Review exercise 1

(E) **1** An engineer is developing a new production process to assemble microswitches. The time taken, t minutes, to produce each switch, and its size, s mm^3, are recorded in the table.

Size, s mm^3	2.0	4.5	6.2	7.3	9.1
Time, t minutes	0.33	1.15	1.76	2.15	2.95

a Calculate the product moment correlation coefficient between $\log s$ and $\log t$. **(2)**

b Use your answer to part **a** to explain why an equation of the form $t = as^n$, where a and n are constants, is likely to be good model for the relationship between s and t. **(1)**

c The regression line of $\log t$ on $\log s$ is given as $\log t = -0.9051 + 1.4437 \log s$. Determine the values of the constants a and n in the equation given in part **b**. **(2)**

← Sections 1.1, 1.2

(E) **2** The table shows some data collected on the pressure, in pascals, of some gases (P) and the temperature (t °C).

Pressure (P)	**Temperature (t)**
45	3.65
73	11.01
81	15.24
90	21.95
102	35.21
115	58.43

The data is coded using the changes of variable $x = P$ and $y = \log t$. The regression line of y on x is found to be $y = -0.2139 + 0.0172x$.

a Given that the data can be modelled by an equation of the form $t = ab^P$ where a and b are constants, find, correct to three significant figures, the values of a and b. **(3)**

b Explain why this model is not reliable for estimating the temperature when the pressure is 250 pascals. **(1)**

← Section 1.1

(E) **3** Over a period of time, researchers took 10 blood samples from one patient with a blood disease. For each sample, they measured the levels of serum magnesium, s mg/dl, in the blood and the corresponding level of the disease protein, d mg/dl. The results are shown in the table.

s	1.2	1.9	3.2	3.9	2.5	4.5	5.7	4.0	1.1	5.9
d	3.8	7.0	11.0	12.0	9.0	12.0	13.5	12.2	2.0	13.9

a Calculate the value of the product moment correlation coefficient between s and d for this sample. **(1)**

b Stating your hypotheses clearly, test, at the 1% significance level, whether there is a positive correlation between serum magnesium and disease protein levels in this patient. **(3)**

← Sections 1.2, 1.3

(E) **4** A sailor from Jacksonville believes there is a negative correlation between the daily mean air temperature and the daily mean windspeed. He collects data from 6 days in June 2015.

Temperature (°C)	Windspeed (knots)
24.8	4.9
23.3	2.6
22.7	7.6
24.7	4.5
24.5	4.3
25.1	4.0

By calculating the product moment correlation coefficient for this data, test at the 10% level of significance, whether there is evidence to support the sailor's claim. State your hypotheses clearly. **(4)**

← Sections 1.2, 1.3

E/P **5** Data about the number of miles done by a sample of one-year-old cars and their value is collected from a dealer. The dealer believes there is a negative correlation between the number of miles done and the value.

Number of miles	2000	3500	4200	6500	7800
Value (£, 000)	12	9.1	8.2	7.7	6.1

a Test at the 2.5% level of significance the dealer's claim. State your hypotheses clearly. **(4)**

b State the effect that changing the level of significance to 1% would have on the dealer's conclusion. **(1)**

← Sections 1.2, 1.3

E/P **6** At a college, there are 148 students studying either engineering, childcare or tourism. Of these students, 89 wear glasses and the others do not. There are 30 engineering students of whom 18 wear glasses. There are 68 childcare students, of whom 44 wear glasses.

A student is chosen at random.

Find the probability that this student:

a is studying tourism **(1)**

b does not wear glasses, given that the student is studying tourism. **(2)**

Amongst the engineering students, 80% are right-handed. Corresponding percentages for childcare and tourism students are 75% and 70% respectively.

A student is again chosen at random.

c Find the probability that this student is right-handed. **(2)**

d Given that this student is right-handed, find the probability that the student is studying engineering. **(2)**

← Sections 2.2, 2.4

E **7** A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left-handed and having dark hair.

Glasses were worn by 36 people, 28 were left-handed and 36 had dark hair. There were 17 who wore glasses and were left-handed, 19 who wore glasses and had dark hair and 15 who were left-handed and had dark hair. Only 10 people wore glasses, were left-handed and had dark hair.

a Represent these data on a Venn diagram. **(3)**

A person was selected at random from this group.

Find the probability that this person:

b wore glasses but was not left-handed and did not have dark hair **(1)**

c did not wear glasses, was not left-handed and did not have dark hair **(1)**

d had only two of the attributes **(2)**

e wore glasses given they were left-handed and had dark hair. **(2)**

← Sections 2.3, 2.4

E **8** A survey of the reading habits of some students revealed that, on a regular basis, 25% read broadsheet newspapers, 45% read tabloid newspapers and 40% do not read newspapers at all.

a Find the proportion of students who read both broadsheet and tabloid newspapers. **(2)**

b Draw a Venn diagram to represent this information. **(3)**

A student is selected at random. Given that this student reads newspapers on a regular basis,

c find the probability that this student only reads broadsheet newspapers. **(2)**

←Sections 2.3, 2.4

E **9** A bag contains 3 blue counters and 5 red counters. One counter is drawn at random from the bag and not replaced. A second counter is then drawn.

a Draw a tree diagram to represent this situation. **(2)**

b Find the probability that:

i the second counter drawn is blue. **(2)**

ii both counters selected are blue, given that the second counter is blue. **(2)**

← Sections 2.4, 2.5

E **10** For the events A and B, $P(A \cap B') = 0.34$, $P(A' \cap B) = 0.13$ and $P(A \cup B) = 0.62$.

a Draw a Venn diagram to illustrate the complete sample space for the events A and B. **(2)**

b Write down the values of $P(A)$ and $P(B)$. **(2)**

c Find $P(A|B')$. **(2)**

d Determine whether or not A and B are independent. **(2)**

← Sections 2.1, 2.3, 2.4

E/P **11** Two events A and B are such that $P(B) = 0.3$ and $P(A \cap B) = 0.15$. If A and B are independent, find:

a $P(A)$ **(1)**

b $P(A' \cap B')$ **(1)**

A third event C has $P(C) = 0.4$. Given that B and C are mutually exclusive and $P(A \cap C) = 0.1$,

c Draw a Venn diagram to illustrate this situation. **(2)**

d Find:

i $P(A|C)$ **(2)**

ii $P(A \cap (B \cup C'))$ **(2)**

iii $P(A|(B \cup C'))$ **(2)**

← Sections 2.1, 2.3, 2.4

E/P **12** The probability that Joanna oversleeps is 0.15. If she oversleeps, the probability that she is late to college is 0.75. If she gets up on time, the probability that she is late to college is 0.1.

a Find the probability that Joanna is late to college on any particular day. **(2)**

b Find the probability that Joanna overslept given that she is late to college. **(2)**

← Sections 2.4, 2.5

E/P **13** A random variable X is assumed to be normally distributed with mean 100 and standard deviation 15.

a Find the probability that X is less than 91. **(1)**

The probability that X is at least $100 + k$ is 0.2090.

b Find, to the nearest integer, the value of k. **(2)**

← Sections 3.2, 3.3

E/P **14** The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg.

Find the probability that a randomly chosen athlete:

a is taller than 188 cm **(1)**

b weighs less than 97 kg. **(1)**

c Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg. **(2)**

d Comment on the assumption that height and weight are independent. **(1)**

←Section 3.2

E/P **15** From experience a high jumper knows that he can clear a height of at least 1.78 m once in five attempts. He also knows that he can clear a height of at least 1.65 m on seven out of 10 attempts.

Assuming that the heights cleared by the high jumper follow a normal distribution,

a find, to three decimal places, the mean and the standard deviation of the heights cleared by the high jumper **(3)**

b calculate the probability that the high jumper will clear a height of 1.74 m. **(1)**

← Sections 3.2, 3.5

E/P **16** A company makes dinner plates with an average diameter of 22 cm. The diameter, D cm, can be modelled using a normal distribution. Given that 32% of plates are less than 21.5 cm in diameter, find

a the standard deviation of the diameter **(2)**

b the proportion of plates with diameter between 21 cm and 22.5 cm. **(2)**

A plate can be used in a restaurant if the diameter is between 21 cm and 22.5 cm. A sample of 30 plates is taken.

c Find the probability that at least 10 of these plates can be used. **(2)**

← Section 3.5

E/P **17** For a particular type of plant 45% have white flowers and the remainder have coloured flowers. Gardenmania sells plants in batches of 12. A batch is selected at random.

Calculate the probability this batch contains:

a exactly 5 plants with white flowers **(1)**

b more plants with white flowers than coloured ones. **(2)**

Gardenmania takes a random sample of 10 batches of plants.

c Find the probability that exactly 3 of these batches contain more plants with white flowers than coloured ones. **(2)**

Due to an increasing demand for these plants by large companies, Gardenmania decides to sell them in batches of 150.

d Use a suitable approximation to calculate the probability that a batch of 150 plants contains more than 75 with white flowers. **(3)**

←Section 3.6

E/P **18** At a school fair, the probability of getting a winning ticket on the tombola is 0.48. A random sample of 80 tickets is taken.

a Find the probability that there are exactly 35 winning tickets, giving your answer to 5 decimal places. **(2)**

b Find the percentage error when using a normal approximation to calculate the probability that there are exactly 35 winning tickets. **(4)**

← Section 3.6

E **19** The time, in minutes, it takes Robert to complete the puzzle in his morning newspaper each day is normally distributed with mean 18 and standard deviation 3. After taking a holiday, Robert records the times taken to complete a random sample of 15 puzzles and he finds that the mean time is 16.5 minutes. You may assume that the holiday has not changed the standard deviation of times taken to complete the puzzle.

Stating your hypotheses clearly test, at the 5% level of significance, whether or not there has been a reduction in the mean time Robert takes to complete the puzzle. **(4)**

← Section 3.7

P **20** The length of adult male rattlesnakes, L metres, is normally distributed with standard deviation 0.4 metres.

Given that 5% of rattlesnakes have a length less than 1.7 metres,

a find the mean length of a rattlesnake. **(3)**

Ten rattlesnakes are chosen at random.

b Find the probability that at least 6 have a length greater than 2.3 metres. **(3)**

Adult female rattlesnakes are of length M metres, where M is normally distributed with standard deviation 0.3 metres.

Adult female rattlesnakes are thought to have a mean length of 1.9 metres. Given that a random sample of 20 female rattlesnakes is taken,

c find, at the 5% level of significance, the critical regions for a hypothesis test that the mean length is not equal to 1.9 metres. State your hypotheses clearly. **(4)**

d The mean length of the sample of 20 female rattlesnakes is found to be 2.09 metres. Comment on this observation in the light of your answer to part **c**. **(2)**

← Sections 3.5, 3.7

E **21** Daily mean temperature in Hurn is modelled as being normally distributed with a standard deviation of 2.3 °C.

A random sample of 20 recorded daily mean temperatures is taken in 2015.

Given that the mean of the sample is 11.1 °C, test at the 5% level of significance whether the mean of the daily mean temperatures is less than 12 °C. State your hypotheses clearly. **(4)**

← Section 3.7

Challenge

1 $P(A) = 0.7$ and $P(B) = 0.3$.

a Given that $P(A \cap B') = p$, find the range of possible values of p.

$P(C) = 0.5$ and $P(A \cap B \cap C) = 0.05$.

b Given that $P(A' \cap B \cap C) = q$, find the range of possible values of q. ← Sections 2.3, 2.4

2 A politician claims to have the support of 53% of constituents. A random sample of 300 constituents was taken, and is used to test the politician's claim at the 10% significance level.

a Use a suitable approximation to find the critical regions for this test. Round your answers to 2 decimal places.

It was found that 173 constituents said they supported the politician.

b Using your answer to part **a**, comment on the politician's claim. ← Sections 3.6, 3.7