Review exercise 1

(E) **1** **a** Write down the value of $8^{\frac{1}{3}}$. **(1)**

b Find the value of $8^{-\frac{2}{3}}$. **(2)**

← Section 1.4

2 **a** Find the value of $125^{\frac{4}{3}}$. **(2)**

b Simplify $24x^2 \div 18x^{\frac{4}{3}}$. **(2)**

← Sections 1.1, 1.4

(E) **3** **a** Express $\sqrt{80}$ in the form $a\sqrt{5}$, where a is an integer. **(2)**

b Express $(4 - \sqrt{5})^2$ in the form $b + c\sqrt{5}$, where b and c are integers. **(2)**

← Section 1.5

(E) **4** **a** Expand and simplify $(4 + \sqrt{3})(4 - \sqrt{3})$. **(2)**

b Express $\dfrac{26}{4 + \sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers. **(3)**

← Sections 1.5, 1.6

(E/P) **5** Here are three numbers:

$1 - \sqrt{k}$, $2 + 5\sqrt{k}$ and $2\sqrt{k}$

Given that k is a positive integer, find:

a the mean of the three numbers. **(2)**

b the range of the three numbers. **(1)**

← Section 1.5

(E) **6** Given that $y = \dfrac{1}{25}x^4$, express each of the following in the form kx^n, where k and n are constants.

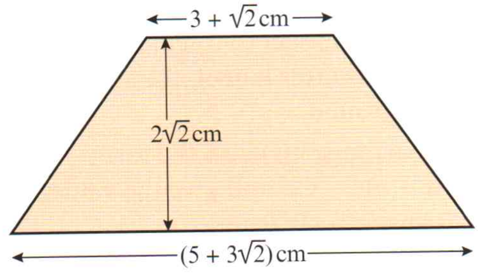
a y^{-1} **(1)**

b $5y^{\frac{1}{2}}$ **(1)**

← Section 1.4

(E/P) **7** Find the area of this trapezium in cm^2. Give your answer in the form $a + b\sqrt{2}$, where a and b are integers to be found. **(4)**

← Section 1.5



(E) **8** Given that $p = 3 - 2\sqrt{2}$ and $q = 2 - \sqrt{2}$, find the value of $\dfrac{p + q}{p - q}$.

Give your answer in the form $m + n\sqrt{2}$, where m and n are rational numbers to be found. **(4)**

← Sections 1.5, 1.6

(E/P) **9** **a** Factorise the expression $x^2 - 10x + 16$. **(1)**

b Hence, or otherwise, solve the equation $8^{2y} - 10(8^y) + 16 = 0$. **(2)**

← Sections 1.3, 2.1

(E) **10** $x^2 - 8x - 29 \equiv (x + a)^2 + b$, where a and b are constants.

a Find the value of a and the value of b. **(2)**

b Hence, or otherwise, show that the roots of $x^2 - 8x - 29 = 0$ are $c \pm d\sqrt{5}$, where c and d are integers. **(3)**

← Sections 2.1, 2.2

E/P **11** The functions f and g are defined as
$f(x) = x(x - 2)$ and $g(x) = x + 5$, $x \in \mathbb{R}$.
Given that $f(a) = g(a)$ and $a > 0$,
find the value of a to three significant figures. **(3)**

← Sections 2.1, 2.3

P **12** An athlete launches a shot put from shoulder height. The height of the shot put, in metres, above the ground t seconds after launch, can be modelled by the following function:

$h(t) = 1.7 + 10t - 5t^2 \qquad t \geqslant 0$

a Give the physical meaning of the constant term 1.7 in the context of the model.

b Use the model to calculate how many seconds after launch the shot put hits the ground.

c Rearrange $h(t)$ into the form $A - B(t - C)^2$ and give the values of the constants A, B and C.

d Using your answer to part **c** or otherwise, find the maximum height of the shot put, and the time at which this maximum height is reached.

← Section 2.6

E/P **13** Given that $f(x) = x^2 - 6x + 18$, $x \geqslant 0$,

a express $f(x)$ in the form $(x - a)^2 + b$, where a and b are integers. **(2)**

The curve C with equation $y = f(x)$, $x \geqslant 0$, meets the y-axis at P and has a minimum point at Q.

b Sketch the graph of C, showing the coordinates of P and Q. **(3)**

The line $y = 41$ meets C at the point R.

c Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers. **(2)**

← Sections 2.2, 2.4

E **14** The function $h(x) = x^2 + 2\sqrt{2}x + k$ has equal roots.

a Find the value of k. **(1)**

b Sketch the graph of $y = h(x)$, clearly labelling any intersections with the coordinate axes. **(3)**

← Sections 1.5, 2.4, 2.5

E/P **15** The function $g(x)$ is defined as
$g(x) = x^9 - 7x^6 - 8x^3$, $x \in \mathbb{R}$.

a Write $g(x)$ in the form $x^3(x^3 + a)(x^3 + b)$, where a and b are integers. **(1)**

b Hence find the three roots of $g(x)$. **(1)**

← Section 2.3

E/P **16** Given that

$x^2 + 10x + 36 \equiv (x + a)^2 + b,$

where a and b are constants,

a find the value of a and the value of b. **(2)**

b Hence show that the equation $x^2 + 10x + 36 = 0$ has no real roots. **(2)**

The equation $x^2 + 10x + k = 0$ has equal roots.

c Find the value of k. **(2)**

d For this value of k, sketch the graph of $y = x^2 + 10x + k$, showing the coordinates of any points at which the graph meets the coordinate axes. **(3)**

← Sections 2.2, 2.4, 2.5

E/P **17** Given that $x^2 + 2x + 3 \equiv (x + a)^2 + b$,

a find the value of the constants a and b **(2)**

b Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes. **(3)**

c Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part **b**. **(2)**

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

d Find the set of possible values of k, giving your answer in surd form. **(2)**

← Section 2.2, 2.4, 2.5

E **18** **a** By eliminating y from the equations:

$y = x - 4,$
$2x^2 - xy = 8,$

show that

$x^2 + 4x - 8 = 0.$ **(2)**

b Hence, or otherwise, solve the simultaneous equations:

$y = x - 4,$
$2x^2 - xy = 8,$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers. **(4)**

← Section 3.2

E **19** Find the set of values of x for which:

a $3(2x + 1) > 5 - 2x,$ **(2)**

b $2x^2 - 7x + 3 > 0,$ **(3)**

c **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0.$ **(1)**

← Sections 3.4, 3.5

E/P **20** The functions p and q are defined as $\mathrm{p}(x) = -2(x + 1)$ and $\mathrm{q}(x) = x^2 - 5x + 2$, $x \in \mathbb{R}$. Show algebraically that there is no value of x for which $\mathrm{p}(x) = \mathrm{q}(x)$. **(3)**

← Sections 2.3, 2.5

E **21** **a** Solve the simultaneous equations:

$y + 2x = 5$
$2x^2 - 3x - y = 16.$ **(5)**

b Hence, or otherwise, find the set of values of x for which:

$2x^2 - 3x - 16 > 5 - 2x.$ **(2)**

← Sections 3.2, 3.5

E/P **22** The equation $x^2 + kx + (k + 3) = 0$, where k is a constant, has different real roots.

a Show that $k^2 - 4k - 12 > 0.$ **(2)**

b Find the set of possible values of k. **(2)**

← Sections 2.5, 3.5

E **23** Find the set of values for which

$\frac{6}{x + 5} < 2, x \neq -5.$ **(6)**

← Section 3.5

E **24** The functions f and g are defined as $\mathrm{f}(x) = 9 - x^2$ and $\mathrm{g}(x) = 14 - 6x$, $x \in \mathbb{R}$.

a On the same set of axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. Indicate clearly the coordinates of any points where the graphs intersect with each other or the coordinate axes. **(5)**

b On your sketch, shade the region that satisfies the inequalities $y > 0$ and $\mathrm{f}(x) > \mathrm{g}(x)$. **(1)**

← Sections 3.2, 3.3, 3.7

E/P **25** **a** Factorise completely $x^3 - 4x$. **(1)**

b Sketch the curve with equation $y = x^3 - 4x$, showing the coordinates of the points where the curve crosses the x-axis. **(2)**

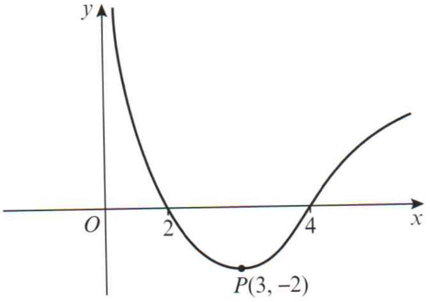
c On a separate diagram, sketch the curve with equation

$y = (x - 1)^3 - 4(x - 1)$

showing the coordinates of the points where the curve crosses the x-axis. **(2)**

← Sections 1.3, 4.1, 4.5

E **26**



The figure shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the points (2, 0) and (4, 0). The minimum point on the curve is $P(3, -2)$.

In separate diagrams, sketch the curves with equation

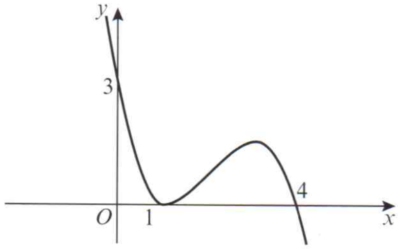
a $y = -\mathrm{f}(x)$ **(2)**

b $y = \mathrm{f}(2x)$ **(2)**

On each diagram, give the coordinates of the points at which the curve crosses the x-axis, and the coordinates of the image of P under the given transformation.

← Sections 4.6, 4.7

E **27**



The figure shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams, sketch the curves with equations

a $y = \mathrm{f}(x + 1)$ **(2)**

b $y = 2\mathrm{f}(x)$ **(2)**

c $y = \mathrm{f}\left(\frac{1}{2}x\right)$ **(2)**

On each diagram, show clearly the coordinates of all the points where the curve meets the axes.

← Sections 4.5, 4.6, 4.7

E **28** Given that $\mathrm{f}(x) = \frac{1}{x}, x \neq 0$,

a sketch the graph of $y = \mathrm{f}(x) + 3$ and state the equations of the asymptotes **(2)**

b find the coordinates of the point where $y = \mathrm{f}(x) + 3$ crosses a coordinate axis. **(2)**

← Sections 4.3, 4.5

E **29** The quartic function t is defined as $\mathrm{t}(x) = (x^2 - 5x + 2)(x^2 - 5x + 4), x \in \mathbb{R}$.

a Find the four roots of $\mathrm{t}(x)$, giving your answers to 3 significant figures where necessary. **(3)**

b Sketch the graph of $y = \mathrm{t}(x)$, showing clearly the coordinates of all the points where the curve meets the axes. **(2)**

← Sections 4.2, 2.1

E **30** The point (6, −8) lies on the graph of $y = \mathrm{f}(x)$. State the coordinates of the point to which P is transformed on the graph with equation:

a $y = -\mathrm{f}(x)$ **(1)**

b $y = \mathrm{f}(x - 3)$ **(1)**

c $2y = \mathrm{f}(x)$ **(1)**

← Section 4.7

E/P **31** The curve C_1 has equation $y = -\frac{a}{x}$, where a is a positive constant.

The curve C_2 has equation $y = (x - b)^2$, where b is a positive constant.

a Sketch C_1 and C_2 on the same set of axes. Label any points where either curve meets the coordinate axes, giving your coordinates in terms of a and b. **(4)**

b Using your sketch, state the number of real solutions to the equation $x(x - 5)^2 = -7$. **(1)**

← Sections 4.3, 4.4

E/P **32** **a** Sketch the graph of $y = \frac{1}{x^2} - 4$, showing clearly the coordinates of the points where the curve crosses the coordinate axes and stating the equations of the asymptotes. **(4)**

b The curve with $y = \frac{1}{(x + k)^2} - 4$ passes through the origin. Find the two possible values of k. **(2)**

← Sections 4.1, 4.5, 4.7

Challenge

1 **a** Solve the equation $x^2 - 10x + 9 = 0$

b Hence, or otherwise, solve the equation $3^{x-2}(3^x - 10) = -1$ ← Sections 1.1, 1.3, 2.1

2 A rectangle has an area of 6 cm² and a perimeter of $8\sqrt{2}$ cm. Find the dimensions of the rectangle, giving your answers as surds in their simplest form. ← Sections 1.5, 2.2

3 Show algebraically that the graphs of $y = 3x^3 + x^2 - x$ and $y = 2x(x - 1)(x + 1)$ have only one point of intersection, and find the coordinates of this point. ← Section 3.3

4 The quartic function $\mathrm{f}(x) = (x^2 + x - 20)(x^2 + x - 2)$ has three roots in common with the function $\mathrm{g}(x) = \mathrm{f}(x - k)$, where k is a constant. Find the two possible values of k. ← Sections 4.2, 4.5, 4.7

5 Find all real solutions to the equation $16m^2 = 54\sqrt{m}$ ← Section 1.4