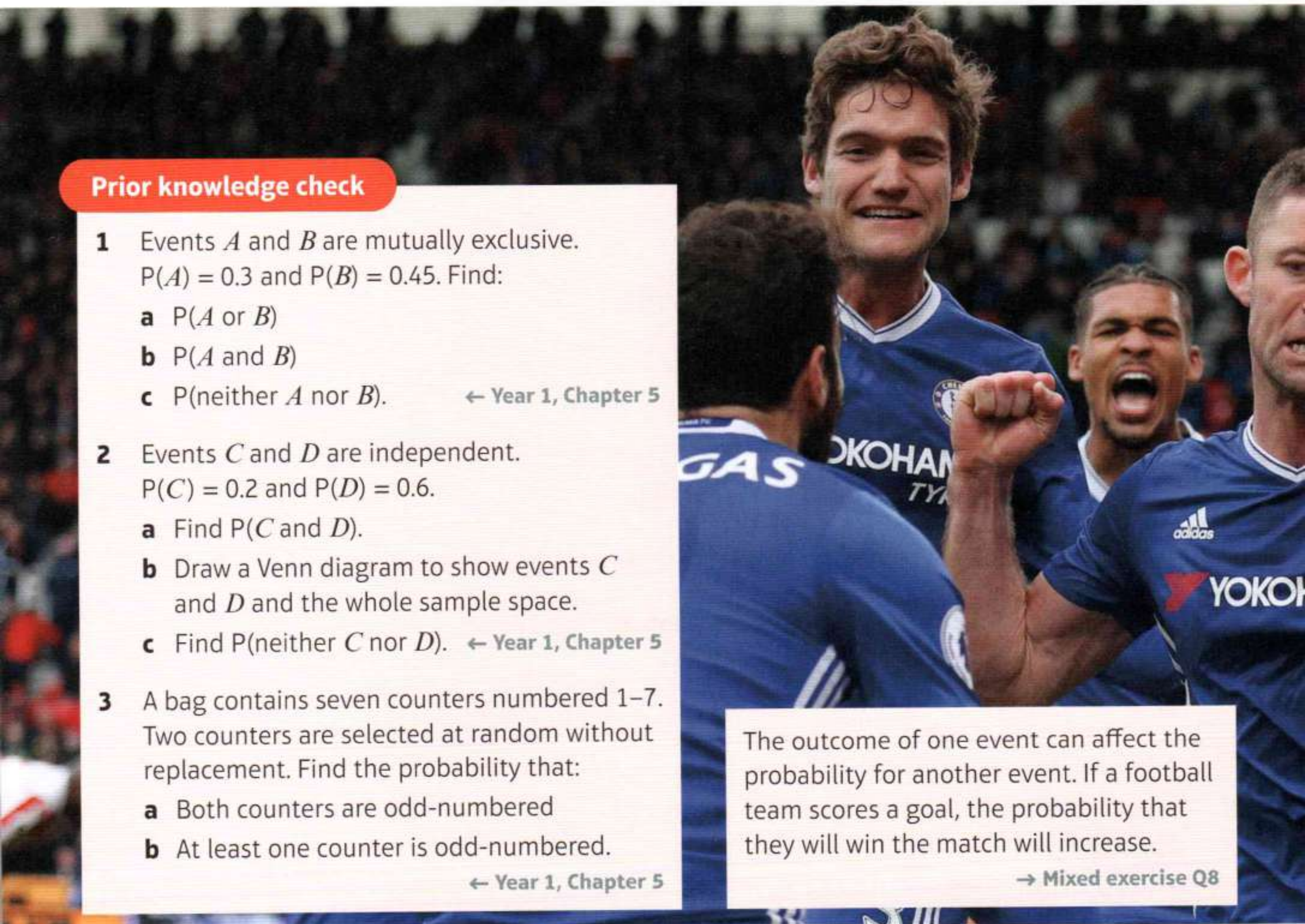
2 Conditional probability

Objectives

After completing this chapter you should be able to:

- Understand set notation in probability → pages 17–21
- Understand conditional probability → pages 21–24
- Solve conditional probability problems using two-way tables and Venn diagrams → pages 24–27
- Use probability formulae to solve problems → pages 27–30
- Solve conditional probability using tree diagrams → pages 30–31



Prior knowledge check

1 Events A and B are mutually exclusive. $P(A) = 0.3$ and $P(B) = 0.45$. Find:

a $P(A \text{ or } B)$

b $P(A \text{ and } B)$

c P(neither A nor B). ← Year 1, Chapter 5

2 Events C and D are independent. $P(C) = 0.2$ and $P(D) = 0.6$.

a Find $P(C \text{ and } D)$.

b Draw a Venn diagram to show events C and D and the whole sample space.

c Find P(neither C nor D). ← Year 1, Chapter 5

3 A bag contains seven counters numbered 1–7. Two counters are selected at random without replacement. Find the probability that:

a Both counters are odd-numbered

b At least one counter is odd-numbered. ← Year 1, Chapter 5

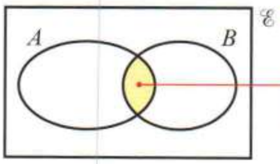
The outcome of one event can affect the probability for another event. If a football team scores a goal, the probability that they will win the match will increase. → Mixed exercise Q8

2.1 Set notation

You can use **set notation** to describe events within a sample space. This can help you abbreviate probability statements.

For example:

- **The event A and B can be written as $A \cap B$. The '$\cap$' symbol is the symbol for intersection.**



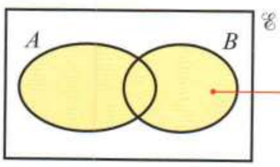
The symbol ℰ is used to represent the **whole sample space**.

The **intersection** of A and B is written as $A \cap B$.

If A and B are independent, $P(A \cap B) = P(A) \times P(B)$.

Notation If two events, A and B, are mutually exclusive, then their intersection is the **empty set**, $\varnothing$. You can write $A \cap B = \varnothing$.

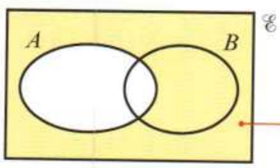
- **The event A or B can be written as $A \cup B$. The '$\cup$' symbol is the symbol for union.**



The **union** of A and B is written as $A \cup B$.

If A and B are mutually exclusive then,
$P(A \cup B) = P(A) + P(B)$.

- **The event not A can be written as A'. This is also called the complement of A.**



$P(A') = 1 - P(A)$

Events A and A' are always mutually exclusive.

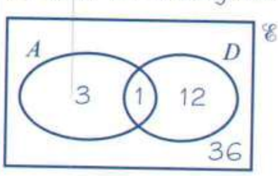
Online Explore set notation on a Venn diagram using GeoGebra.

Example 1

A card is selected at random from a pack of 52 playing cards. Let A be the event that the card is an ace and D the event that the card is a diamond. Find:

a $P(A \cap D)$ **b** $P(A \cup D)$ **c** $P(A')$ **d** $P(A' \cap D)$

Draw a Venn diagram:



Notation Venn diagrams can show either probabilities or the number of outcomes in each event.
$n(A)$ is the notation used to indicate the number of outcomes. For example there are four aces so $n(A) = 4$ and there is one ace of diamonds so $n(A \cap D) = 1$.

a $A \cap D$ is the event 'the card chosen is the ace of diamonds'.

$P(A \cap D) = \frac{1}{52}$

There is one outcome in $A \cap D$ and 52 outcomes in $\mathscr{E}$ so probability is $\frac{1}{52}$.

b $A \cup D$ is the event 'the card chosen is an ace or a diamond or both'.

$P(A \cup D) = \frac{16}{52} = \frac{4}{13}$

$n(A \cup D) = 3 + 12 + 1 = 16$

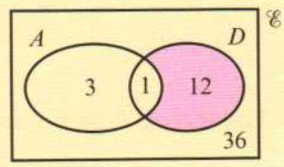
c A' is the event 'the card chosen is not an ace'.

$P(A') = \frac{48}{52} = \frac{12}{13}$

d $A' \cap D$ is the event 'the card chosen is not an ace and is a diamond'.

$P(A' \cap D) = \frac{12}{52} = \frac{3}{13}$

This is the set of all outcomes that are not in A but **are** in D.



Example 2

Given that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$,

a explain why events A and B are not independent.

Given also that $P(C) = 0.2$, that events A and C are mutually exclusive and that events B and C are independent,

b draw a Venn diagram to illustrate the events A, B and C, showing the probabilities for each region.

c Find $P((A \cap B') \cup C)$.

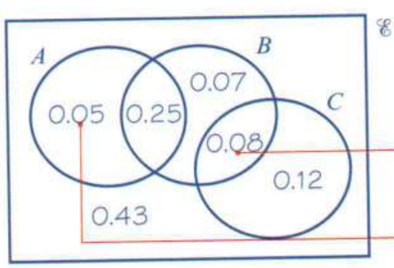
a $P(A) \times P(B) = 0.3 \times 0.4 = 0.12$

$P(A) \times P(B) \neq P(A \cap B)$ so A and B are not independent.

Problem-solving

When transferring information onto a Venn diagram, work from the intersections outwards if possible.

b



Since B and C are independent, $P(B \cap C) = 0.4 \times 0.2 = 0.08$.

Since A and C are mutually exclusive, A overlaps only with B. This region representing just A is $0.3 - 0.25$.

c $P(A \cap B') = 0.05$

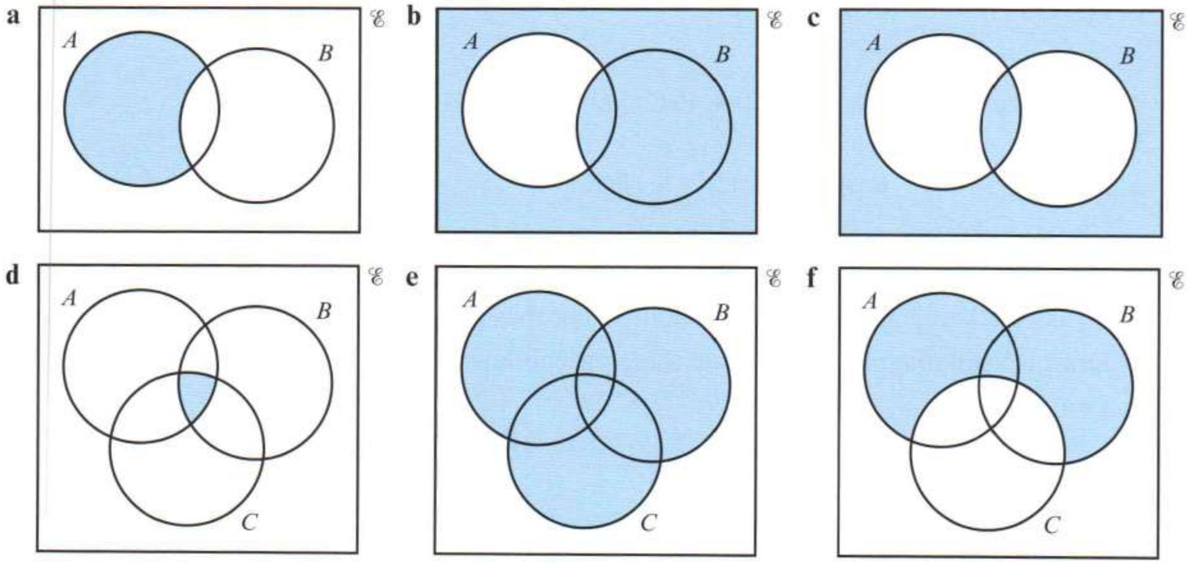
This is the region inside set A but outside set B.

$P((A \cap B') \cup C) = 0.05 + 0.2 = 0.25$

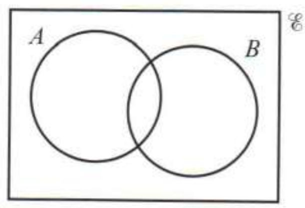
Add the two probabilities, since it is a union relationship and there is no overlap.

Exercise 2A

1 Use set notation to describe the shaded area in each of these Venn diagrams:



2 On copies of this Venn diagram, shade:

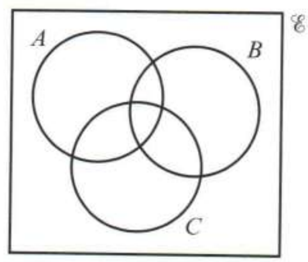


a $A \cup B'$

b $A' \cap B'$

c $(A \cap B)'$

3 On copies of this Venn diagram, shade:

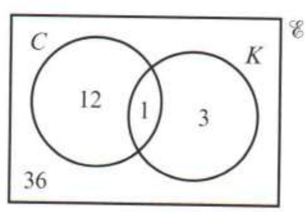


a $(A \cap B) \cup C$

b $(A' \cup B') \cap C$

c $(A \cap B \cap C')'$

4 A card is chosen at random from a pack of 52 playing cards. C is the event 'the card chosen is a club' and K is the event 'the card chosen is a King'.
The Venn diagram shows the number of outcomes for each event.



Find:

a $P(K)$ **b** $P(C)$ **c** $P(C \cap K)$

d $P(C \cup K)$ **e** $P(C')$ **f** $(K' \cap C)$

5 A and B are two events. $P(A) = 0.5$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$.
Find:

a $P(A \cup B)$ **b** $P(B')$
c $P(A \cap B')$ **d** $P(A \cup B')$

Hint Draw a Venn diagram.

6 C and D are two events. $P(D) = 0.4$, $P(C \cap D) = 0.15$ and $P(C' \cap D') = 0.1$.
Find:

a $P(C' \cap D)$ **b** $P(C \cap D')$ **c** $P(C)$ **d** $P(C' \cup D')$

7 The probability that a member of a sports club plays hockey (H) is 0.5 and the probability that they play cricket (C) is 0.4. The probability that they play both is 0.25.

a Draw a Venn diagram to illustrate these probabilities.

b Find:
i $P(H \cup C)$ **ii** $P(H' \cap C)$ **iii** $P(H \cup C')$

(P) **8** A bag contains 50 counters numbered from 1 to 50. The counters are either red or blue. A counter is picked at random. The two events R and E are the events 'counter is red' and 'counter is even-numbered' respectively. Given that $n(R) = 17$, $n(E) = 30$ and $n(R \cup E) = 40$,

a draw a Venn diagram to illustrate the outcomes.

b Find:
i $n(R \cap E)$
ii $P(R' \cap E')$
iii $P((R \cap E)')$

Watch out $n(R)$ represents the **number** of outcomes in the event R, whereas $P(R)$ represents the **probability** that the event R occurs.

(E/P) **9** A, B and C are three events with $P(A) = 0.55$, $P(B) = 0.35$ and $P(C) = 0.4$. $P(A \cap C) = 0.2$. Given that A and B are mutually exclusive and B and C are independent,

a draw a Venn diagram to illustrate the probabilities. **(4 marks)**

b Find:
i $P(A' \cap B')$ **(1 mark)**
ii $P(A \cup (B \cap C'))$ **(1 mark)**
iii $P((A \cap C)' \cup B')$ **(1 mark)**

(E/P) **10** A, B and C are three events with $P(A) = 0.25$, $P(B) = 0.4$, $P(C) = 0.45$ and $P(A \cap B \cap C) = 0.1$.
Given that A and B are independent, B and C are independent, and $A \cap B' \cap C = \varnothing$,

Problem-solving $\varnothing$ is the empty set. $P(\varnothing) = 0$.

a draw a Venn diagram to illustrate the probabilities. **(4 marks)**

b Find:
i $P(A' \cap (B' \cup C))$ **(1 mark)**
ii $P((A \cup B) \cap C)$ **(1 mark)**

c State, with reasons, whether events A' and C independent. **(2 marks)**

E/P **11** Members of a school book club read either murder mysteries (M), ghost stories (G) or epic fiction (E). $P(M) = 0.5$, $P(G) = 0.4$ and $P(E) = 0.6$. Given that no one reads both ghost stories and epic fiction and that $P(M \cap G) = 0.3$,

a draw a Venn diagram to illustrate these probabilities. **(4 marks)**

b Find:

i $P(M \cup G)$ **ii** $P((M \cap G) \cup (M \cap E))$ **(2 marks)**

c Are the events G' and M independent? You must justify your answer. **(2 marks)**

E/P **12** Given that events A and B are independent and that $P(A) = x$ and $P(B) = y$, find, in terms of x and y:

a $P(A \cap B)$ **(2 marks)**

b $P(A \cup B)$ **(2 marks)**

c $P(A \cup B')$ **(2 marks)**

Challenge

Given that events A, B and C are all independent and that $P(A) = x$, $P(B) = y$ and $P(C) = z$, find, in terms of x, y and z:

a $P(A \cap B \cap C)$ **b** $P(A \cup B \cup C)$ **c** $P((A \cup B') \cap C)$

2.2 Conditional probability

The probability of an event can change depending on the outcome of a previous event. For example, the probability of your being late for work may change depending on whether you oversleep or not.

Situations like this can be modelled using **conditional probability**. You use a vertical line to indicate conditional probabilities.

- **The probability that B occurs given that A has already occurred is written as $P(B|A)$.**

Similarly, $P(B|A')$ describes the probability of B occurring given that A has not occurred.

- **For independent events, $P(A|B) = P(A|B') = P(A)$, and $P(B|A) = P(B|A') = P(B)$.**

You can use this condition to determine independence.

You can solve some problems involving conditional probability by considering a **restricted sample space** of the outcomes where one event has already occurred.

Example 3

A school has 75 students in year 12. Of these students, 25 study only humanities subjects (H) and 37 study only science subjects (S). 11 students study both science and humanities subjects.

a Draw a two-way table to show this information.

b Find:

i $P(S' \cap H')$ **ii** $P(S|H)$ **iii** $P(H|S')$

a

	H	H'	Total
S	**11**	**37**	48
S'	**25**	2	27
Total	36	39	**75**

Put the information from the question in the table. These values are shown in bold.

$11 + 37 = 48$

$75 - 48 = 27$

b i $P(S' \cap H') = \frac{2}{75}$

There are two students who study neither science nor humanities out of a total of 75.

ii $P(S|H) = \frac{11}{36}$

Given that H is already true, you need to **restrict the sample space** to those 36 students. 11 of them also study science.

iii $P(H|S') = \frac{25}{27}$

There are 25 humanities students out of the 27 students who do not study science.

Example 4

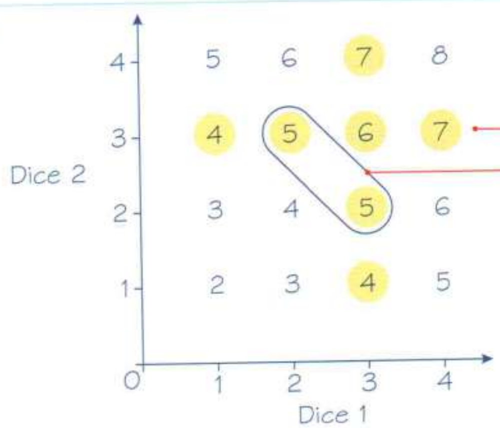
Two four-sided dice are thrown together, and the sum of the numbers shown is recorded.

a Draw a sample-space diagram showing the possible outcomes.

b Given that at least one dice lands on a 3, find the probability that the sum on the two dice is exactly 5.

c State one modelling assumption used in your calculations.

a



There are seven outcomes where one of the dice lands on a 3. The yellow circles show the restricted sample space.

Two of these outcomes have a total of 5.

b P(sum is 5|one dice lands on 3) $= \frac{2}{7}$

There are 7 equally likely outcomes in the restricted sample space.

c All outcomes are equally likely (i.e. both dice are fair).

Exercise 2B

1 The two-way table shows the fast-food preferences of 60 students in a sixth-form.

	Pizza	Curry	Total
Male	11	18	29
Female	14	17	31
Total	25	35	60

Find:

a P(Male) **b** P(Curry|Male) **c** P(Male|Curry) **d** P(Pizza|Female)

2 In a sports club, there are 75 members of whom 32 are female. Of the female members, 15 play badminton and 17 play squash. There are 22 men who play squash and the rest play badminton.

a Draw a two-way table to illustrate this situation.

b Find:

i P(Male|Squash) **ii** P(Female|Badminton) **iii** P(Squash|Female)

3 A group of 80 children are asked about their favourite ice-cream flavour. Of the 45 girls, 13 like vanilla, 12 like chocolate and the rest like strawberry. Of the boys, 2 like vanilla and 23 like strawberry. The rest like chocolate.

a Draw a two-way table to show this situation.

b Find:

i P(Boy|Strawberry) **ii** P(Girl|Vanilla) **iii** P(Chocolate|Boy)

4 A red and a blue spinner each have four equally likely outcomes, numbered 1 to 4. The two spinners are spun at the same time, and the sum of the numbers shown, X, is recorded.

a Draw a sample space diagram for X.

b Find:

i $P(X = 5)$ **ii** $P(X = 3|\text{Red spinner is } 2)$ **iii** $P(\text{Blue spinner is } 3|X = 5)$

5 Two fair six-sided dice are thrown and the product is recorded.

a Draw a sample-space diagram to illustrate the possible outcomes.

b Given that the first dice shows a 5, find the probability that the product is 20.

c Given that the product is 12, find the probability that the second dice shows a 6.

d Explain the importance of the word 'fair' in this context.

6 A card is drawn at random from a pack of 52 playing cards. Given that the card is a diamond, find the probability that the card is an ace.

7 Two coins are flipped and the results are recorded. Given that at least one of the coins landed on a head, find the probability that there were:

a two heads **b** a head and a tail.

c State one modelling assumption used in your calculations.

E 8 120 students are asked about their viewing habits. 56 say they watch sports (S) and 77 say they watch dramas (D). Of those who watch dramas, 18 also watch sports.

a Draw a two-way table to show this information. **(2 marks)**

b One student is chosen at random. Find:

i $P(D')$ **(1 mark)**

ii $P(S' \cap D')$ **(1 mark)**

iii $P(S|D)$ **(1 mark)**

iv $P(D'|S)$ **(1 mark)**

E 9 A rambling group is made up of 63 women and 47 men. 26 of the women and 18 of the men use a walking stick.

a Draw a two-way table to show this information. **(2 marks)**

b One rambler is chosen at random. Find:

i P(Uses a stick) **(1 mark)**

ii P(Uses a stick|Female) **(1 mark)**

iii P(Male|Uses a stick) **(1 mark)**

P 10 A veterinary surgery has 750 registered pet owners. Of these 450 are female. 320 of the pet owners own a cat and 250 own a dog. Of the remaining pet owners, 25 are males who own another type of pet. No one owns more than one type of pet. 175 female owners have a cat. One owner is chosen at random. Given that:

F is the event that an owner is female

D is the event that an owner has a dog

C is the event that an owner has a cat.

Find:

a $P(D' \cap C')$ **b** $P(D|F')$ **c** $P(F'|C)$ **d** $P((D' \cap C')|F)$

2.3 Conditional probabilities in Venn diagrams

You can find conditional probabilities from a Venn diagram by considering the section of the Venn diagram that corresponds to the restricted sample space.

Example 5

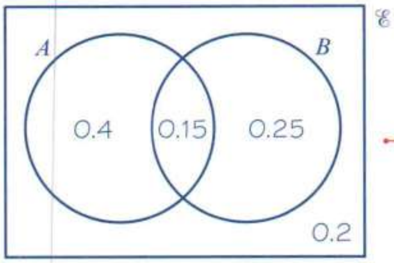
A and B are two events such that $P(A) = 0.55$, $P(B) = 0.4$ and $P(A \cap B) = 0.15$.

a Draw a Venn diagram showing the probabilities for events A and B.

b Find:

i $P(A|B)$ **ii** $P(B|(A \cup B))$ **iii** $P(A'|B')$

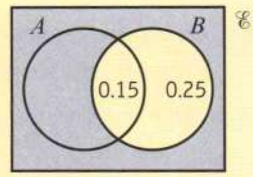
a



Use the information given to fill in the probabilities on each of the four regions in the Venn diagram.

b i $P(A|B) = \frac{0.15}{0.15 + 0.25} = \frac{3}{8}$

The sample space is restricted to just circle B. The part of circle A inside B has probability 0.15.

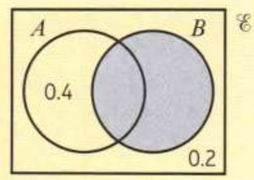


ii $P(B|A \cup B) = \frac{0.15 + 0.25}{0.4 + 0.15 + 0.25} = \frac{1}{2}$

The sample space is restricted to just the union of A and B.

iii $P(A'|B') = \frac{0.2}{0.4 + 0.2} = \frac{1}{3}$

Consider the restricted sample space first. This is everything **not** inside circle B.



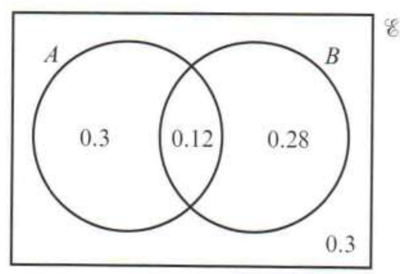
Exercise 2C

1 The Venn diagram shows the probabilities for two events, A and B.

Find:

a $P(A \cup B)$ **b** $P(A|B)$

c $P(B|A')$ **d** $P(B|A \cup B)$



2 C and D are two events such that $P(C) = 0.8$, $P(D) = 0.4$ and $P(C \cap D) = 0.25$.

a Draw a Venn diagram showing the probabilities for events C and D.

b Find:

i $P(C \cup D)$ **ii** $P(C|D)$ **iii** $P(D|C)$ **iv** $P(D'|C')$

3 S and T are two events such that $P(S) = 0.5$ and $P(T) = 0.7$. Given that S and T are independent,

a draw a Venn diagram showing the probabilities for events S and T.

b Find:

i $P(S \cap T)$ **ii** $P(S|T)$ **iii** $P(T|S')$ **iv** $P(S|S' \cup T')$

4 120 members of a youth club play either snooker (A), pool (B), both or neither. Given that 65 play snooker, 50 play pool and 20 play both, find:

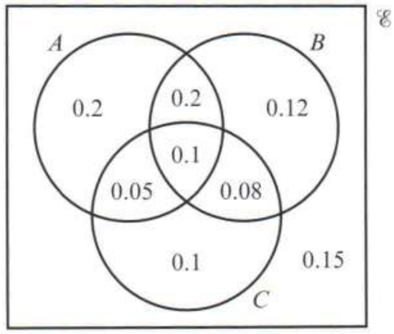
Hint Draw a Venn diagram.

a $P(A \cap B')$ **b** $P(A|B)$ **c** $P(B|A')$ **d** $P(A|A \cup B)$

5 The eating tastes of 80 cats are recorded. 45 like Feskers (F) and 32 like Whilix (W). 12 like neither. One cat is chosen at random. Find:

a $P(F \cap W)$ **b** $P(F|W)$ **c** $P(W|F)$ **d** $P(W'|F')$

6 The Venn diagram shows the probabilities of three events, A, B and C.

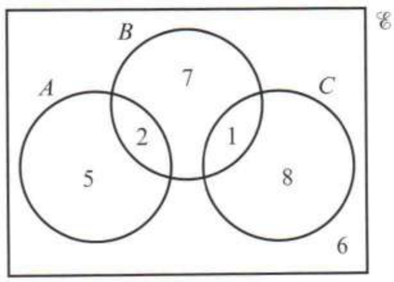


Find:

a $P(A|B)$ **b** $P(C|A')$

c $P((A \cap B)|C')$ **d** $P(C|(A' \cup B'))$

E/P 7 The Venn diagram shows the number of students in a class who watch any of 3 popular TV programmes A, B and C.



One of these students is selected at random. Given that the student watches at least one of the TV programmes, find the probability that the student watches:

a programme C **(2 marks)**

b exactly two of the programmes. **(2 marks)**

c Determine whether or not watching programme B and watching programme C are statistically independent. **(3 marks)**

Problem-solving

If $P(A|B) = P(A)$ then events A and B are independent.

E/P 8 Three events, A, B and C are such that A and B are mutually exclusive and B and C are independent. $P(A) = 0.2$, $P(B) = 0.6$ and $P(C) = 0.5$. Given that $P(A' \cap B' \cap C') = 0.1$,

a draw a Venn diagram to show the probabilities for events A, B and C. **(4 marks)**

b Find:

i $P(A|C)$ **(1 mark)**

ii $P(B|C')$ **(1 mark)**

iii $P(C|(A \cup B))$ **(1 mark)**

E/P **9** A doctor completes a medical study of 100 people, 5 of whom are known to have an illness and 95 of whom are known not to. A diagnostic test is applied. All 5 of the people with the illness test positive, and 10 people without the illness also test positive. Given that event A = person has the disease and event B = person tests positive,

a draw a Venn diagram to represent this situation. **(3 marks)**

b Calculate $P(A|B)$. **(2 marks)**

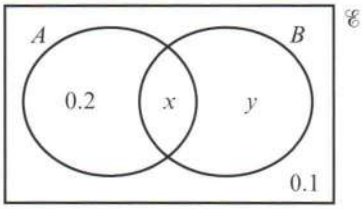
c With reference to your answer to part **b**, comment on the usefulness of the diagnostic test. **(2 marks)**

P **10** Events A and B are such that $P(A) = 0.6$ and $P(B) = 0.7$. Given that $P(A' \cap B') = 0.12$, find:

a $P(B|A')$ **b** $P(B|A)$

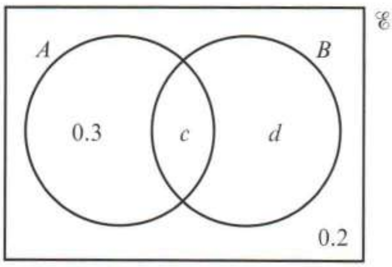
c Explain what your answers to parts **a** and **b** tell you about events A and B.

E/P **11** The Venn diagram shows the probabilities for two events, A and B. Given that $P(A|B) = P(B')$, find the values of x and y.



(3 marks)

E/P **12** The Venn diagram shows the probabilities for two events, A and B. Given that $P(A|B) = P(A')$, find the values of c and d.



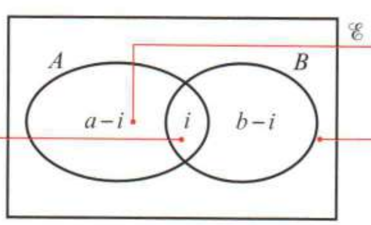
(3 marks)

2.4 Probability formulae

There is a formula you can use for two events that links the probability of the union and the probability of the intersection.

If $P(A) = a$ and $P(B) = b$

The probability of the intersection, $P(A \cap B)$, is i.



Subtract this probability from a and b and write the probabilities on the Venn diagram as shown.

The probability of $A \cup B$ is

$P(A \cup B) = (a - i) + (b - i) + i$
$= a + b - i$

Since $i = P(A \cap B)$ you can write the following **addition formula** for two events A and B:

- $\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

Example 6

A and B are two events, with $P(A) = 0.6$, $P(B) = 0.7$ and $P(A \cup B) = 0.9$.

Find $P(A \cap B)$.

Watch out You do not know whether A and B are independent so you can't use $(A \cap B) = P(A) \times P(B)$. Use the addition formula.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

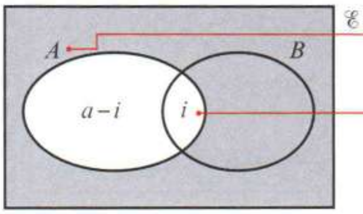
So $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.6 + 0.7 - 0.9$

$= 0.4$

Rearrange the addition formula to make $P(A \cap B)$ the subject.

You can also use the Venn diagram in the explanation above to find a formula for $P(B|A)$:



To find $P(B|A)$ restrict the sample space to the set of outcomes in which A has already occurred.

This is the subset of outcomes in the restricted sample space in which B occurs.

So $P(B|A) = \frac{i}{(a-i)+i} = \frac{i}{a}$

Since $P(B \cap A) = i$ and $P(A) = a$, you can write the following **multiplication formula** for conditional probability.

- $\mathbf{P(B|A) = \frac{P(B \cap A)}{P(A)}}$ **so** $\mathbf{P(B \cap A) = P(B|A) \times P(A)}$

Example 7

C and D are two events such that $P(C) = 0.2$, $P(D) = 0.6$ and $P(C|D) = 0.3$.

Find:

a $P(C \cap D)$ **b** $P(D|C)$ **c** $P(C \cup D)$

a $P(C \cap D) = P(C|D) \times P(D)$

$= 0.3 \times 0.6 = 0.18$

b $P(D|C) = \frac{P(D \cap C)}{P(C)}$

$= \frac{0.18}{0.2} = 0.9$

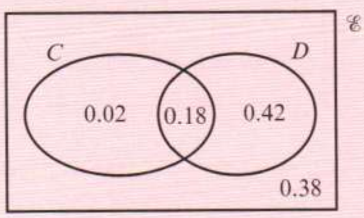
c $P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$= 0.2 + 0.6 - 0.18 = 0.62$

Use the multiplication formula.

Problem-solving

If you wanted to draw a Venn diagram to show these events it would help to find $P(C \cap D)$ first using the multiplication formula.



Exercise 2D

1 A and B are two events where $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.6$.
Find:

a $P(A \cap B)$ b $P(A')$ c $P(A \cup B')$ d $P(A' \cup B)$

(P) 2 C and D are two events where $P(C) = 0.55$, $P(D) = 0.65$ and $P(C \cap D) = 0.4$.

a Find $P(C \cup D)$.

b Draw a Venn diagram and use it to find:
i $P(C' \cap D')$ ii $P(C|D)$ iii $P(C|D')$

c Explain why events C and D are not statistically independent.

3 E and F are two events where $P(E) = 0.7$, $P(F) = 0.8$ and $P(E \cap F) = 0.6$.

a Find $P(E \cup F)$.

b Draw a Venn diagram and use it to find:
i $P(E \cup F')$ ii $P(E' \cap F)$ iii $P(E|F')$

(P) 4 There are two events T and Q where $P(T) = P(Q) = 3P(T \cap Q)$ and $P(T \cup Q) = 0.75$.
Find:

a $P(T \cap Q)$ b $P(T)$ c $P(Q')$ d $P(T' \cap Q')$ e $P(T \cap Q')$

5 A survey of all the households in the town of Bury was carried out. The survey showed that 70% have a freezer and 20% have a dishwasher and 80% have either a dishwasher or a freezer or both appliances. Find the probability that a randomly chosen household in Bury has both appliances.

6 A and B are two events such that $P(A) = 0.4$, $P(B) = 0.5$ and $P(A|B) = 0.4$. Find:

a $P(B|A)$ b $P(A' \cap B')$ c $P(A' \cap B)$.

7 Let A and B be events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$.
Find:

a $P(A|B)$ b $P(A' \cap B)$ c $P(A' \cap B')$

8 C and D are two events where $P(C|D) = \frac{1}{3}$, $P(C|D') = \frac{1}{5}$ and $P(D) = \frac{1}{4}$. Find:

a $P(C \cap D)$ b $P(C \cap D')$ c $P(C)$

d $P(D|C)$ e $P(D'|C)$ f $P(D'|C')$

(E) 9 Given that $P(A) = 0.42$, $P(B) = 0.37$ and $P(A \cap B) = 0.12$. Find:

a $P(A \cup B)$ **(2 marks)**

b $P(A \mid B')$ **(2 marks)**

The event C has $P(C) = 0.3$.

The events B and C are mutually exclusive and the events A and C are independent.

c Find $P(A \cap C)$. **(2 marks)**

d Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region. **(4 marks)**

e Find $P((A' \cup C)')$. **(2 marks)**

E/P **10** Three events A, B and C are such that $P(A) = 0.4$, $P(B) = 0.7$, $P(C) = 0.4$ and $P(A \cap B) = 0.3$. Given that A and C are mutually exclusive and that B and C are independent, find:

a $P(B \cap C)$ **(1 mark)**

b $P(B|C)$ **(1 mark)**

c $P(A|B')$ **(1 mark)**

d $P((B \cap C)|A')$ **(1 mark)**

E/P **11** Anna and Bella are sometimes late for school. The events A and B are defined as follows:
A is the event that Anna is late for school
B is the event that Bella is late for school
$P(A) = 0.3$, $P(B) = 0.7$ and $P(A' \cap B') = 0.1$. On a randomly selected day, find the probability that:

a both Anna and Bella are late to school **(1 mark)**

b Anna is late to school given that Bella is late to school. **(2 marks)**

Their teacher suspects that Anna and Bella being late for school is linked in some way.

c Comment on his suspicion, showing your working. **(2 marks)**

E/P **12** John and Kayleigh play darts in the same team. The events J and K are defined as follows:
J is the event that John wins his match
K is the event that Kayleigh wins her match
$P(J) = 0.6$, $P(K) = 0.7$ and $P(J \cup K) = 0.8$.
Find the probability that:

a both John and Kayleigh win their matches **(1 mark)**

b John wins his match given that Kayleigh loses hers **(2 marks)**

c Kayleigh wins her match given that John wins his. **(2 marks)**

d Determine whether the events J and K are statistically independent. You must show all your working. **(2 marks)**

Challenge

The discrete random variable X has probability function:

$P(X = x) = kx, x = 1, 2, 3, 4, 5$

Find:

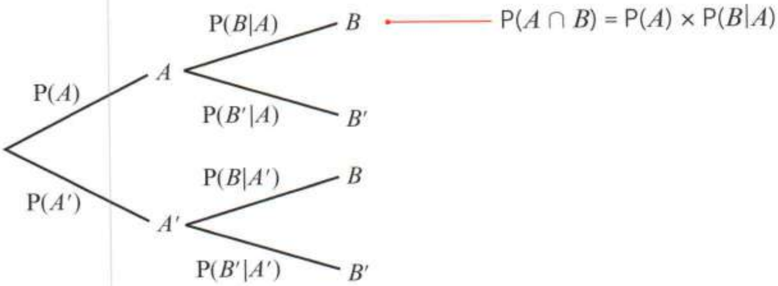
a the value of k

b $P(X = 5|X > 2)$

c $P(X \text{ is odd}|X \text{ is prime})$

2.5 Tree diagrams

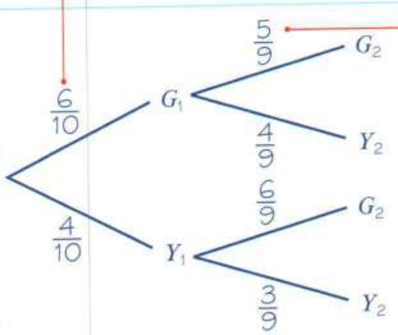
Conditional probabilities can be represented on a tree diagram.



The probabilities on the second set of branches represent the conditional probabilities of B given that A has, or has not, happened.

Example 8

A bag contains 6 green beads and 4 yellow beads. A bead is taken from the bag at random, the colour is recorded and it is not replaced. A second bead is then taken from the bag and its colour recorded. Given that both balls are the same colour, find the probability that they are both yellow.



Initially there are 10 beads in the bag and 6 are green. $P(G_1) = \frac{6}{10}$.

Since a green bead is removed and not replaced, the total number of beads is reduced to 9 and there are just 5 green beads remaining.

P(both yellow|same colour)

$$= \frac{\text{P(both yellow and same colour)}}{\text{P(same colour)}}$$

$$= \frac{\frac{4}{10} \times \frac{3}{9}}{\frac{4}{10} \times \frac{3}{9} + \frac{6}{10} \times \frac{5}{9}} = \frac{\frac{12}{90}}{\frac{42}{90}} = \frac{12}{42} = \frac{2}{7}$$

Use $P(B|A) = \frac{P(B \cap A)}{P(A)}$

Exercise 2E

1 A bag contains five red and four blue tokens. A token is chosen at random, the colour recorded and the token is not replaced. A second token is chosen and the colour recorded.

 a Draw a tree diagram to illustrate this situation.

 Find the probability that:

 b the second token is red given that the first token is blue

c the first token is red given that the second token is blue

d the first token is blue given that the tokens are different colours

e the tokens are the same colour given that the second token is red.

2 A and B are two events such that $P(B|A) = 0.45$, $P(B|A') = 0.35$ and $P(A) = 0.7$.

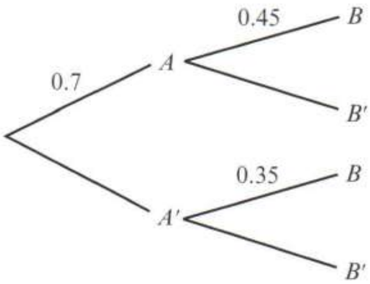
a Copy and complete the tree diagram representing this information.

b Find:

i $P(A \cap B)$

ii $P(A' \cap B')$

iii $P(A|B)$



3 A box of 24 chocolates contains 10 dark and 14 milk chocolates. Linda chooses a chocolate at random and eats it, followed by another one.

a Draw a tree diagram to represent this information.

Find the probability that Linda eats:

b two dark chocolates

c one dark and one milk chocolate

d two dark chocolates given that she eats at least one dark chocolate.

(P) **4** Jean always goes to work by bus or takes a taxi. If one day she goes to work by bus, the probability she goes to work by taxi the next day is 0.4. If one day she goes to work by taxi, the probability she goes to work by bus the next day is 0.7.

Given that Jean takes the bus to work on Monday, find the probability that she takes a taxi to work on Wednesday.

(P) **5** Sue has two coins. One is fair, with a head on one side and a tail on the other. The second is a trick coin and has a tail on both sides. Sue picks up one of the coins at random and flips it.

a Find the probability that it lands heads up.

b Given that it lands tails up, find the probability that she picked up the fair coin.

(E) **6** A bag contains 4 blue balls and 7 green balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

a Draw a tree diagram to represent the information. **(3 marks)**

Find the probability that:

b the second ball selected is green **(2 marks)**

c both balls selected are green, given that the second ball selected is green. **(2 marks)**

(E) **7** In an engineering company, factories A, B and C are all producing tin sheets of the same type. Factory A produces 25% of the sheets, factory B produces 45% and the rest are produced by factory C. Factories A, B and C produce flawed sheets with probabilities 0.02, 0.07 and 0.04 respectively.

a Draw a tree diagram to represent this information. **(3 marks)**

b Find the probability that a randomly selected sheet is:

i produced by factory B and flawed **(2 marks)**

ii flawed. **(3 marks)**

c Given that a randomly selected sheet is flawed, find the probability that it was produced by factory A. **(3 marks)**

(E/P) **8** A genetic condition is known to be present in 4% of a population. A test is developed to help determine whether or not someone has the genetic condition.
If a person has the condition, the test is positive with probability 0.9.
If a person does not have the condition, the test is positive with probability 0.02.

a Draw a tree diagram to represent this information. **(3 marks)**

A person is selected at random from the population and tested for this condition.

b Find the probability that the test is negative. **(3 marks)**

A doctor randomly selects a person from the population and tests them for the condition.

Given that the test is negative,

c find the probability that they do have the condition. **(2 marks)**

d Comment on the effectiveness of this test. **(1 mark)**

(E) **9** On a randomly chosen day the probabilities that Bill travels to work by car, by bus or by train are 0.1, 0.6 and 0.3 respectively. The probabilities of being late when using these methods of travel are 0.55, 0.3 and 0.05 respectively.

a Draw a tree diagram to represent this information. **(3 marks)**

b Find the probability that on a randomly chosen day,

i Bill travels by train and is late **(2 marks)**

ii Bill is late. **(2 marks)**

c Given that Bill is late, find the probability that he did not travel by car. **(4 marks)**

(E/P) **10** A box A contains 7 counters of which 4 are green and 3 are blue.
A box B contains 5 counters of which 2 are green and 3 are blue.
A counter is drawn at random from box A and placed in box B. A second counter is drawn at random from box A and placed in box B.
A third counter is then drawn at random from the counters in box B.

a Draw a tree diagram to show this situation. **(4 marks)**

The event C occurs when the 2 counters drawn from box A are of the same colour.
The event D occurs when the counter drawn from box B is blue.

b Find P(C). **(3 marks)**

c Show that $P(D) = \frac{27}{49}$ **(3 marks)**

d Show that $P(C \cap D) = \frac{11}{49}$ **(2 marks)**

e Hence find $P(C \cup D)$. **(2 marks)**

f Given that all three counters drawn are the same colour, find the probability that they are all green. **(3 marks)**

(E/P) **11** A box of jelly beans contains 7 sweet flavours and 3 sour flavours. Two of the jelly beans are taken one after the other and eaten. Emilia wants to find the probability that both jelly beans eaten are sweet given that at least one of them is. Her solution is shown below:

> P(both jelly beans are sweet) $= \frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$
>
> P(at least one jelly bean is sweet)
>
> $= 1 -$ P(neither are sweet) $= 1 - \frac{3}{10} \times \frac{3}{10} = \frac{91}{100}$
>
> P(both are sweet given at least one is sweet)
>
> $= \frac{\frac{49}{100}}{\frac{91}{100}} = \frac{49}{91}$

Identify Emilia's mistake and find the correct probability. **(4 marks)**

Mixed exercise 2

(E) **1** A and B are two events such that $P(A) = 0.4$ and $P(B) = 0.35$. If $P(A \cap B) = 0.2$, find:

a $P(A \cup B)$ **(1 mark)**

b $P(A' \cap B')$ **(1 mark)**

c $P(B|A)$ **(2 marks)**

d $P(A'|B)$ **(2 marks)**

(E/P) **2** J, K and L are three events such that $P(J) = 0.25$, $P(K) = 0.45$ and $P(L) = 0.15$. Given that K and L are independent, J and L are mutually exclusive and $P(J \cap K) = 0.1$

a draw a Venn diagram to illustrate this situation. **(2 marks)**

b Find:

i $P(J \cup K)$ **(1 mark)**

ii $P(J' \cap L')$ **(1 mark)**

iii $P(J|K)$ **(2 marks)**

iv $P(K|J' \cap L')$ **(2 marks)**

(E/P) **3** Of 60 students in a high-school sixth form, 35 study French and 45 study Spanish. If 27 students study both, find the probability that a student chosen at random:

a studies only one subject **(1 mark)**

b studies French given that they study Spanish **(2 marks)**

c studies Spanish given that they do not study French. **(2 marks)**

It is found that 75% of the students who study just French wear glasses and half of the students who study just Spanish wear glasses. Find the probability that a student chosen at random:

d studies one language and wears glasses **(2 marks)**

e wears glasses given that they study one language. **(2 marks)**

E/P **4** A bag contains 6 red balls and 9 green balls. A ball is chosen at random from that bag, its colour noted and the ball placed to one side. A second ball is chosen at random and its colour noted.

a Draw a tree diagram to illustrate this situation. **(2 marks)**

b Find the probability that:

i both balls are green **(1 mark)**

ii the balls are different colours. **(2 marks)**

Further balls are drawn from the bag and not replaced. Find the probability that:

c the third ball is red **(2 marks)**

d it takes just four selections to get four green balls. **(2 marks)**

E **5** In a tennis match, the probability that Anne wins the first set against Colin is 0.7. If Anne wins the first set, the probability that she wins the second set is 0.8. If Anne loses the first set, the probability that she wins the second set is 0.4. A match is won when one player wins two sets.

a Find the probability that the game is over after two sets. **(2 marks)**

b Find the probability that Anne wins given that the game is over after two sets. **(2 marks)**

If the game is tied at one set all, a tiebreaker is played and the probability of Anne winning it is 0.55.

c Find the probability of Anne winning the entire match. **(3 marks)**

E/P **6** The colours of the paws of 75 kittens are recorded. 26 kittens have all black paws and 14 kittens have all white paws. 15 have a combination of black and white paws. One kitten is chosen at random. Find the probability that the kitten has:

a neither white nor black paws **(1 mark)**

b a combination of black and white paws given that they have some black paws. **(2 marks)**

Two kittens are now chosen. Find the probability that:

c both kittens have all black paws **(2 marks)**

d both kittens have some white paws. **(2 marks)**

E/P **7** Two events A and B are such that $P(A) = 0.4$ and $P(A \cap B) = 0.12$. If A and B are independent, find:

a $P(B)$ **(1 mark)**

b $P(A' \cap B')$ **(1 mark)**

A third event C has $P(C) = 0.4$. Given that A and C are mutually exclusive and $P(B \cap C) = 0.1$,

c draw a Venn diagram to illustrate this situation. **(2 marks)**

d Find:

i $P(B|C)$ **(2 marks)**

ii $P(A \cap (B' \cup C))$ **(2 marks)**

E/P **8** In a football match, the probability that team A scores first is 0.6, and the probability that team B scores first is 0.35.

a Suggest a reason why these probabilities do not add up to 1. **(1 mark)**

The probability that team A scores first and wins the match is 0.48.

b Find the probability that team A scores first and does not win the match. **(3 marks)**

If team B scores first, the probability that team A will win the match is 0.3.

c Given that team A won the match, find the probability that they did not score first. **(3 marks)**

Challenge

$P(A) = 0.6$ and $P(B) = 0.2$

a Given that $P(A \cap B') = p$, find the range of possible values of p.

$P(C) = 0.7$ and $P(A \cap B \cap C) = 0.1$

b Given $P(A \cap B' \cap C) = q$, find the range of possible values of q.

Summary of key points

1 The event A and B can be written as $A \cap B$. The '$\cap$' symbol is the symbol for **intersection**.
The event A or B can be written as $A \cup B$. The '$\cup$' symbol is the symbol for **union**.
The event not A can be written as A'. This is also called the **complement** of A.

2 The probability that B occurs given that A has already occurred is written as $P(B|A)$.
For independent events, $P(A|B) = P(A|B') = P(A)$, and $P(B|A) = P(B|A') = P(B)$.

3 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4 $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$ so $P(B \cap A) = P(B|A) \times P(A)$