4 Moments

Objectives

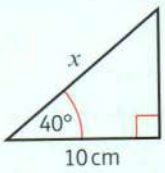
After completing this chapter you should be able to:

- Calculate the turning effect of a force applied to a rigid body → pages 71–72
- Calcualte the resultant moment of a set of forces acting on a rigid body → pages 73–76
- Solve problems involving uniform rods in equilibrium → pages 76–80
- Solve problems involving non-uniform rods → pages 80–83
- Solve problems involving rods on the point of tilting → pages 83–85

Prior knowledge check

1 Find the value of x in each of the following:

a



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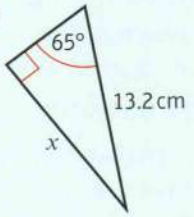
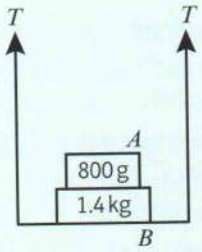


← GCSE Mathematics

2 Masses A and B rest on a light scale-pan supported by two strings, each with tension T.



Find:

a the value of T

b the normal reaction of the scale-pan on mass B

c the normal reaction of mass B on mass A.

← Year 1, Chapter 10

Moments measure the turning effect of a force. Engineers use moments to work out how much load can be applied safely to a crane. → Mixed exercise Q12

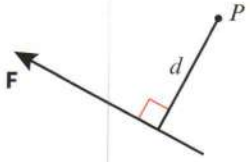
4.1 Moments

So far you have looked mostly at situations involving particles. This means you can ignore rotational effects. In this chapter, you begin to model objects as **rigid bodies**. This allows you to consider the size of the object as well as where forces are applied.

The moment of a force measures the turning effect of the force on a rigid body.

It is the product of the magnitude of the force and the perpendicular distance from the axis of rotation.

- **Clockwise moment of F about $P = |\mathbf{F}| \times d$**

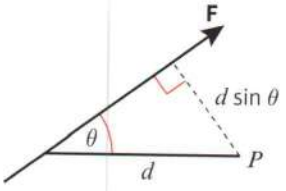


The moment of the force, **F**, is acting about the point P.

Watch out When you describe a moment, you need to give the direction of rotation.

In the diagram above, the distance given is perpendicular to the line of action of the force. When this is not the case, you need to use trigonometry to find the perpendicular distance.

- **Clockwise moment of F about $P = |\mathbf{F}| \times d \sin\theta$**



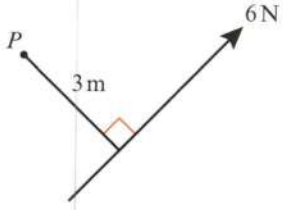
Notation A moment is a force multiplied by a distance, so its units are **newton metres (N m)** or **newton centimetres (N cm)**.

Online Explore the moment of a force acting about a point using GeoGebra.

Example 1

Find the moment of each force about the point P.

a



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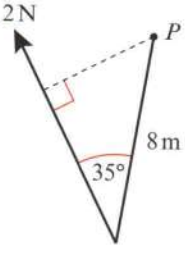


a Moment of the 6 N force about P
= magnitude of force × perpendicular distance
= $6 \times 3 = 18$ N m anticlockwise

b Moment of the 12 N force about P
= magnitude of force × perpendicular distance
= $12 \times 8 \sin 35° = 55.1$ N m clockwise (3 s.f.)

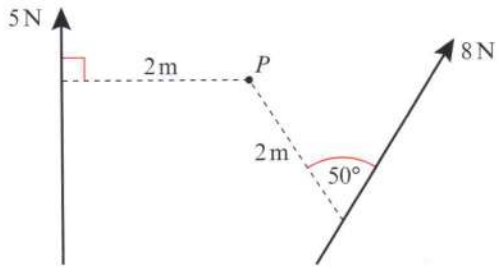
The distance given on the diagram is the perpendicular distance, so you can substitute the given values directly into the formula.

Don't forget to include the direction of the rotation when you describe the moment of the force.

This time you need to use the perpendicular distance $8 \sin 35°$.

Example 2

The diagram shows two forces acting on a lamina. Find the moment of each of the forces about the point P.



Moment of the 5 N force
= magnitude of force × perpendicular distance
= 5 × 2 = 10 N m clockwise

Moment of the 8 N force
= magnitude of force × perpendicular distance
= 8 × 2 sin 50° = 12.3 N m anticlockwise (3 s.f.)

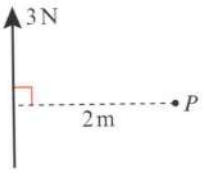
The moments act in opposite directions.

Notation A lamina is a 2D object whose thickness can be ignored.

Exercise 4A

1 Calculate the moment about P of each of these forces acting on a lamina.

a



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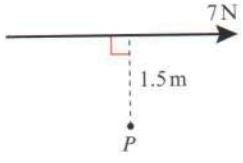

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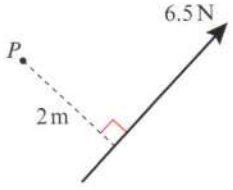

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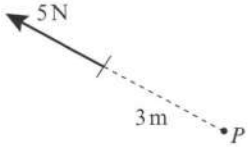
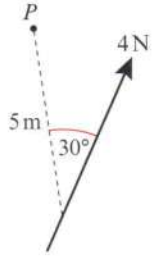
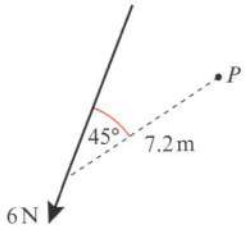


2 Calculate the moment about P of each of these forces acting on a lamina.

a



b



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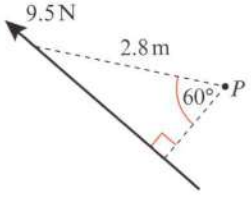
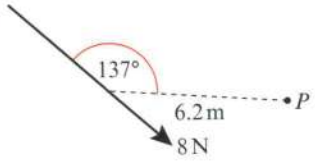
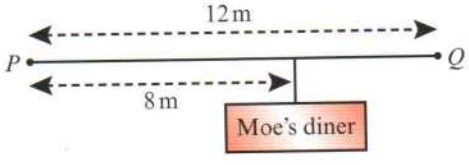


d



3 The diagram shows a sign hanging from a wooden beam. The sign has a mass of 4 kg.

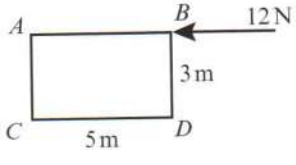


a Calculate the moment of the weight of the mass:

i about P **ii** about Q

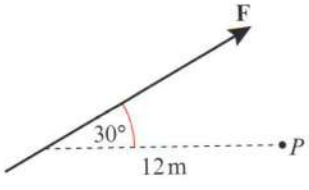
b Comment on any modelling assumptions you have made.

(P) **4** $ABCD$ is a rectangular lamina. A force of 12 N acts horizontally at B, as shown in the diagram. Find the moment of this force about:



a A **b** B **c** C **d** D

(P) **5** In the diagram, the force **F** produces a moment of 15 N m clockwise about the pivot P. Calculate the magnitude of **F**.



4.2 Resultant moments

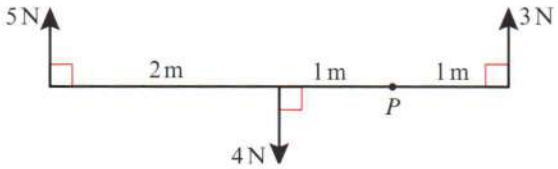
When you have several **coplanar** forces acting on a body, you can determine the turning effect around a given point by choosing a positive direction (clockwise or anticlockwise) and then finding the sum of the moments produced by each force.

Notation **Coplanar forces** are forces that act in the same plane.

- **The sum of the moments acting on a body is called the resultant moment.**

Example 3

The diagram shows a set of forces acting on a light rod. Calculate the resultant moment about the point P.



The moment of the 5 N force is
$5 \times (2 + 1) = 15$ N m clockwise.

The moment of the 4 N force is
$4 \times 1 = 4$ N m anticlockwise.

The moment of the 3 N force is
$3 \times 1 = 3$ N m anticlockwise.

Choosing clockwise as positive:

Resultant moment $= 15 + (-4) + (-3) = 8$ N m

$\therefore$ resultant moment is 8 N m clockwise.

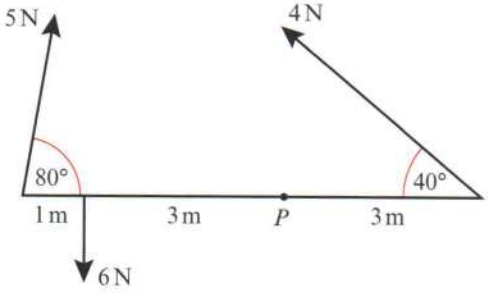
Your positive direction is clockwise, so the anticlockwise moments are negative.

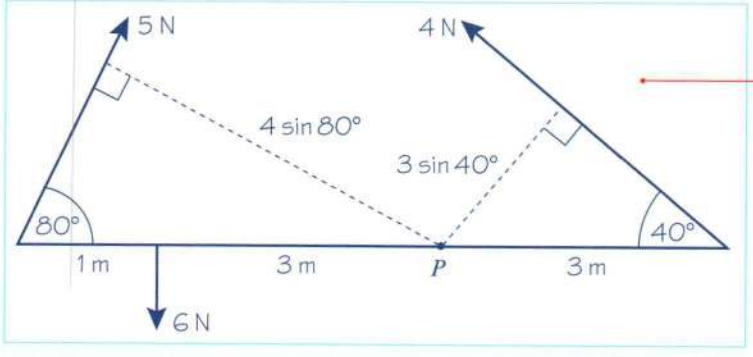
Problem-solving

You could also solve this problem by considering the clockwise and anticlockwise moments separately.
Sum of clockwise moments = 15 N m
Sum of anticlockwise moments = 3 + 4 = 7 N m
Resultant moment = 15 − 7 = 8 N m clockwise

Example 4

The diagram shows a set of forces acting on a light rod. Calculate the resultant moment about the point P.





Draw a diagram to find the perpendicular distances.

Moment of 6 N force
$= 6 \times 3 = 18$ N m anticlockwise

Moment of 5 N force
$= 5 \times 4\sin 80° = 19.696...$ N m clockwise

Moment of 4 N force
$= 4 \times 3\sin 40° = 7.713...$ N m anticlockwise

Take clockwise as the positive direction:

Resultant moment $= (-18) + 19.696... + (-7.713...) = -6.017...$

$\therefore$ resultant moment is 6.02 N m anticlockwise.

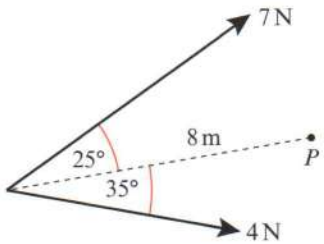
Find the moment of each force.

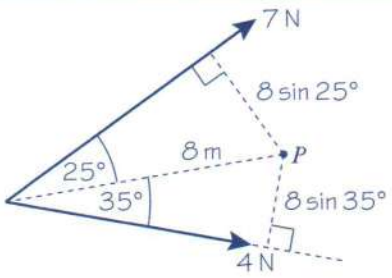
Choose a positive direction.

Add the unrounded values then round your answer to three significant figures.

Example 5

The diagram shows two forces acting on a lamina.
Calculate the resultant moment about the point P.





Draw a diagram and find the perpendicular distances.

Moment of 7 N force
$= 7 \times 8\sin 25° = 23.66...$ N m clockwise.

Moment of 4 N force
$= 4 \times 8\sin 35° = 18.35...$ N m anticlockwise.

Find the moments of both forces.

Take clockwise as the positive direction:

Choose a positive direction.

Resultant moment $= 23.66... + (-18.35...) = 5.31...$

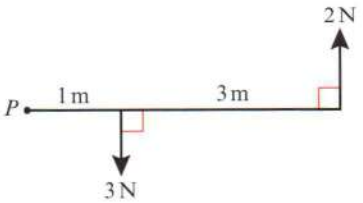
$\therefore$ resultant moment is 5.31 N m clockwise.

Sum together your moments to find the resultant moment.

Exercise 4B

1 These diagrams show sets of forces acting on a light rod. In each case, calculate the resultant moment about P.

a



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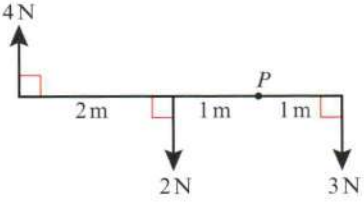

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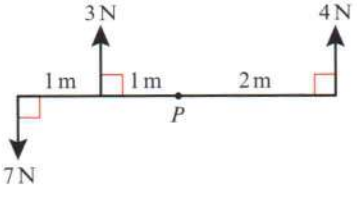

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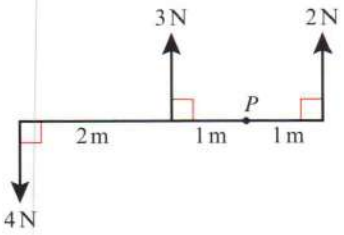

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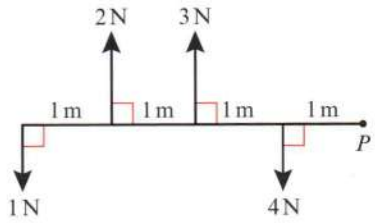

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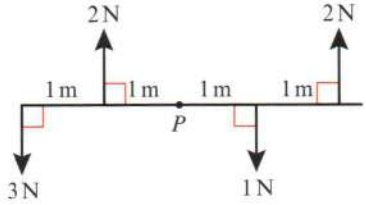
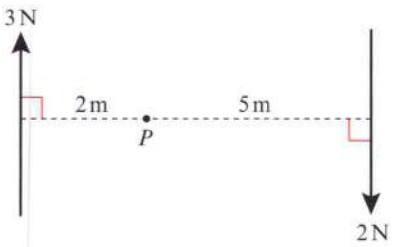


2 These diagrams show forces acting on a lamina. In each case, find the resultant moment about P.

a



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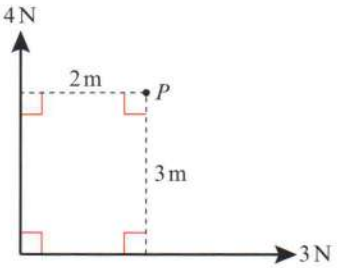

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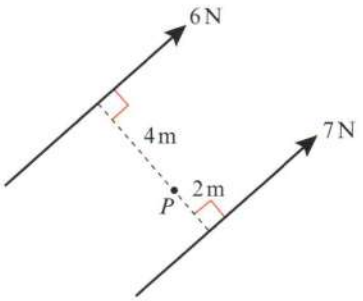

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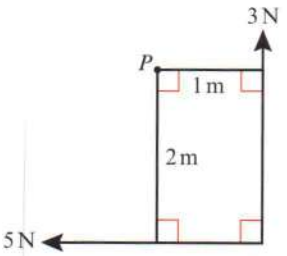

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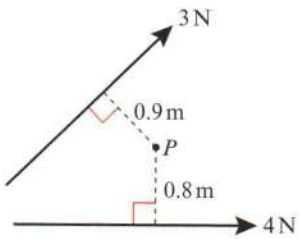

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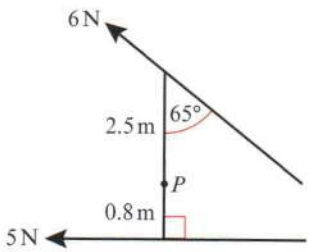
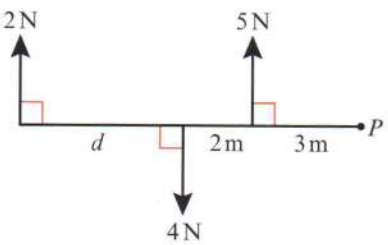
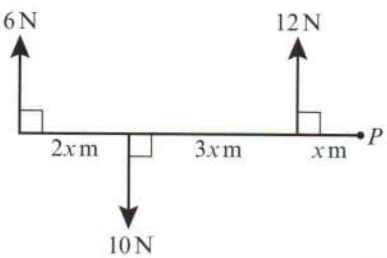


(P) **3** The diagram shows a set of forces acting on a light rod. The resultant moment about P is 17 N m clockwise. Find the length, d.



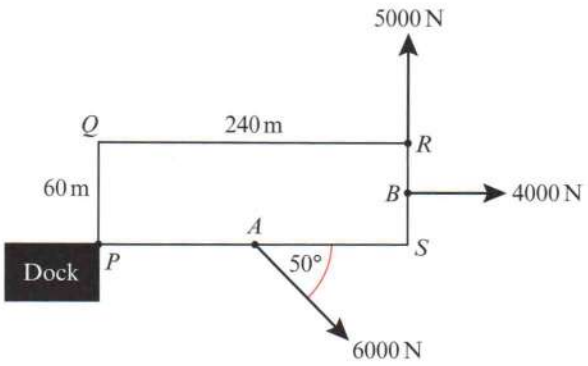
(E/P) **4** The diagram shows a set of forces acting on a light rod. The resultant moment about P is 12.8 N m clockwise. Find the value of x.



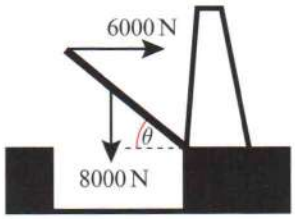
(3 marks)

E/P **5** A cruise ship is tethered to a dock and is being moved by three tugs. The cruise ship is modelled as a rectangular lamina $PQRS$ fixed at P under the action of three coplanar forces. A is the midpoint of PS and B is the midpoint of RS.

Determine the direction of the rotation of the cruise ship and the magnitude of the resultant moment about P. **(5 marks)**



E/P **6** The diagram shows a drawbridge inclined at an angle of θ to the horizontal, where $0 < \theta < 90°$. The drawbridge is modelled as a uniform rod of weight 8000 N. A horizontal force of magnitude 6000 N is applied at the top of the drawbridge. Given that the drawbridge is rising, prove that $\tan\theta > \frac{2}{3}$ **(5 marks)**



Hint The drawbridge is modelled as a uniform rod so its weight acts at its midpoint.

4.3 Equilibrium

- **When a rigid body is in equilibrium, the resultant force in any direction is 0 N and the resultant moment about any point is 0 N m.**

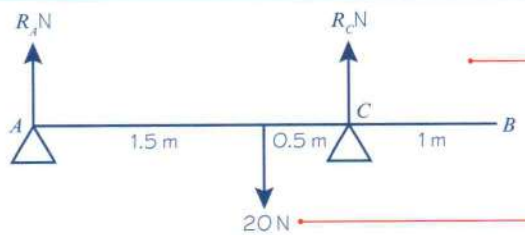
Hint If the resultant moment is zero then the sum of the clockwise moments equals the sum of the anticlockwise moments.

You can simplify many problems involving rigid bodies by choosing which point(s) to take moments about. When you take moments at a given point, you can ignore the rotational effect of any forces acting at that point.

Example 6

The diagram shows a uniform rod AB, of length 3 m and weight 20 N, resting horizontally on supports at A and C, where $AC = 2$ m.

Calculate the magnitude of the reaction at each of the supports.



Draw a diagram showing all the forces acting.

The weight of the rod acts at its centre of mass. You are told that this is a uniform rod, so the weight acts at the midpoint of the rod.

Total of forces acting upwards = total of forces acting downwards.

The rod is in equilibrium.

Resolving vertically: $R_A + R_C = 20$

Considering the moments about point A:

$20 \times 1.5 = R_C \times (1.5 + 0.5)$

$30 = 2R_C$

$15 = R_C$

$R_A + 15 = 20$

$R_A = 5$

Therefore the reaction at A is 5 N and the reaction at C is 15 N.

Clockwise moment = anticlockwise moment

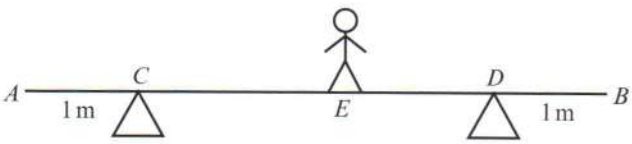
Problem-solving

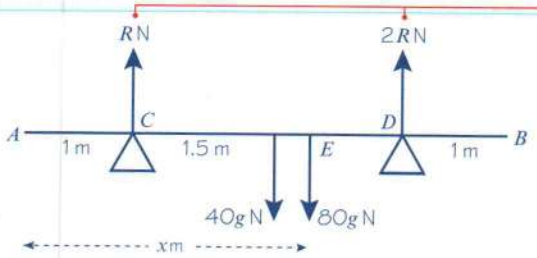
Take moments about the point that makes the algebra as simple as possible. Taking moments about A results in an equation with just one unknown.

Substituting the value of R_C into the first equation.

Example 7

A uniform beam AB, of mass 40 kg and length 5 m, rests horizontally on supports at C and D, where $AC = DB = 1$ m. When a man of mass 80 kg stands on the beam at E the magnitude of the reaction at D is twice the magnitude of the reaction at C. By modelling the beam as a rod and the man as a particle, find the distance AE.





Draw a diagram showing the forces.

Because you are told a relationship between the reaction at C and the reaction at D you can use this on your diagram.

Resolving vertically:

$R + 2R = 40g + 80g$

$3R = 120g$

$R = 40g$

The rod is in equilibrium so there is no resultant force.

Let the distance AE be x m.

Taking moments about A:

$40g \times 2.5 + 80g \times x = 40g \times 1 + 80g \times 4$

$100g + 80g \times x = 360g$

$80g \times x = 260g$

$\Rightarrow x = \frac{260g}{80g} = \frac{26}{8} = 3.25$

Distance $AE = 3.25$ m

Clockwise moment = anticlockwise moment

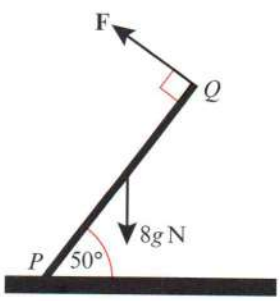
Problem-solving

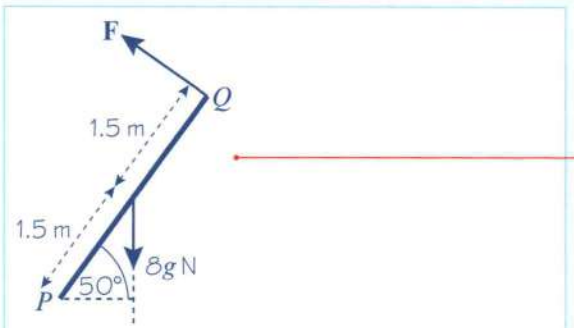
How have you used the modelling assumptions in the question?

- Since the beam is a rod, you can ignore its width
- Since the man is a particle, his weight acts at the point E

Example 8

A uniform rod PQ is hinged at the point P, and is held in equilibrium at an angle of 50° to the horizontal by a force of magnitude **F** acting perpendicular to the rod at Q. Given that the rod has a length of 3 m and a mass of 8 kg, find the value of **F**.





Taking moments about P:

$3 \times F = 8g \times 1.5 \cos 50°$

$3F = 75.5918...$

$F = 25$ N (2 s.f.)

If you take moments about P then you can ignore the reaction at the hinge.

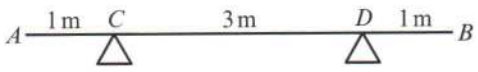
Find the perpendicular distance from P to the line of action of the weight of the rod.

You have used $g = 9.8\ \text{N kg}^{-1}$ in your calculations, so round your final answer to 2 s.f.

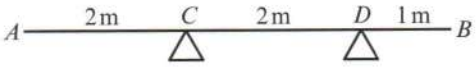
Exercise 4C

1 AB is a uniform rod of length 5 m and weight 20 N. In these diagrams AB is resting in a horizontal position on supports at C and D. In each case, find the magnitudes of the reactions at C and D.

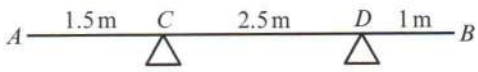
a



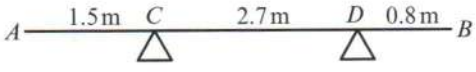
b



c

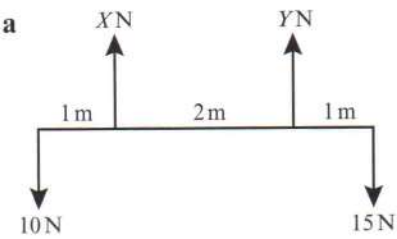


d

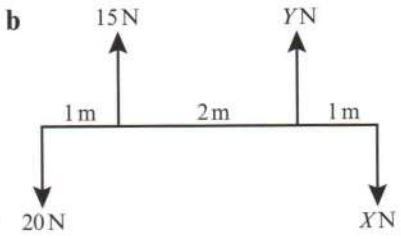


2 Each of these diagrams shows a light rod in equilibrium in a horizontal position under the action of a set of forces. Find the values of the unknown forces and distances.

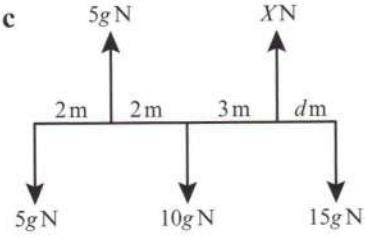
a



b



c



3 Jack and Jill are playing on a seesaw made from a uniform plank AB, of length 5 m pivoted at M, the midpoint of AB. Jack has mass 35 kg and Jill has mass 28 kg. Jill sits at A and Jack sits at a distance x m from B. The plank is in equilibrium. Find the value of x.

4 A uniform rod AB, of length 3 m and mass 12 kg, is pivoted at C, where $AC = 1$ m. A vertical force **F** applied at A maintains the rod in horizontal equilibrium. Calculate the magnitude of **F**.

5 A broom consists of a broomstick of length 130 cm and mass 5 kg and a broomhead of mass 5.5 kg attached at one end. By modelling the broomstick as a uniform rod and the broomhead as a particle, find where a support should be placed so that the broom will balance horizontally.

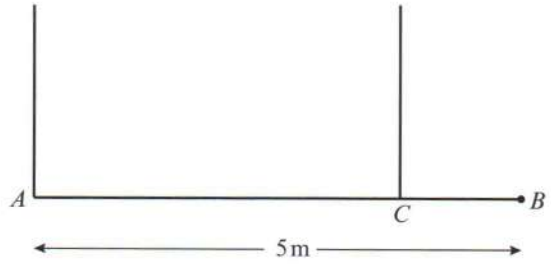
(P) **6** A uniform rod AB, of length 4 m and weight 20 N, is suspended horizontally by two vertical strings attached at A and at B. A particle of weight 10 N is attached to the rod at point C, where $AC = 1.5$ m.

a Find the magnitudes of the tensions in the two strings.

The particle is moved so that the rod remains in horizontal equilibrium with the tension in the string at B 1.5 times the tension in the string at A.

b Find the new distance of the particle from A.

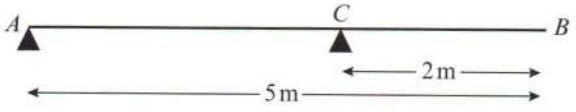
(E/P) **7** A uniform beam AB, of length 5 m and mass 60 kg, has a load of 40 kg attached at B. It is then held horizontally in equilibrium by two vertical wires attached at A and C. The tension in the wire at C is four times the tension in the wire at A. By modelling the beam as a uniform rod and the load as a particle, calculate:



a the tension in the wire at C **(2 marks)**

b the distance CB. **(5 marks)**

(E) **8** A uniform plank AB has length 5 m and mass 15 kg. The plank is held in equilibrium horizontally by two smooth supports A and C as shown in the diagram, where $BC = 2$ m.

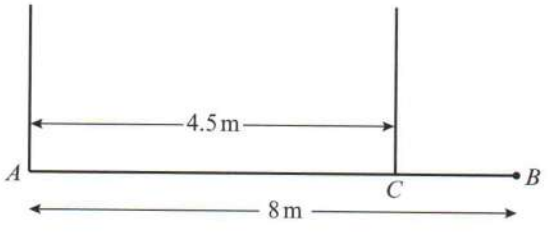


a Find the reaction on the plank at C. **(3 marks)**

A person of mass 45 kg stands on the plank at the point D and it remains in equilibrium. The reactions on the plank at A and C are now equal.

b Find the distance AD. **(7 marks)**

(E/P) **9** A uniform beam AB has weight W N and length 8 m. The beam is held in a horizontal position in equilibrium by two vertical light inextensible wires attached to the beam at the points A and C where $AC = 4.5$ m, as shown in the diagram. A particle of weight 30 N is attached to the beam at B.



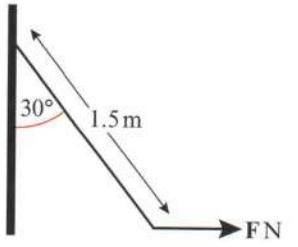
a Show that the tension in the wire attached to the beam at C is $\left(\frac{8}{9}W + \frac{160}{3}\right)$ N. **(4 marks)**

b Find, in terms of W, the tension in the wire attached to the beam at A. **(3 marks)**

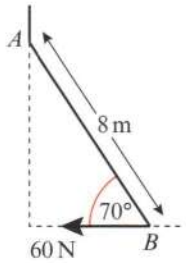
Given that the tension in the wire attached to the beam at C is twelve times the tension in the wire attached to the beam at A,

c find the value of W. **(3 marks)**

(E/P) **10** A metal lever of mass 5 kg and length 1.5 m is attached by a smooth hinge to a vertical wall. The lever is held at an angle of 30° to the vertical by a horizontal force of magnitude **F** N applied at the other end of the lever. By modelling the lever as a uniform rod, find the value of **F**. **(4 marks)**



(E) **11** A uniform ladder, AB, is leaning against a smooth vertical wall on rough horizontal ground at an angle of 70° to the horizontal. The ladder has length 8 m, and is held in equilibrium by a frictional force of magnitude 60 N acting horizontally at B, as shown in the diagram.



a Write down the magnitude of the normal reaction of the wall on the ladder at A. **(1 mark)**

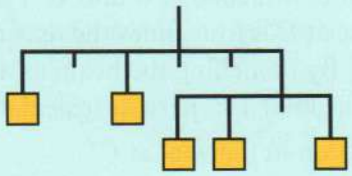
b Find the mass of the ladder. **(4 marks)**

Problem-solving

In part **b** you can ignore the normal reaction at B by taking moments about that point.

Challenge

The diagram shows a kinetic sculpture made from hanging rods. The distances between the points marked on each rod are equal. Arrange 1 kg, 2 kg, 3 kg, 4 kg and 5 kg weights onto the marked squares, using each weight once, so that the sculpture hangs in equilibrium with the rods horizontal.



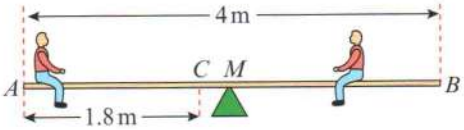
4.4 Centres of mass

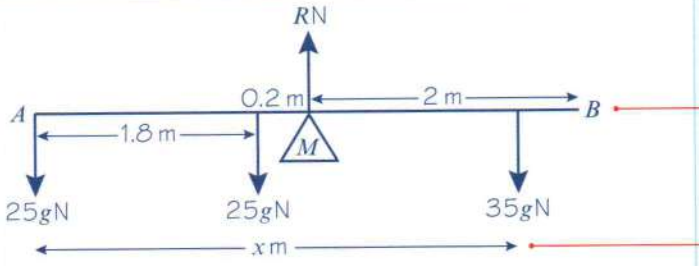
So far you have only considered **uniform** rods, where the centre of mass is always at the midpoint. If a rod is **non-uniform** the centre of mass is not necessarily at the midpoint of the rod.

You might need to consider the moment due to the weight of a non-uniform rod, or find the position of its centre of mass.

Example 9

Sam and Tamsin are sitting on a non-uniform plank AB of mass 25 kg and length 4 m. The plank is pivoted at M, the midpoint of AB. The centre of mass of AB is at C where AC is 1.8 m. Sam has mass 35 kg.
Tamsin has mass 25 kg and sits at A.
Where must Sam sit for the plank to be horizontal?





Model the plank as a rod and the children as particles. Then draw a diagram to represent the situation.

Suppose that Sam sits at a point x m from A.

Take moments about M:

$25g \times 2 + 25g \times 0.2 = 35g \times (x - 2)$

Take moments about M to eliminate the reaction at M from your calculations.

$50 + 5 = 35x - 70$

$35x = 125$

Divide both sides of the equation by g.

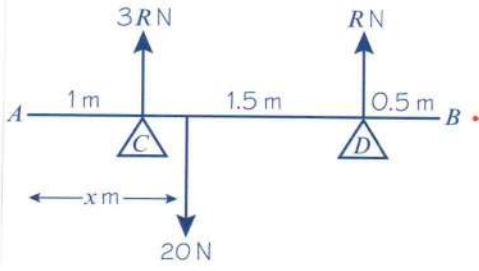
$x = 3.57$

Sam should sit 3.57 m from end A (or 0.43 m from end B).

Online Explore the moment acting about pivot M using GeoGebra.

Example 10

A non-uniform rod AB is 3 m long and has weight 20 N. It is in a horizontal position resting on supports at points C and D, where $AC = 1$ m and $AD = 2.5$ m. The magnitude of the reaction at C is three times the magnitude of the reaction at D. Find the distance of the centre of mass of the rod from A.



Draw a diagram. Make sure that you have used all the information given in the question.

Suppose that the centre of mass acts at a point x m from A.

Resolving vertically, $3R + R = 20$

$R = 5$

Whichever point you choose to take moments about, you are going to need to know the magnitude of R.

Taking moments about A:

$20 \times x = 15 \times 1 + 5 \times 2.5$

$20x = 27.5$

$x = 1.38$ (3 s.f.)

Use your value of R.

The centre of mass is 1.38 m from A, to 3 s.f.

Exercise 4D

1 A non-uniform rod AB, of length 4 m and weight 6 N, rests horizontally on two supports, A and B. Given that the centre of mass of the rod is 2.4 m from the end A, find the reactions at the two supports.

2 A non-uniform bar AB of length 5 m is supported horizontally on supports, A and B. The reactions at these supports are $3g$ N and $7g$ N respectively.

a State the weight of the bar.

b Find the distance of the centre of mass of the bar from A.

3 A non-uniform plank AB, of length 4 m and weight 120 N, is pivoted at its midpoint. The plank is in equilibrium in a horizontal position with a child of weight 200 N sitting at A and a child of weight 300 N sitting at B. By modelling the plank as a rod and the two children as particles, find the distance of the centre of mass of the plank from A.

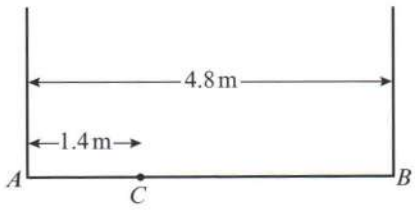
(P) **4** A non-uniform rod AB, of length 5 m and mass 15 kg, rests horizontally suspended from the ceiling by two vertical strings attached to C and D, where $AC = 1$ m and $AD = 3.5$ m.

a Given that the centre of mass is at E where $AE = 3$ m, find the magnitudes of the tensions in the strings.

When a particle of mass 9 kg is attached to the rod at F the tension in the string at D is twice the tension in the string at C.

b Find the distance AF.

E/P **5** A plank AB has mass 24 kg and length 4.8 m. A load of mass 15 kg is attached to the plank at the point C, where $AC = 1.4$ m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B, as shown in the diagram. The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.

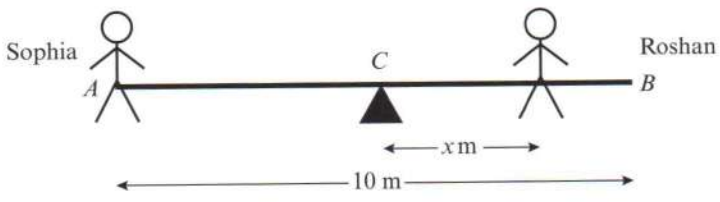


a Find the tension in the rope attached at B. **(4 marks)**

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 25 N greater than the tension in the rope attached at B.

b Find the distance of the centre of mass of the plank from A. **(6 marks)**

E **6** A seesaw in a playground consists of a beam AB of length 10 m which is supported by a smooth pivot at its centre C. Sophia has mass 30 kg and sits on the end A. Roshan has mass 50 kg and sits at a distance x metres from C, as shown in the diagram. The beam is initially modelled as a uniform rod. Using this model,



a find the value of x for which the seesaw can rest in equilibrium in a horizontal position. **(3 marks)**

b State what is implied by the modelling assumption that the beam is uniform. **(1 mark)**

Roshan finds he must sit at a distance 4 m from C for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 25 kg. Using this model,

c find the distance of the centre of mass of the beam from C. **(4 marks)**

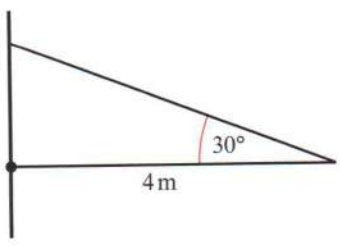
E/P **7** A non-uniform rod AB, of length 25 m and weight 80 N, rests horizontally in equilibrium on supports C and D as shown in the diagram. The centre of mass of the rod is 10 m from A.

A particle of weight W newtons is attached to the rod at a point E, where E is x metres from A.

The rod remains in equilibrium and the magnitude of the reaction at C is five times the magnitude of the reaction at D.

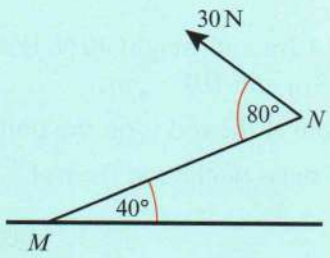
Show that $W = \dfrac{400}{25 - 3x}$ **(5 marks)**

E/P **8** A non-uniform rod of weight 100 N and length 4 m is freely hinged to a vertical wall, and held in place by a cable attached at an angle of 30° to the end of the rod. Given that the tension in the cable is 80 N and that the rod is held in horizontal equilibrium, find the distance of the centre of mass of the rod from the wall. **(8 marks)**



Challenge

A non-uniform beam of weight 120 N and length 5 m is smoothly pivoted at a point M and is held at an angle of 40° by a cable attached at point N. Given that the tension in the cable is 30 N and it makes an angle of 80° with the beam, find the distance of the centre of mass of the beam from M.

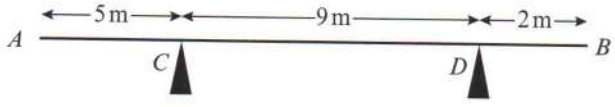


4.5 Tilting

You need to be able to answer questions involving rods that are on the point of tilting.

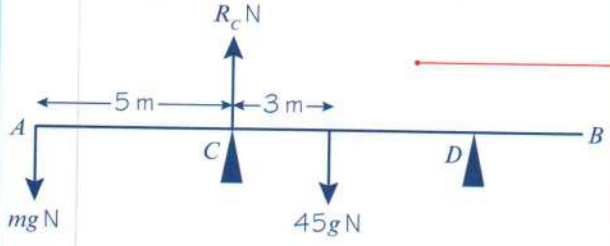
- **When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or the tension in any other wire or string) is zero.**

Example 11



A uniform beam AB, of mass 45 kg and length 16 m, rests horizontally on supports C and D where $AC = 5$ m and $CD = 9$ m.

When a child stands at A, the beam is on the point of tilting about C.
Find the mass of the child.



Taking moments about C:

$mg \times 5 = 3 \times 45g$

$5mg = 135g$

$$m = \frac{135g}{5g} = 27$$

The mass of the child is 27 kg.

Draw a diagram showing the forces. Remember, as the beam is 'on the point of tilting', C is a pivot and the reaction force at D is zero.

Take moments about C. This means you can ignore the reaction R_C.

Anticlockwise moment = Clockwise moment

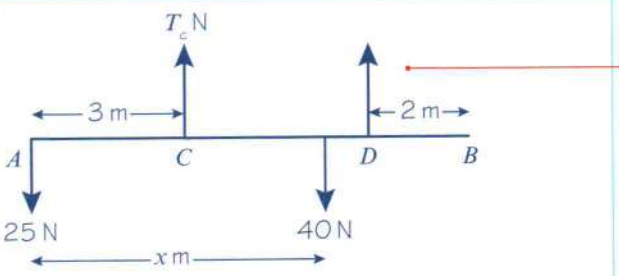
Online See the point at which the beam starts to tilt due to the weight of the child and explore the problem with different forces and distances using GeoGebra.

Example 12

A non-uniform rod AB, of length 10 m and weight 40 N, is suspended from a pair of light cables attached to C and D where $AC = 3$ m and $BD = 2$ m.

When a weight of 25 N is hung from A the rod is on the point of rotating.

Find the distance of the centre of mass of the rod from A.



Taking moments about C:

$25 \times 3 = 40 \times (x - 3)$

$75 = 40x - 120$

$40x = 195$

$x = 4.875$

Distance of the centre of mass from A is 4.875 m.

Draw a diagram showing the forces about C. As the rod is 'on the point of rotating', C is a pivot and the tension force at D is zero. As the rod is non-uniform you don't know the distance from A to the centre of mass, so you can label it as x.

You don't require the tension force, T_C, so take moments about C.

Problem-solving

The distance from C to the centre of mass is $(x - 3)$ m. Equate the clockwise and anticlockwise moments then solve the equation to find the value of x.

Exercise 4E

1 A uniform rod AB has length 4 m and mass 8 kg. It is resting in a horizontal position on supports at points C and D where $AC = 1$ m and $AD = 2.5$ m. A particle of mass m kg is placed at point E where $AE = 3.3$ m. Given that the rod is about to tilt about D, calculate the value of m.

2 A uniform bar AB, of length 6 m and weight 40 N, is resting in a horizontal position on supports at points C and D where $AC = 2$ m and $AD = 5$ m. When a particle of weight 30 N is attached to the bar at point E the bar is on the point of tilting about C. Calculate the distance AE.

3 A plank AB, of mass 12 kg and length 3 m, is in equilibrium in a horizontal position resting on supports at C and D where $AC = 0.7$ m and $DB = 1.1$ m. A boy of mass 32 kg stands on the plank at point E. The plank is about to tilt about D. By modelling the plank as a uniform rod and the boy as a particle, calculate the distance AE.

(P) **4** A uniform rod AB has length 5 m and weight 20 N. The rod is resting on supports at points C and D where $AC = 2$ m and $BD = 1$ m.

a Find the magnitudes of the reactions at C and D.

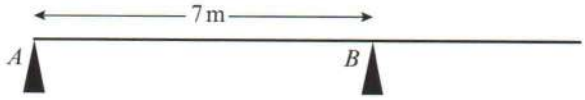
A particle of weight 12 N is placed on the rod at point A.

b Show that this causes the rod to tilt about C.

A second particle of weight 100 N is placed on the rod at E to hold it in equilibrium.

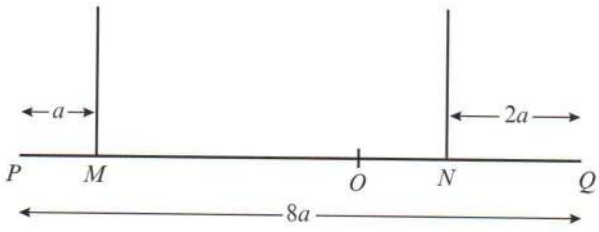
c Find the minimum and maximum possible distances of E from A.

E **5** A uniform plank of mass 100 kg and length 10 m rests horizontally on two smooth supports, A and B, as shown in the diagram. A man of mass 80 kg starts walking from one end of the plank, A, to the other end.



Find the distance he can walk past B before the plank starts to tip. **(4 marks)**

E/P **6** A non-uniform beam PQ, of mass m kg and length $8a$, hangs horizontally in equilibrium from two wires at M and N, where $PM = a$ and $QN = 2a$, as shown in the diagram. The centre of mass of the beam is at the point O. A particle of mass $\frac{3}{4}m$ kg is placed on the beam at Q and the beam is on the point of tipping about N.

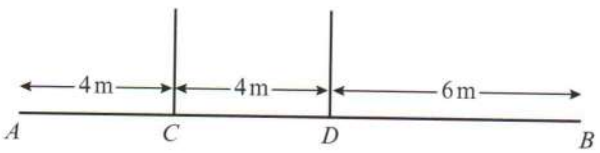


a Show that $ON = \frac{3}{2}a$. **(3 marks)**

The particle is removed and replaced at the midpoint of the beam and the beam remains in equilibrium.

b Find the magnitude of the tension in the wire attached at point N in terms of m. **(5 marks)**

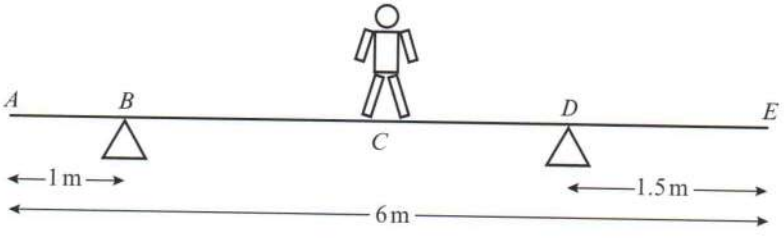
E/P **7** A uniform beam AB, of weight W and length 14 m, hangs in equilibrium in a horizontal position from two vertical cables attached at points C and D where $AC = 4$ m and $BD = 6$ m.



A weight of 180 N is hung from A and the beam is about to tilt. The weight is removed and a different weight, V, is hung from B and the beam does not tilt. Find the maximum value of V. **(6 marks)**

Mixed exercise 4

E **1** A plank AE, of length 6 m and weight 100 N, rests in a horizontal position on supports at B and D, where $AB = 1$ m and $DE = 1.5$ m. A child of weight 145 N stands at C, the midpoint of AE, as shown in the diagram. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate:



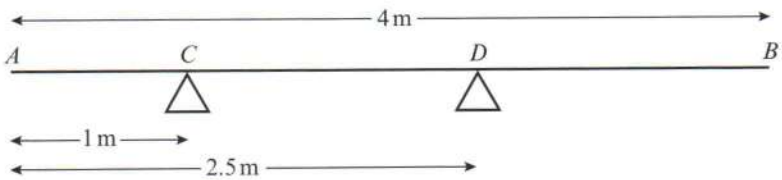
a the magnitude of the force exerted by the support on the plank at B **(3 marks)**

b the magnitude of the force exerted by the support on the plank at D. **(2 marks)**

The child now stands at a different point F on the plank. The plank is in equilibrium and on the point of tilting about D.

c Calculate the distance DF. **(4 marks)**

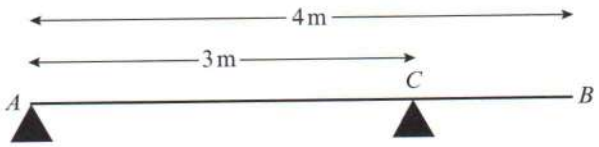
E/P **2** A uniform rod AB has length 4 m and weight 150 N. The rod rests in equilibrium in a horizontal position, smoothly supported at points C and D, where $AC = 1$ m and $AD = 2.5$ m as shown in the diagram. A particle of weight W N is attached to the rod at a point E where $AE = x$ metres. The rod remains in equilibrium and the magnitude of the reaction at C is now equal to the magnitude of the reaction at D.



a Show that $W = \dfrac{150}{7 - 4x}$ **(6 marks)**

b Hence deduce the range of possible values of x. **(3 marks)**

E **3** A uniform plank AB has mass 40 kg and length 4 m. It is supported in a horizontal position by two smooth pivots. One pivot is at the end A and the other is at the point C where $AC = 3$ m, as shown in the diagram.



A man of mass 80 kg stands on the plank which remains in equilibrium. The magnitude of the reaction at A is twice the magnitude of the reaction at C. The magnitude of the reaction at C is R N. The plank is modelled as a rod and the man is modelled as a particle.

a Find the value of R. **(2 marks)**

b Find the distance of the man from A. **(3 marks)**

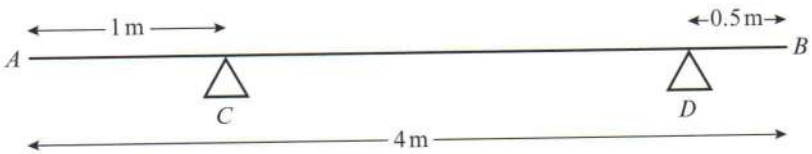
c State how you have used the modelling assumption that:

i the plank is uniform

ii the plank is a rod

iii the man is a particle. **(3 marks)**

E/P **4** A non-uniform rod AB has length 4 m and weight 150 N. The rod rests horizontally in equilibrium on two smooth supports C and D, where $AC = 1$ m and $DB = 0.5$ m, as shown in the diagram. The centre of mass of AB is x metres from A. A particle of weight W N is placed on the rod at A. The rod remains in equilibrium and the magnitude of the reaction of C on the rod is 100 N.



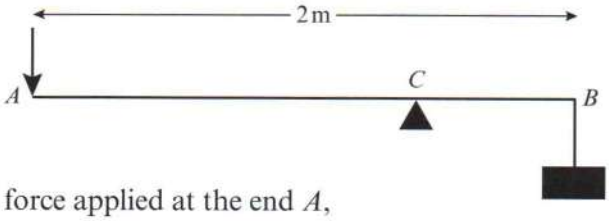
a Show that $550 + 7W = 300x$. **(4 marks)**

The particle is now removed from A and placed on the rod at B. The rod remains in equilibrium and the reaction of C on the rod now has magnitude 52 N.

b Obtain another equation connecting W and x. **(4 marks)**

c Calculate the value of x and the value of W. **(3 marks)**

(E) **5** A lever consists of a uniform steel rod AB, of weight 100 N and length 2 m, which rests on a small smooth pivot at a point C. A load of weight 1700 N is suspended from the end B of the rod by a rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at the end A, as shown in the diagram. The rope is modelled as a light string.



a Given that $BC = 0.25$ m find the magnitude of the force applied at A. **(4 marks)**

The position of the pivot is changed so that the rod remains in equilibrium when the force at A has magnitude 150 N.

b Find, to the nearest centimetre, the new distance of the pivot from B. **(4 marks)**

(E) **6** A plank AB has length 4 m. It lies on a horizontal platform, with the end A lying on the platform and the end B projecting over the edge, as shown in the diagram. The edge of the platform is at the point C.

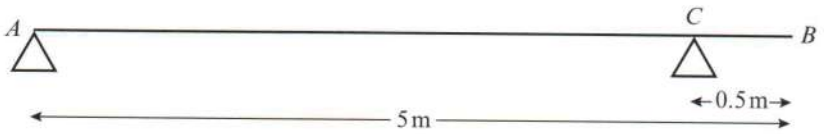
Jack and Jill are experimenting with the plank. Jack has mass 48 kg and Jill has mass 36 kg. They discover that if Jack stands at B and Jill stands at A and $BC = 1.8$ m, the plank is in equilibrium and on the point of tilting about C.

a By modelling the plank as a uniform rod, and Jack and Jill as particles, find the mass of the plank. **(4 marks)**

They now alter the position of the plank in relation to the platform so that, when Jill stands at B and Jack stands at A, the plank is again in equilibrium and on the point of tilting about C.

b Find the distance BC in this position. **(4 marks)**

(E) **7** A plank of wood AB has mass 12 kg and length 5 m. It rests in a horizontal position on two smooth supports. One support is at the end A. The other is at the point C, 0.5 m from B, as shown in the diagram. A girl of mass 30 kg stands at B with the plank in equilibrium.

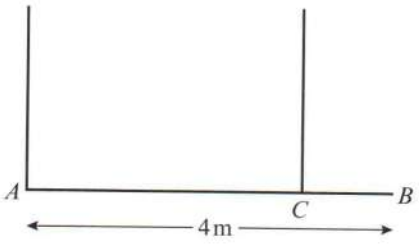


a By modelling the plank as a uniform rod and the girl as a particle, find the reaction on the plank at A. **(4 marks)**

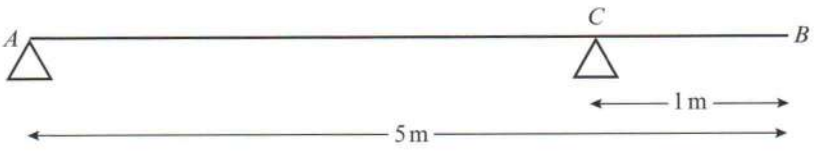
The girl gets off the plank. A boulder of mass m kg is placed on the plank at A and a man of mass 93 kg stands on the plank at B. The plank remains in equilibrium and is on the point of tilting about C.

b By modelling the plank again as a uniform rod, and the man and the boulder as particles, find the value of m. **(5 marks)**

(P) **8** A plank AB has mass 50 kg and length 4 m. A load of mass 25 kg is attached to the plank at B. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes attached at A and C, as shown in the diagram. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at C is four times the tension in the rope at A, calculate the distance CB. **(7 marks)**



(E) **9** A beam AB has weight 200 N and length 5 m. The beam rests in equilibrium in a horizontal position on two smooth supports.



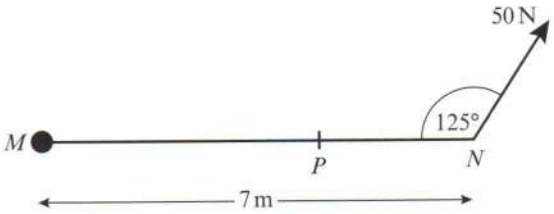
One support is at end A and the other is at a point C on the beam, where $BC = 1$ m, as shown in the diagram. The beam is modelled as a uniform rod.

a Find the reaction on the beam at C. **(4 marks)**

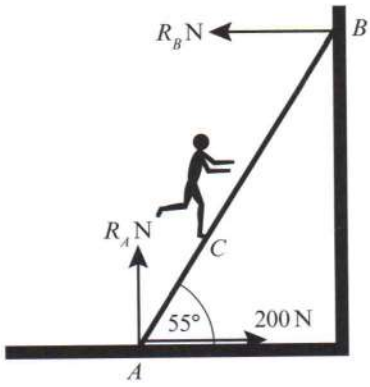
A woman of weight 500 N stands on the beam at the point D. The beam remains in equilibrium. The reactions on the beam at A and C are now equal.

b Find the distance AD. **(5 marks)**

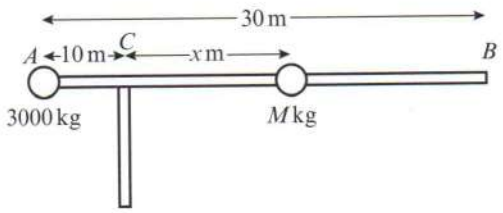
(E/P) **10** A non-uniform plank MN of length 7 m is attached to a pivot at M and is held in a horizontal position by a force of 50 N applied at N at an angle of 125° to the plank as shown in the diagram. The centre of mass of the plank is at the point P. Given that the plank is in equilibrium and has a mass of 6 kg, find the distance MP. **(4 marks)**



(E/P) **11** A ladder, AB, is leaning against a smooth vertical wall and on rough horizontal ground at an angle of 55° to the horizontal. The ladder has length 10 m and mass 20 kg. A man of mass 80 kg is standing at the point C on the ladder. Given that the magnitude of the frictional force at A is 200 N, find the distance AC. Model the ladder as a uniform rod and the man as a particle. **(5 marks)**



(E/P) **12** The beam of a crane is modelled as a uniform rod AB, of length 30 m and weight 4000 kg, resting in horizontal equilibrium. The beam is supported by a tower at C, where $AC = 10$ m. A counterbalance mass of weight 3000 kg is placed at A and a load of mass M is placed a variable distance x m from the supporting tower, where $x \geqslant 5$.



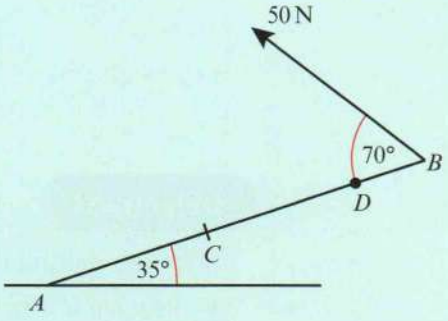
a Find an expression for M in terms of x. **(4 marks)**

b Hence determine the maximum and minimum loads that can be lifted by the crane. **(2 marks)**

c Criticise this model in relation to the beam. **(1 mark)**

Challenge

1 A non-uniform beam AB, of mass 10 kg and length 8 m, is pivoted at a point A. A particle of mass 2 kg is attached to the beam at a point D which is 1 m from B. The beam is held in equilibrium at an angle of 35° to the horizontal by a rope attached at point B. Given that the tension in the rope is 50 N and it makes an angle of 70° to the beam, find the distance of the centre of mass of the beam from A.

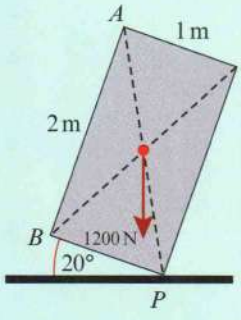


2 A builder is attempting to tip over a refrigerator. The refrigerator is modelled as a rectangular lamina of weight 1200 N. The centre of mass of the lamina is at the point of intersection of the diagonals of the rectangle, as shown in the diagram.

Given that the refrigerator is pivoting at vertex P and that the base of the refrigerator makes an angle of 20° to the floor, find the minimum force needed to tip the refrigerator if the force is applied:

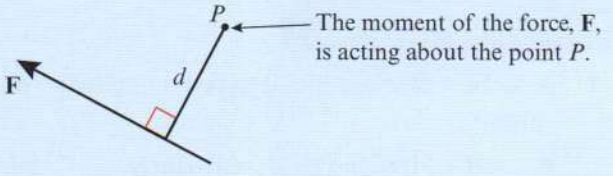
a horizontally at A

b vertically at B.

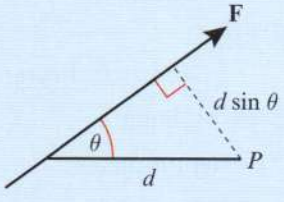


Summary of key points

1 Moment of **F** about $P = |\mathbf{F}| \times d$ clockwise



2 Moment of **F** about $P = |\mathbf{F}| \times d\sin\theta$ clockwise



3 The sum of the moments acting on a body is called the resultant moment.

4 When a rigid body is in equilibrium the resultant force in any direction is 0 N and the resultant moment about any point is 0 N m.

5 When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or the tension in any other wire or string) is zero.