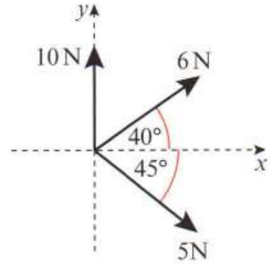


2 For each of the following systems of forces, find the sum of the components in
i the x-direction, **ii** the y-direction.

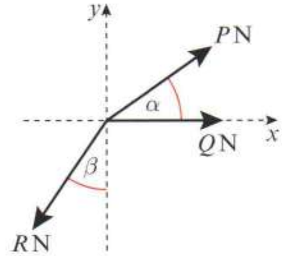
a



b

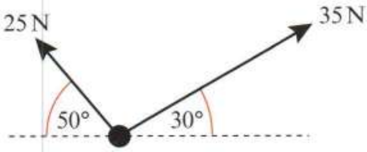


c

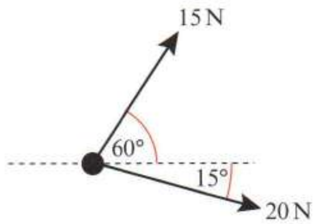


3 Find the magnitude and direction of the resultant force acting on each of the particles shown below.

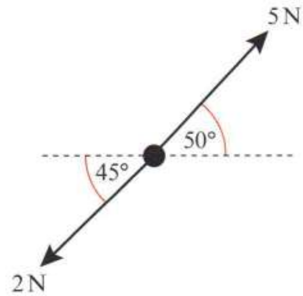
a



b

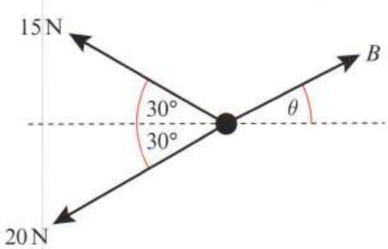


c

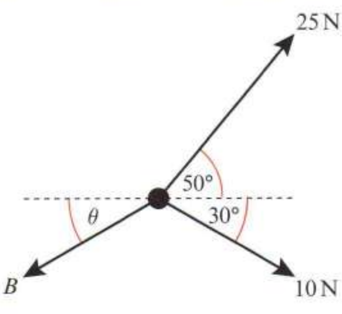


(P) **4** Three forces act upon a particle as shown in the diagrams below.
Given that the particle is in equilibrium, calculate the magnitude of B and the value of θ.

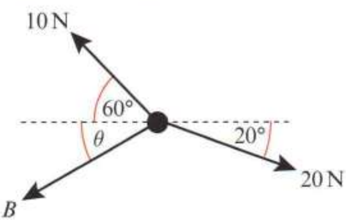
a



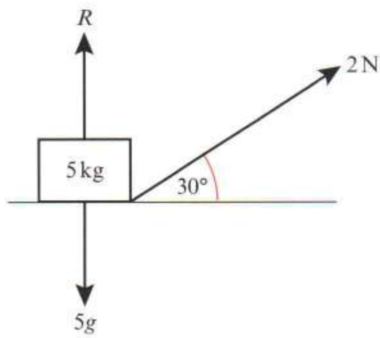
b



c



5 A box of mass 5 kg lies on a smooth horizontal floor. The box is pulled by a force of 2 N applied at an angle of 30° to the horizontal, causing the box to accelerate horizontally along the floor.



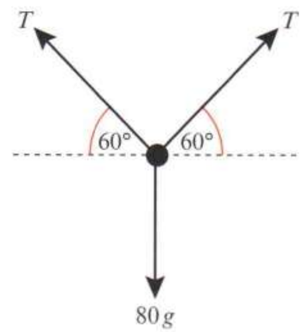
a Work out the acceleration of the box.

b Work out the normal reaction of the box with the floor.

(E) **6** A force P is applied to a box of mass 10 kg causing the box to accelerate at $2\,\text{m}\,\text{s}^{-2}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 45° to the plane, work out the value of P. **(3 marks)**

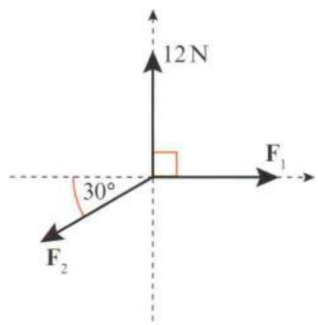
(E) **7** A force of 20 N is applied to a box of mass m kg causing the box to accelerate at $0.5\,\text{m}\,\text{s}^{-2}$ along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 25° to the plane, work out the value of m. **(3 marks)**

E/P **8** A parachutist of mass 80 kg is attached to a canopy by two lines, each with tension T. The parachutist is falling with constant velocity, and experiences a resistance to motion due to air resistance equal to one quarter of her weight. Show that the tension in each line, T, is $20\sqrt{3}\,g$ N.



(3 marks)

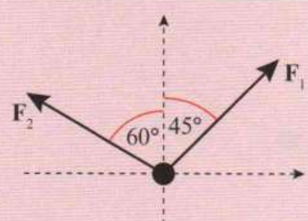
E/P **9** A system of forces act upon a particle as shown in the diagram. The resultant force on the particle is $(2\sqrt{3}\mathbf{i} + 2\mathbf{j})$ N. Calculate the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.



(3 marks)

Challenge

Two forces act upon a particle as shown in the diagram. The resultant force on the particle is $(3\mathbf{i} + 5\mathbf{j})$ N. Calculate the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.



5.2 Inclined planes

Force diagrams may be used to model situations involving objects on inclined planes.

- **To solve problems involving inclined planes, it is usually easier to resolve parallel to and at right angles to the plane.**

Example 5

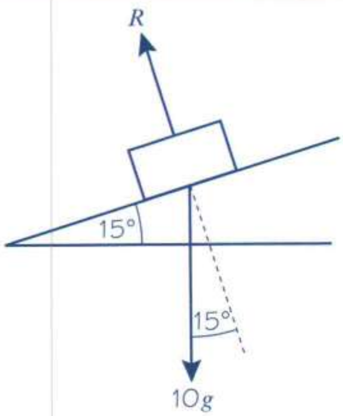
A block of mass 10 kg slides down a smooth slope angled at 15° to the horizontal.

a Draw a force diagram to show all the forces acting on the block.

b Calculate the magnitude of the normal reaction of the slope on the block.

c Find the acceleration of the block.

a



b $R(\nwarrow)$, $R = 10g\cos 15°$
$= 95\text{ N (2 s.f.)}$

c $R(\swarrow)$, $10g\cos 75° = 10a$
$a = 2.5\text{ m s}^{-2}\text{(2 s.f.)}$

Watch out The normal reaction force acts at right angles to the plane, not vertically.

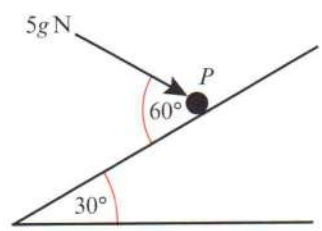
Your working will be easier if you resolve at right angles to the plane. The weight of the block acts at an angle of 15° to this direction.

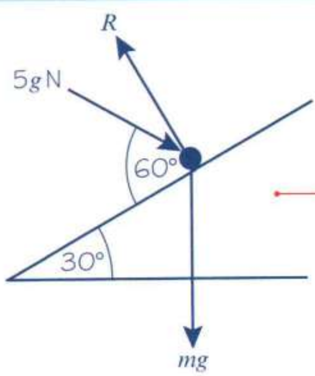
Notation The diagonal arrows, **R**(↖) and **R**(↙), show that you are resolving perpendicular to the slope and down the slope. You can also use **R**(//) to show resolution parallel to the slope and **R**(⊥) to show resolution perpendicular to the slope.

Resolve down the slope and use $F = ma$.

Example 6

A particle of mass m is pushed up a smooth slope by a force of magnitude $5g$ N acting at an angle of 60° to the slope, causing the particle to accelerate up the slope at 0.5 m s^{-2}. Show that the mass of the particle is $\left(\frac{5g}{1+g}\right)$kg.





Draw a diagram to show all the forces acting on the particle.

$R(\nearrow)$, $5g\cos 60° - mg\sin 30° = 0.5m$
$2.5g - 0.5mg = 0.5m$
$2.5g = 0.5m + 0.5mg$
$5g = m + mg$
$5g = m(1 + g)$

Resolve up the slope, in the direction of the acceleration, and write an equation of motion for the particle.

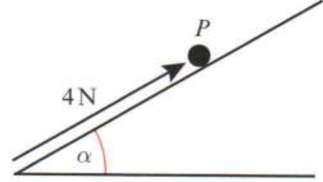
$m = \left(\frac{5g}{1+g}\right)$ kg as required

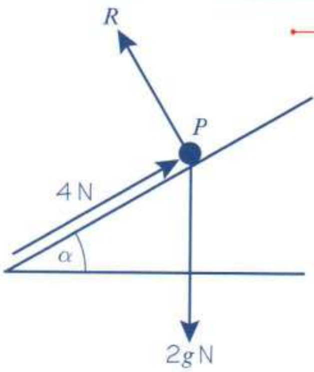
You need to find the mass of the particle in terms of g, so you don't need to use $g = 9.8\text{ m s}^{-2}$ in your working.

Example 7

A particle P of mass 2 kg is moving on a smooth slope and is being acted on by a force of 4 N that acts parallel to the slope as shown.

The slope is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. Work out the acceleration of the particle.





Draw a diagram to show all the forces acting on the particle.

R(↗), $4 - 2g\sin\alpha = 2a$

$4 - 2 \times 9.8 \times \frac{3}{5} = 2a$

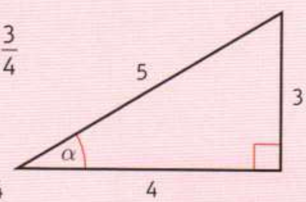
$2a = -7.76$

$a = -3.9\text{ m s}^{-2}$ (2 s.f.)

Resolve up the slope, in the direction of the acceleration, and use Newton's second law.

Problem-solving

You know that $\tan\alpha = \frac{3}{4}$ so you can draw a triangle to work out $\sin\alpha$ and $\cos\alpha$.



$\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

You can use these exact values in your calculations.

The particle accelerates down the slope at 3.9 m s^{-2}.

You resolved up the slope and the acceleration is negative, so the particle is accelerating down the slope.

Exercise 5B

1 A particle of mass 3 kg slides down a smooth slope that is inclined at 20° to the horizontal.

  a Draw a force diagram to represent all the forces acting on the particle.

  b Work out the normal reaction between the particle and the plane.

  c Find the acceleration of the particle.

2 A force of 50 N is pulling a particle of mass 5 kg up a smooth plane that is inclined at 30° to the horizontal. Given that the force acts parallel to the plane,

  a draw a force diagram to represent all the forces acting on the particle

  b work out the normal reaction between the particle and the plane

  c find the acceleration of the particle.

3 A particle of mass 0.5 kg is held at rest on a smooth slope that is inclined at an angle α to the horizontal. The particle is released. Given that $\tan\alpha = \frac{3}{4}$, calculate:

  a the normal reaction between the particle and the plane

  b the acceleration of the particle.

(E) **4** A force of 30 N is pulling a particle of mass 6 kg up a rough slope that is inclined at 15° to the horizontal. The force acts in the direction of motion of the particle and the particle experiences a constant resistance due to friction.

a Draw a force diagram to represent all the forces acting on the particle. **(4 marks)**

Given that the particle is moving with constant speed,

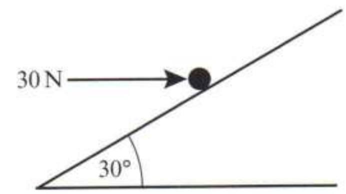
b calculate the magnitude of the resistance due to friction. **(5 marks)**

(E) **5** A particle of mass m kg is sliding down a smooth slope that is angled at 30° to the horizontal. The normal reaction between the plane and the particle is 5 N.

a Calculate the mass m of the particle. **(3 marks)**

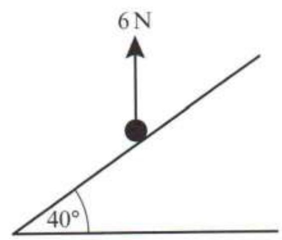
b Calculate the acceleration of the particle. **(3 marks)**

(E/P) **6** A force of 30 N acts horizontally on a particle of mass 5 kg that rests on a smooth slope that is inclined at 30° to the horizontal as shown in the diagram.
Find the acceleration of the particle.



(4 marks)

(E/P) **7** A particle of mass 3 kg is moving on a rough slope that is inclined at 40° to the horizontal. A force of 6 N acts vertically upon the particle. Given that the particle is moving at a constant velocity, calculate the value of F, the constant resistance due to friction.

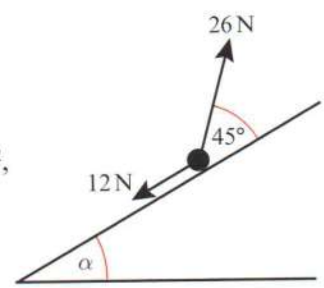


(4 marks)

(E/P) **8** A particle of mass m kg is pulled up a rough slope by a force of 26 N that acts at an angle of 45° to the slope. The particle experiences a constant frictional force of magnitude 12 N.

Given that $\tan \alpha = \frac{1}{\sqrt{3}}$ and that the acceleration of the particle is $1\,\text{m s}^{-2}$,

show that $m = 1.08$ kg (3 s.f.).



(5 marks)

Challenge

A particle is sliding down a smooth slope inclined at an angle θ to the horizontal, where $0 < \theta < 30°$. The angle of inclination of the slope is increased by 60°, and the magnitude of the acceleration of the particle increases from a to $4a$.

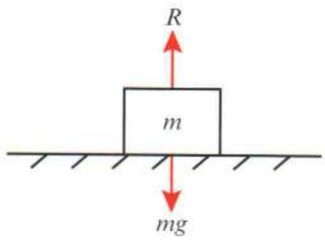
a Show that $\tan \theta = \frac{\sqrt{3}}{7}$

b Hence find θ, giving your answer to 3 significant figures.

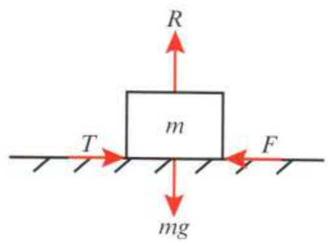
5.3 Friction

Friction is a force which opposes motion between two rough surfaces. It occurs when the two surfaces are moving relative to one another, or when there is a **tendency** for them to move relative to one another.

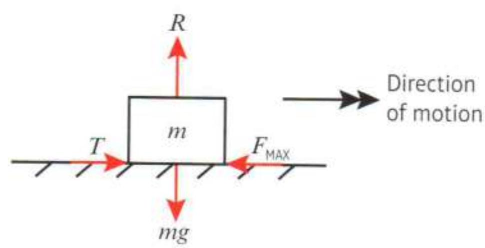
This block is stationary. There is no horizontal force being applied, so there is no tendency for the block to move. There is no frictional force acting on the block.



This block is also stationary. There is a horizontal force being applied which is not sufficient to move the block. There is a tendency for the block to move, but it doesn't because the force of friction is equal and opposite to the force being applied.



As the applied force increases, the force of friction increases to prevent the block from moving. If the magnitude of the applied force exceeds a certain **maximum** or **limiting value**, the block will move. While the block moves, the force of friction will remain constant at its maximum value.



The limiting value of the friction depends on two things:

- the normal reaction R between the two surfaces in contact
- the roughness of the two surfaces in contact.

You can measure roughness using the **coefficient of friction**, which is represented by the letter μ (pronounced *myoo*). The rougher the two surfaces, the larger the value of μ. For smooth surfaces there is no friction and $\mu = 0$.

- **The maximum or limiting value of the friction between two surfaces, F_{MAX}, is given by**

$$F_{MAX} = \mu R$$

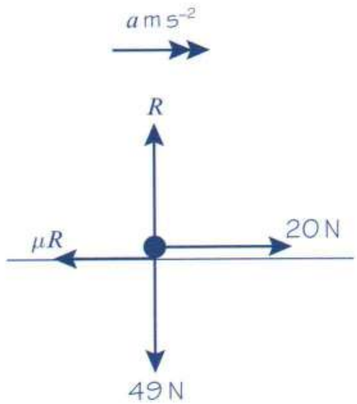
where μ is the coefficient of friction and R is the normal reaction between the two surfaces.

Example 8

A particle of mass 5 kg is pulled along a rough horizontal surface by a horizontal force of magnitude 20 N. The coefficient of friction between the particle and the floor is 0.2. Calculate:

a the magnitude of frictional force

b the acceleration of the particle.



a R(↑), $R = 49$

$F = \mu R = 0.2 \times 49 = 9.8$ N

b R(→), $20 - \mu R = 5a$

$5a = 20 - 9.8$

$= 10.2$

$a = 2.0$ m s^{-2} (2 s.f.)

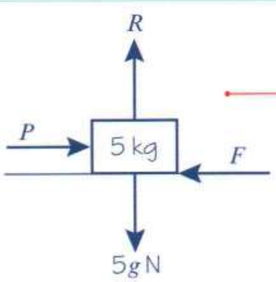
For a moving particle $F = F_{MAX}$ so you can use $F = \mu R$ to find the magnitude of the frictional force.

Write an equation of motion for the particle, resolving horizontally. Note that the frictional force always acts in a direction so as to **oppose** the motion of the particle.

Example 9

A block of mass 5 kg lies on rough horizontal ground. The coefficient of friction between the block and the ground is 0.4. A horizontal force P is applied to the block. Find the magnitude of the frictional force acting on the block and the acceleration of the block when the magnitude of P is

a 10 N **b** 19.6 N **c** 30 N.



First draw a diagram showing all the forces acting on the block.

R(↑), $R = 5g = 49$ N

So $F_{MAX} = \mu R = 0.4 \times 49$

$= 19.6$ N

The normal reaction will equal the weight as the force P has no vertical component and there is no vertical acceleration.

You then need to calculate what the maximum possible frictional force is in this situation.

Do not round this value as you will need to use it in your calculations.

The maximum available frictional force is 19.6 N.

a When $P = 10$ N, the friction will only need to be 10 N to prevent the block from sliding and the block will remain at rest in equilibrium.

b When $P = 19.6$ N, the friction will need to be at its maximum value of 19.6 N to prevent the block from sliding, and the block will remain at rest in **limiting equilibrium**.

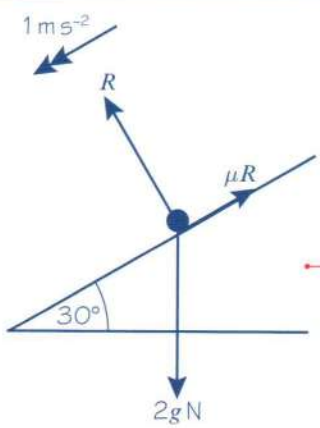
c When $P = 30$ N, the friction will be unable to prevent the block from sliding, and it will remain at its maximum value of 19.6 N. The block will accelerate from rest along the plane in the direction of P with acceleration a, where

$$30 - 19.6 = 5a$$
$$a = 2.1\text{ m s}^{-2} \text{ (2 s.f.)}$$

Watch out An object in limiting equilibrium can either be at rest or moving with constant velocity.

Example 10

A particle of mass 2 kg is sliding down a rough slope that is inclined at 30° to the horizontal. Given that the acceleration of the particle is 1 m s^{-2}, find the coefficient of friction, μ, between the particle and the slope.



Online Explore this problem with different masses, slopes and frictional coefficients using GeoGebra.

Draw a diagram showing the weight, the frictional force and the normal reaction.

R(↖), $R = 2g\cos 30°$
$= 16.974...$

R(↙), $2g\sin 30° - \mu R = 2a$

$9.8 - (16.974...)\mu = 2$

$$\mu = \frac{7.8}{16.974...}$$
$$= 0.46 \text{ (2 s.f.)}$$

Use $F = ma$ to write an equation of motion for the particle

Watch out Make sure you use the normal reaction, not the weight, when substituting into $F_{MAX} = \mu R$.

Exercise 5C

1 Each of the following diagrams shows a body of mass 5 kg lying initially at rest on rough horizontal ground. The coefficient of friction between the body and the ground is $\frac{1}{7}$. In each diagram R is the normal reaction of the ground on the body and F is the frictional force exerted on the body. Any other forces applied to the body are as shown on the diagrams.

Hint The forces acting on the body can affect the magnitude of the normal reaction. In part **d** the normal reaction is $(5g + 14)$ N, so $F_{MAX} = \mu(5g + 14)$ N.

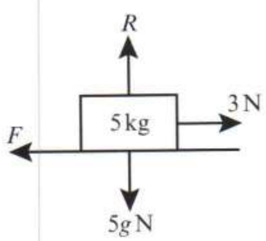
In each case

i find the magnitude of F,

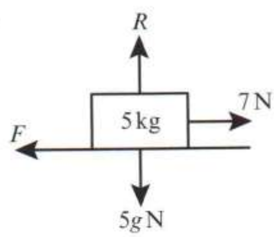
ii state whether the body will remain at rest or accelerate from rest along the ground,

iii find, when appropriate, the magnitude of this acceleration.

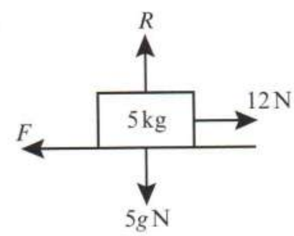
a



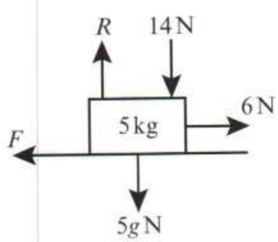
b



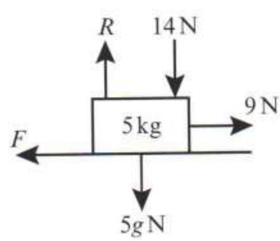
c



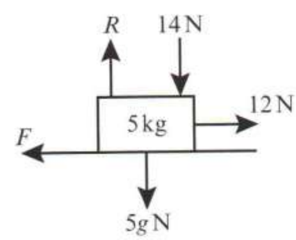
d



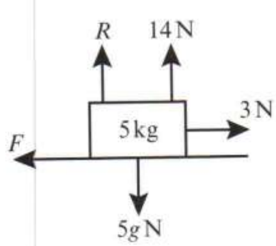
e



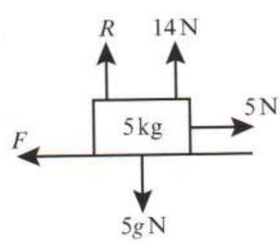
f



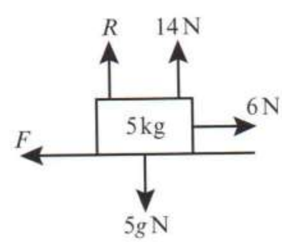
g



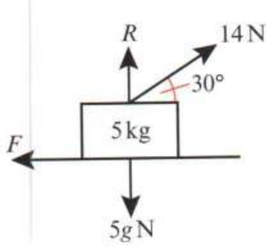
h



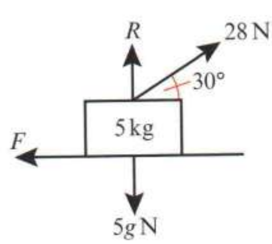
i



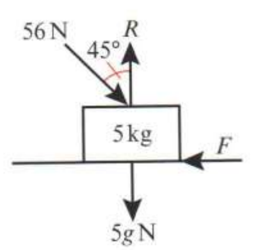
j



k

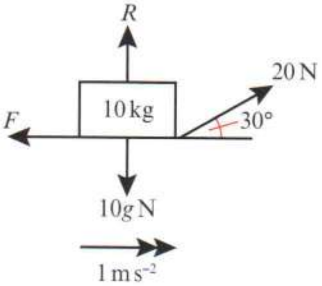


l

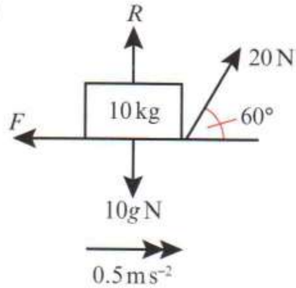


2 In each of the following diagrams, the forces shown cause the body of mass 10 kg to accelerate as shown along the rough horizontal plane. R is the normal reaction and F is the frictional force. Find the normal reaction and the coefficient of friction in each case.

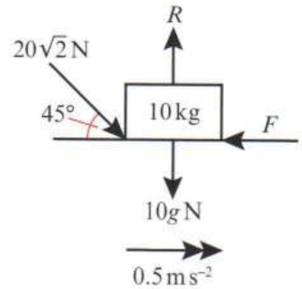
a



b



c

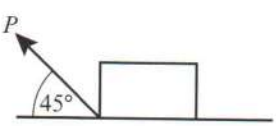


(E) **3** A particle of mass 0.5 kg is sliding down a rough slope that is angled at 15° to the horizontal. The acceleration of the particle is $0.25\,\text{m s}^{-2}$. Calculate the coefficient of friction between the particle and the slope. **(3 marks)**

(E) **4** A particle of mass 2 kg is sliding down a rough slope that is angled at 20° to the horizontal. A force of magnitude P acts parallel to the slope and is attempting to pull the particle up the slope. The acceleration of the particle is $0.2\,\text{m s}^{-2}$ down the slope and the coefficient of friction between the particle and the slope is 0.3. Find the value of P. **(4 marks)**

5 A particle of mass 5 kg is being pushed up a rough slope that is angled at 30° to the horizontal by a horizontal force P. Given that the coefficient of friction is 0.2 and the acceleration of the particle is $2\,\text{m s}^{-2}$ calculate the value of P.

(E/P) **6** A sled of mass 10 kg is being pulled along a rough horizontal plane by a force P that acts at an angle of 45° to the horizontal. The coefficient of friction between the sled and the plane is 0.1. Given that the sled accelerates at $0.3\,\text{m s}^{-2}$, find the value of P.



(7 marks)

(P) **7** A train of mass m kg is travelling at $20\,\text{m s}^{-1}$ when it applies its brakes, causing the wheels to lock up. The train decelerates at a constant rate, coming to a complete stop in 30 seconds.

Problem-solving

Use the formulae for constant acceleration.

← Year 1, Chapter 9

a By modelling the train as a particle, show that, in this model, the coefficient of friction between the railway track and the

wheels of the train is $\mu = \dfrac{2}{3g}$.

The train is no longer modelled as a particle, so that the effects of air resistance can be taken into account.

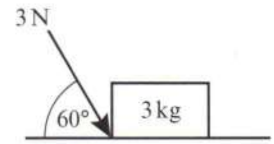
b State, with a reason, whether the coefficient of friction between the track and the wheels will increase or decrease in this revised model.

Challenge

A particle of mass m kg is sliding down a rough slope that is angled at α to the horizontal. The coefficient of friction between the particle and the slope is μ. Show that the acceleration of the particle is independent of its mass.

Mixed exercise 5

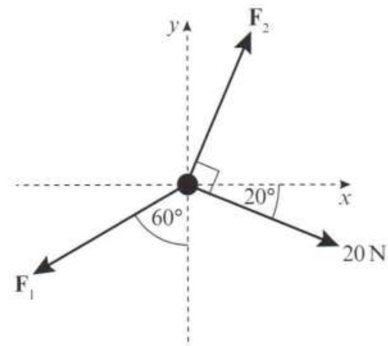
1 A box of mass 3 kg lies on a smooth horizontal floor. A force of 3 N is applied at an angle of 60° to the horizontal, causing the box to accelerate horizontally along the floor.



a Find the magnitude of the normal reaction of the floor on the box.

b Find the acceleration of the box.

(P) **2** A system of forces acts upon a particle as shown in the diagram. The resultant force on the particle is $(3\mathbf{i} + 2\mathbf{j})$ N. Calculate the magnitudes $\mathbf{F}_1$ and $\mathbf{F}_2$.



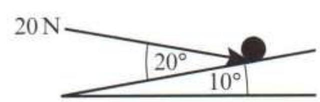
3 A force of 20 N is pulling a particle of mass 2 kg up a rough slope that is inclined at 45° to the horizontal. The force acts parallel to the slope, and the resistance due to friction is constant and has magnitude 4 N.

a Draw a force diagram to represent all the forces acting on the particle.

b Work out the normal reaction between the particle and the plane.

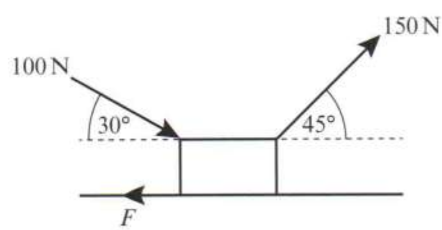
c Show that the acceleration of the particle is $1.1\ \text{m s}^{-2}$ (2 s.f.).

(E) **4** A particle of mass 5 kg sits on a smooth slope that is inclined at 10° to the horizontal. A force of 20 N acts on the particle at an angle of 20° to the plane, as shown in the diagram. Find the acceleration of the particle.



(5 marks)

(E/P) **5** A box is being pushed and pulled across a rough surface by constant forces as shown in the diagram. The box is moving at a constant speed. By modelling the box as a particle, show that the magnitude of the resistance due to friction F is $25(3\sqrt{2} + 2\sqrt{3})$ N. **(4 marks)**



(E/P) **6** A trailer of mass 20 kg sits at rest on a rough horizontal plane. A force of 20 N pulls the trailer at an angle of 30° above the horizontal. Given that the trailer is in limiting equilibrium, work out the value of the coefficient of friction. **(6 marks)**

(E/P) **7** A particle of mass 2 kg is moving down a rough plane that is inclined at α to the horizontal, where $\tan \alpha = \frac{3}{4}$. A force of P N acts horizontally upon the particle towards the plane. Given that the coefficient of friction is 0.3 and that the particle is moving at a constant velocity, calculate the value of P. **(7 marks)**

(E/P) **8** A particle of mass 0.5 kg is being pulled up a rough slope that is angled at 30° to the horizontal by a force of 5 N. The force acts at an angle of 30° above the slope. Given that the coefficient of friction is 0.1, calculate the acceleration of the particle. **(7 marks)**

(E/P) **9** A car of mass 2150 kg is travelling down a rough road that is inclined at 10° to the horizontal. The engine of the car applies a constant driving force of magnitude 700 N, which acts in the direction of travel of the car. Any friction between the road and the tyres is initially ignored, and air resistance is modelled as a single constant force of magnitude F N that acts to oppose the motion of the car.

a Given that the car is travelling in a straight line at a constant speed of $22\,\text{m}\,\text{s}^{-1}$, find the magnitude of F. **(3 marks)**

The driver brakes suddenly. In the subsequent motion the car continues to travel in a straight line, and the tyres skid along the road, bringing the car to a standstill after 40 m. The driving force is removed, and the force due to air resistance is modelled as remaining constant.

b Find the coefficient of friction between the tyres and the road. **(7 marks)**

c Criticise this model with relation to

i the frictional forces acting on the car

ii the motion of the car. **(2 marks)**

Challenge

A boat of mass 400 kg is being pulled up a rough slipway at a constant speed of $5\,\text{m}\,\text{s}^{-1}$ by a winch. The slipway is modelled as a plane inclined at an angle of 15° to the horizontal, and the boat is modelled as a particle. The coefficient of friction between the boat and the slipway is 0.2.

At the point when the boat is 8 m from the water-line, as measured along the line of greatest slope of the slipway, the winch cable snaps. Show that the boat will slide back down into the water, and calculate the total time from the winch cable breaking to the boat reaching the water-line.

Summary of key points

1 If a force is applied at an angle to the direction of motion, you can resolve it to find the component of the force that acts in the direction of motion.

2 The component of a force of magnitude F in a certain direction is $F\cos\theta$, where θ is the size of the angle between the force and the direction.

3 To solve problems involving inclined planes, it is usually easier to resolve parallel to and at right angles to the plane.

4 The maximum or limiting value of the friction between two surfaces, F_{MAX}, is given by $\boldsymbol{F_{\text{MAX}} = \mu R}$ where μ is the coefficient of friction and R is the normal reaction between the two surfaces.