Answers

CHAPTER 1

Prior knowledge check

1 a $2m^2n + 3mn^2$ b $6x^2 - 12x - 10$
2 a 2^8 b 2^4 c 2^6
3 a $3x + 12$ b $10 - 15x$ c $12x - 30y$
4 a 8 b $2x$ c xy
5 a $2x$ b $10x$ c $\frac{5x}{3}$

Exercise 1A

1 a x^7 b $6x^5$ c k d $2p^2$
e x f y^{10} g $5x^2$ h p^2
i $2a^3$ j $2p$ k $6a^9$ l $3a^2b^3$
m $27x^8$ n $24x^{11}$ o $63a^{12}$ p $32y^6$
q $4a^6$ r $6a^{12}$
2 a $9x - 18$ b $x^2 + 9x$
c $-12y + 9y^2$ d $xy + 5x$
e $-3x^2 - 5x$ f $-20x^2 - 5x$
g $4x^2 + 5x$ h $-15y + 6y^3$
i $-10x^2 + 8x$ j $3x^3 - 5x^2$
k $4x - 1$ l $2x - 4$
m $9d^2 - 2c$ n $13 - r^2$
o $3x^3 - 2x^2 + 5x$ p $14y^2 - 35y^3 + 21y^4$
q $-10y^2 + 14y^3 - 6y^4$ r $4x + 10$
s $11x - 6$ t $7x^2 - 3x + 7$
u $-2x^2 + 26x$ v $-9x^3 + 23x^2$
3 a $3x^3 + 5x^5$ b $3x^4 - x^6$ c $\frac{x^3}{2} - x$
d $4x^2 + \frac{5}{2}$ e $\frac{7x^6}{5} + x$ f $3x^4 - \frac{5x^2}{3}$

Exercise 1B

1 a $x^2 + 11x + 28$
b $x^2 - x - 6$
c $x^2 - 4x + 4$
d $2x^2 + 3x - 2xy - 3y$
e $4x^2 + 11xy - 3y^2$
f $6x^2 - 10xy - 4y^2$
g $2x^2 - 11x + 12$
h $9x^2 + 12xy + 4y^2$
i $4x^2 + 6x + 16xy + 24y$
j $2x^2 + 3xy + 5x + 15y - 25$
k $3x^2 - 4xy - 8x + 4y + 5$
l $2x^2 + 5x - 7xy - 4y^2 - 20y$
m $x^2 + 2x + 2xy + 6y - 3$
n $2x^2 + 15x + 2xy + 12y + 18$
o $13y - 4x + 12 - 4y^2 + xy$
p $12xy - 4y^2 + 3y + 15x + 10$
q $5xy - 20y - 2x^2 + 11x - 12$
r $22y - 4y^2 - 5x + xy - 10$
2 a $5x^2 - 15x - 20$
b $14x^2 + 7x - 70$
c $3x^2 - 18x + 27$
d $x^3 - xy^2$
e $6x^3 + 8x^2 + 3x^2y + 4xy$
f $x^2y - 4xy - 5y$
g $12x^2y + 6xy - 8xy^2 - 4y^2$
h $19xy - 35y - 2x^2y$
i $10x^3 - 4x^2 + 5x^2y - 2xy$
j $x^3 + 3x^2y - 2x^2 + 6xy - 8x$
k $2x^2y + 9xy + xy^2 + 5y^2 - 5y$
l $6x^2y + 4xy^2 + 2y^2 - 3xy - 3y$
m $2x^3 + 2x^2y - 7x^2 + 3xy - 15x$
n $24x^3 - 6x^2y - 26x^2 + 2xy + 6x$
o $6x^3 + 15x^2 - 3x^2y - 18xy^2 - 30xy$
p $x^3 + 6x^2 + 11x + 6$
q $x^3 + x^2 - 14x - 24$
r $x^3 - 3x^2 - 13x + 15$
s $x^3 - 12x^2 + 47x - 60$
t $2x^3 - x^2 - 5x - 2$
u $6x^3 + 19x^2 + 11x - 6$
v $18x^3 - 15x^2 - 4x + 4$
w $x^3 - xy^2 - x^2 + y^2$
x $8x^3 - 36x^2y + 54xy^2 - 27y^3$
3 $2x^2 - xy + 29x - 7y + 24$
4 $4x^3 + 12x^2 + 5x - 6\text{ cm}^3$
5 $a = 12, b = 32, c = 3, d = -5$

Challenge
$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Exercise 1C

1 a $4(x + 2)$ b $6(x - 4)$
c $5(4x + 3)$ d $2(x^2 + 2)$
e $4(x^2 + 5)$ f $6x(x - 3)$
g $x(x - 7)$ h $2x(x + 2)$
i $x(3x - 1)$ j $2x(3x - 1)$
k $5y(2y - 1)$ l $7x(5x - 4)$
m $x(x + 2)$ n $y(3y + 2)$
o $4x(x + 3)$ p $5y(y - 4)$
q $3xy(3y + 4x)$ r $2ab(3 - b)$
s $5x(x - 5y)$ t $4xy(3x + 2y)$
u $5y(3 - 4z^2)$ v $6(2x^2 - 5)$
w $xy(y - x)$ x $4y(3y - x)$
2 a $x(x + 4)$ b $2x(x + 3)$
c $(x + 8)(x + 3)$ d $(x + 6)(x + 2)$
e $(x + 8)(x - 5)$ f $(x - 6)(x - 2)$
g $(x + 2)(x + 3)$ h $(x - 6)(x + 4)$
i $(x - 5)(x + 2)$ j $(x + 5)(x - 4)$
k $(2x + 1)(x + 2)$ l $(3x - 2)(x + 4)$
m $(5x - 1)(x - 3)$ n $2(3x + 2)(x - 2)$
o $(2x - 3)(x + 5)$ p $2(x^2 + 3)(x^2 + 4)$
q $(x + 2)(x - 2)$ r $(x + 7)(x - 7)$
s $(2x + 5)(2x - 5)$ t $(3x + 5y)(3x - 5y)$
u $4(3x + 1)(3x - 1)$ v $2(x + 5)(x - 5)$
w $2(3x - 2)(x - 1)$ x $3(5x - 1)(x + 3)$
3 a $x(x^2 + 2)$ b $x(x^2 - x + 1)$
c $x(x^2 - 5)$ d $x(x + 3)(x - 3)$
e $x(x - 4)(x + 3)$ f $x(x + 5)(x + 6)$
g $x(x - 1)(x - 6)$ h $x(x + 8)(x - 8)$
i $x(2x + 1)(x - 3)$ j $x(2x + 3)(x + 5)$
k $x(x + 2)(x - 2)$ l $3x(x + 4)(x + 5)$
4 $(x^2 + y^2)(x + y)(x - y)$
5 $x(3x + 5)(2x - 1)$

Challenge
$(x - 1)(x + 1)(2x + 3)(2x - 3)$

Exercise 1D

1 a x^5 b x^{-2} c x^4 d x^3
e x^5 f $12x^0 = 12$ g $3x^{\frac{1}{2}}$ h $5x$
i $6x^{-1}$ j $x^{\frac{5}{6}}$ k $x^{\frac{17}{6}}$ l $x^{\frac{1}{6}}$

2 **a** 5 **b** 729 **c** 3 **d** $\frac{1}{16}$
e $\frac{1}{3}$ **f** $\frac{-1}{125}$ **g** 1 **h** 216
i $\frac{125}{64}$ **j** $\frac{9}{4}$ **k** $\frac{5}{6}$ **l** $\frac{64}{49}$

3 **a** $8x^5$ **b** $\frac{5}{x^2} - \frac{2}{x^3}$ **c** $5x^4$
d $\frac{1}{x^2} + 4$ **e** $\frac{2}{x^3} + \frac{1}{x^2}$ **f** $\frac{8}{27}x^6$
g $\frac{3}{x} - 5x^2$ **h** $\frac{1}{3x^2} + \frac{1}{5x}$

4 **a** 3 **b** $\frac{16}{\sqrt[3]{x}}$

5 **a** $\frac{x}{2}$ **b** $\frac{32}{x^6}$

6 **a** $x = 49$ **b** $y = 27$
c $x = \frac{1}{4}$ **d** $z = \frac{1}{10\,000}$

7 $x = \frac{1}{9}$

Exercise 1E

1 **a** $2\sqrt{7}$ **b** $6\sqrt{2}$ **c** $5\sqrt{2}$ **d** $4\sqrt{2}$
e $3\sqrt{10}$ **f** $\sqrt{3}$ **g** $\sqrt{3}$ **h** $6\sqrt{5}$
i $7\sqrt{2}$ **j** $12\sqrt{7}$ **k** $-3\sqrt{7}$ **l** $9\sqrt{5}$
m $23\sqrt{5}$ **n** 2 **o** $19\sqrt{3}$

2 **a** $2\sqrt{3} + 3$ **b** $3\sqrt{5} - \sqrt{15}$
c $4\sqrt{2} - \sqrt{10}$ **d** $6 + 2\sqrt{5} - 3\sqrt{2} - \sqrt{10}$
e $6 - 2\sqrt{7} - 3\sqrt{3} + \sqrt{21}$ **f** $13 + 6\sqrt{5}$
g $8 - 6\sqrt{3}$ **h** $5 - 2\sqrt{3}$
i $3 + 5\sqrt{11}$

3 $3\sqrt{3}$

Exercise 1F

1 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$ **c** $\frac{\sqrt{2}}{2}$ **d** $\frac{\sqrt{5}}{5}$
e $\frac{1}{2}$ **f** $\frac{1}{4}$ **g** $\frac{\sqrt{13}}{13}$ **h** $\frac{1}{3}$

2 **a** $\frac{1 - \sqrt{3}}{-2}$ **b** $\sqrt{5} - 2$ **c** $\frac{3 + \sqrt{7}}{2}$
d $3 + \sqrt{5}$ **e** $\frac{\sqrt{5} + \sqrt{3}}{2}$ **f** $\frac{(3 - \sqrt{2})(4 + \sqrt{5})}{11}$
g $5(\sqrt{5} - 2)$ **h** $5(4 + \sqrt{14})$ **i** $\frac{11(3 - \sqrt{11})}{-2}$
j $\frac{5 - \sqrt{21}}{-2}$ **k** $\frac{14 - \sqrt{187}}{3}$ **l** $\frac{35 + \sqrt{1189}}{6}$
m -1

3 **a** $\frac{11 + 6\sqrt{2}}{49}$ **b** $9 - 4\sqrt{5}$ **c** $\frac{44 + 24\sqrt{2}}{49}$
d $\frac{81 - 30\sqrt{2}}{529}$ **e** $\frac{13 + 2\sqrt{2}}{161}$ **f** $\frac{7 - 3\sqrt{3}}{11}$

4 $-\frac{7}{4} + \frac{\sqrt{5}}{4}$

Mixed exercise 1

1 **a** y^8 **b** $6x^7$ **c** $32x$ **d** $12b^9$

2 **a** $x^2 - 2x - 15$ **b** $6x^2 - 19x - 7$
c $6x^2 - 2xy + 19x - 5y + 10$

3 **a** $x^3 + 3x^2 - 4x$ **b** $x^3 + 6x^2 - 13x - 42$
c $6x^3 - 5x^2 - 17x + 6$

4 **a** $15y + 12$ **b** $15x^2 - 25x^3 + 10x^4$
c $16x^2 + 13x$ **d** $9x^3 - 3x^2 + 4x$

5 **a** $x(3x + 4)$ **b** $2y(2y + 5)$
c $x(x + y + y^2)$ **d** $2xy(4y + 5x)$

6 **a** $(x + 1)(x + 2)$ **b** $3x(x + 2)$
c $(x - 7)(x + 5)$ **d** $(2x - 3)(x + 1)$
e $(5x + 2)(x - 3)$ **f** $(1 - x)(6 + x)$

7 **a** $2x(x^2 + 3)$ **b** $x(x + 6)(x - 6)$
c $x(2x - 3)(x + 5)$

8 **a** $3x^6$ **b** 2 **c** $6x^2$ **d** $\frac{1}{2}x^{-\frac{1}{3}}$

9 **a** $\frac{4}{9}$ **b** $\frac{3375}{4913}$

10 **a** $\frac{\sqrt{7}}{7}$ **b** $4\sqrt{5}$

11 **a** $21\,877$
b $(5x + 6)(7x - 8)$
When $x = 25$, $5x + 6 = 131$ and $7x - 8 = 167$; both 131 and 167 are prime numbers.

12 **a** $3\sqrt{2} + \sqrt{10}$ **b** $10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{15}$
c $24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{14}$

13 **a** $\frac{\sqrt{3}}{3}$ **b** $\sqrt{2} + 1$ **c** $-3\sqrt{3} - 6$
d $\frac{30 - \sqrt{851}}{-7}$ **e** $7 - 4\sqrt{3}$ **f** $\frac{23 + 8\sqrt{7}}{81}$

14 **a** $b = -4$ and $c = -5$ **b** $(x + 3)(x - 5)(x + 1)$

15 **a** $\frac{1}{4}x$ **b** $256x^{-3}$

16 $\frac{5}{\sqrt{75} - \sqrt{50}} = \frac{1}{\sqrt{3} - \sqrt{2}} = \sqrt{3} + \sqrt{2}$

17 $-36 + 10\sqrt{11}$

18 $x(1 + 8x)(1 - 8x)$

19 $y = 6x + 3$

20 $4\sqrt{3}$

21 $3 - \sqrt{3}$ cm

22 $\frac{4 - 4x^{\frac{1}{2}} + x^1}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$

23 **a** $a = \frac{11}{2}$ **b** $y = \frac{11 - 2x}{6}$

24 $4x^{\frac{5}{2}} + x^2$, $a = \frac{5}{2}$ $b = 2$

Challenge

a $a - b$

b $\frac{(\sqrt{1} - \sqrt{2}) + (\sqrt{2} - \sqrt{3}) + \ldots + (\sqrt{24} - \sqrt{25})}{-1} = \sqrt{25} - \sqrt{1} = 4$

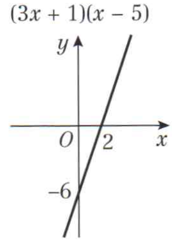
CHAPTER 2

Prior knowledge check

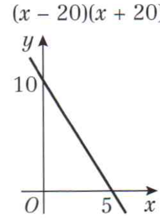
1 **a** $x = -5$ **b** $x = 3$
c $x = 5$ or $x = -5$ **d** $x = 16$ or $x = 0$

2 **a** $(x + 3)(x + 5)$ **b** $(x + 5)(x - 2)$
c $(3x + 1)(x - 5)$ **d** $(x - 20)(x + 20)$

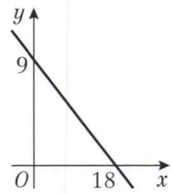
3 **a**



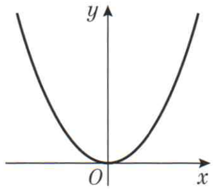
b



3 **c**



d



4 **a** $x < 3$ **b** $x \geqslant 9$ **c** $x \leqslant 2.5$ **d** $x > -7$

Exercise 2A

1 **a** $x = -1$ or $x = -2$ **b** $x = -1$ or $x = -4$
c $x = -5$ or $x = -2$ **d** $x = 3$ or $x = -2$
e $x = 3$ or $x = 5$ **f** $x = 4$ or $x = 5$
g $x = 6$ or $x = -1$ **h** $x = 6$ or $x = -2$

2 **a** $x = 0$ or $x = 4$ **b** $x = 0$ or $x = 25$
c $x = 0$ or $x = 2$ **d** $x = 0$ or $x = 6$
e $x = -\frac{1}{2}$ or $x = -3$ **f** $x = -\frac{1}{3}$ or $x = \frac{3}{2}$
g $x = -\frac{2}{3}$ or $x = \frac{3}{2}$ **h** $x = \frac{3}{2}$ or $x = \frac{5}{2}$

3 **a** $x = \frac{1}{3}$ or $x = -2$ **b** $x = 3$ or $x = 0$
c $x = 13$ or $x = 1$ **d** $x = 2$ or $x = -2$
e $x = \pm\sqrt{\frac{5}{3}}$ **f** $x = 3 \pm \sqrt{13}$
g $x = \dfrac{1 \pm \sqrt{11}}{3}$ **h** $x = 1$ or $x = -\frac{7}{6}$
i $x = -\frac{1}{2}$ or $x = \frac{7}{3}$ **j** $x = 0$ or $x = -\frac{11}{6}$

4 $x = 4$

5 $x = -1$ or $x = -\frac{2}{25}$

Exercise 2B

1 **a** $x = \frac{1}{2}(-3 \pm \sqrt{5})$ **b** $x = \frac{1}{2}(3 \pm \sqrt{17})$
c $x = -3 \pm \sqrt{3}$ **d** $x = \frac{1}{2}(5 \pm \sqrt{33})$
e $x = \frac{1}{3}(-5 \pm \sqrt{31})$ **f** $x = \frac{1}{2}(1 \pm \sqrt{2})$
g $x = 2$ or $x = -\frac{1}{4}$ **h** $x = \frac{1}{11}(-1 \pm \sqrt{78})$

2 **a** $x = -0.586$ or $x = -3.41$ **b** $x = 7.87$ or $x = 0.127$
c $x = 0.765$ or $x = -11.8$ **d** $x = 8.91$ or $x = -1.91$
e $x = 0.105$ or $x = -1.90$ **f** $x = 3.84$ or $x = -2.34$
g $x = 4.77$ or $x = 0.558$ **h** $x = 4.89$ or $x = -1.23$

3 **a** $x = -6$ or $x = -2$ **b** $x = 1.09$ or $x = -10.1$
c $x = 9.11$ or $x = -0.110$ **d** $x = -\frac{1}{2}$ or $x = -2$
e $x = 1$ or $x = -9$ **f** $x = 1$
g $x = 4.68$ or $x = -1.18$ **h** $x = 3$ or $x = 5$

4 Area $= \frac{1}{2}(2x)(x + (x + 10)) = 50\,\text{m}^2$
So $x^2 + 5x - 25 = 0$
Using the quadratic formula:
$x = \frac{1}{2}(-5 \pm 5\sqrt{5})$
Height $= 2x = 5(\sqrt{5} - 1)\,\text{m}$

Challenge
$x = 13$

Exercise 2C

1 **a** $(x + 2)^2 - 4$ **b** $(x - 3)^2 - 9$
c $(x - 8)^2 - 64$ **d** $(x + \frac{1}{2})^2 - \frac{1}{4}$
e $(x - 7)^2 - 49$

2 **a** $2(x + 4)^2 - 32$ **b** $3(x - 4)^2 - 48$
c $5(x + 2)^2 - 20$ **d** $2(x - \frac{5}{4})^2 - \frac{25}{8}$
e $-2(x - 2)^2 + 8$

3 **a** $2(x + 2)^2 - 7$ **b** $5(x - \frac{3}{2})^2 - \frac{33}{4}$
c $3(x + \frac{1}{3})^2 - \frac{4}{3}$ **d** $-4(x + 2)^2 + 26$
e $-8(x - \frac{1}{8})^2 + \frac{81}{8}$

4 $a = \frac{3}{2}$, $b = \frac{15}{4}$

5 $A = 6$, $B = 0.04$, $C = -10$

Exercise 2D

1 **a** $x = -3 \pm 2\sqrt{2}$ **b** $x = -6 \pm \sqrt{33}$
c $x = -2 \pm \sqrt{6}$ **d** $x = 5 \pm \sqrt{30}$

2 **a** $x = \frac{1}{2}(-3 \pm \sqrt{15})$ **b** $x = \frac{1}{5}(-4 \pm \sqrt{26})$
c $x = \frac{1}{8}(1 \pm \sqrt{129})$ **d** $x = \frac{1}{2}(-3 \pm \sqrt{39})$

3 **a** $p = -7$, $q = -48$
b $(x - 7)^2 = 48$
$x = 7 \pm \sqrt{48} = 7 \pm 4\sqrt{3}$
$r = 7$, $s = 4$

4 $x^2 + 2bx + c = (x + b)^2 - b^2 + c$
$(x + b)^2 = b^2 - c$
$x = -b \pm \sqrt{b^2 - c}$

Challenge

a $ax^2 + 2bx + c = 0$

$$x^2 + \frac{2b}{a}x + \frac{c}{a} = 0$$
$$\left(x + \frac{b}{a}\right)^2 - \frac{b^2}{a^2} + \frac{c}{a} = 0$$
$$\left(x + \frac{b}{a}\right)^2 = \frac{b^2 - ac}{a^2}$$
$$x = -\frac{b}{a} \pm \sqrt{\frac{b^2 - ac}{a^2}}$$

b $ax^2 + bx + c = 0$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise 2E

1 **a** 8 **b** 7 **c** 3 **d** 10.5 **e** 0
f 0 **g** 25 **h** 2 **i** 7

2 $a = 4$ or $a = -2$

3 **a** $\frac{2}{3}$ **b** 2 and -9 **c** -10 and 4
d 12 and -12 **e** 0, -5 and -7 **f** 0, 3 and -8

4 $x = 3$ and $x = 2$

5 $x = 0$, 2.5 and 6

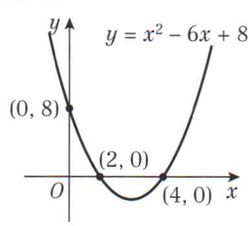
6 **a** $(x - 1)^2 + 1$
$p = -1$, $q = 1$
b Squared terms are always $\geqslant 0$, so the minimum value is $0 + 1 = 1$

7 **a** -2 and -1 **b** 2, -2, $2\sqrt{2}$ and $-2\sqrt{2}$
c -1 and $\frac{1}{3}$ **d** $\frac{1}{2}$ and 1
e 4 and 25 **f** 8 and -27

8 **a** $(3^x - 27)(3^x - 1)$ **b** 0 and 3

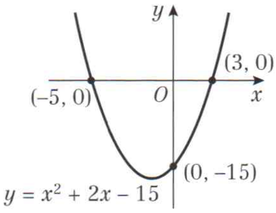
Exercise 2F

1 **a**



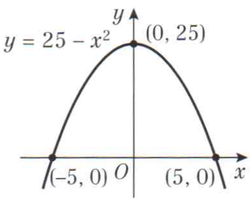
Turning point: $(3, -1)$
Line of symmetry: $x = 3$

b



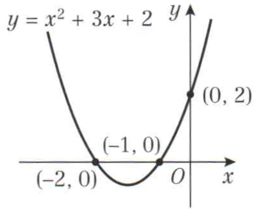
Turning point: (−1, −16)
Line of symmetry: $x = -1$

c



Turning point: (0, 25)
Line of symmetry: $x = 0$

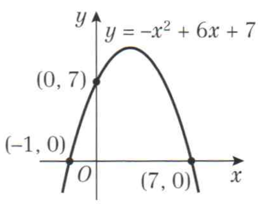
d



Turning point: $\left(-\frac{3}{2}, -\frac{1}{4}\right)$

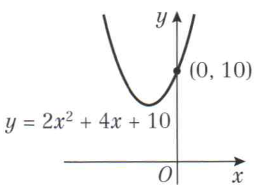
Line of symmetry: $x = -\frac{3}{2}$

e



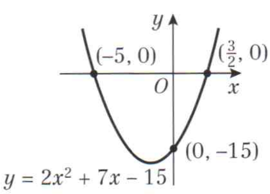
Turning point: (3, 16)
Line of symmetry: $x = 3$

f



Turning point: (−1, 8)
Line of symmetry: $x = -1$

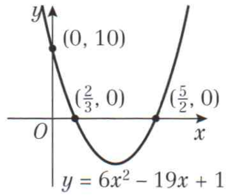
g



Turning point: $\left(-\frac{7}{4}, -\frac{169}{8}\right)$

Line of symmetry: $x = -\frac{7}{4}$

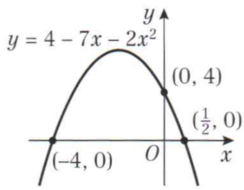
h



Turning point: $\left(\frac{19}{12}, -\frac{121}{24}\right)$

Line of symmetry: $x = \frac{19}{12}$

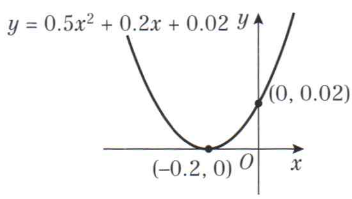
i



Turning point: $\left(-\frac{7}{4}, \frac{81}{8}\right)$

Line of symmetry: $x = -\frac{7}{4}$

j



Turning point: (−0.2, 0)
Line of symmetry: $x = -0.2$

2 **a** $a = 1, b = -8, c = 15$
b $a = -1, b = 3, c = 10$
c $a = 2, b = 0, c = -18$
d $a = \frac{1}{4}, b = -\frac{3}{4}, c = -1$

3 $a = 3, b = -30, c = 72$

Exercise 2G

1 **a** **i** 52 **ii** −23 **iii** 37 **iv** 0 **v** −44
b **i** h(x) **ii** f(x) **iii** k(x) **iv** j(x) **v** g(x)

2 $k < 9$

3 $t = \frac{9}{8}$

4 $s = 4$

5 $k > \frac{4}{3}$

6 **a** $p = 6$ **b** $x = -9$

7 **a** $k^2 + 16$
b k^2 is always positive so $k^2 + 16 > 0$

Challenge

a Need $b^2 > 4ac$. If $a, c > 0$ or $a, c < 0$, choose b such that $b > \sqrt{4ac}$. If $a > 0$ and $c < 0$ (or vice versa), then $4ac < 0$, so $4ac < b^2$ for all b.

b Not if one of a or c are negative as this would require b to be the square root of a negative number. Possible if both negative or both positive.

Exercise 2H

1 a The height of the bridge above water level
  b $x = 1103$ and $x = -1103$
  c 2206 m

2 a 21.8 mph and 75.7 mph
  b $A = 39.77$, $B = 0.01$, $C = 48.75$
  c 48.75 mph
  d −11 mpg; a negative answer is impossible so this model is not valid for very high speeds

3 a 6 tonnes
  b 39.6 kilograms per hectare.

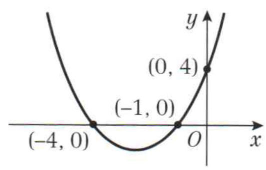
4 a $M = 40\,000$
  b $r = 400\,000 - 1000(p - 20)^2$
    $A = 400\,000$, $B = 1000$, $C = 20$
  c £20

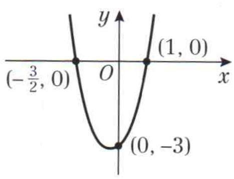
Challenge

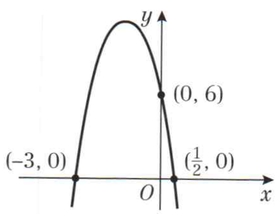
a $a = 0.01$, $b = 0.3$, $c = -4$
b 36.2 mph

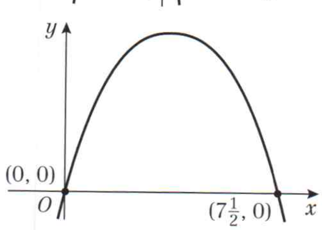
Mixed exercise 2

1 a $y = -1$ or -2 b $x = \frac{2}{3}$ or -5
  c $x = -\frac{1}{5}$ or 3   d $x = \frac{5 \pm \sqrt{7}}{2}$

2 a


  b


  c


  d


3 a $k = 1$ b $x = 3$ and $x = -2$

4 a $k = 0.0902$ or $k = -11.1$
  b $t = 2.28$ or $t = 0.219$
  c $x = -2.30$ or $x = 1.30$
  d $x = 0.839$ or $x = -0.239$

5 a $(x + 6)^2 - 45$; $p = 1$, $q = 6$, $r = -45$
  b $5(x - 4)^2 - 67$; $p = 5$, $q = -4$, $r = -67$
  c $-2(x - 2)^2 + 8$; $p = -2$, $q = -2$, $r = 8$
  d $2\left(x - \frac{1}{2}\right)^2 - \frac{3}{2}$; $p = 2$, $q = -\frac{1}{2}$, $r = -\frac{3}{2}$

6 $k = \frac{1}{5}$

7 a $p = 3$, $q = 2$, $r = -7$ b $-2 \pm \sqrt{\frac{7}{3}}$

8 a $f(x) = (2^x - 16)(2^x - 4)$ b 4 and 2

9 $1 \pm \sqrt{13}$

10 $x = -5$ or $x = 4$

11 a 10 m   b 1.28 s
  c $h(t) = 10.625 - 10(t - 0.25)^2$
    $A = 10.625$, $B = 10$, $C = 0.25$
  d 10.625 m at 0.25 s

12 a $16k^2 + 4$
  b $k^2 \geqslant 0$ for all k, so $16k^2 + 4 > 0$
  c When $k = 0$, $f(x) = 2x + 1$; this is a linear function with only one root

13 a $\frac{9}{4}$ b 1, −1, 2 and −2

14 a $H = 10$
  b $r = 1322.5 - 10(p - 11.5)^2$
    $A = 1322.5$, $B = 10$, $C = 11.5$
  c Old revenue is 80 × £15 = £1200; new revenue is £1322.50; difference is £122.50. The best selling price of a cushion is £11.50.

Challenge

a $\frac{b + c}{b} = \frac{b}{c}$
  $b^2 - cb - c^2 = 0$
  Using quadratic formula: $b = \frac{c + \sqrt{5c^2}}{2}$
  So $b : c$ is $\frac{c + \sqrt{5c^2}}{2} : c$
  Dividing by c: $\frac{1 + \sqrt{5}}{2} : 1$

b Let $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$
  So $x = \sqrt{1 + x} \Rightarrow x^2 - x - 1 = 0$
  Using quadratic formula: $x = \frac{1 + \sqrt{5}}{2}$

CHAPTER 3

Prior knowledge check

1 a $A \cap B = \{1, 2, 4\}$ b $(A \cup B)' = \{7, 9, 11, 13\}$
2 a $5\sqrt{3}$ b $\sqrt{5} + 2\sqrt{2}$
3 a graph **ii**   b graph **iii**   c graph **i**

Exercise 3A

1 a $x = 4$, $y = 2$ b $x = 1$, $y = 3$
  c $x = 2$, $y = -2$ d $x = 4\frac{1}{2}$, $y = -3$
  e $x = -\frac{2}{3}$, $y = 2$ f $x = 3$, $y = 3$

2 a $x = 5$, $y = 2$ b $x = 5\frac{1}{2}$, $y = -6$
  c $x = 1$, $y = -4$ d $x = 1\frac{3}{4}$, $y = \frac{1}{4}$

3 a $x = -1$, $y = 1$ b $x = 4$, $y = -4$
  c $x = 0.5$, $y = -2.5$

4 a $3x + ky = 8$ (1); $x - 2ky = 5$ (2)
    (1) × 2: $6x + 2ky = 16$ (3)
    (2) + (3) $7x = 21$ so $x = 3$
  b −2

5 $p = 3$, $q = 1$

Exercise 3B

1 a $x = 5$, $y = 6$ or $x = 6$, $y = 5$
  b $x = 0$, $y = 1$ or $x = \frac{4}{5}$, $y = -\frac{3}{5}$
  c $x = -1$, $y = -3$ or $x = 1$, $y = 3$

d $a = 1, b = 5$ or $a = 3, b = -1$

e $u = 1\frac{1}{2}, v = 4$ or $u = 2, v = 3$

f $x = -1\frac{1}{2}, y = 5\frac{3}{4}$ or $x = 3, y = -1$

2 a $x = 3, y = \frac{1}{2}$ or $x = 6\frac{1}{3}, y = -2\frac{5}{6}$

b $x = 4\frac{1}{2}, y = 4\frac{1}{2}$ or $x = 6, y = 3$

c $x = -19, y = -15$ or $x = 6, y = 5$

3 a $x = 3 + \sqrt{13}, y = -3 + \sqrt{13}$ or
$x = 3 - \sqrt{13}, y = -3 - \sqrt{13}$

b $x = 2 - 3\sqrt{5}, y = 3 + 2\sqrt{5}$ or $x = 2 + 3\sqrt{5}$,
$y = 3 - 2\sqrt{5}$

4 $x = -5, y = 8$ or $x = 2, y = 1$

5 a $3x^2 + x(2 - 4x) + 11 = 0$
$3x^2 + 2x - 4x^2 + 11 = 0$
$x^2 - 2x - 11 = 0$

b $x = 1 + 2\sqrt{3}, y = -2 - 8\sqrt{3}$
or $x = 1 - 2\sqrt{3}, y = -2 + 8\sqrt{3}$

6 a $k = 3, p = -2$

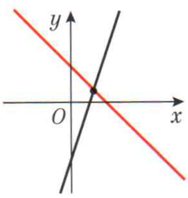
b $x = -6, y = -23$

Challenge

$y = x + k$
$x^2 + (x + k)^2 = 4$
$x^2 + x^2 + 2kx + k^2 - 4 = 0$
$2x^2 + 2kx + k^2 - 4 = 0$ for one solution $b^2 - 4ac = 0$
$4k^2 - 4 \times 2(k^2 - 4) = 0$
$4k^2 - 8k^2 + 32 = 0$ $4k^2 = 32$ $k^2 = 8$ $k = \pm 2\sqrt{2}$

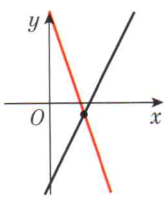
Exercise 3C

1 a i



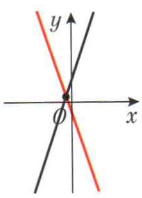
ii (2, 1)

b i



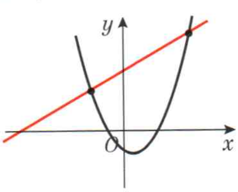
ii (3, –1)

c i



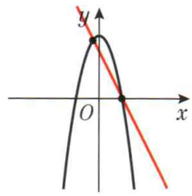
ii (–0.5, 0.5)

2 a



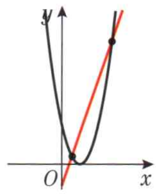
b (3.5, 9) and (–1.5, 4)

3 a



b (–1, 8) and (3, 0)

4 a



b (6, 16) and (1, 1)

5 (–11, –15) and (3, –1)

6 $(-1\frac{1}{6}, -4\frac{1}{2})$ and (2, 5)

7 a 2 points b 1 point c 0 points

8 a $y = 2x - 1$
$x^2 + 4k(2x - 1) + 5k = 0$
$x^2 + 8kx - 4k + 5k = 0$ $x^2 + 8kx + k = 0$

b $k = \frac{1}{16}$ c $x = -\frac{1}{4}, y = -\frac{3}{2}$

9 If swimmer reaches the bottom of the pool
$0.5x^2 - 3x = 0.3x - 6$
$0.5x^2 - 3.3x + 6 = 0$
$b^2 - 4ac = (-3.3)^2 - 4 \times 0.5 \times 6 = -1.11$
negative so no points of intersection and swimmer does not reach the bottom of the pool

Exercise 3D

1 a $x < 4$ b $x \geqslant 7$ c $x > 2\frac{1}{2}$ d $x \leqslant -3$
e $x < 11$ f $x < 2\frac{3}{5}$ g $x > -12$ h $x < 1$
i $x \leqslant 8$ j $x > 1\frac{1}{7}$

2 a $x \geqslant 3$ b $x < 1$ c $x \leqslant -3\frac{1}{4}$ d $x < 18$
e $x > 3$ f $x \geqslant 4\frac{2}{5}$ g $x < 4$ h $x > -7$
i $x \leqslant -\frac{1}{2}$ j $x \geqslant \frac{3}{4}$ k $x \geqslant -\frac{10}{3}$ l $x \geqslant \frac{9}{11}$

3 a $\{x: x > 2\frac{1}{2}\}$ b $\{x: 2 < x < 4\}$
c $\{x: 2\frac{1}{2} < x < 3\}$ d No values
e $x = 4$ f $\{x: x < 1.2\} \cup \{x: x > 2.2\}$
g $\{x: x \leqslant -\frac{2}{3}\} \cup \{x: x \geqslant \frac{3}{2}\}$

Challenge

$p = -1, q = 4, r = 6$

Exercise 3E

1 a $3 < x < 8$ b $-4 < x < 3$
c $x < -2, x > 5$ d $x \leqslant -4, x \geqslant -3$
e $-\frac{1}{2} < x < 7$ f $x < -2, x > 2\frac{1}{2}$
g $\frac{1}{2} \leqslant x \leqslant 1\frac{1}{2}$ h $x < \frac{1}{3}, x > 2$
i $-3 < x < 3$ j $x < -2\frac{1}{2}, x > \frac{2}{3}$
k $x < 0, x > 5$ l $-1\frac{1}{2} \leqslant x \leqslant 0$

2 a $-5 < x < 2$ b $x < -1, x > 1$
c $\frac{1}{2} < x < 1$ d $-3 < x < \frac{1}{4}$

3 a $\{x: 2 < x < 4\}$ b $\{x: x > 3\}$
c $\{x: -\frac{1}{4} < x < 0\}$ d No values

e $\{x: -5 < x < -3\} \cup \{x: x > 4\}$
f $\{x: -1 < x < 1\} \cup \{x: 2 < x < 3\}$

4 a $x < 0$ or $x > 2$ b $x < 0$ or $x > 0.8$
c $x < -1$ or $x > 0$ d $x < 0$ or $x > 0.5$
e $x < -\frac{1}{5}$ or $x > \frac{1}{5}$ f $x \leqslant -\frac{2}{3}$ or $x \geqslant 3$

5 a $-2 < k < 6$ b $p \leqslant -8$ or $p \geqslant 0$

6 $\{x: x < -2\} \cup \{x: x > 7\}$

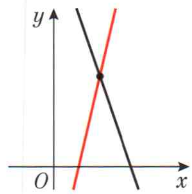
7 a $\{x: x < \frac{2}{3}\}$ b $\{x: -\frac{1}{2} < x < 3\}$
c $\{x: -\frac{1}{2} < x < \frac{2}{3}\}$

8 $x < 3$ or $x > 5.5$

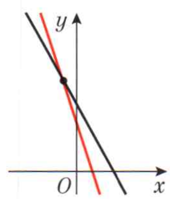
9 No real roots $b^2 - 4ac < 0$ $(-2k)^2 - 4 \times k \times 3 < 0$
$4k^2 - 12k = 0$ when $k = 0$ and $k = 3$
solution $0 \leqslant k < 3$
note when $k = 0$ equation gives $3 = 0$

Exercise 3F

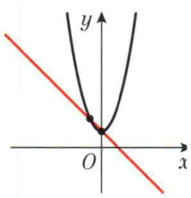
1 a $P(3.2, -1.8)$ b $x < 3.2$

2 a i


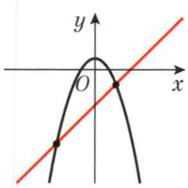
ii (4, 5) iii $x \leqslant 4$

b i


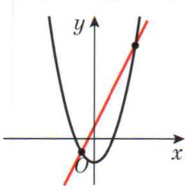
ii (−3, 23) iii $x \geqslant -3$

c i


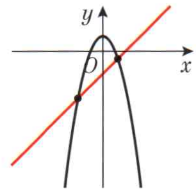
ii (−2, 9), (0, 5) iii $-2 \leqslant x \leqslant 0$

d i


ii (−5, −22), (3, −6) iii $x \leqslant -5$ or $x \geqslant 3$

e i


ii (−2, −1), (9, 76) iii $-2 \leqslant x \leqslant 9$

f i


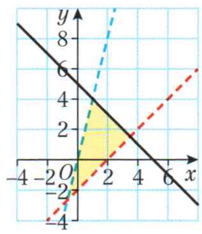
ii (−5, −18), (3, −2) iii $x \leqslant -5$ or $x \geqslant 3$

3 a $-1 < x < 2$ b $0.5 < x < 3$
c $x < 0.5$ or $x > 3$ d $x < 0$ or $x > 2$
e $1 < x < 3$ f $x < -1$ or $x > -0.75$

Challenge

a (−1.5, −3.75), (6, 0)
b $\{x: -1.5 < x < 6\}$

Exercise 3G

1


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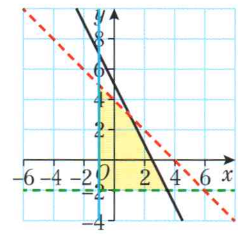

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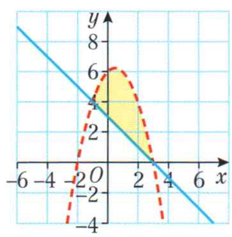

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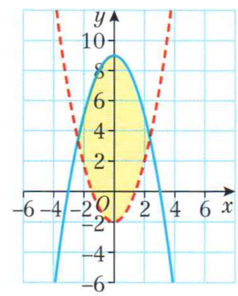

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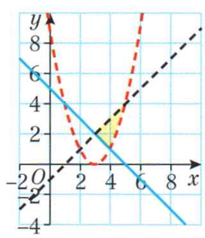
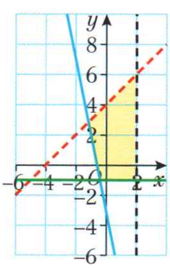


6 a (1, 6), (3, 4), (1, 2)
b $x \geqslant 1, y \leqslant 7 - x, y \geqslant x + 1$

7 $y < 2 - 5x - x^2, 2x + y \geqslant 0, x + y \leqslant 4$

8 a


b $(-\frac{7}{6}, \frac{17}{6})$, (2, 6), (2, −1), (−0.4, −1)
c (−0.4, −1) d $\frac{941}{60}$

Mixed exercise 3

1 **a** $4kx - 2y = 8$
$4kx + 3y = -2$
$-5y = 10$
$y = -2$

b $x = \frac{1}{k}$

2 $x = -4, y = 3\frac{1}{2}$

3 **a** Substitute $x = 1 + 2y$ into $3xy - y^2 = 8$
b $(3, 1)$ and $\left(-\frac{11}{5}, -\frac{8}{5}\right)$

4 **a** Substitute $y = 2 - x$ into $x^2 + xy - y^2 = -1$
b $x = 3 \pm \sqrt{6}, y = -1 \mp \sqrt{6}$

5 **a** $3^x = (3^2)^{y-1} = 3^{2y-2} \Rightarrow x = 2y - 2$
b $x = 4, y = 3$ and $x = -2\frac{2}{3}, y = -\frac{1}{3}$

6 $x = -1\frac{1}{2}, y = 2\frac{1}{4}$ and $x = 4, y = -\frac{1}{2}$

7 **a** $k = -2$ **b** $(-1, 2)$

8 Yes, the ball will hit the ceiling

9 **a** $\{x: x > 10\frac{1}{2}\}$ **b** $\{x: x < -2\} \cup \{x: x > 7\}$

10 $3 < x < 4$

11 **a** $x = -5, x = 4$ **b** $\{x: x < -5\} \cup \{x: x > 4\}$

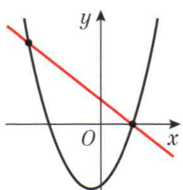
12 **a** $x < 2\frac{1}{2}$ **b** $\frac{1}{2} < x < 5$
c $0 < x < 4$ **d** $\frac{1}{2} < x < 2\frac{1}{2}$

13 $1 \leqslant x \leqslant 8$

14 $k \leqslant 3\frac{1}{5}$

15 $b^2 < 4ac$ so $16k^2 < -40k$
$8k(2k + 5) < 0$ so $-\frac{5}{2} < k < 0$

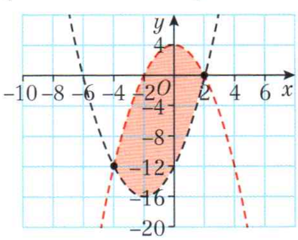
16 **a**



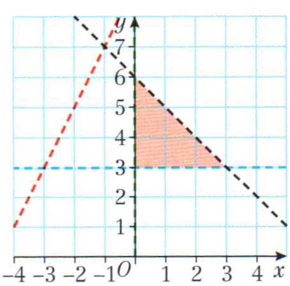
b $(-7, 20), (3, 0)$ **c** $x < -7, x > 3$

17 $\frac{1}{4}(-1 - \sqrt{185}) < x < \frac{1}{4}(-1 + \sqrt{185})$

18



19 **a**



b $\frac{9}{2}$

Challenge

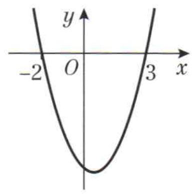
1 $0 < k \leqslant 1.6$ **2** $-2 \leqslant k \leqslant 7$

CHAPTER 4

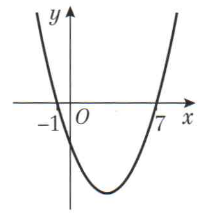
Prior knowledge check

1 **a** $(x + 5)(x + 1)$ **b** $(x - 3)(x - 1)$

2 **a**



b

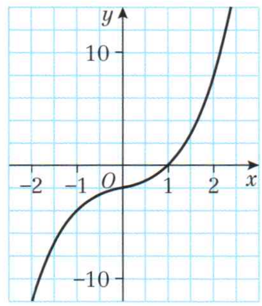


3 **a**

x	−2	−1.5	−1	−0.5	0
y	−12	−6.875	−4	−2.625	−2

x	0.5	1	1.5	2
y	−1.375	0	2.875	8

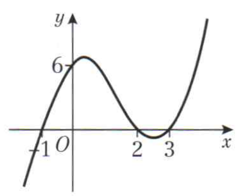
b



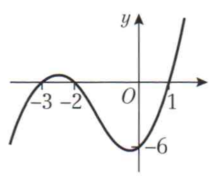
4 **a** $x = 2, y = 4$ **b** $x = 1, y = 1$

Exercise 4A

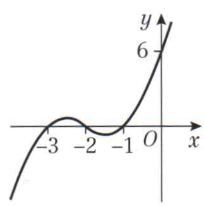
1 **a**



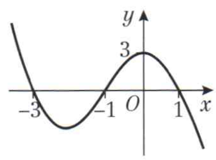
b



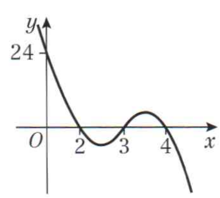
c



d



e



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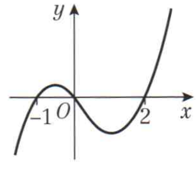

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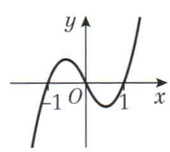

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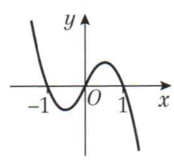
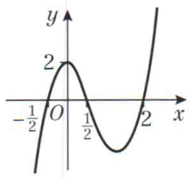
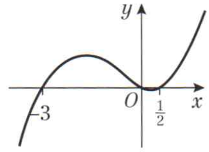


Online Full worked solutions are available in SolutionBank.

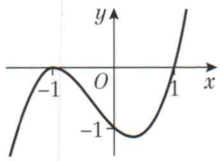
i



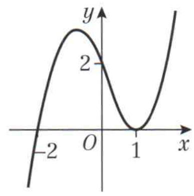
j



2 **a**



b



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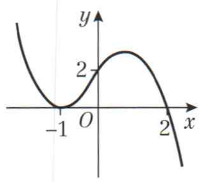
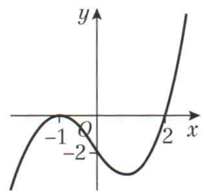
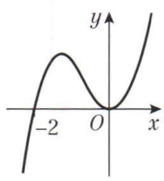


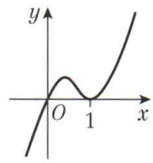
d



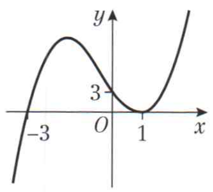
e



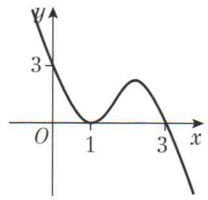
f



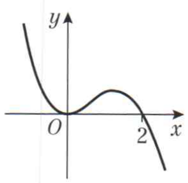
g



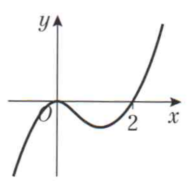
h



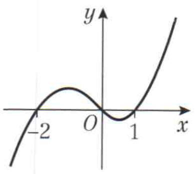
i



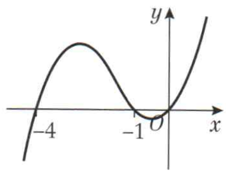
j



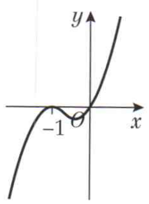
3 **a** $y = x(x + 2)(x - 1)$



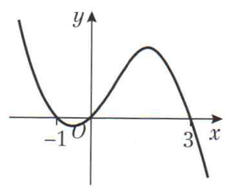
b $y = x(x + 4)(x + 1)$



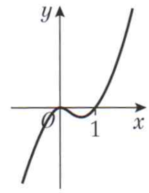
c $y = x(x + 1)^2$



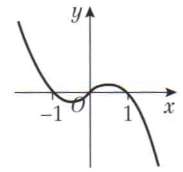
d $y = x(x + 1)(3 - x)$



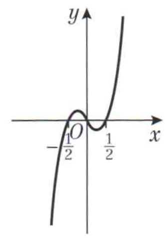
e $y = x^2(x - 1)$



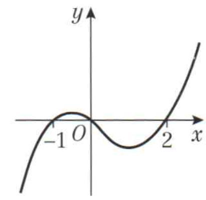
f $y = x(1 - x)(1 + x)$



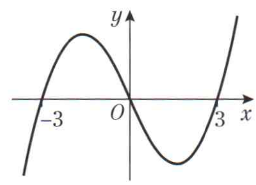
g $y = 3x(2x - 1)(2x + 1)$



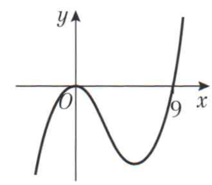
h $y = x(x + 1)(x - 2)$



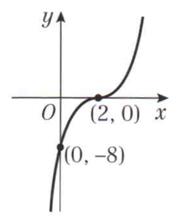
i $y = x(x - 3)(x + 3)$



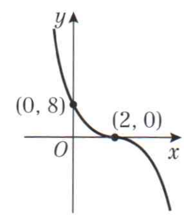
j $y = x^2(x - 9)$



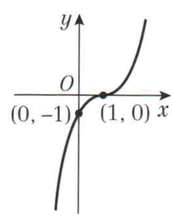
4 **a**



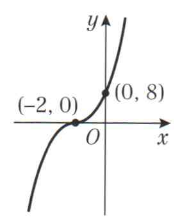
b



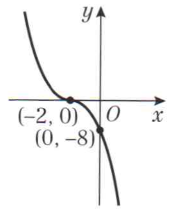
c



d



e



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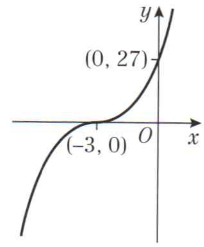
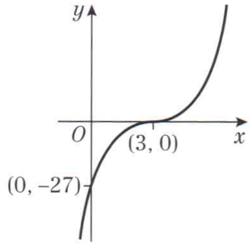


g


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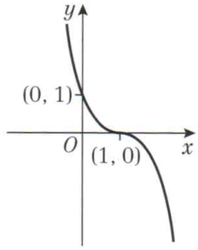
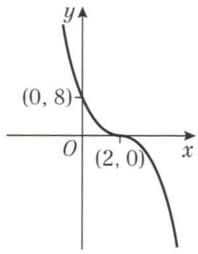


i


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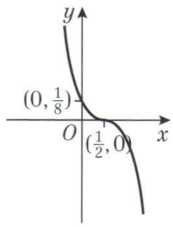
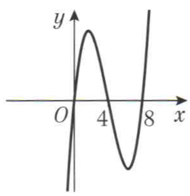


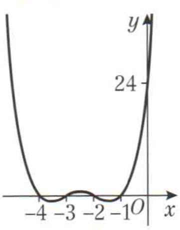
5 a $b = 4, c = 1, d = -6$ b $(0, -6)$

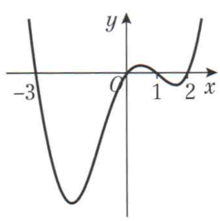
6 $a = \frac{1}{3}, b = -\frac{4}{3}, c = \frac{1}{3}, d = 2$

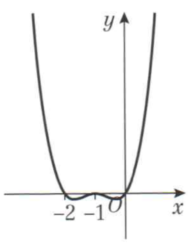
7 a $x(x^2 - 12x + 32)$ b $x(x - 8)(x - 4)$

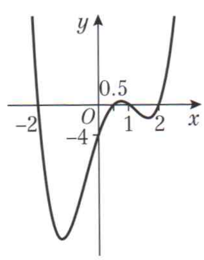
c


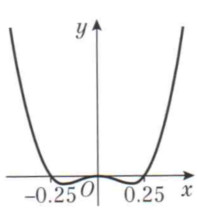
Exercise 4B

1 a


b


c


d


e


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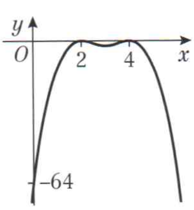
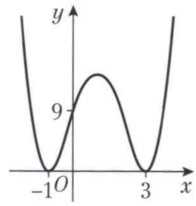
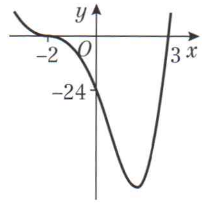
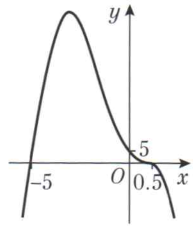
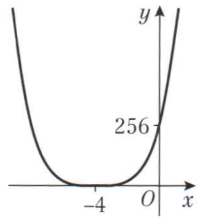
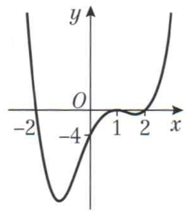


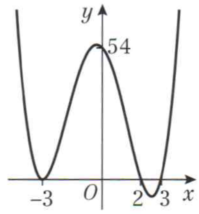
g


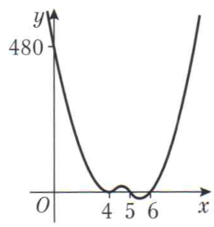
h


i


j


2 a


b


c


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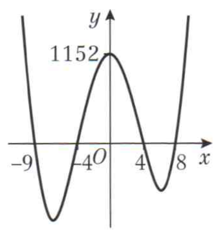
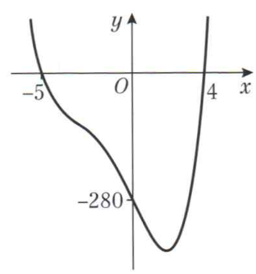


3 a $(0, 12)$

b $b = -2, c = -7, d = 8, e = 12$

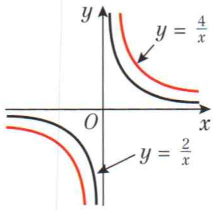
4


Challenge

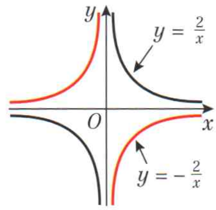
$a = \frac{1}{3}, b = -\frac{4}{3}, c = -\frac{2}{3}, d = 4, e = 3$

Exercise 4C

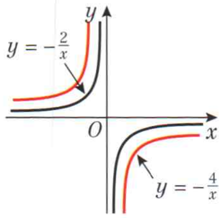
1 a



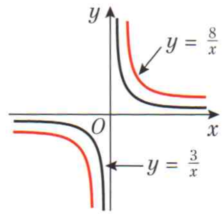
b



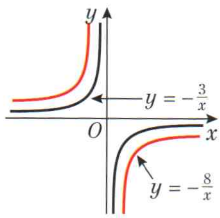
c



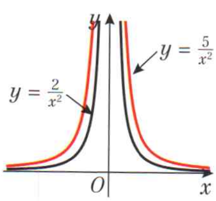
d



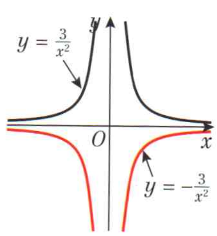
e



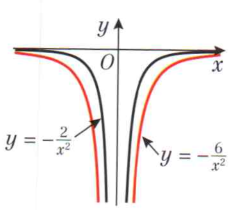
2 a



b

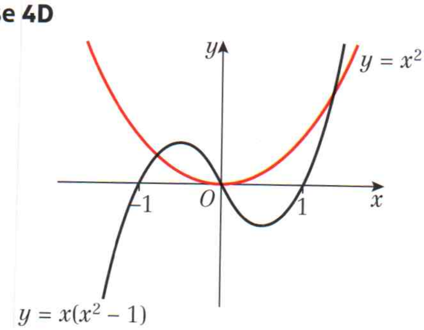


c



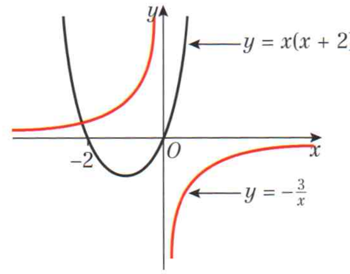
Exercise 4D

1 a i



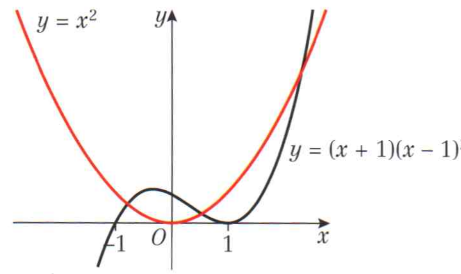
ii 3 iii $x^2 = x(x^2 - 1)$

b i



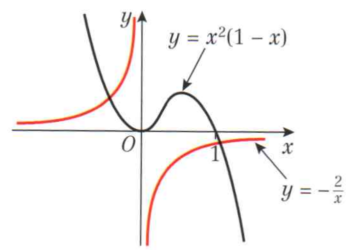
ii 1 iii $x(x + 2) = -\frac{3}{x}$

c i



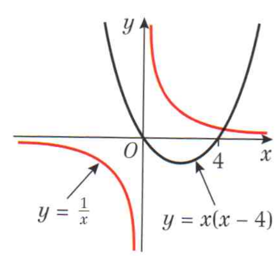
ii 3 iii $x^2 = (x + 1)(x - 1)^2$

d i



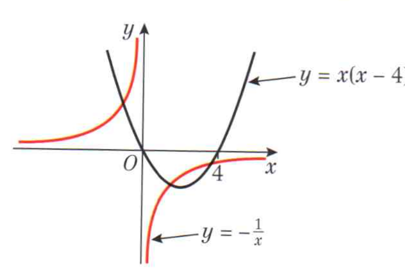
ii 2 iii $x^2(1 - x) = -\frac{2}{x}$

e i



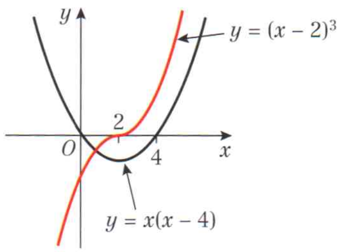
ii 1 iii $x(x - 4) = \frac{1}{x}$

f i



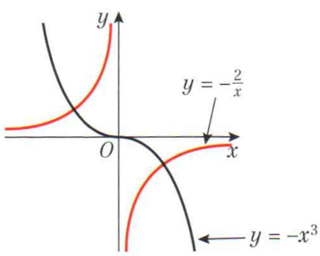
ii 3 iii $x(x - 4) = -\frac{1}{x}$

g **i**



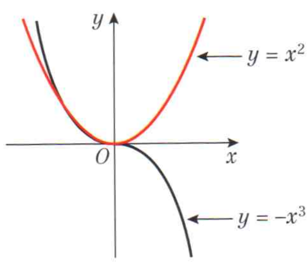
ii 1 **iii** $x(x - 4) = (x - 2)^3$

h **i**



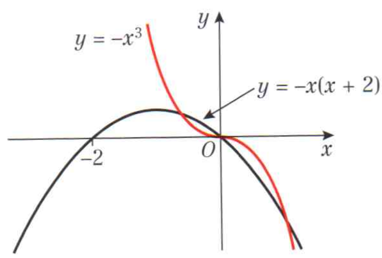
ii 2 **iii** $-x^3 = -\frac{2}{x}$

i **i**



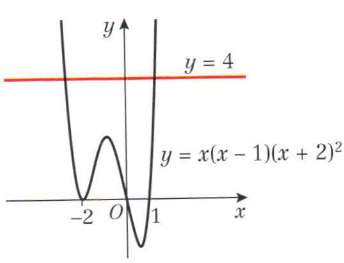
ii 2 **iii** $-x^3 = x^2$

j **i**



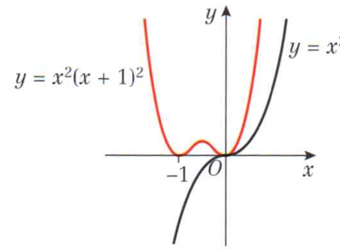
ii 3 **iii** $-x^3 = -x(x + 2)$

k **i**



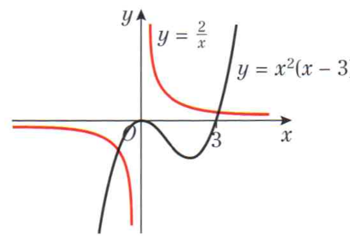
ii 2 **iii** $x(x - 1)(x + 2)^2 = 4$

l **i**



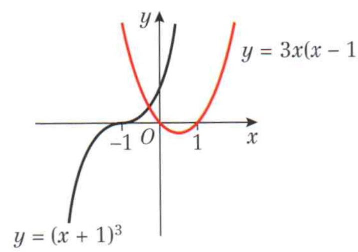
ii 1 **iii** $x^3 = x^2(x + 1)^2$

2 **a**



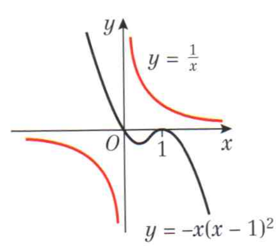
b Only 2 intersections

3 **a**



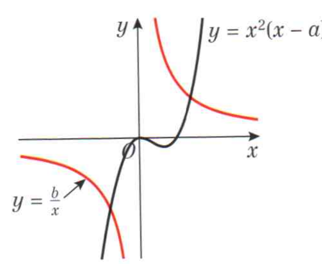
b Only 1 intersection

4 **a**



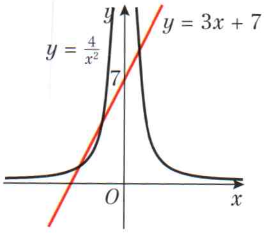
b Graphs do not intersect

5 **a**



b 2; the graphs cross in two places so there are two solutions.

6 a

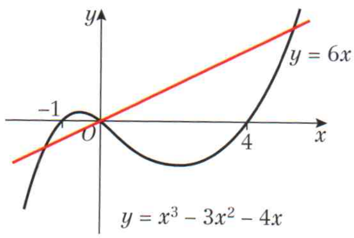


b 3

c Expand brackets and rearrange.

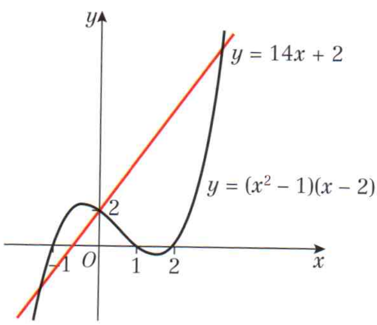
d $(-2, 1)$, $(-1, 4)$, $(\frac{2}{3}, 9)$

7 a



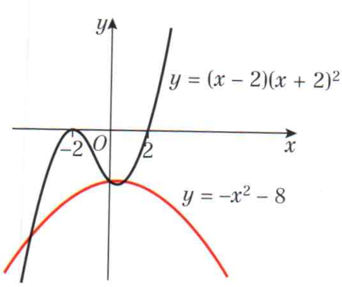
b $(0, 0)$; $(-2, -12)$; $(5, 30)$

8 a



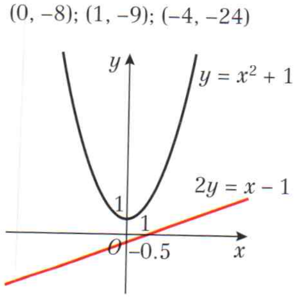
b $(0, 2)$; $(-3, -40)$; $(5, 72)$

9 a



b $(0, -8)$; $(1, -9)$; $(-4, -24)$

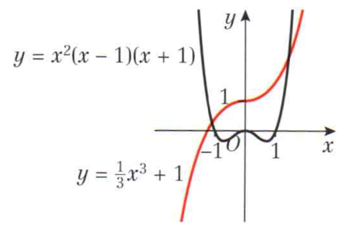
10 a



b Graphs do not intersect.

c $a < -\frac{7}{16}$

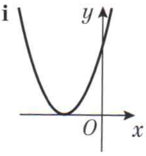
11 a



b 2

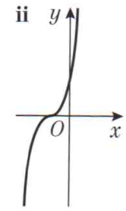
Exercise 4E

1 a i



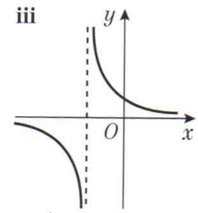
$(-2, 0)$, $(0, 4)$

ii



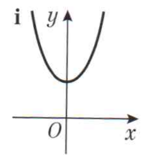
$(-2, 0)$, $(0, 8)$

iii



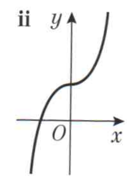
$(0, \frac{1}{2})$, $x = -2$, $y = 0$

b i



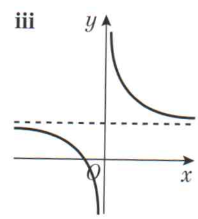
$(0, 2)$

ii



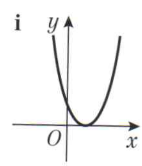
$(-\sqrt[3]{2}, 0)$, $(0, 2)$

iii



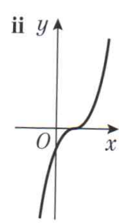
$(-\frac{1}{2}, 0)$, $y = 2$, $x = 0$

c i



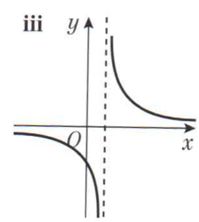
$(0, 1)$, $(1, 0)$

ii



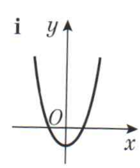
$(0, -1)$, $(1, 0)$

iii



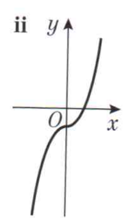
$(0, -1)$, $x = 1$, $y = 0$

d i



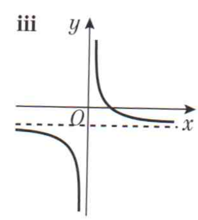
$(-1, 0)$,
$(0, -1)$,
$(1, 0)$

ii



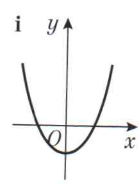
$(0, -1)$, $(1, 0)$

iii



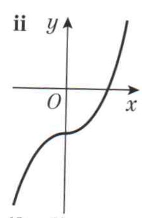
$(1, 0)$, $y = -1$, $x = 0$

e i



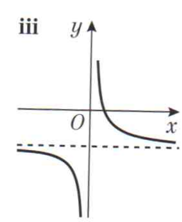
$(-\sqrt{3}, 0)$,
$(0, -3)$,
$(\sqrt{3}, 0)$

ii



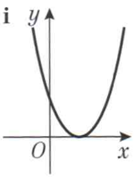
$(0, -3)$,
$(\sqrt[3]{3}, 0)$

iii



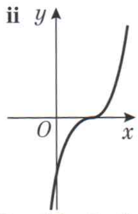
$(\frac{1}{3}, 0)$, $y = -3$, $x = 0$

f **i**



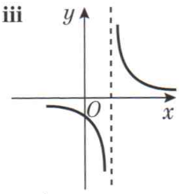
(0, 9), (3, 0)

ii



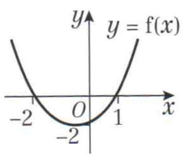
(0, −27), (3, 0)

iii

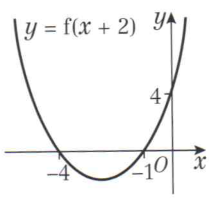


$(0, -\frac{1}{3})$, $x = 3$, $y = 0$

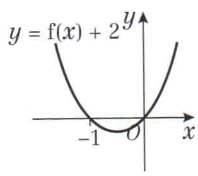
2 **a**



b **i**

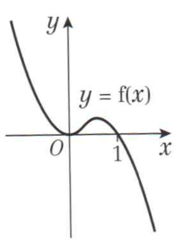


ii

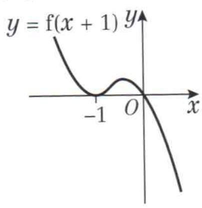


c $f(x + 2) = (x + 1)(x + 4)$; (0, 4)
$f(x) + 2 = (x - 1)(x + 2) + 2$; (0, 0)

3 **a**

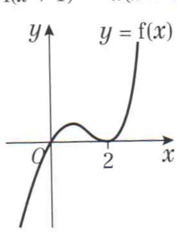


b

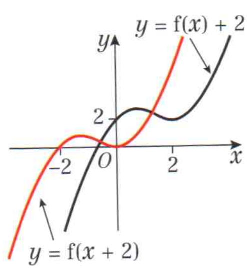


c $f(x + 1) = -x(x + 1)^2$; (0, 0)

4 **a**

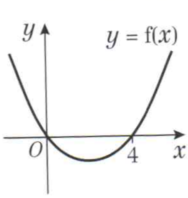


b

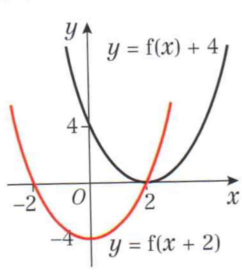


c $f(x + 2) = (x + 2)x^2$; (0, 0); (−2, 0)

5 **a**

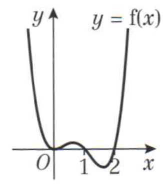


b

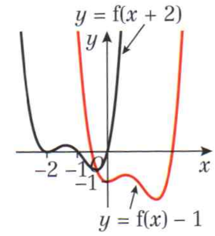


c $f(x + 2) = (x + 2)(x - 2)$; (2, 0); (−2, 0); (0, −4)
$f(x) + 4 = (x - 2)^2$; (2, 0); (0, 4)

6 **a**



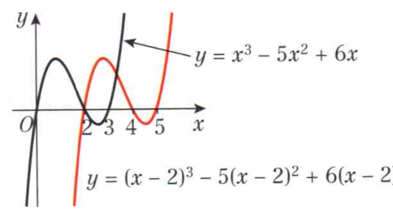
b



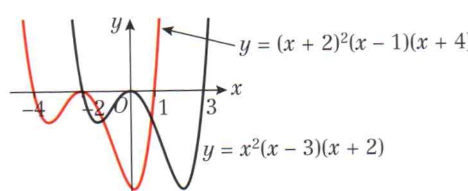
7 **a** (6, −1) **b** (4, 2)

8 $y = \dfrac{1}{x - 4}$

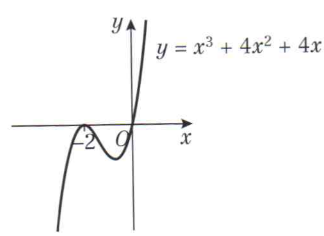
9



10

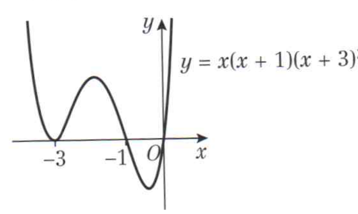


11 **a**



b −1 or 1

12 **a**



b −2, −3 or −5

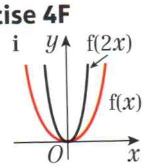
Challenge

1 (3, 2)

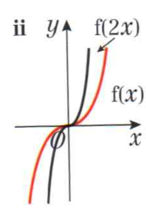
2 **a** (−7, −12) **b** $f(x - 2) + 1$

Exercise 4F

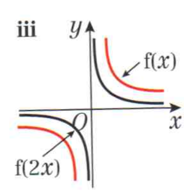
1 **a** **i**



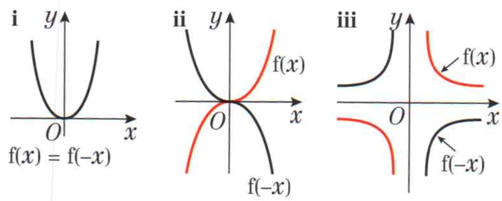
ii



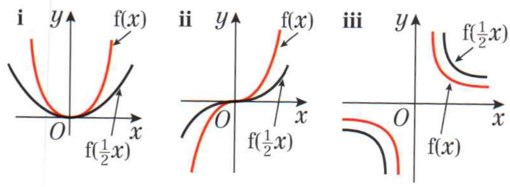
iii



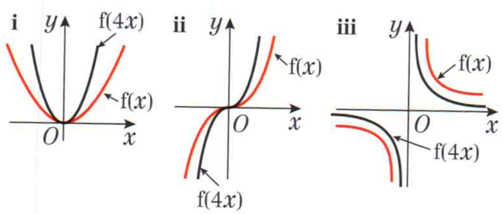
b



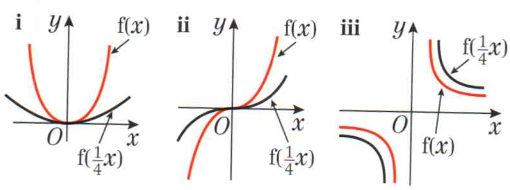
c



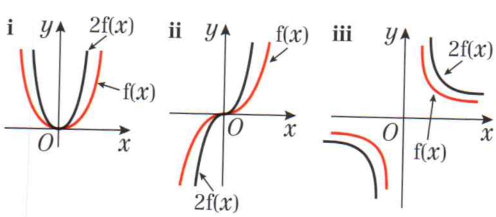
d



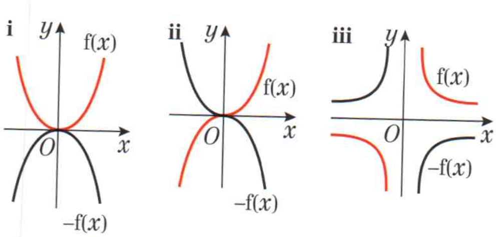
e



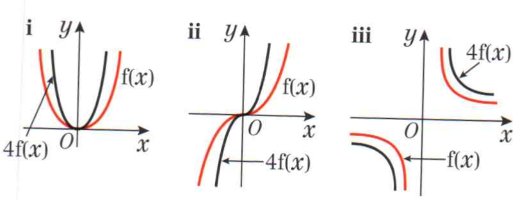
f



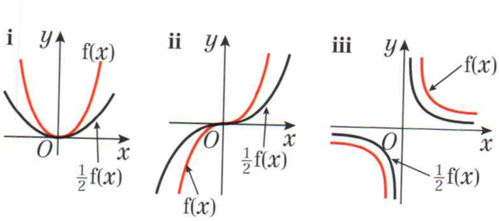
g



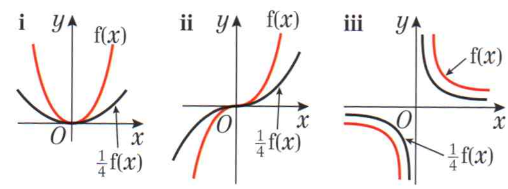
h



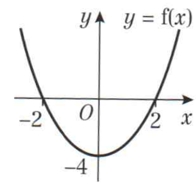
i



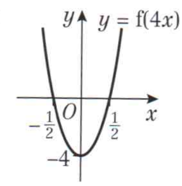
j

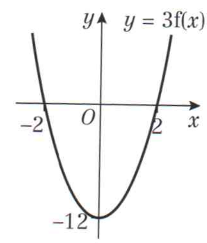


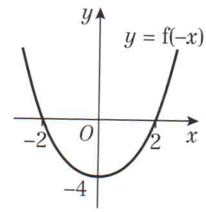
2 **a**

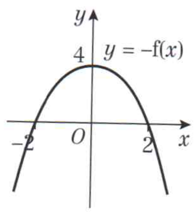


b

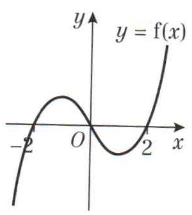




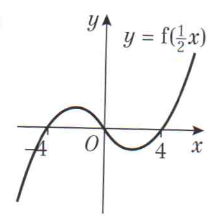


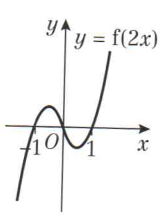


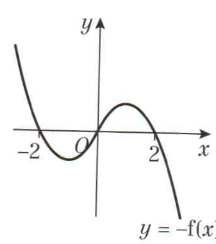
3 **a**



b







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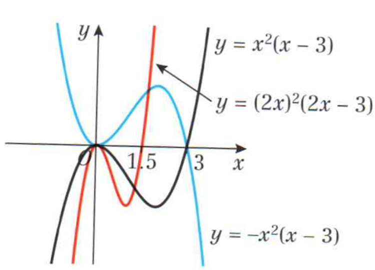
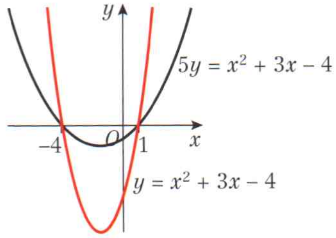
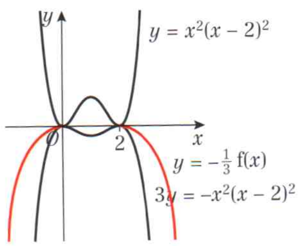
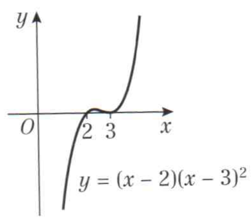


5 

6 

7 a (1, −3) b (2, −12)

8 (−4, 8)

9 a 

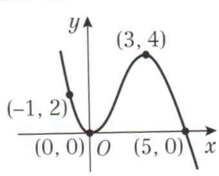
b 2 and 3

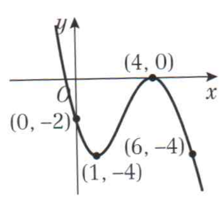
Challenge

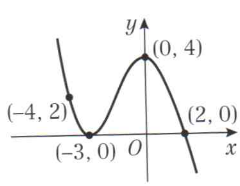
1 (2, −2)

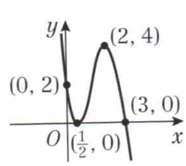
2 $\frac{1}{4}f(\frac{1}{2}x)$

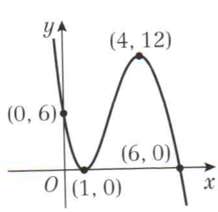
Exercise 4G

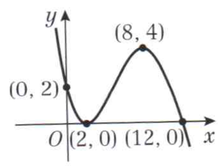
1 a 

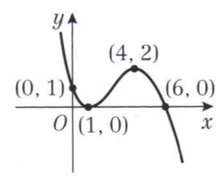
b 

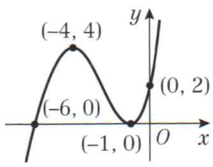
c 

d 

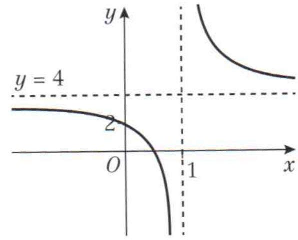
e 

f 

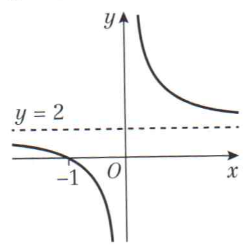
g 

h 

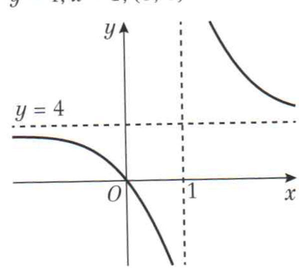
2 a $y = 4$, $x = 1$, (0, 2)



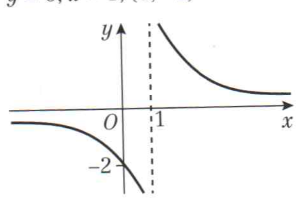
b $y = 2$, $x = 0$, (−1, 0)



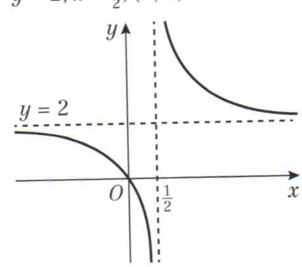
c $y = 4$, $x = 1$, (0, 0)



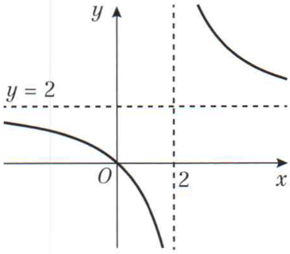
d $y = 0$, $x = 1$, (0, −2)



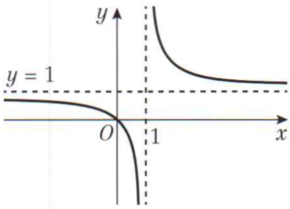
e $y = 2$, $x = \frac{1}{2}$, (0, 0)



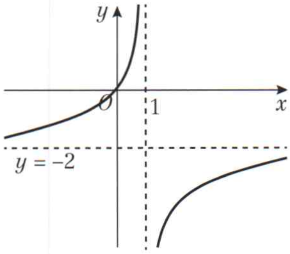
f $y = 2$, $x = 2$, $(0, 0)$



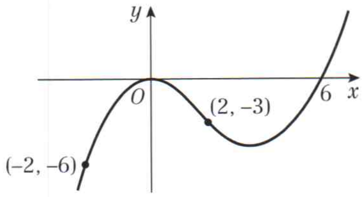
g $y = 1$, $x = 1$, $(0, 0)$



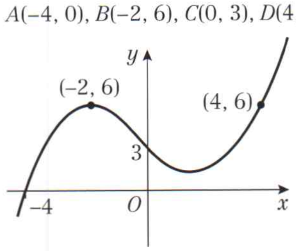
h $y = -2$, $x = 1$, $(0, 0)$



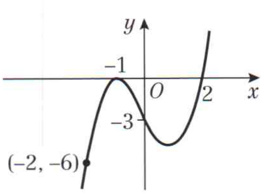
3 **a** $A(-2, -6)$, $B(0, 0)$, $C(2, -3)$, $D(6, 0)$



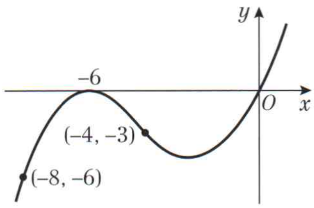
b $A(-4, 0)$, $B(-2, 6)$, $C(0, 3)$, $D(4, 6)$



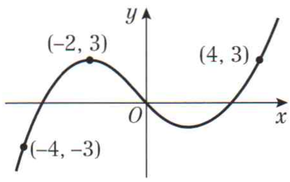
c $A(-2, -6)$, $B(-1, 0)$, $C(0, -3)$, $D(2, 0)$



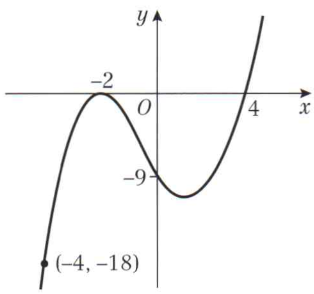
d $A(-8, -6)$, $B(-6, 0)$, $C(-4, -3)$, $D(0, 0)$



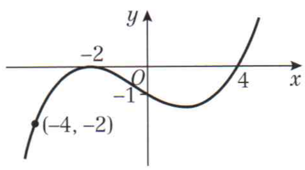
e $A(-4, -3)$, $B(-2, 3)$, $C(0, 0)$, $D(4, 3)$



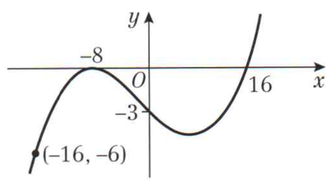
f $A(-4, -18)$, $B(-2, 0)$, $C(0, -9)$, $D(4, 0)$



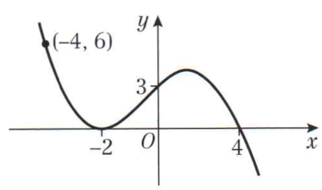
g $A(-4, -2)$, $B(-2, 0)$, $C(0, -1)$, $D(4, 0)$



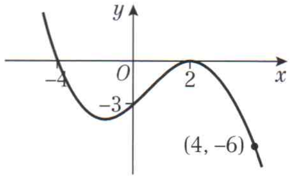
h $A(-16, -6)$, $B(-8, 0)$, $C(0, -3)$, $D(16, 0)$



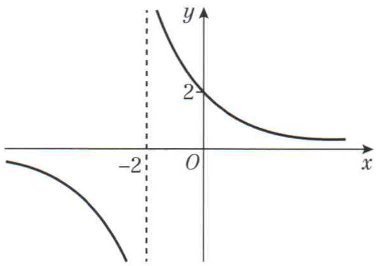
i $A(-4, 6)$, $B(-2, 0)$, $C(0, 3)$, $D(4, 0)$



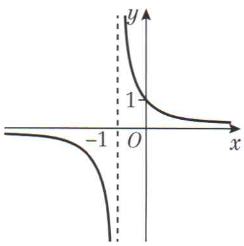
j $A(4, -6)$, $B(2, 0)$, $C(0, -3)$, $D(-4, 0)$



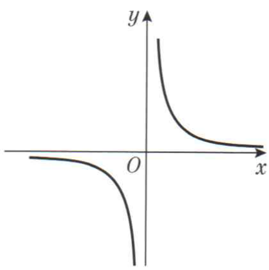
4 a i $x = -2$, $y = 0$, $(0, 2)$



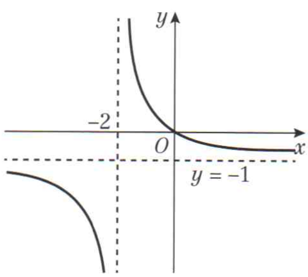
ii $x = -1$, $y = 0$, $(0, 1)$



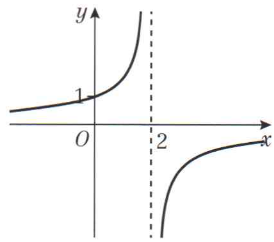
iii $x = 0$, $y = 0$



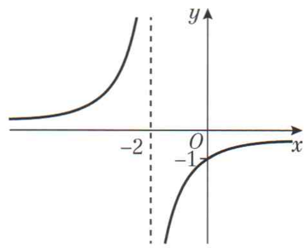
iv $x = -2$, $y = -1$, $(0, 0)$



v $x = 2$, $y = 0$, $(0, 1)$



vi $x = -2$, $y = 0$, $(0, -1)$

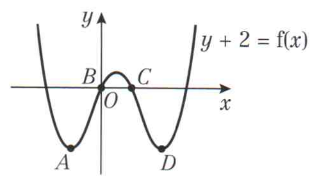


b $f(x) = \dfrac{2}{x + 2}$

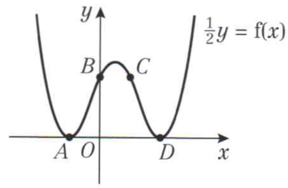
5 a $\frac{1}{2}$

b **i** $(6, 1)$ **ii** $(2, 3)$ **iii** $(2, -3.5)$

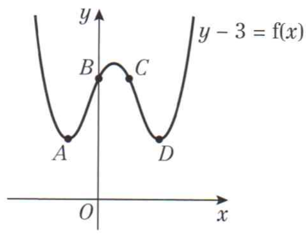
6 a $A(-1, -2)$ $B(0, 0)$ $C(1, 0)$ $D(2, -2)$



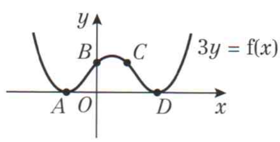
b $A(-1, 0)$ $B(0, 4)$ $C(1, 4)$ $D(2, 0)$



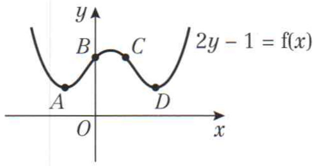
c $A(-1, 3)$ $B(0, 5)$ $C(1, 5)$ $D(2, 3)$



d $A(-1, 0)$ $B(0, \frac{2}{3})$ $C(1, \frac{2}{3})$ $D(2, 0)$

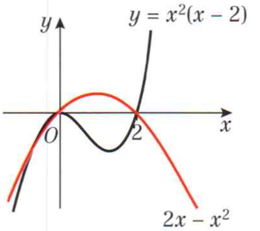


e $A(-1, 0.5)$ $B(0, 1.5)$ $C(1, 1.5)$ $D(2, 0.5)$



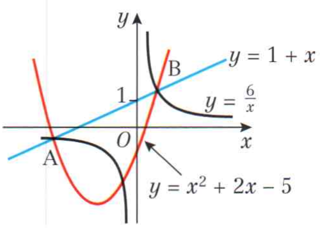
Mixed exercise 4

1 **a**



b $x = 0, -1, 2$; points $(0, 0)$, $(2, 0)$, $(-1, -3)$

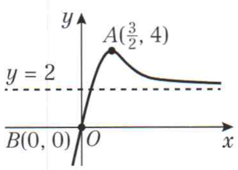
2 **a**



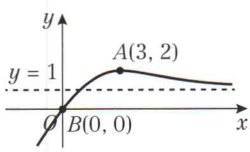
b $A(-3, -2)$, $B(2, 3)$

c $y = x^2 + 2x - 5$

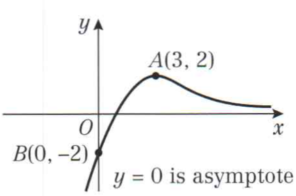
3 **a**



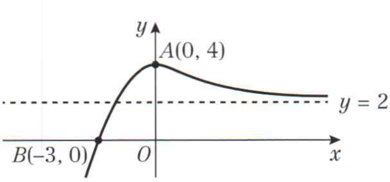
b



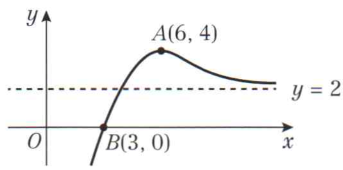
c



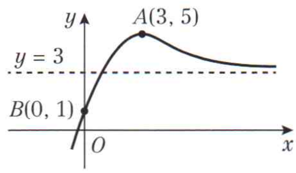
d



e

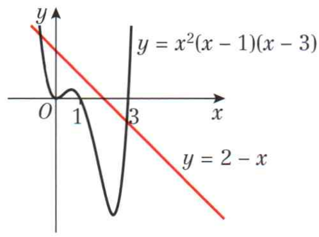


f



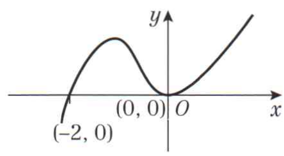
4 **a** $x = -1$ at A, $x = 3$ at B

5 **a, b**

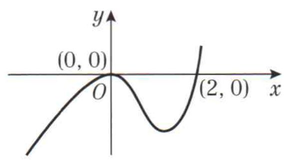


c 2 **d** $(0, 2)$

6 **a**

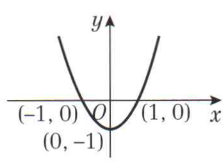


b

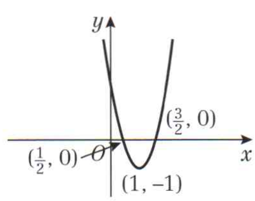


7 **a** $y = x^2 - 4x + 3$

b **i**



ii



8 **a** $(0, 2)$ **b** -2 **c** $-1, 1, 2$

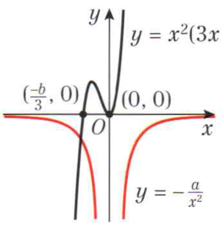
9 **a** **i** $(\frac{4}{3}, 3)$ **ii** $(4, 6)$ **iii** $(9, 3)$

iv $(4, -3)$ **v** $(4, -\frac{1}{2})$

b $f(2x)$, $f(x + 2)$

c **i** $f(x - 4) + 3$ **ii** $2f(\frac{1}{2}x)$

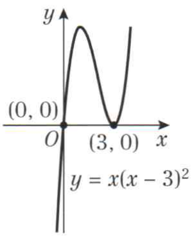
10 a



b 1; only one intersection of the two curves

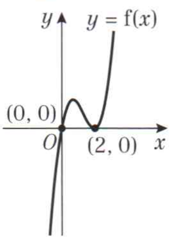
11 a $x(x-3)^2$

b

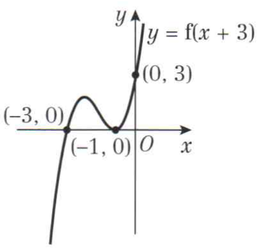


c −4 and −7

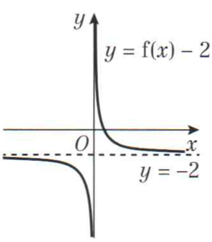
12 a



b

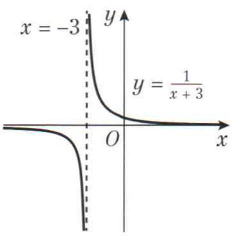


13 a Asymptotes at $x = 0$ and $y = -2$



b $(\frac{1}{2}, 0)$

c



d Asymptotes at $y = 0$ and $x = -3$; intersection at $(0, \frac{1}{3})$

Challenge

$(6 - c, -4 - d)$

Review exercise 1

1 a 2 **b** $\frac{1}{4}$

2 a 625 **b** $\frac{4}{3}x^{\frac{2}{3}}$

3 a $4\sqrt{5}$ **b** $21 - 8\sqrt{5}$

4 a 13 **b** $8 - 2\sqrt{3}$

5 a $1 + 2\sqrt{k}$ **b** $1 + 6\sqrt{k}$

6 a $25x^{-4}$ **b** x^2

7 $8 + 8\sqrt{2}$

8 $1 - 2\sqrt{2}$

9 a $(x - 8)(x - 2)$ **b** $y = 1, y = \frac{1}{3}$

10 a $a = -4, b = -45$ **b** $x = 4 \pm 3\sqrt{5}$

11 4.19 (3 s.f.)

12 a The height of the athlete's shoulder is 1.7 m

b 2.16 s (3 s.f.)

c $6.7 - 5(t - 1)^2$

d 6.7 m after 1 second

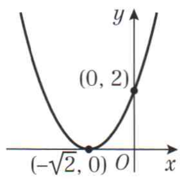
13 a $(x - 3)^2 + 9$

b P is (0, 18), Q is (3, 9)

c $x = 3 + 4\sqrt{2}$

14 a $k = 2$

b



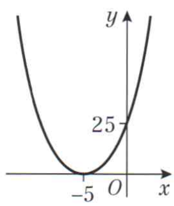
15 a $x^3(x^3 - 8)(x^3 + 1)$ **b** −1, 0, 2

16 a $a = 5, b = 11$

b $(x + 5)^2 = -11$, so no real roots

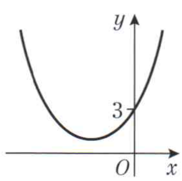
c $k = 25$

d



17 a $a = 1, b = 2$

b



c discriminant = −8, so no real roots

d $-2\sqrt{3} < k < 2\sqrt{3}$

18 a Substitute $y = x - 4$ into $2x^2 - xy = 8$ and rearrange.

b $x = -2 \pm 2\sqrt{3}, y = -6 \pm 2\sqrt{3}$

19 a $x > \frac{1}{4}$

b $x < \frac{1}{2}$ or $x > 3$

c $\frac{1}{4} < x < \frac{1}{2}$ or $x > 3$

20 $-2(x + 1) = x^2 - 5x + 2$

$x^2 - 3x + 4 = 0$

The discriminant of this is $-7 < 0$, so no real solutions.

21 a $x = \frac{7}{2}, y = -2; x = -3, y = 11$

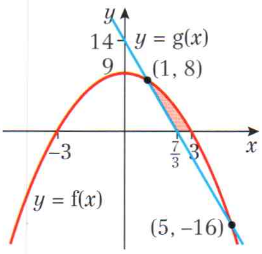
b $x < -3$ or $x > 3\frac{1}{2}$

22 a Different real roots, discriminant > 0 so $k^2 - 4k - 12 > 0$

b $k < -2$ or $k > 6$

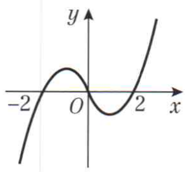
23 $x < -5$ or $x > -2$

24 a, b

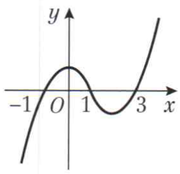


25 a $x(x - 2)(x + 2)$

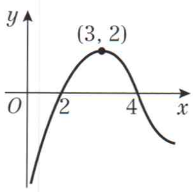
b



c

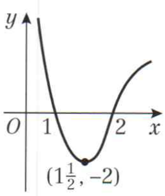


26 a



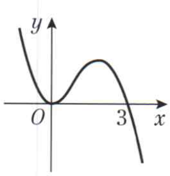
(2, 0) (4, 0) and (3, 2)

b



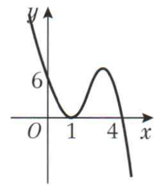
(1, 0) (2, 0) and $(1\frac{1}{2}, -2)$

27 a



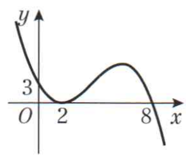
(0, 0) and (3, 0)

b



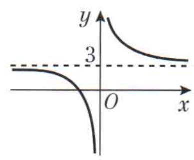
(1, 0) (4, 0) and (0, 6)

c



(2, 0) (8, 0) and (0, 3)

28 a

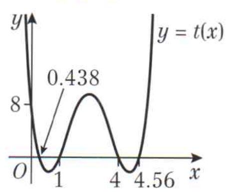


Asymptotes: $y = 3$ and $x = 0$

b $(-\frac{1}{3}, 0)$

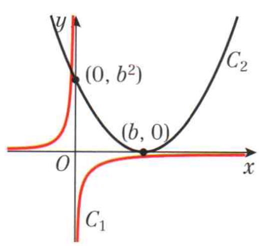
29 a 0.438, 1, 4, 4.56

b



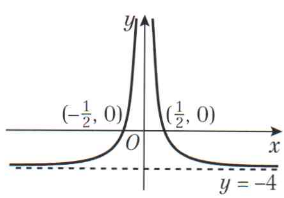
30 a (6, 8) **b** (9, –8) **c** (6, –4)

31 a



b 1

32 a



Asymptotes: $x = 0, y = -4$

b $-\frac{1}{2}, \frac{1}{2}$

Challenge

1 a $x = 1, x = 9$ **b** $x = 0, x = 2$

2 $\sqrt{2}$ cm, $3\sqrt{2}$ cm

3 $3x^3 + x^2 - x = 2x(x - 1)(x + 1)$

$3x^3 + x^2 - x = 2x^3 - 2x$

$x^3 + x^2 + x = 0$

$x(x^2 + x + 1) = 0$

The discriminant of the bracket is $-3 < 0$ so this contributes no real solutions.

The only solution is when $x = 0$ at (0, 0).

4 –3, 3

5 $m = 0$ and $m = \frac{9}{4}$

CHAPTER 5

Prior knowledge check

1 a $(-2, -1)$ b $\left(\frac{9}{19}, \frac{26}{19}\right)$ c $(7, 3)$
2 a $4\sqrt{5}$ b $10\sqrt{2}$ c $5\sqrt{5}$
3 a $y = 5 - 2x$ b $y = \frac{2}{5}x - \frac{9}{5}$ c $y = \frac{3}{7}x + \frac{12}{7}$

Exercise 5A

1 a $\frac{1}{2}$ b $\frac{1}{6}$ c $-\frac{3}{5}$ d 2
e -1 f $\frac{1}{2}$ g $\frac{1}{2}$ h 8
i $\frac{2}{3}$ j -4 k $-\frac{1}{3}$ l $-\frac{1}{2}$
m 1 n $\frac{q^2 - p^2}{q - p} = q + p$
2 7
3 12
4 $4\frac{1}{3}$
5 $2\frac{1}{4}$
6 $\frac{1}{4}$
7 26
8 -5
9 Gradient of AB = gradient of BC = 0.5; point B is common
10 Gradient of AB = gradient of BC = -0.5; point B is common

Exercise 5B

1 a -2 b -1 c 3 d $\frac{1}{3}$
e $-\frac{2}{3}$ f $\frac{5}{4}$ g $\frac{1}{2}$ h 2
i $\frac{1}{2}$ j $\frac{1}{2}$ k -2 l $-\frac{3}{2}$
2 a 4 b -5 c $-\frac{2}{3}$ d 0
e $\frac{7}{5}$ f 2 g 2 h -2
i 9 j -3 k $\frac{3}{2}$ l $-\frac{1}{2}$
3 a $4x - y + 3 = 0$ b $3x - y - 2 = 0$
c $6x + y - 7 = 0$ d $4x - 5y - 30 = 0$
e $5x - 3y + 6 = 0$ f $7x - 3y = 0$
g $14x - 7y - 4 = 0$ h $27x + 9y - 2 = 0$
i $18x + 3y + 2 = 0$ j $2x + 6y - 3 = 0$
k $4x - 6y + 5 = 0$ l $6x - 10y + 5 = 0$
4 $(3, 0)$
5 $(0, 0)$
6 $(0, 5)$, $(-4, 0)$
7 a $\frac{1}{3}$ b $x - 3y + 15 = 0$
8 a $-\frac{2}{5}$ b $2x + 5y - 10 = 0$
9 $ax + by + c = 0$
$by = -ax - c$
$y = \left(-\frac{a}{b}\right)x - \left(\frac{c}{b}\right)$
10 $a = 6$, $c = 10$
11 $P(3,0)$
12 a -16 b -27

Challenge

Gradient $= -\frac{a}{b}$; y-intercept $= a$. So $y = -\frac{a}{b}x + a$
Rearrange to give $ax + by - ab = 0$

Exercise 5C

1 a $y = 2x + 1$ b $y = 3x + 7$ c $y = -x - 3$
d $y = -4x - 11$ e $y = \frac{1}{2}x + 12$ f $y = -\frac{2}{3}x - 5$
g $y = 2x$ h $y = -\frac{1}{2}x + 2b$
2 a $y = 4x - 4$ b $y = x + 2$ c $y = 2x + 4$
d $y = 4x - 23$ e $y = x - 4$ f $y = \frac{1}{2}x + 1$
g $y = -4x - 9$ h $y = -8x - 33$ i $y = \frac{6}{5}x$
j $y = \frac{2}{7}x + \frac{5}{14}$
3 $5x + y - 37 = 0$
4 $y = x + 2$, $y = -\frac{1}{6}x - \frac{1}{3}$, $y = -6x + 23$
5 $a = 3$, $c = -27$
6 $a = -4$, $b = 8$

Challenge

a $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

b $y - y_1 = \frac{(y_2 - y_1)}{(x^2 - y_1)}(x - x_1)$

$\frac{(y - y_1)}{(y_2 - y_1)} = \frac{(x - x_1)}{(x_2 - x_1)}$

c $y = \frac{3}{7}x + \frac{52}{7}$

Exercise 5D

1 $y = 3x - 6$
2 $y = 2x + 8$
3 $2x - 3y + 24 = 0$
4 $-\frac{1}{5}$
5 $(-3, 0)$
6 $(0, 1)$
7 $(0, 3\frac{1}{2})$
8 $y = \frac{2}{5}x + 3$
9 $2x + 3y - 12 = 0$
10 $\frac{8}{5}$
11 $y = \frac{4}{3}x - 4$
12 $6x + 15y - 10 = 0$
13 $y = -\frac{4}{5}x + 4$
14 $x - y + 5 = 0$
15 $y = -\frac{3}{8}x + \frac{1}{2}$
16 $y = 4x + 13$

Exercise 5E

1 a Parallel b Not parallel c Not parallel
2 r: $y = \frac{4}{5}x + 3.2$, s: $y = \frac{4}{5}x - 7$
Gradients equal therefore lines are parallel.
3 Gradient of $AB = \frac{3}{5}$, gradient of $BC = -\frac{7}{2}$, gradient of $CD = \frac{3}{5}$, gradient of $AD = \frac{10}{3}$. The quadrilateral has a pair of parallel sides, so it is a trapezium.
4 $y = 5x + 3$
5 $2x + 5y + 20 = 0$
6 $y = -\frac{1}{2}x + 7$
7 $y = \frac{2}{3}x$
8 $4x - y + 15 = 0$

Exercise 5F

1 a Perpendicular b Parallel
c Neither d Perpendicular
e Perpendicular f Parallel
g Parallel h Perpendicular
i Perpendicular j Parallel
k Neither l Perpendicular
2 $y = -\frac{1}{6}x + 1$
3 $y = \frac{8}{3}x - 8$
4 $y = -\frac{1}{3}x$
5 $y = -\frac{1}{3}x + \frac{13}{3}$
6 $y = -\frac{3}{2}x + \frac{17}{2}$
7 $3x + 2y - 5 = 0$
8 $7x - 4y + 2 = 0$

9 l has gradient $-\frac{1}{3}$ and n has gradient 3. Gradients are negative reciprocals, therefore lines perpendicular.

10 AB: $y = -\frac{1}{2}x + 4\frac{1}{2}$, CD: $y = -\frac{1}{2}x - \frac{1}{2}$, AD: $y = 2x + 7$, BC: $y = 2x - 13$. Two pairs of parallel sides and lines with gradients 2 and $-\frac{1}{2}$ are perpendicular, so $ABCD$ is a rectangle.

11 **a** $A(\frac{7}{5}, 0)$ **b** $55x - 25y - 77 = 0$

12 $-\frac{9}{4}$

Exercise 5G

1 **a** 10 **b** 13 **c** 5 **d** $\sqrt{5}$
e $\sqrt{106}$ **f** $\sqrt{113}$

2 Distance between A and $B = \sqrt{50}$ and distance between B and $C = \sqrt{50}$ so the lines are congruent.

3 Distance between P and $Q = \sqrt{74}$ and distance between Q and $R = \sqrt{73}$ so the lines are not congruent.

4 $x = -8$ or $x = 6$

5 $y = -2$ or $y = 16$

6 **a** Both lines have gradient 2.
b $y = -\frac{1}{2}x + \frac{23}{2}$ or $x + 2y - 23 = 0$
c $\left(\frac{29}{5}, \frac{43}{5}\right)$
d $\frac{7\sqrt{5}}{5}$

7 $P\left(-\frac{3}{5}, \frac{29}{5}\right)$ or $P(3, -5)$

8 **a** $AB = \sqrt{178}$, $BC = 3$ and $AC = \sqrt{205}$. All sides are different lengths, therefore the triangle is a scalene triangle.
b $\frac{39}{2}$ or 19.5

9 **a** $A(2, 11)$
b $B\left(\frac{41}{4}, 0\right)$
c $\frac{451}{8}$

10 **a** $\left(\frac{5}{2}, 0\right)$ **b** $(-5, 0)$
c $(-10, -10)$ **d** $\frac{75}{2}$

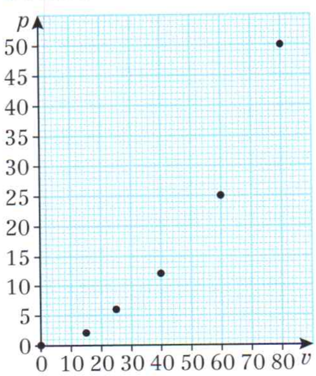
11 **a** $y = \frac{1}{2}x - \frac{9}{2}$ **b** $y = -2x + 8$
c $T(0, 8)$ **d** $RS = 2\sqrt{5}$ and $TR = 5\sqrt{5}$
e 25

12 **a** $x + 4y - 52 = 0$ **b** $A(0,13)$
c $B(4, 12)$ **d** 26

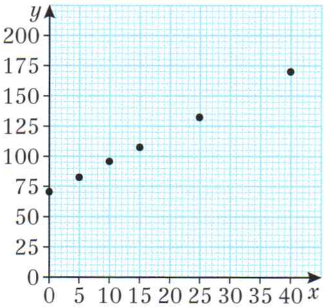
Exercise 5H

1 **a i** $k = 50$ **ii** $d = 50t$
b i $k = 0.3$ or £0.30 **ii** $C = 0.3t$
c i $k = \frac{3}{5}$ **ii** $p = \frac{3}{5}t$

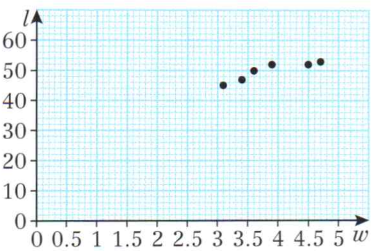
2 **a** not linear



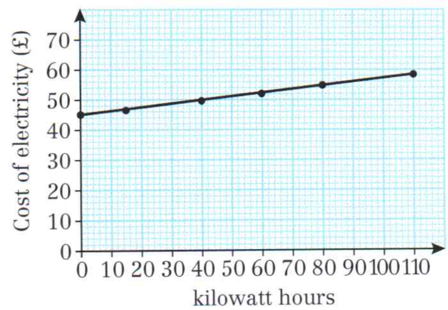
b linear



c not linear

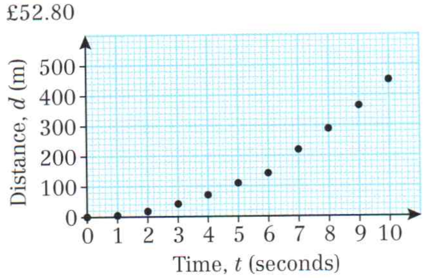


3 **a**



b The data forms a straight line, so a linear model is appropriate.
c $E = 0.12h + 45$
d $a = £0.12$ = cost of 1 kilowatt hour of electricity, $b = £45$ = fixed electricity costs (per month or per quarter)
e £52.80

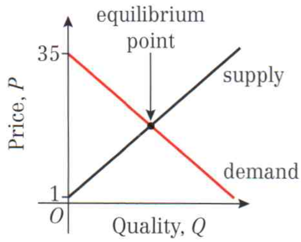
4 **a**



b The data does not follow a straight line. There is a definite curve to the points on the graph.

5 **a** $C = 350d + 5000$
b $a = 350$ = daily fee charged by the website designer. $b = 5000$ = initial cost charged by the website designer.
c 24 days

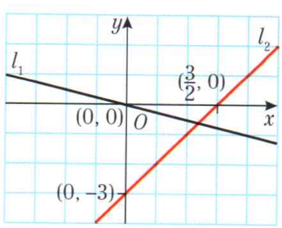
6 a $F = 1.8C + 32$ or $F = \frac{9}{5}C + 32$
b $a = 1.8$ = increase in Fahrenheit temperature when the Celsius temperature increases by 1°C.
$b = 32$ temperature in Fahrenheit when temperature in Celsius is 0°.
c 38.5°C
d −40°C
7 a $n = 750t + 17\,500$
b The increase in the number of homes receiving the internet will be the same each year.
8 a All the points lie close to the straight line shown.
b $h = 4f + 69$
c 175 cm
9 a



b $Q = 24, P = 17$

Mixed exercise 5

1 a $y = -\frac{5}{12}x + \frac{11}{6}$ b −22
2 a $\frac{2k-2}{8-k} = \frac{1}{3}$ therefore $7k = 14$, $k = 2$
b $y = \frac{1}{3}x + \frac{1}{3}$
3 a $L_1 = y = \frac{1}{7}x + \frac{12}{7}$, $L_2 = y = -x + 12$
b (9, 3)
4 a $y = \frac{3}{2}x - \frac{3}{2}$ b (3, 3)
5 $11x - 10y + 19 = 0$
6 a $y = -\frac{1}{2}x + 3$ b $y = \frac{1}{4}x + \frac{9}{4}$
7 Gradient $= \frac{3 + 4\sqrt{3} - 3\sqrt{3}}{2 + \sqrt{3} - 1} = \frac{3 + \sqrt{3}}{1 + \sqrt{3}} = \sqrt{3}$
$y = \sqrt{3}x + c$ and $A(1, 3\sqrt{3})$, so $c = 2\sqrt{3}$
Equation of line is $y = \sqrt{3}x + 2\sqrt{3}$
When $y = 0$, $x = -2$, so the line meets the x-axis at (−2, 0)
8 a $y = -3x + 14$ b (0, 14)
9 a $y = -\frac{1}{2}x + 4$ b Student's own work.
c (1, 1). Note: equation of line n: $y = -\frac{1}{2}x + \frac{3}{2}$
10 20
11 a $2x + y = 20$ b $y = \frac{1}{3}x + \frac{4}{3}$
12 a $\frac{1}{2}$ b 6 c $2x + y - 16 = 0$
d 10
13 a $7x + 5y - 18 = 0$ b $\frac{162}{35}$
14 a



b $\left(\frac{4}{3}, -\frac{1}{3}\right)$ c $12x - 3y - 17 = 0$

15 a $x + 2y - 16 = 0$
b $y = -\frac{2}{3}x$
c $C(-48, 32)$
d Slope of OA is $\frac{3}{2}$. Slope of OC is $-\frac{2}{3}$. Lines are perpendicular.
e $OA = 2\sqrt{13}$ and $OC = 16\sqrt{13}$
f Area = 208
16 a $d = \sqrt{50a^2} = 5a\sqrt{2}$ b $5\sqrt{2}$
c $15\sqrt{2}$ d $25\sqrt{2}$
17 a $d = \sqrt{10x^2 - 28x + 26}$
b $B\left(-\frac{6}{5}, -\frac{18}{5}\right)$ and $C(4, 12)$
c $y = -\frac{1}{3}x + \frac{14}{3}$
d $\left(\frac{7}{5}, \frac{21}{5}\right)$
e 20.8
18 a gradient = 10
b $C = 10P - 9000$
c When the oil production increases by 1 million tonnes, the carbon dioxide emissions increase by 10 million tonnes.
d The model is not valid for small values of P, as it is not possible to have a negative amount of carbon dioxide emissions. It is always dangerous to extrapolate beyond the range on the model in this way.

Challenge

1 130
2 $\left(\frac{78}{19}, \frac{140}{19}\right)$
3 $\left(a, \frac{a(c-a)}{b}\right)$

CHAPTER 6

Prior knowledge check

1 a $(x + 5)^2 + 3$ b $(x - 3)^2 - 8$
c $(x - 6)^2 - 36$ d $(x + \frac{7}{2})^2 - \frac{49}{4}$
2 a $y = \frac{9}{4}x - 6$ b $y = -\frac{1}{2}x - \frac{3}{2}$
c $y = \frac{4}{3}x + \frac{10}{3}$
3 a $b^2 - 4ac = -7$ No real solutions
b $b^2 - 4ac = 89$ Two real solutions
c $b^2 - 4ac = 0$ One real solution
4 $y = -\frac{5}{6}x - \frac{3}{2}$

Exercise 6A

1 a (5, 5) b (6, 4) c (−1, 4) d (0, 0)
e (2, 1) f $(-8, \frac{3}{2})$ g $(4a, 0)$ h $\left(-\frac{u}{2}, -v\right)$
i $(2a, a - b)$ j $(3\sqrt{2}, 4)$ k $(2\sqrt{2}, \sqrt{2} + 3\sqrt{3})$
2 $a = 10, b = 1$
3 $(\frac{3}{2}, 7)$
4 $\left(\frac{3a}{5}, \frac{b}{4}\right)$
5 a $(\frac{3}{2}, 3)$ or (1.5, 3) b $y = 2x$, $3 = 2 \times 1.5$
6 $\frac{2}{3}$
7 Centre is $\left(3, -\frac{7}{2}\right)$. $3 - 2\left(-\frac{7}{2}\right) - 10 = 0$
8 (10, 5)
9 $(-7a, 17a)$

10 $p = 8, q = 7$
11 $a = -2, b = 4$

Challenge

a $p = 9, q = -1$
b $y = -x + 13$
c AC: $y = -x + 8$. Lines have the same slope, so they are parallel.

Exercise 6B

1 **a** $y = 2x + 3$ **b** $y = -\frac{1}{3}x + \frac{47}{3}$ **c** $y = \frac{5}{2}x - 25$
d $y = 3$ **e** $y = -\frac{3}{4}x + \frac{37}{8}$ **f** $x = 9$
2 $y = -x + 7$
3 $2x - y - 8 = 0$
4 **a** $y = -\frac{5}{3}x - \frac{13}{3}$ **b** $y = 3x - 8$ **c** $\left(\frac{11}{14}, -\frac{79}{14}\right)$
5 $q = -\frac{5}{4}, b = -\frac{189}{8}$

Challenge

a PR: $y = -\frac{5}{2}x + \frac{9}{4}$
PQ: $y = -\frac{1}{4}x + \frac{33}{8}$
RQ: $y = 2x + 6$
b $\left(-\frac{5}{6}, \frac{13}{3}\right)$

Exercise 6C

1 **a** $(x - 3)^2 + (y - 2)^2 = 16$
b $(x + 4)^2 + (y - 5)^2 = 36$
c $(x - 5)^2 + (y + 6)^2 = 12$
d $(x - 2a)^2 + (y - 7a)^2 = 25a^2$
e $(x + 2\sqrt{2})^2 + (y + 3\sqrt{2})^2 = 1$
2 **a** $(-5, 4)$, 9 **b** $(7, 1)$, 4
c $(-4, 0)$, 5 **d** $(-4a, -a)$, $12a$
e $(3\sqrt{5}, -\sqrt{5})$, $3\sqrt{3}$
3 **a** $(4 - 2)^2 + (8 - 5)^2 = 4 + 9 = 13$
b $(0 + 7)^2 + (-2 - 2)^2 = 49 + 16 = 65$
c $7^2 + (-24)^2 = 49 + 576 = 625 = 25^2$
d $(6a - 2a)^2 + (-3a + 5a)^2 = 16a^2 + 4a^2 = 20a^2$
e $(\sqrt{5} - 3\sqrt{5})^2 + (-\sqrt{5} - \sqrt{5})^2 = (-2\sqrt{5})^2 + (-2\sqrt{5})^2 = 20 + 20 = 40 = (2\sqrt{10})^2$
4 $(x - 8)^2 + (y - 1)^2 = 25$
5 $(x - \frac{3}{2})^2 + (y - 4)^2 = \frac{65}{4}$
6 $\sqrt{5}$
7 **a** $r = 2$
b Distance $PQ = PR = RQ = 2\sqrt{3}$, three equal length sides therefore triangle is equilateral.
8 **a** $(x - 2)^2 + y^2 = (\sqrt{15})^2$
b Centre (2, 0) and radius $= \sqrt{15}$
9 **a** $(x - 5)^2 + (y + 2)^2 = 7^2$
b Centre (5, −2) and radius = 7
10 **a** Centre (1, −4), radius 5
b Centre (−6, 2), radius 7
c Centre (11, 3), radius $3\sqrt{10}$
d Centre (−2.5, 1.5), radius $\frac{5\sqrt{2}}{2}$
e Centre (2 ,−2), radius $\sqrt{6.5}$
11 **a** Centre (−6, −1)
b $k > -37$
12 $Q(-13, 28)$
13 $k = -2$ and $k = 8$

Challenge

1 $k = 3, (x - 3)^2 + (y - 2)^2 = 50$
$k = 5, (x - 5)^2 + (y - 2)^2 = 50$
2 $(x + f)^2 - f^2 + (y + g)^2 - g^2 + c = 0$
So $(x + f)^2 + (y + g)^2 = f^2 + g^2 - c$
Circle with centre $(-f, -g)$ and radius $\sqrt{f^2 + g^2 - c}$.

Exercise 6D

1 (7, 0), (−5, 0)
2 (0, 2), (0, −8)
3 (6, 10), (−2, 2)
4 (4, −9), (−7, 2)
5 $2x^2 - 24x + 79 = 0$ has no real solutions, therefore lines do not intersect circle.
6 **a** $b^2 - 4ac = 64 - 4 \times 1 \times 16 = 0$. So there is only one point of intersection.
b (4, 7)
7 **a** (0, −2), (4, 6) **b** midpoint of AB is (2, 2)
8 **a** 13 **b** $p = 1$ or 5
9 **a** $A(5, 0)$ and $B(-3, -8)$ (or vice-versa)
b $y = -x - 3$
c (4, −7) is a solution to $y = -x - 3$.
d 20
10 **a** Substitute $y = kx$ to give
$(k^2 + 1)x^2 - (12k + 10)x + 57 = 0$
$b^2 - 4ac > 0, -84k^2 + 240k - 128 > 0,$
$21k^2 - 60k + 32 < 0$
b $0.71 < k < 2.15$
Exact answer is $\frac{10}{7} - \frac{2\sqrt{57}}{21} < k < \frac{10}{7} + \frac{2\sqrt{57}}{21}$
11 $k < \frac{8}{17}$
12 $k = -20 \pm 2\sqrt{105}$

Exercise 6E

1 **a** $3\sqrt{10}$
b Gradient of radius = 3, gradient of line $= -\frac{1}{3}$, gradients are negative reciprocals and therefore perpendicular.
2 **a** $(x - 4)^2 + (y - 6)^2 = 73$ **b** $3x + 8y + 13 = 0$
3 **a** $y = -2x - 1$
b Centre of circle (1, −3) satisfies $y = -2x - 1$.
4 **a** $y = \frac{1}{2}x - 3$
b Centre of circle (2, −2) satisfies $y = \frac{1}{2}x - 3$
5 **a** (−7, −6) satisfies $x^2 + 18x + y^2 - 2y + 29 = 0$
b $y = \frac{2}{7}x - 4$ **c** $R(0, -4)$ **d** $\frac{53}{2}$
6 **a** (0, −17), (17, 0)
b 144.5
7 $y = 2x + 27$ and $y = 2x - 13$
8 **a** $p = 4, p = -6$
b (3, 4) and (3, −6)
9 **a** $(x - 11)^2 + (y + 5)^2 = 100$
b $y = \frac{3}{4}x - \frac{3}{4}$
c $A(8 - 4\sqrt{3}, -1 - 3\sqrt{3})$ and $B(8 + 4\sqrt{3}, -1 + 3\sqrt{3})$
d $10\sqrt{3}$
10 **a** $y = 4x - 22$
b $a = 5$
c $(x - 5)^2 + (y + 2)^2 = 34$
d $A(5 + \sqrt{2}, -2 + 4\sqrt{2})$ and $B(5 - \sqrt{2}, -2 - 4\sqrt{2})$

11 a $P(-2, 5)$ and $Q(4, 7)$
b $y = 2x + 9$ and $y = -\frac{1}{2}x + 9$
c $y = -3x + 9$
d $(0, 9)$

Challenge

1 $y = \frac{1}{2}x - 2$
2 a $\angle CPR = \angle CQR = 90°$ (Angle between tangent and radius)
$CP = CQ = \sqrt{10}$ (Radii of circle)
$CR = \sqrt{(6-2)^2 + (-1-1)^2} = \sqrt{20}$
So using Pythagoras' Theorem,
$PR = QR = \sqrt{20 - 10} = \sqrt{10}$
4 equal sides and two opposite right-angles, so $CPRQ$ is a square
b $y = \frac{1}{3}x - 3$ and $y = -3x + 17$

Exercise 6F

1 a $WV^2 = WU^2 + UV^2$
b $(2, 3)$
c $(x-2)^2 + (y-3)^2 = 41$
2 a $AC^2 = AB^2 + BC^2$
b $(x-5)^2 + (y-2)^2 = 25$
c 15
3 a i $y = \frac{3}{2}x + \frac{21}{2}$ ii $y = -\frac{2}{3}x + 4$
b $(-3, 6)$
c $(x+3)^2 + (y-6)^2 = 169$
4 a i $y = \frac{1}{3}x + \frac{10}{3}$ ii $x = -1$
b $(x+1)^2 + (y-3)^2 = 125$
5 $(x-3)^2 + (y+4)^2 = 50$
6 a $AB^2 + BC^2 = AC^2$
$AB^2 = 400$, $BC^2 = 100$, $AC^2 = 500$
b $(x+2)^2 + (y-5)^2 = 125$
c $D(8, 0)$ satisfies the equation of the circle.
7 a $AB = BC = CD = DA = \sqrt{50}$; can't be a non-square rhombus
b 50
c $(3,6)$
8 a $DE^2 = b^2 + 6b + 13$
$EF^2 = b^2 + 10b + 169$
$DF^2 = 200$
So $b^2 + 6b + 13 + b^2 + 10b + 169 = 200$
$(b+9)(b-1) = 0$; as $b > 0$, $b = 1$
b $(x+5)^2 + (y+4)^2 = 50$
9 a Centre $(-1, 12)$ and radius = 13
b Use distance formula to find $AB = 26$. This is twice radius, so AB is the diameter. Other methods possible.
c $C(-6, 0)$

Mixed exercise 6

1 a $C(3, 6)$
b $r = 10$
c $(x-3)^2 + (y-6)^2 = 100$
d P satisfies the equation of the circle.
2 $(0-5)^2 + (0+2)^2 = 5^2 + 2^2 = 29 < 30$ therefore point is inside the circle
3 a Centre $(0, -4)$ and radius = 3
b $(0, -1)$ and $(0, -7)$
c Students' own work. Equation $x^2 = -7$ has no real solutions.
4 a $P(8, 8)$, $(8+1)^2 + (8-3)^2 = 9^2 + 5^2 = 81 + 25 = 106$
b $\sqrt{106}$
5 a All points satisfy $x^2 + y^2 = 1$, therefore all lie on circle.
b $AB = BC = CA = 3$
6 a $k = 1$, $k = -\frac{2}{5}$
b $(x-1)^2 + (y-3)^2 = 13$
7 Substitute $y = 3x - 9$ into the equation
$x^2 + px + y^2 + 4y = 20$
$x^2 + px + (3x-9)^2 + 4(3x-9) = 20$
$10x^2 + (p-42)x + 25 = 0$
Using the discriminant: $(p-42)^2 - 1000 < 0$
$42 - 10\sqrt{10} < p < 42 + 10\sqrt{10}$
8 $(x-2)^2 + (y+4)^2 = 20$
9 a $2\sqrt{29}$ b 12
10 $(-1, 0)$, $(11, 0)$
11 The values of m and n are $7 - \sqrt{105}$ and $7 + \sqrt{105}$.
12 a $a = 6$ and $b = 8$ b $y = -\frac{4}{3}x + 8$ c 24
13 a $p = 0$, $q = 24$ b $(0, 49)$, $(0, -1)$
14 $x + y + 10 = 0$
15 60
16 l_1: $y = -4x + 12$ and l_2: $y = -\frac{8}{19}x + 12$
17 a $y = \frac{1}{3}x + \frac{8}{3}$
b $(x+2)^2 + (y-2)^2 = 50$
c 20
18 a $P(-3, 1)$ and $Q(9, -7)$
b $3x - 2y + 11 = 0$ and $3x - 2y - 41 = 0$
19 a $y = -4x + 6$ and $y = \frac{1}{4}x + 6$
b $P(-4, 5)$ and $Q(1, 2)$
c 17
20 a $P(5, 16)$ and $Q(13, 8)$
b l_2: $y = \frac{1}{7}x + \frac{107}{7}$ and l_3: $y = 7x - 83$
c l_4: $y = x + 3$
d All 3 equations have solution $x = \frac{43}{3}$, $y = \frac{52}{3}$ so $R(\frac{43}{3}, \frac{52}{3})$
e $\frac{200}{3}$
21 a $(4,0)$, $(0,12)$
b $(2,6)$
c $(x-2)^2 + (y-6)^2 = 40$
22 a $q = 4$
b $(x+\frac{5}{2})^2 + (y-2)^2 = \frac{65}{4}$
23 a $RS^2 + ST^2 = RT^2$
b $(x-2)^2 + (y+2)^2 = 61$
24 $(x-1)^2 + (y-3)^2 = 34$
25 a i $y = -4x - 4$ ii $x = -2$
b $(x+2)^2 + (y-4)^2 = 34$

Challenge

a $x + y - 14 = 0$
b $P(7, 7)$ and $Q(9, 5)$
c 10

CHAPTER 7

Prior knowledge check

1 a $15x^7$ b $\frac{x}{3y}$
2 a $(x-6)(x+4)$ b $(3x-5)(x-4)$
3 a 8567 b 1652
4 a $y = 1 - 3x$ b $y = \frac{1}{2}x - 7$
5 a $(x-1)^2 - 21$ b $2(x+1)^2 + 13$

Exercise 7A

1 a $4x^3 + 5x - 7$ b $2x^4 + 9x^2 + x$
c $-x^3 + 4x + \frac{6}{x}$ d $7x^4 - x^2 - \frac{4}{x}$

e $4x^3 - 2x^2 + 3$ f $3x - 4x^2 - 1$
g $\frac{7x^2}{5} - \frac{x^3}{5} - \frac{2}{5x}$ h $2x - 3x^3 + 1$
i $\frac{x^7}{2} - \frac{9x^3}{2} + 2x^2 - \frac{3}{x}$ j $3x^8 + 2x^5 - \frac{4x^3}{3} + \frac{2}{3x}$

2 a $x + 3$ b $x + 4$ c $x + 3$
d $x + 7$ e $x + 5$ f $x + 4$
g $\frac{x - 4}{x - 3}$ h $\frac{x + 2}{x + 4}$ i $\frac{x + 4}{x - 6}$
j $\frac{2x + 3}{x - 5}$ k $\frac{2x - 3}{x + 1}$ l $\frac{x - 2}{x + 2}$
m $\frac{2x + 1}{x - 2}$ n $\frac{x + 4}{3x + 1}$ o $\frac{2x + 1}{2x - 3}$

3 $a = 1, b = 4, c = -2$

Exercise 7B

1 a $(x + 1)(x^2 + 5x + 3)$ b $(x + 4)(x^2 + 6x + 1)$
c $(x + 2)(x^2 - 3x + 7)$ d $(x - 3)(x^2 + 4x + 5)$
e $(x - 5)(x^2 - 3x - 2)$ f $(x - 7)(x^2 + 2x + 8)$

2 a $(x + 4)(6x^2 + 3x + 2)$ b $(x + 2)(4x^2 + x - 5)$
c $(x + 3)(2x^2 - 2x - 3)$ d $(x - 6)(2x^2 - 3x - 4)$
e $(x + 6)(-5x^2 + 3x + 5)$ f $(x - 2)(-4x^2 + x - 1)$

3 a $x^3 + 3x^2 - 4x + 1$ b $4x^3 + 2x^2 - 3x - 5$
c $-3x^3 + 3x^2 - 4x - 7$ d $-5x^4 + 2x^3 + 4x^2 - 3x + 7$

4 a $x^3 + 2x^2 - 5x + 4$ b $x^3 - x^2 + 3x - 1$
c $2x^3 + 5x + 2$ d $3x^4 + 2x^3 - 5x^2 + 3x + 6$
e $2x^4 - 2x^3 + 3x^2 + 4x - 7$ f $4x^4 - 3x^3 - 2x^2 + 6x - 5$
g $5x^3 + 12x^2 - 6x - 2$ h $3x^4 + 5x^3 + 6$

5 a $x^2 - 2x + 5$ b $2x^2 - 6x + 1$
c $-3x^2 - 12x + 2$

6 a $x^2 + 4x + 12$ b $2x^2 - x + 5$
c $-3x^2 + 5x + 10$

7 Divide $x^3 + 2x^2 - 5x - 10$ by $(x + 2)$ to give $(x^2 - 5)$. So $x^3 + 2x^2 - 5x - 10 = (x + 2)(x^2 - 5)$.

8 a -8 b -7 c -12

9 Divide $3x^3 - 2x^2 + 4$ by $(x - 1)$ to get $3x^2 + x + 1$ remainder 5.

10 Divide $3x^4 - 8x^3 + 10x^2 - 3x - 25$ by $(x + 1)$ to get $3x^3 - 11x^2 + 21x - 24$ remainder -1.

11 $(x + 4)(5x^2 - 20x + 7)$

12 $3x^2 + 6x + 4$

13 $x^2 + x + 1$

14 $x^3 - 2x^2 + 4x - 8$

15 14

16 a -200 b $(x + 2)(x - 7)(3x + 1)$

17 a i 30 ii 0 b $x = -3, x = -4, x = 1$

18 a $a = 1, b = 2, c = -3$
b $f(x) = (2x - 1)(x + 3)(x - 1)$
c $x = 0.5, x = -3, x = 1$

19 a $a = 3, b = 2, c = 1$
b Quadratic has no real solutions so only $\frac{1}{4}$ is a solution

Exercise 7C

1 a $f(1) = 0$ b $f(-3) = 0$ c $f(4) = 0$

2 $(x - 1)(x + 3)(x + 4)$

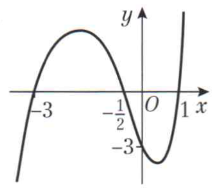
3 $(x + 1)(x + 7)(x - 5)$

4 $(x - 5)(x - 4)(x + 2)$

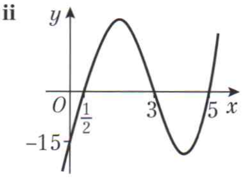
5 $(x - 2)(2x - 1)(x + 4)$

6 a $(x + 1)(x - 5)(x - 6)$ b $(x - 2)(x + 1)(x + 2)$
c $(x - 5)(x + 3)(x - 2)$

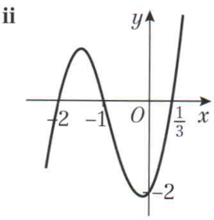
7 a i $(x - 1)(x + 3)(2x + 1)$ ii



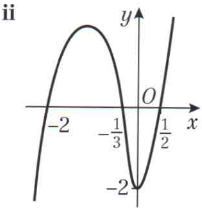
b i $(x - 3)(x - 5)(2x - 1)$ ii



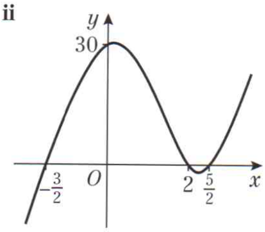
c i $(x + 1)(x + 2)(3x - 1)$ ii



d i $(x + 2)(2x - 1)(3x + 1)$ ii



e i $(x - 2)(2x - 5)(2x + 3)$ ii



8 2

9 -16

10 $p = 3, q = 7$

11 $c = 2, d = 3$

12 $g = 3, h = -7$

13 a $f(4) = 0$
b $f(x) = (x - 4)(3x^2 + 6)$
For $3x^2 + 6 = 0$, $b^2 - 4ac = -72$ so there are no real roots. Therefore, 4 is the only real root of $f(x) = 0$.

14 a $f(-2) = 0$ b $(x + 2)(2x + 1)(2x - 3)$
c $x = -2, x = -\frac{1}{2}$ and $x = 1\frac{1}{2}$

15 a $f(2) = 0$ b $x = 0, x = 2, x = -\frac{1}{3}$ and $x = \frac{1}{3}$

Challenge

a $f(1) = 2 - 5 - 42 - 9 + 54 = 0$
$f(-3) = 162 + 135 - 378 + 27 + 54 = 0$

b $2x^4 - 5x^3 - 42x^2 - 9x + 54$
$= (x - 1)(x + 3)(x - 6)(2x + 3)$
$x = 1$ $x = -3, x = 6, x = -1.5$

Exercise 7D

1 $n^2 - n = n(n-1)$
If n is even, $n-1$ is odd and even × odd = even
If n is odd, $n-1$ is even and odd × even = even

2 $\frac{x}{(1+\sqrt{2})} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})} = \frac{x(1-\sqrt{2})}{(1-2)} = \frac{x - x\sqrt{2}}{-1} = x\sqrt{2} - x$

3 $(x+\sqrt{y})(x-\sqrt{y}) = x^2 - x\sqrt{y} + x\sqrt{y} - y = x^2 - y$

4 $(2x-1)(x+6)(x-5) = (2x-1)(x^2+x-30)$
$= 2x^3 + x^2 - 61x + 30$

5 LHS $= x^2 + bx$, using completing the square,
$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

6 $x^2 + 2bx + c = 0$, using completing the square
$(x+b)^2 + c - b^2 = 0$
$(x+b)^2 = b^2 - c$
$x + b = \pm\sqrt{b^2 - c}$
$x = -b \pm \sqrt{b^2 - c}$

7 $\left(x - \frac{2}{x}\right)^3 = \left(x - \frac{2}{x}\right)\left(x^2 - 4 + \frac{4}{x^2}\right) = x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}$

8 $\left(x^3 - \frac{1}{x}\right)\left(x^{\frac{3}{2}} + x^{\frac{-5}{2}}\right) = x^{\frac{9}{2}} + x^{\frac{1}{2}} - x^{\frac{1}{2}} - x^{-\frac{7}{2}} = x^{\frac{9}{2}} - x^{-\frac{7}{2}}$
$= x^{\frac{1}{2}}\left(x^4 - \frac{1}{x^4}\right)$

9 $3n^2 - 4n + 10 = 3\left[n^2 - \frac{4}{3}n + \frac{10}{3}\right] = 3\left[\left(n - \frac{2}{3}\right)^2 + \frac{10}{3} - \frac{4}{9}\right]$
$= 3\left(n - \frac{2}{3}\right)^2 + \frac{26}{3}$
The minimum value is $\frac{26}{3}$ so $3n^2 - 4n + 10$ is always positive.

10 $-n^2 - 2n - 3 = -[n^2 + 2n + 3] = -[(n+1)^2 + 3 - 1]$
$= -(n+1)^2 - 2$
The maximum value is -2 so $-n^2 - 2n - 3$ is always negative.

11 $x^2 + 8x + 20 = (x+4)^2 + 4$
The minimum value is 4 so $x^2 + 8x + 20$ is always greater than or equal to 4.

12 $kx^2 + 5kx + 3 = 0$, $b^2 - 4ac < 0$, $25k^2 - 12k < 0$, $k(25k - 12) < 0$, $0 < k < \frac{12}{25}$.
When $k = 0$ there are no real roots, so $0 \leqslant k < \frac{12}{25}$

13 $px^2 - 5x - 6 = 0$, $b^2 - 4ac > 0$, $25 + 24p > 0$, $p > -\frac{25}{24}$

14 Gradient $AB = -\frac{1}{2}$, gradient $BC = 2$,
Gradient AB × gradient $BC = -\frac{1}{2} \times 2 = -1$,
so AB and BC are perpendicular.

15 Gradient $AB = 3$, gradient $BC = \frac{1}{4}$, gradient $CD = 3$, gradient $AD = \frac{1}{4}$
Gradient AB = gradient CD so AB and CD are parallel.
Gradient BC = gradient AD so BC and AD are parallel.

16 Gradient $AB = \frac{1}{3}$, gradient $BC = 3$, gradient $CD = \frac{1}{3}$, gradient $AD = 3$
Gradient AB = gradient CD so AB and CD are parallel.
Gradient BC = gradient AD so BC and AD are parallel.
Length $AB = \sqrt{10}$, $BC = \sqrt{10}$, $CD = \sqrt{10}$ and $AD = \sqrt{10}$, so all four sides are equal.

17 Gradient $AB = -3$, gradient $BC = \frac{1}{3}$,
Gradient AB × gradient $BC = -3 \times \frac{1}{3} = -1$, so AB and BC are perpendicular.
Length $AB = \sqrt{40}$, $BC = \sqrt{40}$, $AB = BC$

18 $(x-1)^2 + y^2 = k$, $y = ax$, $(x-1)^2 + a^2x^2 = k$,
$x^2(1 + a^2) - 2x + 1 - k = 0$
$b^2 - 4ac > 0$, $k > \frac{a^2}{1 + a^2}$.

19 $x = 2$. There is only one solution so the line $4y - 3x + 26 = 0$ only touches the circle in one place so is the tangent to the circle.

20 Area of square $= (a+b)^2 = a^2 + 2ab + b^2$
Shaded area $= 4\left(\frac{1}{2}ab\right)$
Area of smaller square: $a^2 + 2ab + b^2 - 2ab$
$= a^2 + b^2 = c^2$

Challenge

1 The equation of the circle is $(x-3)^2 + (y-5)^2 = 25$ and all four points satisfy this equation.

2 $2k + 1 = 1 \times (2k+1) = ((k+1) - k)((k+1) + k) = (k+1)^2 - k^2$

Exercise 7E

1 3, 4, 5, 6, 7 and 8 are not divisible by 10

2 3, 5, 7, 11, 13, 17, 19, 23 are prime numbers. 9, 15, 21, 25 are the product of two prime numbers.

3 $1^2 + 2^2$ = odd, $2^2 + 3^2$ = odd, $3^2 + 4^2$ = odd, $4^2 + 5^2$ = odd, $5^2 + 6^2$ = odd, $6^2 + 7^2$ = odd, $7^2 + 8^2$ = odd

4 $(3n)^3 = 27n^3 = 9n(3n^2)$ which is a multiple of 9
$(3n+1)^3 = 27n^3 + 27n^2 + 9n + 1 = 9n(3n^2 + 3n + 1) + 1$
which is one more than a multiple of 9
$(3n+2)^3 = 27n^3 + 54n^2 + 36n + 8 = 9n(3n^2 + 6n + 4) + 8$
which is one less than a multiple of 9

5 a For example, when $n = 2$, $2^4 - 2 = 14$, 14 is not divisible by 4.
b Any square number
c For example, when $n = \frac{1}{2}$
d For example, when $n = 1$

6 a Assuming that x and y are positive
b e.g. $x = 0$, $y = 0$

7 $(x+5)^2 \geqslant 0$ for all real values of x, and
$(x+5)^2 + 2x + 11 = (x+6)^2$, so $(x+6)^2 \geqslant 2x + 11$

8 $(a-1)^2 \geqslant 0$, $a^2 - 2a + 1 \geqslant 0$, $a^2 + 1 \geqslant 2a$
a is positive so both sides of the inequality can be divided by a without reversing the inequality.
So $a + \frac{1}{a} \geqslant 2$

9 a $(p+q)^2 = p^2 + 2pq + q^2 = (p-q)^2 + 4pq$
$(p-q)^2 \geqslant 0$ since it is a square, so $(p+q)^2 \geqslant 4pq$
$p > 0, q > 0 \Rightarrow p + q > 0 \Rightarrow p + q \geqslant \sqrt{4pq}$
b e.g. $p = q = -1$: $p + q = -2$, $\sqrt{4pq} = 2$

10 a Starts by assuming the inequality is true: i.e. negative ⩾ positive
b e.g. $x = y = -1$: $x + y = -2$, $\sqrt{x^2 + y^2} = \sqrt{2}$
c $(x+y)^2 = x^2 + 2xy + y^2 > x^2 + y^2$ since $x > 0$, $y > 0 \Rightarrow 2xy > 0$
As $x + y > 0$, can take square roots: $x + y > \sqrt{x^2 + y^2}$

Mixed exercise 7

1 a $x^3 - 7$ b $\frac{x+4}{x-1}$ c $\frac{2x-1}{2x+1}$

2 $3x^2 + 5$

3 $2x^2 - 2x + 5$

4 a When $x = 3$, $2x^3 - 2x^2 - 17x + 15 = 0$
b $A = 2$, $B = 4$, $C = -5$

5 a When $x = 2$, $x^3 + 4x^2 - 3x - 18 = 0$
b $p = 1$, $q = 3$

6 $(x-2)(x+4)(2x-1)$

7 7

8 a $p = 1$, $q = -15$ b $(x+3)(2x-5)$

9 a $r = 3$, $s = 0$ b $x(x+1)(x+3)$

10 a $(x-1)(x+5)(2x+1)$ b $-5, -\frac{1}{2}, 1$

11 **a** When $x = 2$, $x^3 + x^2 - 5x - 2 = 0$

b $2, -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$

12 $\frac{1}{2}$, 3

13 **a** When $x = -4$, $f(x) = 0$

b $x = -4$, $x = 1$ and $x = 5$

14 **a** $f(\frac{2}{3}) = 0$, therefore $(3x - 2)$ is a factor of $f(x)$
$a = 2$, $b = 7$ and $c = 3$

b $(3x - 2)(2x + 1)(x + 3)$

c $x = \frac{2}{3}, -\frac{1}{2}, -3$

15 $\frac{x - y}{(\sqrt{x} - \sqrt{y})} \times \frac{(\sqrt{x} + \sqrt{y})}{(\sqrt{x} + \sqrt{y})} = \frac{(x - y)(\sqrt{x} + \sqrt{y})}{x - y} = \sqrt{x} + \sqrt{y}$

16 $n^2 - 8n + 20 = (n - 4)^2 + 4$, 4 is the minimum value so $n^2 - 8n + 20$ is always positive

17 Gradient $AB = \frac{1}{2}$, gradient $BC = -2$, gradient $CD = \frac{1}{2}$, gradient $AD = -2$
AB and BC, BC and CD, CD and AD and AB and AD are all perpendicular
Length $AB = \sqrt{5}$, $BC = \sqrt{5}$, $CD = \sqrt{5}$ and $AD = \sqrt{5}$, all four sides are equal

18 1 + 3 = even, 3 + 5 = even, 5 + 7 = even, 7 + 9 = even

19 For example when $n = 6$

20 $\left(x - \frac{1}{x}\right)(x^{\frac{4}{3}} + x^{\frac{-2}{3}}) = x^{\frac{7}{3}} + x^{\frac{1}{3}} - x^{\frac{1}{3}} - x^{-\frac{5}{3}} = x^{\frac{1}{3}}\left(x^2 - \frac{1}{x^2}\right)$

21 RHS $= (x + 4)(x - 5)(2x + 3) = (x + 4)(2x^2 - 7x - 15)$
$= 2x^3 + x^2 - 43x - 60 =$ LHS

22 $x^2 - kx + k = 0$, $b^2 - 4ac = 0$, $k^2 - 4k = 0$, $k(k - 4) = 0$, $k = 4$.

23 The distance between opposite edges
$= 2\sqrt{(\sqrt{3})^2 - \left(\frac{\sqrt{3}}{2}\right)^2} = 2\sqrt{3 - \frac{3}{4}} = 3$ which is rational.

24 **a** $(2n + 2)^2 - (2n)^2 = 8n + 4 = 4(2n + 1)$ is always divisible by 4.

b Yes, $(2n + 1)^2 - (2n - 1)^2 = 8n$ which is always divisible by 4.

25 **a** The assumption is that x is positive

b $x = 0$

Challenge

1 **a** Perimeter of inside square $= 4\left(\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}\right) = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Perimeter of outside square = 4
Circumference of circle $= \pi \times 1^2 = \pi$
therefore $2\sqrt{2} < \pi < 4$.

b Perimeter of inside hexagon = 3

Perimeter of outside hexagon $= 6 \times \frac{\sqrt{3}}{3} = 2\sqrt{3}$, therefore $3 < \pi < 2\sqrt{3}$

2 $ax^3 + bx^2 + cx + d \div (x - p) = ax^2 + (b + ap)x + (c + bp + ap^2)$ with remainder $d + cp + bp^2 + ap^3$
$f(p) = ap^3 + bp^2 + cp + d = 0$, which matches the remainder, so $(x - p)$ is a factor of $f(x)$.

CHAPTER 8

Prior knowledge check

1 **a** $4x^2 - 12xy + 9y^2$ **b** $x^3 - 3x^2y + 3xy^2 - y^3$
c $8 + 12x + 6x^2 + x^3$

2 **a** $-8x^3$ **b** $\frac{1}{81x^4}$ **c** $\frac{4}{25}x^2$ **d** $\frac{27}{x^3}$

3 **a** $5\sqrt{x}$ **b** $\frac{1}{16\sqrt[3]{x^2}}$ **c** $\frac{10}{3\sqrt{x}}$ **d** $\frac{16}{81}\sqrt[3]{x^4}$

Exercise 8A

1 **a** 4th row **b** 16th row
c $(n + 1)$th row **d** $(n + 5)$th row

2 **a** $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
b $p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$
c $a^3 - 3a^2b + 3ab^2 - b^3$
d $x^3 + 12x^2 + 48x + 64$
e $16x^4 - 96x^3 + 216x^2 - 216x + 81$
f $a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32$
g $81x^4 - 432x^3 + 864x^2 - 768x + 256$
h $16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$

3 **a** 16 **b** −10 **c** 8 **d** 1280
e 160 **f** −2 **g** 40 **h** −96

4 $1 + 9x + 30x^2 + 44x^3 + 24x^4$

5 $8 + 12y + 6y^2 + y^3$, $8 + 12x - 6x^2 - 11x^3 + 3x^4 + 3x^5 - x^6$

6 ±3

7 $\frac{5}{2}, -1$

8 $12p$

9 $500 + 25X + \frac{X^2}{2}$

Challenge

$\frac{3}{4}$

Exercise 8B

1 **a** 24 **b** 362 880 **c** 720 **d** 210

2 **a** 6 **b** 15 **c** 20 **d** 5
e 45 **f** 126

3 **a** 5005 **b** 120 **c** 184 756 **d** 1140
e 2002 **f** 8568

4 $a = {}^4C_1$, $b = {}^5C_2$, $c = {}^6C_2$, $d = {}^6C_3$

5 330

6 **a** 120, 210 **b** 960

7 **a** 286, 715 **b** 57 915

8 0.1762 to 4 decimal places. Whilst it seems a low probability, there is more chance of the coin landing on 10 heads than any other number of heads.

9 **a** ${}^nC_1 = \frac{n!}{1!(n-1)!}$

$= \frac{1 \times 2 \times \ldots \times (n-2) \times (n-1) \times n}{1 \times 1 \times 2 \times \ldots \times (n-3) \times (n-2) \times (n-1)} = n$

b ${}^nC_2 = \frac{n!}{2!(n-2)!}$

$= \frac{1 \times 2 \times \ldots \times (n-2) \times (n-1) \times n}{1 \times 2 \times 1 \times 2 \times \ldots \times (n-3) \times (n-2)} = \frac{n(n-1)}{2}$

10 $a = 37$

11 $p = 17$

Challenge

a ${}^{10}C_3 = \frac{10!}{3!7!} = 120$ and ${}^{10}C_7 = \frac{10!}{7!3!} = 120$

b ${}^{14}C_5 = \frac{14!}{5!9!} = 2002$ and ${}^{14}C_9 = \frac{14!}{9!5!} = 2002$

c The two answers for part **a** are the same and the two answers for part **b** are the same.

d ${}^nC_r = \frac{n!}{r!(n-r)!}$ and ${}^nC_{n-r} = \frac{n!}{(n-r)!r!}$, therefore ${}^nC_r = {}^nC_{n-r}$

Exercise 8C

1 **a** $1 + 4x + 6x^2 + 4x^3 + x^4$
b $81 + 108x + 54x^2 + 12x^3 + x^4$
c $256 - 256x + 96x^2 - 16x^3 + x^4$

d $x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$
e $1 + 8x + 24x^2 + 32x^3 + 16x^4$
f $1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$

2 a $1 + 10x + 45x^2 + 120x^3$
b $1 - 10x + 40x^2 - 80x^3$
c $1 + 18x + 135x^2 + 540x^3$
d $256 - 1024x + 1792x^2 - 1792x^3$
e $1024 - 2560x + 2880x^2 - 1920x^3$
f $2187 - 5103x + 5103x^2 - 2835x^3$

3 a $64x^6 + 192x^5y + 240x^4y^2 + 160x^3y^3$
b $32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3$
c $p^8 - 8p^7q + 28p^6q^2 - 56p^5q^3$
d $729x^6 - 1458x^5y + 1215x^4y^2 - 540x^3y^3$
e $x^8 + 16x^7y + 112x^6y^2 + 448x^5y^3$
f $512x^9 - 6912x^8y + 41\,472x^7y^2 - 145\,152x^6y^3$

4 a $1 + 8x + 28x^2 + 56x^3$
b $1 - 12x + 60x^2 - 160x^3$
c $1 + 5x + \frac{45}{4}x^2 + 15x^3$
d $1 - 15x + 90x^2 - 270x^3$
e $128 + 448x + 672x^2 + 560x^3$
f $27 - 54x + 36x^2 - 8x^3$
g $64 - 576x + 2160x^2 - 4320x^3$
h $256 + 256x + 96x^2 + 16x^3$
i $128 + 2240x + 16\,800x^2 + 70\,000x^3$

5 $64 - 192x + 240x^2$

6 $243 - 810x + 1080x^2$

7 $x^5 + 5x^3 + 10x + \dfrac{10}{x} + \dfrac{5}{x^3} + \dfrac{1}{x^5}$

Challenge

a $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
$(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$
$(a + b)^4 - (a - b)^4 = 8a^3b + 8ab^3 = 8ab(a^2 + b^2)$

b $82\,896 = 2^4 \times 3 \times 11 \times 157$

Exercise 8D

1 a 90 b 80 c −20
d 1080 e 120 f −4320
g 1140 h −241 920 i −2.5
j 354.375 k −224 l 3.90625

2 $a = \pm\frac{1}{2}$

3 $b = -2$

4 $1, \dfrac{5 \pm \sqrt{105}}{8}$

5 a $p = 5$ b −10 c −80

6 a $5^{30} + 5^{29} \times 30px + 5^{28} \times 435p^2x^2$
b $p = 10$

7 a $1 + 10qx + 45q^2x^2 + 120q^3x^3$
b $q = \pm 3$

8 a $1 + 11px + 55p^2x^2$
b $p = 7, q = 2695$

9 a $1 + 15px + 105p^2x^2$
b $p = -\frac{5}{7}, q = 10\frac{5}{7}$

10 $\dfrac{q}{p} = 2.1$

Challenge

a 314 928 b 43 750

Exercise 8E

1 a $1 - 0.6x + 0.15x^2 - 0.02x^3$
b 0.941 48

2 a $1024 + 1024x + 460.8x^2 + 122.88x^3$
b 1666.56

3 $(1 - 3x)^5 = 1^5 + \binom{5}{1}1^4(-3x)^1 + \binom{5}{2}1^3(-3x)^2 = 1 - 15x + 90x^2$
$(2 + x)(1 - 3x)^5 = (2 + x)(1 - 15x + 90x^2)$
$= 2 - 30x + 180x^2 + x - 15x^2 + 90x^3 \approx 2 - 29x + 165x^2$

4 $a = 162, b = 135, c = 0$

5 a $1 + 16x + 112x^2 + 448x^3$
b $x = 0.01, 1.02^8 \approx 1.171\,648$

6 a $1 - 150x + 10875x^2 - 507500x^3$
b 0.860 368
c 0.860 384, 0.0019%

7 a $59\,049 - 39\,366x + 11\,809.8x^2$
b Substitute $x = 0.1$ into the expansion.

8 a $1 - 15x + 90x^2 - 270x^3$
b $(1 + x)(1 - 3x)^5 \approx (1 + x)(1 - 15x) \approx 1 - 14x$

9 a So that higher powers of p can be ignored as they tend to 0
b $1 - 200p + 19\,900p^2$
c $p = 0.000417$ (3 s.f.)

Mixed exercise 8

1 a 455, 1365 b 3640

2 $a = 28$

3 a 0.0148 b 0.000 000 000 034 9 c 0.166

4 a $p = 16$ b 270 c −1890

5 $A = 8192, B = -53\,248, C = 159\,744$

6 a $1 - 20x + 180x^2 - 960x^3$
b 0.817 04, $x = 0.01$

7 a $1024 - 15\,360x + 103\,680x^2 - 414\,720x^3$
b 880.35

8 a $81 + 216x + 216x^2 + 96x^3 + 16x^4$
b $81 - 216x + 216x^2 - 96x^3 + 16x^4$
c 1154

9 a $n = 8$ b $\frac{35}{8}$

10 a $81 + 1080x + 5400x^2 + 12\,000x^3 + 10\,000x^4$
b 1 012 054 108 081, $x = 100$

11 a $1 + 24x + 264x^2 + 1760x^3$ b 1.268 16, $x = 0.01$
c 1.268 241 795 d 0.006 45% (3 sf)

12 $x^5 - 5x^3 + 10x - \dfrac{10}{x} + \dfrac{5}{x^3} - \dfrac{1}{x^5}$

13 a $\binom{n}{2}(2k)^{n-2} = \binom{n}{3}(2k)^{n-3}$

$$\frac{n!(2k)^{n-2}}{2!(n-2)!} = \frac{n!(2k)^{n-3}}{3!(n-3)!}$$

$$\frac{2k}{n-2} = \frac{1}{3}$$

So $n = 6k + 2$

b $\dfrac{4096}{729} + \dfrac{2048}{81}x + \dfrac{1280}{27}x^2 + \dfrac{1280}{27}x^3$

14 a $64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$
b $k = 1560$

15 a $k = 1.25$ b 3500

16 a $A = 64, B = 160, C = 20$ b $x = \pm\sqrt{\frac{3}{2}}$

17 a $p = 1.5$ b 50.625

18 672

19 a $128 + 448px + 672p^2x^2$
b $p = 5, q = 16\,800$

20 a $1 - 12px + 66p^2x^2$
b $p = -1\frac{1}{11}, q = 13\frac{1}{11}$

21 **a** $128 + 224x + 168x^2$
b Substitute $x = 0.1$ into the expansion.
22 $k = \frac{1}{2}$

Challenge
1 $540 - 405p = 0$, $p = \frac{4}{3}$
2 -4704

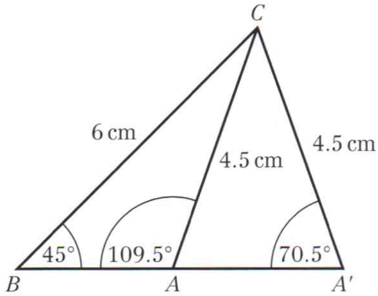
CHAPTER 9

Prior knowledge check
1 **a** 3.10 cm **b** 9.05 cm
2 **a** 25.8° **b** 77.2°
3 **a** graph of $x^2 + 3x$ **b** graph of $(x + 2)^2 + 3(x + 2)$
c graph of $x^2 + 3x - 3$ **b** graph of $(0.5x)^2 + 3(0.5x)$

Exercise 9A
1 **a** 3.19 cm **b** 1.73 cm ($\sqrt{3}$ cm) **c** 9.85 cm
d 4.31 cm **e** 6.84 cm **f** 9.80 cm
2 **a** 108(.2)° **b** 90° **c** 60°
d 52.6° **e** 137° **f** 72.2°
3 192 km
4 11.2 km
5 128.5° or 031.5° (Angle BAC = 48.5°)
6 302 yards (301.5…)
7 Using the cosine rule $\frac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4} = \frac{1}{8}$
8 Using the cosine rule $\frac{2^2 + 3^2 - 4^2}{2 \times 2 \times 3} = -\frac{1}{4}$
9 ACB = 22.3°
10 ABC = 108(.4)°
11 104(.48)°
12 $x = 4.4$
13 $x = 42$
14 **a** $y^2 = (5 - x)^2 + (4 + x)^2 - 2(5 - x)(4 + x)\cos 120°$
$= 25 - 10x + x^2 + 16 + 8x + x^2 - 2(20 + x - x^2)\left(-\frac{1}{2}\right)$
$= x^2 - x + 61$
b Minimum $AC^2 = 60.75$; it occurs for $x = \frac{1}{2}$
15 **a** $\cos \angle ABC = \frac{x^2 + 5^2 - (10 - x)^2}{2x \times 5}$
$= \frac{20x - 75}{10x} = \frac{4x - 15}{2x}$
b 3.5
16 65.3°
17 **a** 28.7 km **b** 056.6°

Exercise 9B
1 **a** 15.2 cm **b** 9.57 cm **c** 8.97 cm **d** 4.61 cm
2 **a** $x = 84°$, $y = 6.32$
b $x = 13.5$, $y = 16.6$
c $x = 85°$, $y = 13.9$
d $x = 80°$, $y = 6.22$ (isosceles triangle)
e $x = 6.27$, $y = 7.16$
f $x = 4.49$, $y = 7.49$ (right-angled)
3 **a** 36.4° **b** 35.8° **c** 40.5° **d** 130°
4 **a** 48.1° **b** 45.6° **c** 14.8° **d** 48.7°
e 86.5° **f** 77.4°
5 **a** 1.41 cm ($\sqrt{2}$ cm) **b** 1.93 cm
6 QPR = 50.6°, PQR = 54.4°
7 **a** $x = 43.2°$, $y = 5.02$ cm **b** $x = 101°$, $y = 15.0$ cm
c $x = 6.58$ cm, $y = 32.1°$ **d** $x = 54.6°$, $y = 10.3$ cm
e $x = 21.8°$, $y = 3.01$ **f** $x = 45.9°$, $y = 3.87°$
8 **a** 6.52 km **b** 3.80 km
9 **a** 7.31 cm **b** 1.97 cm
10 **a** 66.3° **b** 148 m
11 Using the sine rule, $x = \frac{4\sqrt{2}}{2 + \sqrt{2}}$; rationalising
$x = \frac{4\sqrt{2}(2 - \sqrt{2})}{2} = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$.
12 **a** 36.5 m
b That the angles have been measured from ground level

Exercise 9C
1 **a** 70.5°, 109° (109.5°)
b



2 **a** $x = 74.6°$, $y = 65.4°$
$x = 105°$, $y = 34.6°$
b $x = 59.8°$, $y = 48.4$ cm
$x = 120°$, $y = 27.3$cm
c $x = 56.8°$, $y = 4.37$ cm
$x = 23.2°$, $y = 2.06$ cm
3 **a** 5 cm (ACB = 90°) **b** 24.6°
c 45.6°, 134(.4)°
4 2.97 cm
5 In one triangle ABC = 101° (100.9°); in the other BAC = 131° (130.9°)
6 **a** 62.0° **b** The swing is symmetrical

Exercise 9D
1 **a** 23.7 cm² **b** 4.31 cm² **c** 20.2 cm²
2 **a** $x = 41.8°$ or 138(.2)°
b $x = 26.7°$ or 153(.3)°
c $x = 60°$ or 120°
3 275(.3) m (third side = 135.3 m)
4 3.58
5 **a** Area $= \frac{1}{2}(x + 2)(5 - x)\sin 30°$
$= \frac{1}{2}(10 + 3x - x^2) \times \frac{1}{2}$
$= \frac{1}{4}(10 + 3x - x^2)$
b Maximum $A = 3\frac{1}{16}$, when $x = 1\frac{1}{2}$
6 **a** $\frac{1}{2}x(5 + x)\sin 150° = \frac{15}{4}$
$\frac{1}{2}(5x + x^2) \times \frac{1}{2} = \frac{15}{4}$
$5x + x^2 = 15$
$x^2 + 5x - 15 = 0$
b 2.11

Exercise 9E
1 **a** $x = 37.7°$, $y = 86.3°$, $z = 6.86$
b $x = 48°$, $y = 19.5$, $z = 14.6$
c $x = 30°$, $y = 11.5$, $z = 11.5$
d $x = 21.0°$, $y = 29.0°$, $z = 8.09$
e $x = 93.8°$, $y = 56.3°$, $z = 29.9°$
f $x = 97.2°$, $y = 41.4°$, $z = 41.4°$

g $x = 45.3°$, $y = 94.7°$, $z = 14.7$
or $x = 135°$, $y = 5.27°$, $z = 1.36$
h $x = 7.07$, $y = 106°$, $z = 28.7°$
i $x = 49.8°$, $y = 9.39$, $z = 37.0°$

2 **a** $ACB = 32.4°$, $ABC = 108°$, $AC = 15.1$ cm
Area = 41.3 cm^2
b $BAC = 41.5°$, $ABC = 28.5°$, $AB = 9.65$ cm
Area = 15.7 cm^2

3 **a** 8 km **b** 060°

4 107 km

5 12 km

6 **a** 5.44 **b** 7.95 **c** 36.8°

7 **a** $AB + BC > AC \Rightarrow x + 6 > 7 \Rightarrow x > 1$;
$AC + AB > BC \Rightarrow 11 > x + 2 \Rightarrow x < 9$
b **i** $x = 6.08$ from $x^2 = 37$
Area = 14.0 cm^2
ii $x = 7.23$ from $x^2 - 4(\sqrt{2} - 1)x - (29 + 8\sqrt{2}) = 0$
Area = 13.1 cm^2

8 **a** $x = 4$ **b** 4.68 cm^2

9 $AC = 1.93$ cm

10 **a** $AC^2 = (2 - x)^2 + (x + 1)^2 - 2(2 - x)(x + 1)\cos 120°$
$= (4 - 4x + x^2) + (x^2 + 2x + 1) - 2(-x^2 + x + 2)\left(-\frac{1}{2}\right)$
$= x^2 - x + 7$
b $\frac{1}{2}$

11 $4\sqrt{10}$

12 $AC = 1\frac{2}{3}$ cm and $BC = 6\frac{1}{3}$ cm
Area = 5.05 cm^2

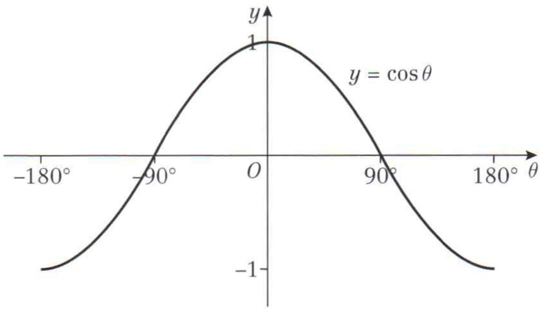
13 **a** 61.3° **b** 78.9 cm^2

14 **a** $DAB = 136.3°$, $BCD = 50.1°$
b 13.1 m^2
c 5.15 m

15 34.2 cm^2

Exercise 9F

1



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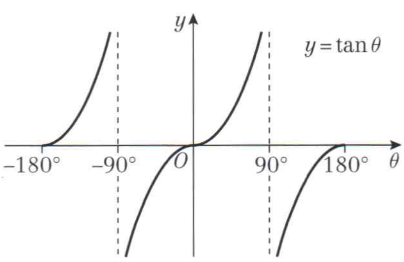
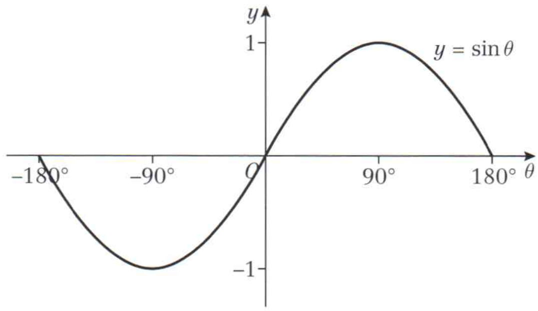


3

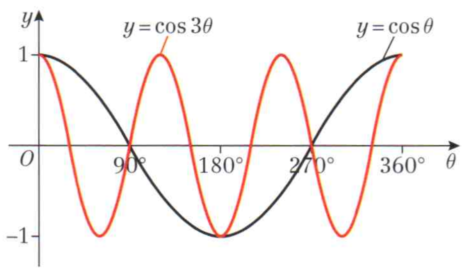


4 **a** –30°
b **i** –120° **ii** –60°, 120°
c **i** 135° **ii** –45°, –135°

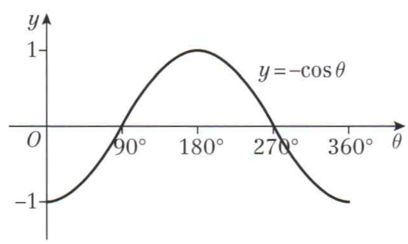
Exercise 9G

1 **a** **i** 1, $x = 0°$ **ii** –1, $x = 180°$
b **i** 4, $x = 90°$ **ii** –4, $x = 270°$
c **i** 1, $x = 0°$ **ii** –1, $x = 180°$
d **i** 4, $x = 90°$ **ii** 2, $x = 270°$
e **i** 1, $x = 270°$ **ii** –1, $x = 90°$
f **i** 1, $x = 30°$ **ii** –1, $x = 90°$

2

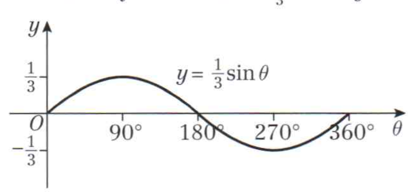


3 **a** The graph of $y = -\cos\theta$ is the graph of $y = \cos\theta$ reflected in the θ-axis



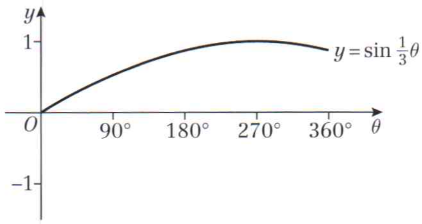
Meets θ-axis at (90°, 0), (270°, 0)
Meets y-axis at (0°, –1)
Maximum at (180°, 1)
Minimum at (0°, –1) and (360°, –1)

b The graph of $y = \frac{1}{3}\sin\theta$ is the graph of $y = \sin\theta$ stretched by a scale factor $\frac{1}{3}$ in the y direction.



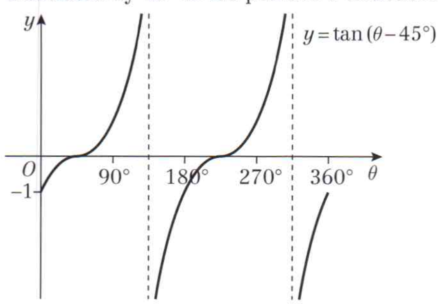
Meets θ-axis at (0°, 0), (180°, 0), (360°, 0)
Meets y-axis at (0°, 0)
Maximum at (90°, $\frac{1}{3}$)
Minimum at (270°, $-\frac{1}{3}$)

c The graph of $y = \sin \frac{1}{3}\theta$ is the graph of $y = \sin \theta$ stretched by a scale factor 3 in the θ direction.



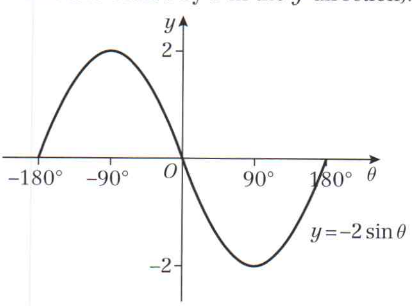
Only meets axis at origin
Maximum at (270°, 1)

d The graph of $y = \tan(\theta - 45°)$ is the graph of $\tan \theta$ translated by 45° in the positive θ direction.



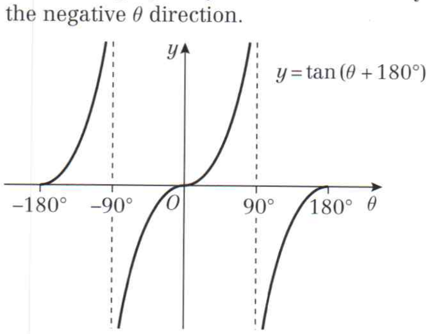
Meets θ-axis at (45°, 0), (225°, 0)
Meets y-axis at (0°, –1)
(Asymptotes at θ = 135° and θ = 315°)

4 a This is the graph of $y = \sin \theta$ stretched by scale factor –2 in the y-direction (i.e. reflected in the θ-axis and scaled by 2 in the y-direction).



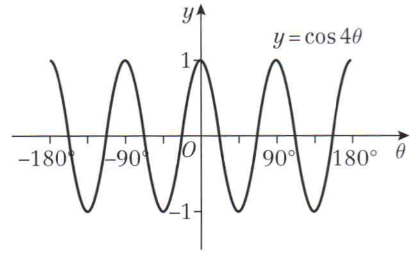
Meets θ-axis at (–180°, 0), (0, 0), (180, 0)
Maximum at (–90°, 2)
Minimum at (90°, –2).

b This is the graph of $y = \tan \theta$ translated by 180° in the negative θ direction.



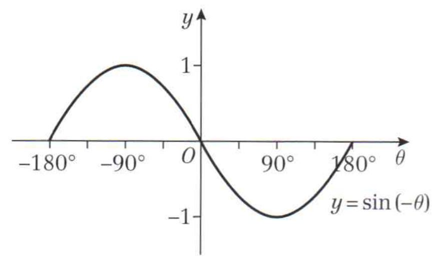
As $\tan \theta$ has a period of 180°
$\tan(\theta + 180°) = \tan \theta$
Meets θ-axis at (–180°, 0), (0, 0), (180°, 0)
Meets y-axis at (0, 0)

c This is the graph of $y = \cos \theta$ stretched by scale factor $\frac{1}{4}$ horizontally.



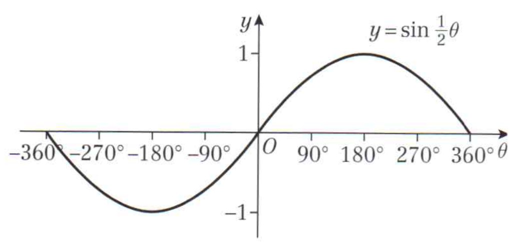
Meets θ-axis at $(-157\frac{1}{2}°, 0)$, $(-112\frac{1}{2}°, 0)$, $(-67\frac{1}{2}°, 0)$, $(-22\frac{1}{2}°, 0)$, $(22\frac{1}{2}°, 0)$, $(67\frac{1}{2}°, 0)$, $(112\frac{1}{2}°, 0)$, $(157\frac{1}{2}°, 0)$
Meets y-axis at (0, 1)
Maxima at (–180°, 1), (–90°, 1), (0, 1), (90°, 1), (180°, 1)
Minima at (–135°, –1), (–45°, –1), (45°, –1), (135°, –1)

d This is the graph of $y = \sin \theta$ reflected in the y-axis. (This is the same as $y = -\sin \theta$.)

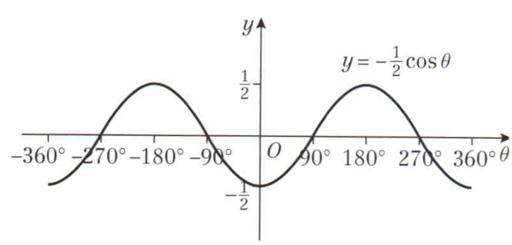


Meets θ-axis at (–180°, 0), (0°, 0), (180°, 0)
Maximum at (–90°, 1)
Minimum at (90°, –1)

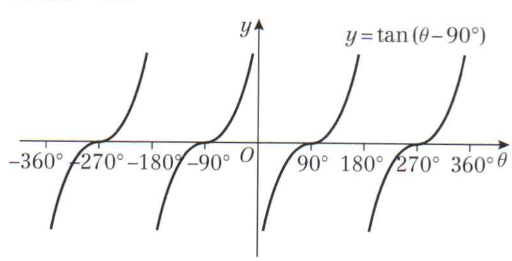
5 a Period = 720°



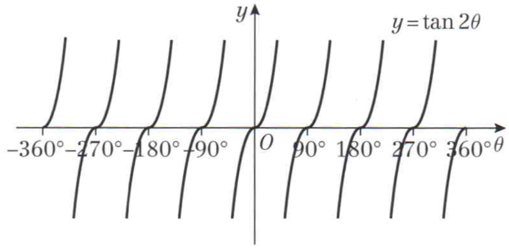
b Period = 360°



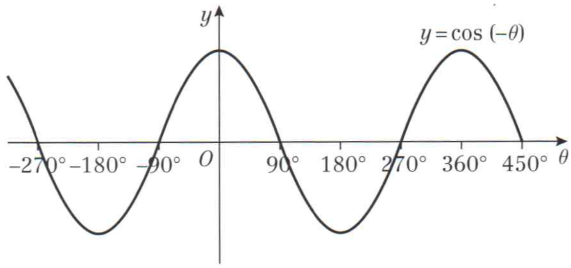
c Period = 180°



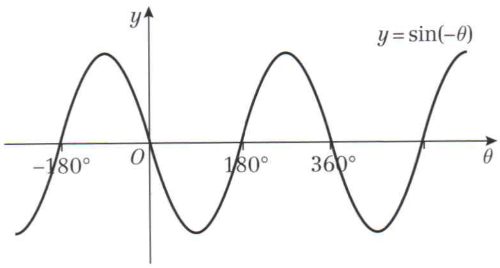
d Period = 90°



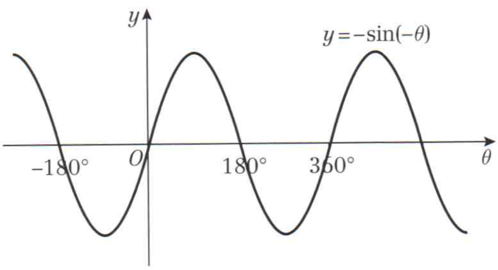
6 a i $y = \cos(-\theta)$ is a reflection of $y = \cos\theta$ in the y-axis, which is the same curve, so $\cos\theta = \cos(-\theta)$.



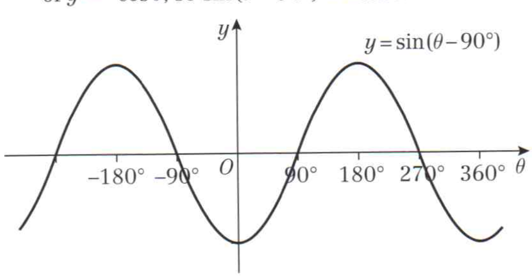
ii $y = \sin(-\theta)$ is a reflection of $y = \sin\theta$ in the y-axis.



$y = -\sin(-\theta)$ is a reflection of $y = \sin(-\theta)$ in the θ-axis, which is the graph of $y = \sin\theta$, so $-\sin(-\theta) = \sin\theta$.



iii $y = \sin(\theta - 90°)$ is the graph of $y = \sin\theta$ translated by 90° to the right, which is the graph of $y = -\cos\theta$, so $\sin(\theta - 90°) = -\cos\theta$.



b $\sin(90° - \theta)$
$= -\sin(-(90° - \theta)) = -\sin(\theta - 90°)$ using (a) (ii)
$= -(-\cos\theta)$ using (a) (iii)
$= \cos\theta$

c Using (a)(i) $\cos(90° - \theta) = \cos(-(90° - \theta))$
$= \cos(\theta - 90°)$, but $\cos(\theta - 90°) = \sin\theta$,
so $\cos(90° - \theta) = \sin\theta$

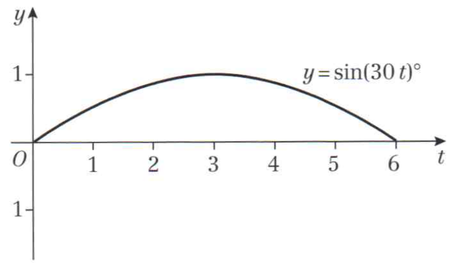
7 a (–300°, 0), (–120°, 0), (60°, 0), (240°, 0)

b $\left(0°, \frac{\sqrt{3}}{2}\right)$

8 a $k = 60°$

b Yes – the graph of $y = \sin\theta$ repeats every 360°, so e.g. $k = 420°$.

9 a



b Between 1 pm and 5 pm

Mixed exercise 9

1 a 155° **b** 13.7 cm

2 a $x = 49.5°$, area = 1.37 cm²

b $x = 55.2°$, area = 10.6 cm²

c $x = 117°$, area = 6.66 cm²

3 6.50 cm²

4 a 50.9 cm² **b** 12.0 cm²

5 a 5 **b** $\frac{25\sqrt{3}}{2}$ cm²

6 area $= \frac{1}{2}ab\sin C$

$1 = \frac{1}{2} \times 2\sqrt{2}\sin C$

$\frac{1}{\sqrt{2}} = \sin C \Rightarrow C = 45°$

Use the cosine rule to find the other side:

$x^2 = 2^2 + (\sqrt{2})^2 - 2 \times 2\sqrt{2}\cos C \Rightarrow x = \sqrt{2}$ cm

So the triangle is isosceles, with two 45° angles, thus is also right-angled.

7 a $AC = \sqrt{5}$, $AB = \sqrt{18}$, $BC = \sqrt{5}$

$$\cos\angle ACB = \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC}$$
$$= \frac{5 + 5 - 18}{2 \times \sqrt{5} \times \sqrt{5}}$$
$$= -\frac{8}{10} = -\frac{4}{5}$$

b $1\frac{1}{2}$ cm²

8 a 4 **b** $\frac{15\sqrt{3}}{4}$ (6.50) cm²

9 a 1.50 km **b** 241° **c** 0.789 km²

10 359 m²

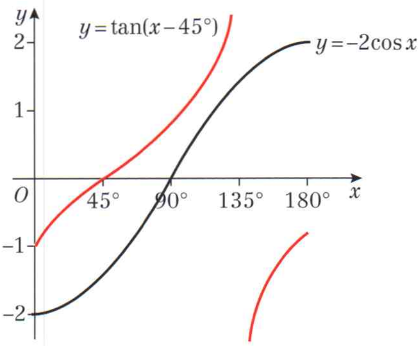
11 35.2 m

12 a A stretch of scale factor 2 in the x direction.

b A translation of +3 in the y direction.

c A reflection in the x-axis.
d A translation of -20 in the x direction.

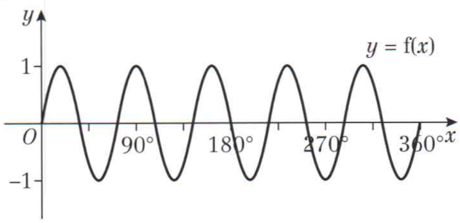
13 a



b There are no solutions.

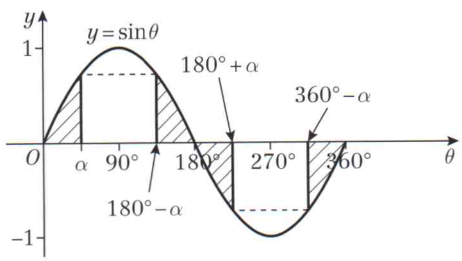
14 a 300° **b** (30°, 1) **c** 60° **d** $\frac{\sqrt{3}}{2}$

15 a $p = 5$



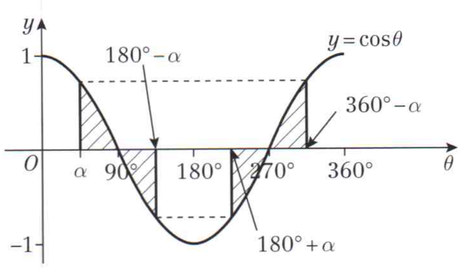
b 72°

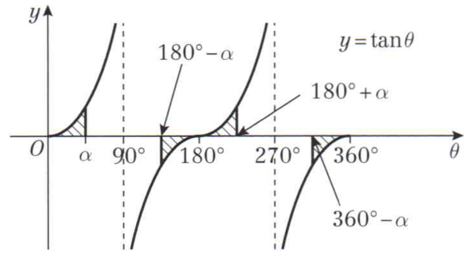
16 a The four shaded regions are congruent.



b $\sin\alpha$ and $\sin(180° - \alpha)$ have the same y value, (call it k)
so $\sin\alpha = \sin(180° - \alpha)$
$\sin(180° + \alpha)$ and $\sin(360° - \alpha)$ have the same y value, (which will be $-k$)
so $\sin\alpha = \sin(180° - \alpha)$
$= -\sin(180° + \alpha) = -\sin(360° - \alpha)$

17 a

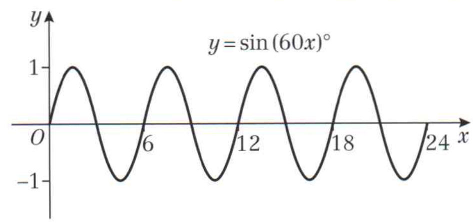




b i From the graph of $y = \cos\theta$, which shows four congruent shaded regions, if the y value at α is k, then y at $180° - \alpha$ is $-k$, y at $180° + \alpha = -k$ and y at $360° - \alpha = +k$
so $\cos\alpha = -\cos(180° - \alpha)$
$= -\cos(180° + \alpha) = \cos(360° - \alpha)$

ii From the graph of $y = \tan\theta$, if the y value at α is k, then at $180° - \alpha$ it is $-k$, at $180° + \alpha$ it is $+k$ and at $360° - \alpha$ it is $-k$,
so $\tan\alpha = -\tan(180° - \alpha)$
$= +\tan(180° + \alpha) = -\tan(360° - \alpha)$

18 a



b 4
c The dunes may not all be the same height.

Challenge
Using the sine rule:

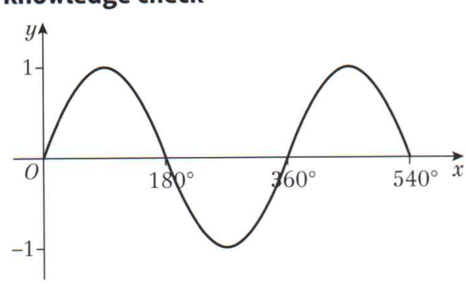
$$\sin(180° - \angle ADB - \angle AEB) = \frac{5\left(\frac{1}{\sqrt{5}}\right)}{\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$180° - \angle ADB - \angle AEB = 135°$ (obtuse)
so $\angle ADB + \angle AEB = 45° = \angle ACB$

CHAPTER 10

Prior knowledge check

1 a



b 4
c 143.1°, 396.9°, 503.1°

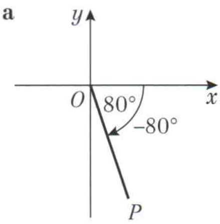
2 a 57.7° **b** 73.0°

3 a $x = 11$ **b** $x = \frac{9}{4}$ **c** $x = -44.4°$

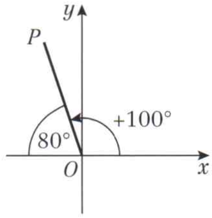
4 a $x = 1$ or $x = 3$
b $x = 1$ or $x = -9$
c $x = \frac{3 \pm \sqrt{65}}{4}$

Exercise 10A

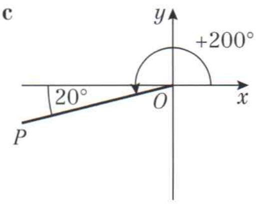
1 **a**



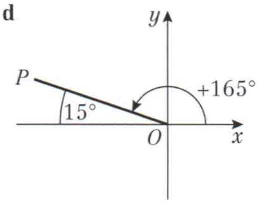
b



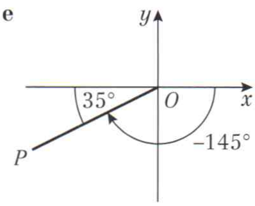
c



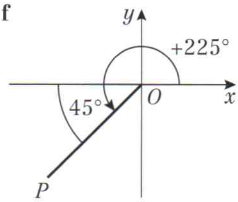
d



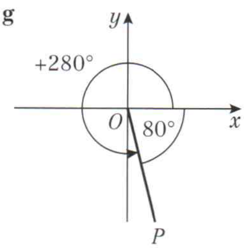
e



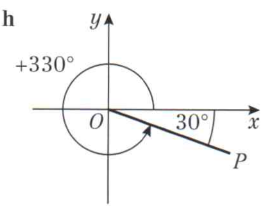
f



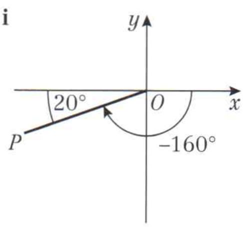
g



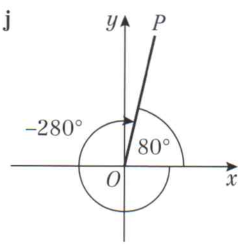
h



i



j



2 **a** First **b** Second **c** Second
d Third **e** Third

3 **a** –1 **b** 1 **c** 0 **d** –1 **e** –1
f 0 **g** 0 **h** 0 **i** 0 **j** 0

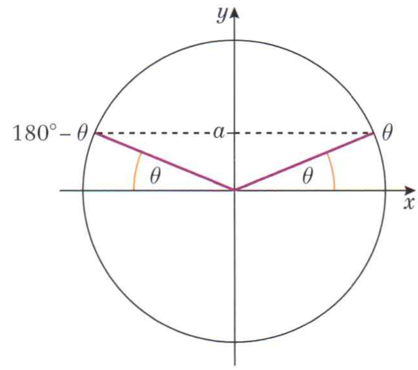
4 **a** $-\sin 60°$ **b** $-\sin 80°$ **c** $\sin 20°$
d $-\sin 60°$ **e** $\sin 80$ **f** $-\cos 70°$
g $-\cos 80°$ **h** $\cos 50°$ **i** $-\cos 20°$
j $-\cos 5°$ **k** $-\tan 80°$ **l** $-\tan 35°$
m $-\tan 30°$ **n** $\tan 5°$ **o** $\tan 60°$

5 **a** $-\sin\theta$ **b** $-\sin\theta$ **c** $-\sin\theta$
d $\sin\theta$ **e** $-\sin\theta$ **f** $\sin\theta$
g $-\sin\theta$ **h** $-\sin\theta$ **i** $\sin\theta$

6 **a** $-\cos\theta$ **b** $-\cos\theta$ **c** $\cos\theta$
d $-\cos\theta$ **e** $\cos\theta$ **f** $-\cos\theta$
g $-\tan\theta$ **h** $-\tan\theta$ **i** $\tan\theta$
j $\tan\theta$ **k** $-\tan\theta$ **l** $\tan\theta$

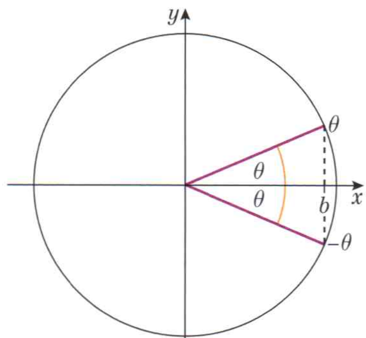
Challenge

a



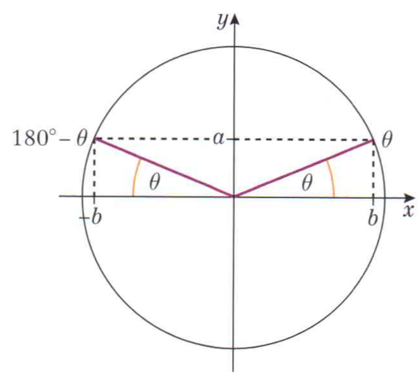
$\sin\theta = \sin(180° - \theta) = a$

b



$\cos\theta = \cos(-\theta) = b$

c



$\tan\theta = \frac{a}{b}$; $\tan(180° - \theta) = \frac{a}{-b} = -\tan\theta$

Exercise 10B

1 **a** $\frac{\sqrt{2}}{2}$ **b** $-\frac{\sqrt{3}}{2}$ **c** $-\frac{1}{2}$ **d** $\frac{\sqrt{3}}{2}$
e $\frac{\sqrt{3}}{2}$ **f** $-\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $-\frac{\sqrt{2}}{2}$
i $-\frac{\sqrt{3}}{2}$ **j** $-\frac{\sqrt{2}}{2}$ **k** –1 **l** –1
m $\frac{\sqrt{3}}{3}$ **n** $-\sqrt{3}$ **o** $\sqrt{3}$

Challenge

a **i** $\sqrt{3}$ **ii** 2 **iii** $\sqrt{2+\sqrt{3}}$ **iv** $\sqrt{2+\sqrt{3}} - \sqrt{2}$
b 15°
c **i** $\frac{\sqrt{2+\sqrt{3}} - \sqrt{2}}{2}$ **ii** $\frac{\sqrt{2+\sqrt{3}}}{2}$

Exercise 10C

1 a $\sin^2 \frac{\theta}{2}$ b 5 c $-\cos^2 A$
d $\cos\theta$ e $\tan x$ f $\tan 3A$
g 4 h $\sin^2\theta$ i 1

2 $1\frac{1}{2}$

3 $3 \tan y$

4 a $1 - \sin^2\theta$ b $\frac{\sin^2\theta}{1-\sin^2\theta}$ c $\sin\theta$
d $\frac{1-\sin^2\theta}{\sin\theta}$ e $1 - 2\sin^2\theta$

5 (One outline example of a proof is given)
a $\text{LHS} = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$
$= 1 + 2\sin\theta\cos\theta$
$= \text{RHS}$
b $\text{LHS} = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = \sin\theta \times \frac{\sin\theta}{\cos\theta}$
$= \sin\theta\tan\theta = \text{RHS}$
c $\text{LHS} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$
$= \frac{1}{\sin x \cos x} = \text{RHS}$
d $\text{LHS} = (1 - \sin^2 A) - \sin^2 A$
$= 1 - 2\sin^2 A = \text{RHS}$
e $\text{LHS} = (4\sin^2\theta - 4\sin\theta\cos\theta + \cos^2\theta)$
$+ (\sin^2\theta + 4\sin\theta\cos\theta + 4\cos^2\theta)$
$= 5(\sin^2\theta + \cos^2\theta) = 5 = \text{RHS}$
f $\text{LHS} = 2 - (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$
$= 2(\sin^2\theta + \cos^2\theta) - (\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta)$
$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$
$= (\sin\theta + \cos\theta)^2 = \text{RHS}$
g $\text{LHS} = \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x)\sin^2 y$
$= \sin^2 x - \sin^2 y = \text{RHS}$

6 a $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$
b $\sin\theta = \frac{4}{5}$, $\tan\theta = -\frac{4}{3}$
c $\cos\theta = \frac{24}{25}$, $\tan\theta = -\frac{7}{24}$

7 a $-\frac{\sqrt{5}}{3}$ b $-\frac{2\sqrt{5}}{5}$

8 a $-\frac{\sqrt{3}}{2}$ b $\frac{1}{2}$

9 a $-\frac{\sqrt{7}}{4}$ b $-\frac{\sqrt{7}}{3}$

10 a $x^2 + y^2 = 1$
b $4x^2 + y^2 = 4$ $\left(\text{or } x^2 + \frac{y^2}{4} = 1\right)$
c $x^2 + y = 1$
d $x^2 = y^2(1 - x^2)$ $\left(\text{or } x^2 + \frac{x^2}{y^2} = 1\right)$
e $x^2 + y^2 = 2$ $\left(\text{or } \frac{(x+y)^2}{4} + \frac{(x-y)^2}{4} = 1\right)$

11 a Using cosine rule: $\cos B = \frac{8^2 + 12^2 - 10^2}{2 \times 8 \times 12} = \frac{9}{16}$
b $\frac{\sqrt{175}}{16}$

12 a Using sine rule: $\sin Q = \frac{\sin 30}{6} \times 8 = \frac{2}{3}$
b $-\frac{\sqrt{5}}{3}$

Exercise 10D

1 a $-63.4°$ b $116.6°, 296.6°$

2 a $66.4°$ b $66.4°, 113.6°, 246.4°, 293.6°$

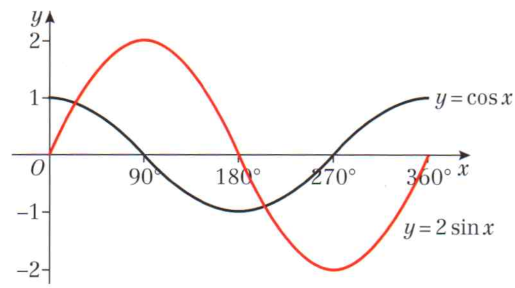
3 a $270°$ b $60°, 240°$
c $60°, 300°$ d $15°, 165°$
e $140°, 220°$ f $135°, 315°$
g $90°, 270°$ h $230°, 310°$

4 a $45.6°, 134.4°$ b $135°, 225°$
c $132°, 228°$ d $229°, 311°$
e $8.13°, 188°$ f $61.9°, 242°$
g $105°, 285°$ h $41.8°, 318°$

5 a $30°, 210°$ b $135°, 315°$
c $53.1°, 233°$ d $56.3°, 236°$
e $54.7°, 235°$ f $148°, 328°$

6 a $-120°, -60°, 240°, 300°$ b $-171°, -8.63°$
c $-144°, 144°$ d $-327°, -32.9°$
e $150°, 330°, 510°, 690°$ f $251°, 431°$

7 a $\tan x$ should be $\frac{2}{3}$
b Squaring both sides creates extra solutions
c $-146.3°, 33.7°$

8 a


b 2 c $26.6°, 206.6°$

9 $71.6°, 108.4°, 251.6°, 288.4°$

10 a $4\sin^2 x - 3(1 - \sin^2 x) = 2$.
Rearrange to get $7\sin^2 x = 5$
b $57.7°, 122.3°, 237.7°, 302.3°$

11 a $2\sin^2 x + 5(1 - \sin^2 x) = 1$.
Rearrange to get $3\sin^2 x = 4$
b $\sin x > 1$

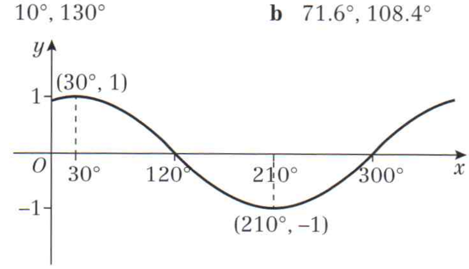
Exercise 10E

1 a $0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°$
b $60°, 180°, 300°$
c $22\frac{1}{2}°, 112\frac{1}{2}°, 202\frac{1}{2}°, 292\frac{1}{2}°$
d $30°, 150°, 210°, 330°$
e $300°$
f $225°, 315°$

2 a $90°, 270°$ b $50°, 170°$ c $165°, 345°$
d $250°, 310°$ e $65°, 245°$
f $-77.3°, -17.3°, 42.7°, 103°, 163°$

3 a $11.2°, 71.2°, 131.2°$ b $6.3°, 186.3°, 366.3°$
c $37.0°, 127.0°$ d $-150°, 30°$

4 a $10°, 130°$ b $71.6°, 108.4°$

5 a


b $\left(0°, \frac{\sqrt{3}}{2}\right)$, (120°, 0), (300°, 0)
c 86.6°, 333.4°

6 a 0.75
b 18.4°, 108.4°, 198.4°, 288.4°

7 a 2.5
b No: increasing k will bring another 'branch' of the tan graph into place.

8 25°, 65°, 145°

Exercise 10F

1 a 60°, 120°, 240°, 300°
b 45°, 135°, 225°, 315°
c 0°, 180°, 199°, 341°, 360°
d 77.0°, 113°, 257°, 293°
e 60°, 300°
f 204°, 336°
g 30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°

2 a ±45°, ±135° b −180°, −117°, 0°, 63.4°, 180°
c ±114° d 0°, ±75.5°, ±180°

3 a 72°, 144° b 0°, 60°
c No solutions in range

4 a ±41.8°, ±138° b 38.2°, 142°

5 60°, 75.5°, 284.5°, 300°

6 48.2°, 131.8°, 228.2°, 311.8°

7 $2\cos^2 x + \cos x - 6 = (2\cos x - 3)(\cos x + 2)$
There are no solutions to $\cos x = -2$ or to $\cos x = \frac{3}{2}$

8 a $1 - \sin^2 x = 2 - \sin x$
Rearrange to get $\sin^2 x - \sin x + 1 = 0$
b The equation has no real roots as $b^2 - 4ac < 0$

9 a $p = 1$, $q = 5$
b 72.8°, 129.0°, 252.8°, 309.0°, 432.8°, 489.0°

Challenge

1 −180°, −60°, 60°, 180°

2 0°, 90°, 180°, 270°, 360°

Mixed exercise 10

1 a −cos 57° b −sin 48° c +tan 10°

2 a 0 b $-\frac{\sqrt{2}}{2}$ c −1
d $\sqrt{3}$ e −1

3 Using $\sin^2 A = 1 - \cos^2 A$, $\sin^2 A = 1 - \left(-\sqrt{\frac{7}{11}}\right)^2 = \frac{4}{11}$.
Since angle A is obtuse, it is in the second quadrant and sin is positive, so $\sin A = \frac{2}{\sqrt{11}}$.
Then $\tan A = \frac{\sin A}{\cos A} = \frac{2}{\sqrt{11}} \times \left(-\sqrt{\frac{11}{7}}\right) = -\frac{2}{\sqrt{7}} = -\frac{2}{7}\sqrt{7}$.

4 a $-\frac{\sqrt{21}}{5}$ b $-\frac{2}{5}$

5 a $\cos^2\theta - \sin^2\theta$ b $\sin^4 3\theta$ c 1

6 a 1 b $\tan y = \frac{4 + \tan x}{2\tan x - 3}$

7 a LHS $= (1 + 2\sin\theta + \sin^2\theta) + \cos^2\theta$
$= 1 + 2\sin\theta + 1$
$= 2 + 2\sin\theta$
$= 2(1 + \sin\theta)$ = RHS
b LHS $= \cos^4\theta + \sin^2\theta$
$= (1 - \sin^2\theta)^2 + \sin^2\theta$
$= 1 - 2\sin^2\theta + \sin^4\theta + \sin^2\theta$
$= (1 - \sin^2\theta) + \sin^4\theta$
$= \cos^2\theta + \sin^4\theta$ = RHS

8 a No solutions: $-1 \leqslant \sin\theta \leqslant 1$
b 2 solutions: $\tan\theta = -1$ has two solutions in the interval.
c No solutions: $2\sin\theta + 3\cos\theta > -5$ so $2\sin\theta + 3\cos\theta + 6$ can never be equal to 0.
d No solutions: $\tan^2\theta = -1$ has no real solutions.

9 a $(4x - y)(y + 1)$ b 14.0°, 180°, 194°

10 a $3\cos 3\theta$ b 16.1°, 104°, 136°, 224°, 256°, 344°

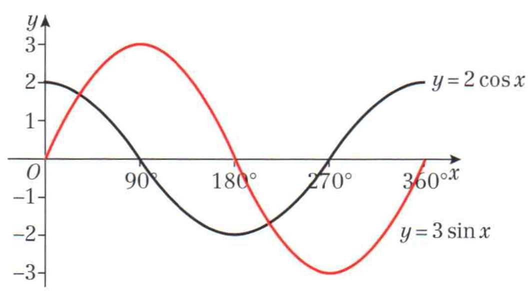
11 a $2\sin 2\theta = \cos 2\theta \Rightarrow \frac{2\sin 2\theta}{\cos 2\theta} = 1$
$\Rightarrow 2\tan 2\theta = 1 \Rightarrow \tan 2\theta = 0.5$
b 13.3°, 103.3°, 193.3°, 283.3°

12 a 225°, 345°
b 22.2°, 67.8°, 202.2°, 247.8°

13 30°, 150°, 210°

14 131.8°, 228.2°

15 a Found additional solutions after dividing by three rather than before. Not applied the full interval for solutions.
b −350°, −310°, −230°, −190°, −110°, −70°, 10°, 50°, 130°, 170°, 250°, 290°

16 a


b 2 c 33.7°, 213.7°

17 a $\frac{9}{11}$ b $\frac{\sqrt{40}}{11}$

18 a Using sine rule: $\sin Q = \sin 45° \times \frac{6}{5} = \frac{\sqrt{2}}{2} \times \frac{6}{5} = \frac{3\sqrt{2}}{5}$
b $-\frac{\sqrt{7}}{5}$

19 a $3\sin^2 x - (1 - \sin^2 x) = 2$.
Rearrange to give $4\sin^2 x = 3$.
b −120°, −60°, 60°, 120°

20 −318.2°, −221.8°, 41.8°, 138.2°

21 a 90° b x = 120° or 300°

Challenge

45°, 54.7°, 125.3°, 135°, 225°, 234.7°, 305.3°, 315°

Review exercise 2

1 $x + 3y - 22 = 0$

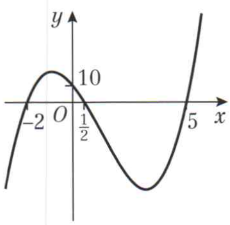
2 $x - 3y - 21 = 0$

3 4, −2.5

4 a 0.45
b $l = 0.45h$
c The model may not be valid for young people who are still growing.

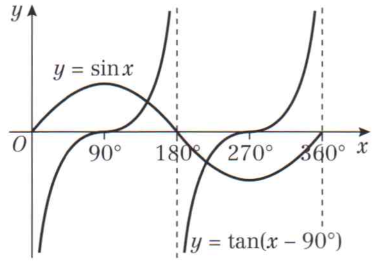
5 a $y = -\frac{1}{3}x + 4$ b C is (3, 3) c 15

6 $3\sqrt{5}$
7 $(-6, 0)$
8 $(x + 3)^2 + (y - 8)^2 = 10$
9 a $(x - 3)^2 + (y + 1)^2 = 20$ ($a = 3$, $b = -1$, $r = \sqrt{20}$)
b Centre $(3, -1)$, radius $\sqrt{20}$
10 a $(3, 5)$ and $(4, 2)$
b $\sqrt{10}$
11 $0 < r < \sqrt{\frac{2}{5}}$
12 a $(x - 1)^2 + (y - 5)^2 = 58$
b $7y - 3x + 26 = 0$
13 a $AB = \sqrt{32}$; $BC = \sqrt{8}$; $AC = \sqrt{40}$; $AC^2 = AB^2 + BC^2$
b AC is a diameter of the circle.
c $(x - 5)^2 + (y - 2)^2 = 10$
14 $a = 3$, $b = -2$, $c = -8$
15 a $2(\frac{1}{2})^3 - 7(\frac{1}{2})^2 - 17(\frac{1}{2}) + 10 = 0$
b $(2x - 1)(x - 5)(x + 2)$
c



16 a 24
b $(x - 3)(3x - 2)(x + 4)$
17 a $g(3) = 3^3 - 13(3) + 12 = 0$
b $(x - 3)(x + 4)(x - 1)$
18 a $a = 0$, $b = 0$
b $a > 0$, $b > 0$ or $a < 0$, $b < 0$
19 a $5^2 = 24 + 1$; $7^2 = 2(24) + 1$; $11^2 = 5(24) + 1$; $13^2 = 7(24) + 1$; $17^2 = 12(24) + 1$; $19^2 = 15(24) + 1$
b $3(24) + 1 = 73$ which is not a square of a prime number
20 a $(x - 5)^2 + (y - 4)^2 = 3^2$
b $\sqrt{41}$
c Sum of radii $= 3 + 3 < \sqrt{41}$ so circles do not touch
21 a $1 - 20x + 180x^2 - 960x^3$
b 0.817
22 $a = 2$, $b = 19$, $c = 70$
23 4
24 $\sqrt{10}$ cm
25 a $\cos 60° = \frac{1}{2} = (5^2 + (2x - 3)^2 - (x + 1)^2) \div 2(5)(2x - 3)$
$5(2x - 3) = (25 + 4x^2 - 12x + 9 - x^2 - 2x - 1)$
$0 = 3x^2 - 24x + 48$
$x^2 - 8x + 16 = 0$
b 4
c 10.8 cm²
26 a 11.93 km
b 100.9°
27 a $AB = BC = 10$ cm, $AC = 6\sqrt{10}$ cm
b 143.1°
28 19.4 cm²
29 a $(x - 5)^2 + (y - 2)^2 = 25$
b 6
c $XY = \sqrt{90}$; $YZ = \sqrt{20}$; $XZ = \sqrt{98}$
$\cos XYZ = (20 + 90 - 98) \div (2 \times \sqrt{20} \times \sqrt{90})$
$\cos XYZ = 12 \div 60\sqrt{2} = \sqrt{2} \div 10$

30 a



b 2
31 a $(-225, 0)$, $(-45, 0)$, $(135, 0)$ and $(315, 0)$
b $\left(0, \frac{\sqrt{2}}{2}\right)$
32 Area of triangle $= \frac{1}{2} \times s \times s \times \sin 60° = \frac{\sqrt{3}}{4}s^2$
Area of square $= s^2$
Total surface area $= 4 \times \left(\frac{\sqrt{3}}{4}s^2\right) + s^2 = (\sqrt{3} + 1)s^2$ cm²
33 a 1
b 45°, 225°
34 30°, 150°, 210°, 330°
35 90°, 150°
36 a $2(1 - \sin^2 x) = 4 - 5\sin x$
$2 - 2\sin^2 x = 4 - 5\sin x$
$2\sin^2 x - 5\sin x + 2 = 0$
b $x = 30°, 150°$
37 72.3°, 147.5°, 252.3°, 327.5°
38 0°, 78.5°, 281.5°
39 $\cos^2 x(\tan^2 x + 1)$
$= \cos^2 x\left(\frac{\sin^2 x}{\cos^2 x} + 1\right)$
$= \sin^2 x + \cos^2 x = 1$

Challenge

1 a 160
b $\left(-\frac{28}{3}, 0\right)$
2 The equation of the second circle is $(x + 4)^2 + (y - 5)^2 = 10^2$ so it has the same centre but a larger radius
3 $$\binom{n}{k} + \binom{n}{k+1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!}$$
$$= \frac{n!(k+1)}{(k+1)!(n-k)!} + \frac{n!(n-k)}{(k+1)!(n-k)!}$$
$$= \frac{n!((k+1) + (n-k))}{(k+1)!(n-k)!}$$
$$= \frac{n!(n+1)}{(k+1)!(n-k)!}$$
$$= \frac{(n+1)!}{(k+1)!(n-k)!}$$
$$= \binom{n+1}{k+1}$$
4 0°, 30°, 150°, 180°, 270°, 360°

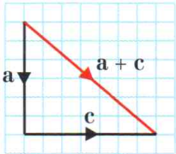
CHAPTER 11

Prior knowledge check

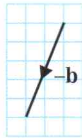
1 a $\begin{pmatrix}4\\2\end{pmatrix}$ b $\begin{pmatrix}5\\-2\end{pmatrix}$ c $\begin{pmatrix}-1\\-3\end{pmatrix}$
2 a $\frac{7}{9}$ b $\frac{2}{9}$ c $\frac{7}{2}$
3 a 123.2° b 13.6 c 5.3 d 21.4°

Exercise 11A

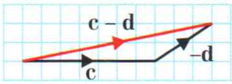
1 **a**



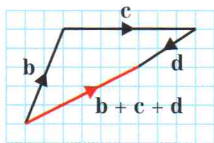
b



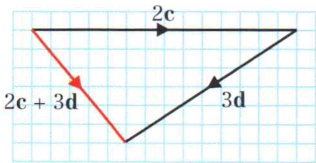
c



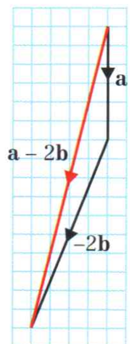
d



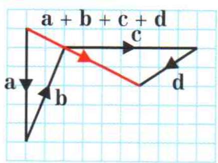
e



f



g



2 **a** $2\mathbf{b}$ **b** $\mathbf{d}$ **c** $\mathbf{b}$
d $2\mathbf{b}$ **e** $\mathbf{d}+\mathbf{b}$ **f** $\mathbf{d}+\mathbf{b}$
g $-2\mathbf{d}$ **h** $-\mathbf{b}$ **i** $2\mathbf{d}+\mathbf{b}$
j $-\mathbf{b}+2\mathbf{d}$ **k** $-\mathbf{b}+\mathbf{d}$ **l** $-\mathbf{d}-\mathbf{b}$

3 **a** $2\mathbf{m}$ **b** $2\mathbf{p}$ **c** $\mathbf{m}$
d $\mathbf{m}$ **e** $\mathbf{p}+\mathbf{m}$ **f** $\mathbf{p}+\mathbf{m}$
g $\mathbf{p}+2\mathbf{m}$ **h** $\mathbf{p}-\mathbf{m}$ **i** $-\mathbf{m}-\mathbf{p}$
j $-2\mathbf{m}+\mathbf{p}$ **k** $-2\mathbf{p}+\mathbf{m}$ **l** $-\mathbf{m}-2\mathbf{p}$

4 **a** $\mathbf{d}-\mathbf{a}$ **b** $\mathbf{a}+\mathbf{b}+\mathbf{c}$
c $\mathbf{a}+\mathbf{b}-\mathbf{d}$ **d** $\mathbf{a}+\mathbf{b}+\mathbf{c}-\mathbf{d}$

5 **a** $2\mathbf{a}+2\mathbf{b}$ **b** $\mathbf{a}+\mathbf{b}$ **c** $\mathbf{b}-\mathbf{a}$

6 **a** $\mathbf{b}$ **b** $\mathbf{b}-3\mathbf{a}$ **c** $\mathbf{a}-\mathbf{b}$
d $2\mathbf{a}-\mathbf{b}$

7 **a** $\overrightarrow{OB}=\mathbf{a}+\mathbf{b}$ **b** $\overrightarrow{OP}=\frac{5}{8}(\mathbf{a}+\mathbf{b})$ **c** $\overrightarrow{AP}=\frac{5}{8}\mathbf{b}-\frac{3}{8}\mathbf{a}$

8 **a** Yes ($\lambda=2$) **b** Yes ($\lambda=4$) **c** No
d Yes ($\lambda=-1$) **e** Yes ($\lambda=-3$) **f** No

9 **a** **i** $\mathbf{b}-\mathbf{a}$ **ii** $\frac{1}{2}\mathbf{a}$ **iii** $\frac{1}{2}\mathbf{b}$ **iv** $\frac{1}{2}\mathbf{b}-\frac{1}{2}\mathbf{a}$
b $\overrightarrow{BC}=\mathbf{b}-\mathbf{a}$, $\overrightarrow{PQ}=\frac{1}{2}(\mathbf{b}-\mathbf{a})$ so PQ is parallel to BC.

10 **a** **i** $2\mathbf{b}$ **ii** $\mathbf{a}-\mathbf{b}$
b $\overrightarrow{AB}=2\mathbf{b}$, $\overrightarrow{OC}=3\mathbf{b}$ so AB is parallel to OC.

11 1.2

Exercise 11B

1 $\mathbf{v}_1$: $8\mathbf{i}$, $\begin{pmatrix}8\\0\end{pmatrix}$ $\mathbf{v}_2$: $9\mathbf{i}+3\mathbf{j}$, $\begin{pmatrix}9\\3\end{pmatrix}$ $\mathbf{v}_3$: $-4\mathbf{i}+2\mathbf{j}$, $\begin{pmatrix}-4\\2\end{pmatrix}$
$\mathbf{v}_4$: $3\mathbf{i}+5\mathbf{j}$, $\begin{pmatrix}3\\5\end{pmatrix}$ $\mathbf{v}_5$: $-3\mathbf{i}-2\mathbf{j}$, $\begin{pmatrix}-3\\-2\end{pmatrix}$ $\mathbf{v}_6$: $-5\mathbf{j}$, $\begin{pmatrix}0\\-5\end{pmatrix}$

2 **a** $8\mathbf{i}+12\mathbf{j}$ **b** $\mathbf{i}+1.5\mathbf{j}$ **c** $-4\mathbf{i}+\mathbf{j}$
d $10\mathbf{i}+\mathbf{j}$ **e** $-2\mathbf{i}+11\mathbf{j}$ **f** $-2\mathbf{i}-10\mathbf{j}$
g $14\mathbf{i}-7\mathbf{j}$ **h** $-8\mathbf{i}+9\mathbf{j}$

3 **a** $\begin{pmatrix}45\\35\end{pmatrix}$ **b** $\begin{pmatrix}4\\0.5\end{pmatrix}$ **c** $\begin{pmatrix}12\\3\end{pmatrix}$
d $\begin{pmatrix}-1\\16\end{pmatrix}$ **e** $\begin{pmatrix}-21\\-29\end{pmatrix}$ **f** $\begin{pmatrix}10\\2\end{pmatrix}$

4 **a** $\lambda=5$ **b** $\mu=-\frac{3}{2}$

5 **a** $\lambda=\frac{1}{3}$ **b** $\mu=-1$
c $s=-1$ **d** $t=-\frac{1}{17}$

6 $\mathbf{i}-\mathbf{j}$

7 **a** $\overrightarrow{AC}=5\mathbf{i}-4\mathbf{j}=\begin{pmatrix}5\\-4\end{pmatrix}$ **b** $\overrightarrow{AP}=3\mathbf{i}-\frac{12}{5}\mathbf{j}=\begin{pmatrix}3\\-\frac{12}{5}\end{pmatrix}$
c $\overrightarrow{OP}=5\mathbf{i}+\frac{8}{5}\mathbf{j}=\begin{pmatrix}5\\\frac{8}{5}\end{pmatrix}$

8 $j=4$, $k=11$

9 $p=3$, $q=2$

10 **a** $p=5$ **b** $8\mathbf{i}-12\mathbf{j}$

Exercise 11C

1 **a** 5 **b** 10 **c** 13
d 4.47 (3 s.f.) **e** 5.83 (3 s.f.) **f** 8.06 (3 s.f.)
g 5.83 (3 s.f.) **h** 4.12 (3 s.f.)

2 **a** $\sqrt{26}$ **b** $5\sqrt{2}$ **c** $\sqrt{101}$

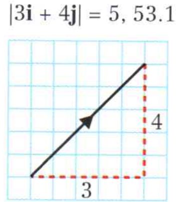
3 **a** $\frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}$ **b** $\frac{1}{13}\begin{pmatrix}5\\-12\end{pmatrix}$
c $\frac{1}{25}\begin{pmatrix}-7\\24\end{pmatrix}$ **d** $\frac{1}{\sqrt{10}}\begin{pmatrix}1\\-3\end{pmatrix}$

4 **a** 53.1° above **b** 53.1° below
c 67.4° above **d** 63.4° above

5 **a** 149° to the right **b** 29.7° to the right
c 31.0° to the left **d** 104° to the left

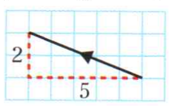
6 **a** $\frac{15\sqrt{2}}{2}\mathbf{i}+\frac{15\sqrt{2}}{2}\mathbf{j}$, $\begin{pmatrix}\frac{15\sqrt{2}}{2}\\\frac{15\sqrt{2}}{2}\end{pmatrix}$ **b** $7.52\mathbf{i}+2.74\mathbf{j}$, $\begin{pmatrix}7.52\\2.74\end{pmatrix}$
c $18.1\mathbf{i}-8.45\mathbf{j}$, $\begin{pmatrix}18.1\\-8.45\end{pmatrix}$ **d** $\frac{5\sqrt{3}}{2}\mathbf{i}-2.5\mathbf{j}$, $\begin{pmatrix}\frac{5\sqrt{3}}{2}\\-2.5\end{pmatrix}$

7 **a** $|3\mathbf{i}+4\mathbf{j}|=5$, 53.1° above



b $|2\mathbf{i}-\mathbf{j}|=\sqrt{5}$, 26.6° below

c $|-5\mathbf{i}+2\mathbf{j}|=\sqrt{29}$, 158.2° above

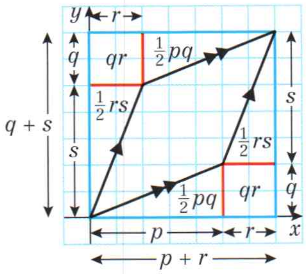


8 $k=\pm6$

9 $p=8$, $q=\pm6$

10 **a** 36.9° **b** 33.7° **c** 70.6°
11 **a** 67.2° **b** 19.0

Challenge
Possible solution:



Area of parallelogram = area of large rectangle – 2(area of small rectangle) – 2 (area triangle 1) – 2(area triangle 2)

Area of parallelogram = $(p + r)(q + s) - 2qr - 2(\frac{1}{2}pq)$
$- 2(\frac{1}{2}rs) = ps - qr$

Exercise 11D

1 **a** **i** $\overrightarrow{OA} = 3\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} + 5\mathbf{j}$, $\overrightarrow{OC} = -2\mathbf{i} + 6\mathbf{j}$
ii $\mathbf{i} + 6\mathbf{j}$ **iii** $-5\mathbf{i} + 7\mathbf{j}$
b **i** $\sqrt{40} = 2\sqrt{10}$ **ii** $\sqrt{37}$ **iii** $\sqrt{74}$

2 **a** $-\mathbf{i} + 5\mathbf{j}$ or $\begin{pmatrix}-1\\5\end{pmatrix}$
b **i** 5 **ii** $\sqrt{13}$ **iii** $\sqrt{26}$

3 **a** $-\mathbf{i} - 9\mathbf{j}$ or $\begin{pmatrix}-1\\-9\end{pmatrix}$
b **i** $\sqrt{82}$ **ii** 5 **iii** $\sqrt{61}$

4 **a** $-2\mathbf{a} + 2\mathbf{b}$ **b** $-3\mathbf{a} + 2\mathbf{b}$ **c** $-2\mathbf{a} + \mathbf{b}$

5 $\begin{pmatrix}-1\\1\end{pmatrix}$, $\begin{pmatrix}7\\9\end{pmatrix}$ or $\begin{pmatrix}9\\3\end{pmatrix}$

6 **a** $2\mathbf{i} + 8\mathbf{j}$ **b** $2\sqrt{17}$

7 $\dfrac{3\sqrt{5}}{5}$

Challenge
$\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{j}$ or $\overrightarrow{OB} = \frac{46}{13}\mathbf{i} + \frac{9}{13}\mathbf{j}$

Exercise 11E

1 $\overrightarrow{XY} = \mathbf{b} - \mathbf{a}$ and $\overrightarrow{YZ} = \mathbf{c} - \mathbf{b}$, so $\mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{b}$.
Hence $\mathbf{a} + \mathbf{c} = 2\mathbf{b}$.

2 **a** **i** $2\mathbf{r}$ **ii** $\mathbf{r}$
b PQ and OB are parallel, so $\angle APQ = \angle AOB$ and $\angle AQP = \angle ABO$, and $\angle PAQ$ is common to both triangles.

3 **a** $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$
b $\overrightarrow{AN} = \frac{1}{3}(\mathbf{b} - \mathbf{a})$, $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{NB} = \frac{2}{3}(\mathbf{b} - \mathbf{a})$
so $AN : NB = 1 : 2$.

4 **a** $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}$
b $\overrightarrow{AP} = -\mathbf{a} + \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c} = \frac{2}{5}(\mathbf{c} - \mathbf{a})$,
$\overrightarrow{PC} = \mathbf{c} - (\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{c}) = \frac{3}{5}(\mathbf{c} - \mathbf{a})$ so $AP : PC = 2 : 3$

5 **a** $\sqrt{26}$ **b** $2\sqrt{2}$ **c** $3\sqrt{2}$
d $\angle BAC = 56°$, $\angle ABC = 34°$, $\angle ACB = 90°$

6 **a** $\overrightarrow{OR} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$,
$\overrightarrow{OS} = 3\overrightarrow{OR} = 3(\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}) = 2\mathbf{a} + \mathbf{b}$
b $\overrightarrow{TP} = \overrightarrow{TO} + \overrightarrow{OP} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{PS} = \overrightarrow{PO} + \overrightarrow{OS} = -\mathbf{a} + 2\mathbf{a} + \mathbf{b}$
$= \mathbf{a} + \mathbf{b}$
$\overrightarrow{TP}$ is parallel (and equal) to $\overrightarrow{PS}$ and they have a point, P, in common so T, P and S lie on a straight line.

Challenge:
a $\overrightarrow{PR} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{PX} = j(\mathbf{b} - \mathbf{a}) = -j\mathbf{a} + j\mathbf{b}$
b $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}\mathbf{b}$, $\overrightarrow{PX} = -\mathbf{a} + k(\mathbf{a} + \frac{1}{2}\mathbf{b}) = (k - 1)\mathbf{a} + \frac{1}{2}k\mathbf{b}$
c Coefficients of **a** and **b** must be the same in both expressions for $\overrightarrow{PX}$
Coefficients of **a**: $k - 1 = -j$; Coefficients of **b**: $j = \frac{1}{2}k$
d Solving simultaneously gives $j = \frac{1}{3}$ and $k = \frac{2}{3}$
e $\overrightarrow{PX} = \frac{1}{3}\overrightarrow{PR}$.
By symmetry, $\overrightarrow{PX} = \overrightarrow{YR} = \overrightarrow{XY}$, so ON and OM divide PR into 3 equal parts.

Exercise 11F

1 **a** $5\,\mathrm{m\,s^{-1}}$ **b** $25\,\mathrm{km\,h^{-1}}$
c $5.39\,\mathrm{m\,s^{-1}}$ **d** $8.06\,\mathrm{cm\,s^{-1}}$
2 **a** 50 km **b** 51.0 m
c 4.74 km **d** 967 cm
3 **a** $5\,\mathrm{m\,s^{-1}}$, 75 m **b** $5.39\,\mathrm{m\,s^{-1}}$, 16.2 m
c $5.39\,\mathrm{km\,h^{-1}}$, 16.2 km **d** $13\,\mathrm{km\,h^{-1}}$, 6.5 km
4 $(2.8\mathbf{i} - 1.6\mathbf{j})\,\mathrm{m\,s^{-2}}$
5 **a** 54.5° **b** $0.3\sqrt{74} = 2.58$ N
6 **a** 26.6° below **i**
b $\mathbf{R} = (3 + p)\mathbf{i} + (q - 4)\mathbf{j}$, $3 + p = 2\lambda$ and
$q - 4 = -\lambda \Rightarrow \lambda = 4 - q$
$3 + p = 2(4 - q) \Rightarrow 3 + p = 8 - 2q$ so $p + 2q = 5$
c $|\mathbf{R}| = 2\sqrt{5}$ newtons
7 **a** $10\mathbf{i} - 100\mathbf{j}$ metres **b** 109.4°
c $1700\,\mathrm{m^2}$
8 **a** $\sqrt{41} = 6.40$ km **b** 321.3°
c $\overrightarrow{AB} = 4\mathbf{i} - 5\mathbf{j}$, $\mathbf{v} = 2(4\mathbf{i} - 5\mathbf{j})$ so the boat is travelling directly towards the buoy.
d $2\sqrt{41} = 12.8\,\mathrm{km\,h^{-1}}$ **e** 30 minutes

Mixed exercise 11

1 **a** $2\sqrt{10}$ newtons **b** 18° to the left
2 **a** 108° **b** $9.49\,\mathrm{km\,h^{-1}}$
3 **a** $9.85\,\mathrm{m\,s^{-1}}$ **b** 59.1 m
c The model ignores friction and air resistance.
The model will become less accurate as t increases.
4 **a** $\mathbf{b} - \frac{3}{5}\mathbf{a}$ **b** $\mathbf{b} - 4\mathbf{a}$ **c** $\frac{8}{5}\mathbf{a} - \mathbf{b}$ **d** $3\mathbf{a} - \mathbf{b}$
5 1.25
6 **a** $\begin{pmatrix}12\\-1\end{pmatrix}$ **b** $\begin{pmatrix}-18\\5\end{pmatrix}$ **c** $\begin{pmatrix}49\\13\end{pmatrix}$
7 **a** $3\mathbf{i} - 2\mathbf{j}$ **b** 32.5° **c** 10.5
8 **a** $p = -1.5$ **b** $\mathbf{i} - 1.5\mathbf{j}$
9 **a** **i** $\frac{1}{17}(8\mathbf{i} + 15\mathbf{j})$ **ii** 61.9° above
b **i** $\frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$ **ii** 16.3° below
c **i** $\frac{1}{41}(-9\mathbf{i} + 40\mathbf{j})$ **ii** 102.7° above
d **i** $\frac{1}{\sqrt{13}}(3\mathbf{i} - 2\mathbf{j})$ **ii** 33.7° below
10 $p = 8.6$, $q = 12.3$
11 ±6
12 **a** $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ **b** $\frac{2}{5}\mathbf{b}$
c $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{AN} = \frac{2}{5}(\mathbf{b} - \mathbf{a})$ so $AN : NB = 2 : 3$

13 a 18.4° below
b $\mathbf{R} = (4 + p)\mathbf{i} + (-5 + q)\mathbf{j}$, $4 + p = 3\lambda$ and $-5 + q = -\lambda$
$4 + p = 3(q - 5)$ so $p + 3q = 11$
c $2\sqrt{10} = 6.32$ newtons

14 $\dfrac{\sqrt{193}}{2} = 6.95\text{ m s}^{-2}$

Challenge

$\overrightarrow{OB} = \frac{3}{2}\mathbf{i} + \frac{5}{2}\mathbf{j}$ or $\frac{99}{34}\mathbf{i} + \frac{5}{34}\mathbf{j}$

CHAPTER 12

Prior knowledge check

1 a 5 b $-\frac{2}{3}$ c $\frac{1}{3}$
2 a x^{10} b $x^{\frac{2}{3}}$ c x^{-1} d $x^{\frac{3}{4}}$
3 a $y = \frac{1}{2}x - 2$ b $y = -\frac{1}{2}x + 8\frac{1}{2}$ c $y = -\frac{1}{4}x + 7\frac{1}{2}$
4 $y = -\frac{1}{2}x$

Exercise 12A

1 a

x-coordinate	−1	0	1	2	3
Estimate for gradient of curve	−4	−2	0	2	4

b Gradient = $2p - 2$ c 1

2 a $\sqrt{1 - 0.6^2} = \sqrt{0.64} = 0.8$
b Gradient = −0.75
c i −1.21 (3 s.f.) ii −1 iii −0.859 (3 s.f.)
d As other point moves closer to A, gradient tends to −0.75.

3 a i 7 ii 6.5 iii 6.1 iv 6.01 v $h + 6$
b Gradient of tangent = 6

4 a i 9 ii 8.5 iii 8.1 iv 8.01 v $8 + h$
b Gradient of tangent = 8

Exercise 12B

1 a $f'(2) = \lim_{h\to0}\dfrac{f(2+h) - f(2)}{h} = \lim_{h\to0}\dfrac{(2+h)^2 - 2^2}{h}$
$= \lim_{h\to0}\dfrac{4h + h^2}{h} = \lim_{h\to0}(4 + h) = 4$

b $f'(-3) = \lim_{h\to0}\dfrac{f(-3+h) - f(-3)}{h} = \lim_{h\to0}\dfrac{(-3+h)^2 - 3^2}{h}$
$= \lim_{h\to0}\dfrac{-6h + h^2}{h} = \lim_{h\to0}(-6 + h) = -6$

c $f'(0) = \lim_{h\to0}\dfrac{f(0+h) - f(0)}{h} = \lim_{h\to0}\dfrac{h^2 - 0^2}{h} = \lim_{h\to0} h = 0$

d $f'(50) = \lim_{h\to0}\dfrac{f(50+h) - f(50)}{h} = \lim_{h\to0}\dfrac{(50+h)^2 - 50^2}{h}$
$= \lim_{h\to0}\dfrac{100h + h^2}{h} = \lim_{h\to0}(100 + h) = 100$

2 a $f'(x) = \lim_{h\to0}\dfrac{f(x+h) - f(x)}{h} = \lim_{h\to0}\dfrac{(x+h)^2 - x^2}{h}$
$= \lim_{h\to0}\dfrac{2xh + h^2}{h} = \lim_{h\to0}(2x + h)$

b As $h \to 0$, $f'(x) = \lim_{h\to0}(2x + h) = 2x$

3 a $g = \lim_{h\to0}\dfrac{(-2+h)^3 - (-2)^3}{h}$
$= \lim_{h\to0}\dfrac{-8 + 3(-2)^2h + 3(-2)h^2 + h^3 + 8}{h}$
$= \lim_{h\to0}\dfrac{12h - 6h^2 + h^3}{h} = \lim_{h\to0}(12 - 6h + h^2)$
b $g = 12$

4 a Gradient of $AB = \dfrac{(-1+h)^3 - 5(-1+h) - 4}{(-1+h) - (-1)}$
$= \dfrac{-1 + 3h - 3h^2 + h^3 + 5 - 5h - 4}{h}$
$= \dfrac{h^3 - 3h^2 - 2h}{h} = h^2 - 3h - 2$
b gradient = −2

5 $\dfrac{dy}{dx} = \lim_{h\to0}\dfrac{6(x+h) - 6x}{h} = \lim_{h\to0}\dfrac{6h}{h} = 6$

6 $\dfrac{dy}{dx} = \lim_{h\to0}\dfrac{4(x+h)^2 - 4x^2}{h} = \lim_{h\to0}\dfrac{8xh + 4h^2}{h}$
$= \lim_{h\to0}(8x + 4h) = 8x$

7 $\dfrac{dy}{dz} = \lim_{h\to0}\dfrac{a(z+h)^2 - az^2}{h} = \lim_{h\to0}\dfrac{(a-a)z^2 + 2azh + ah^2}{h}$
$= \lim_{h\to0}\dfrac{2azh + ah^2}{h} = \lim_{h\to0}(2az + ah) = 2az$

Challenge

a $f'(x) = \lim_{h\to0}\dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to0}\dfrac{x - (x+h)}{xh(x+h)} = \lim_{h\to0}\dfrac{-1}{x(x+h)}$
$= \lim_{h\to0}\dfrac{-1}{x^2 + xh}$

b $f'(x) = \lim_{h\to0}\dfrac{-1}{x(x+h)} = \dfrac{-1}{x^2 + xh} = \dfrac{-1}{x^2 + 0} = -\dfrac{1}{x^2}$

Exercise 12C

1 a $7x^6$ b $8x^7$ c $4x^3$ d $\frac{1}{3}x^{-\frac{2}{3}}$
e $\frac{1}{4}x^{-\frac{3}{4}}$ f $\frac{1}{3}x^{-\frac{2}{3}}$ g $-3x^{-4}$ h $-4x^{-5}$
i $-2x^{-3}$ j $-5x^{-6}$ k $-\frac{1}{2}x^{-\frac{3}{2}}$ l $-\frac{1}{3}x^{-\frac{4}{3}}$
m $9x^8$ n $5x^4$ o $3x^2$ p $-2x^{-3}$
q 1 r $3x^2$

2 a $6x$ b $54x^8$ c $2x^3$ d $5x^{-\frac{3}{4}}$
e $\frac{15}{2}x^{\frac{1}{4}}$ f $-10x^{-2}$ g $6x^2$ h $-\dfrac{1}{2x^5}$
i $x^{-\frac{3}{2}}$ j $\frac{15}{2}\sqrt{x}$

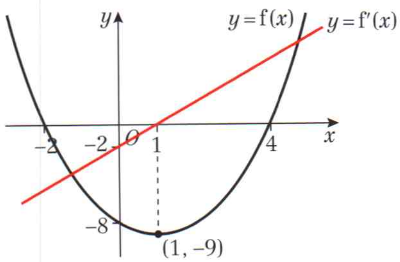
3 a $\frac{3}{4}$ b $\frac{1}{2}$ c 3 d 2

4 $\dfrac{dy}{dx} = \dfrac{3}{2}\sqrt{\dfrac{x}{2}}$

Exercise 12D

1 a $4x - 6$ b $x + 12$ c $8x$ d $16x + 7$
e $4 - 10x$
2 a 12 b 6 c 7 d $2\frac{1}{2}$
e −2 f 4
3 4, 0
4 (−1, −8)
5 −1, 1
6 −4, 6

7 a, b



c At the turning point, the gradient of $y = \mathrm{f}(x)$ is zero, i.e. $\mathrm{f}'(x) = 0$.

Exercise 12E

1 a $4x^3 - x^{-2}$ b $10x^4 - 6x^{-3}$ c $9x^{\frac{1}{2}} - x^{-\frac{3}{2}}$

2 a 0 b $11\frac{1}{2}$

3 a $(2\frac{1}{2}, -6\frac{1}{4})$ b $(4, -4)$ and $(2, 0)$
c $(16, -31)$ d $(\frac{1}{2}, 4)$ $(-\frac{1}{2}, -4)$

4 a $x^{-\frac{1}{2}}$ b $-6x^{-3}$ c $-x^{-4}$
d $\frac{4}{3}x^3 - 2x^2$ e $\frac{1}{2}x^{-\frac{1}{2}} - 6x^{-4}$ f $\frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-2}$
g $-3x^{-2}$ h $3 + 6x^{-2}$ i $5x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}}$
j $3x^2 - 2x + 2$ k $12x^3 + 18x^2$ l $24x - 8 + 2x^{-2}$

5 a 1 b $\frac{2}{9}$ c -4 d 4

6 $-\frac{3}{4}\sqrt{2}$

7 a $512 - 2304x + 4608x^2$
b $\mathrm{f}'(x) \approx \dfrac{\mathrm{d}}{\mathrm{d}x}(512 - 2304x + 4608x^2)$
$= -2304 + 2 \times 4608x$
$= 9216x - 2304$

Exercise 12F

1 a $y + 3x - 6 = 0$ b $4y - 3x - 4 = 0$
c $3y - 2x - 18 = 0$ d $y = x$
e $y = 12x + 14$ f $y = 16x - 22$

2 a $7y + x - 48 = 0$ b $17y + 2x - 212 = 0$

3 $(1\frac{2}{9}, 1\frac{8}{9})$

4 $y = -x$, $4y + x - 9 = 0$; $(-3, 3)$

5 $y = -8x + 10$, $8y - x - 145 = 0$

6 $\left(-\frac{3}{4}, \frac{9}{8}\right)$

Challenge
L has equation $y = 12x - 8$.

Exercise 12G

1 a $x \geqslant -\frac{4}{3}$ b $x \leqslant \frac{2}{3}$ c $x \leqslant -2$
d $x \leqslant 2, x \geqslant 3$ e $x \in \mathbb{R}$ f $x \in \mathbb{R}$
g $x \geqslant 0$ h $x \geqslant 6$

2 a $x \leqslant 4.5$ b $x \geqslant 2.5$
c $x \geqslant -1$ d $-1 \leqslant x \leqslant 2$
e $-3 \leqslant x \leqslant 3$ f $-5 \leqslant x < 0, 0 < x \leqslant 5$
g $0 < x \leqslant 9$ h $-2 \leqslant x \leqslant 0$

3 $\mathrm{f}'(x) = -6x^2 - 3$
$x^2 \geqslant 0$ for all $x \in \mathbb{R}$, so $-6x^2 - 3 \leqslant 0$ for all $x \in \mathbb{R}$.
$\therefore \mathrm{f}(x)$ is decreasing for all $x \in \mathbb{R}$.

4 a Any $p \geqslant 2$
b No. Can be any $p \geqslant 2$.

Exercise 12H

1 a $24x + 3$, 24
b $15 - 3x^{-2}$, $6x^{-3}$
c $\frac{9}{2}x^{-\frac{1}{2}} + 6x^{-3}$, $-\frac{9}{4}x^{-\frac{3}{2}} - 18x^{-4}$
d $30x + 2$, 30
e $-3x^{-2} - 16x^{-3}$, $6x^{-3} + 48x^{-4}$

2 Acceleration $= \frac{3}{4}t^{-\frac{1}{2}} + \frac{3}{2}t^{-\frac{5}{2}}$

3 $\frac{3}{2}$

4 $-\frac{1}{2}$

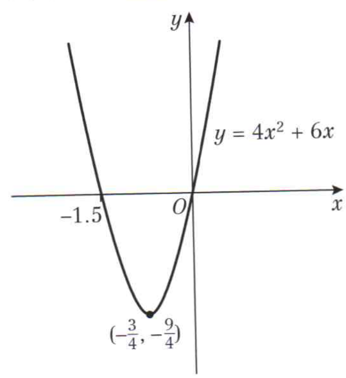
Exercise 12I

1 a -28 b -17 c $-\frac{1}{5}$

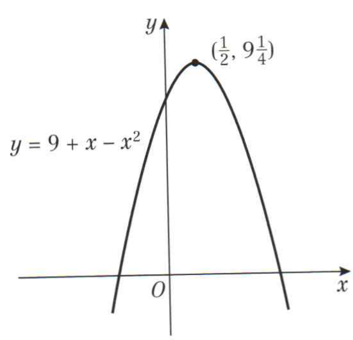
2 a 10 b 4 c 12.25

3 a $\left(-\frac{3}{4}, -\frac{9}{4}\right)$ minimum
b $\left(\frac{1}{2}, 9\frac{1}{4}\right)$ maximum
c $\left(-\frac{1}{3}, 1\frac{5}{27}\right)$ maximum, $(1, 0)$ minimum
d $(3, -18)$ minimum, $\left(-\frac{1}{3}, \frac{14}{27}\right)$ maximum
e $(1, 2)$ minimum, $(-1, -2)$ maximum
f $(3, 27)$ minimum
g $\left(\frac{9}{4}, -\frac{9}{4}\right)$ minimum
h $(2, -4\sqrt{2})$ minimum
i $(\sqrt{6}, -36)$ minimum, $(-\sqrt{6}, -36)$ minimum, $(0, 0)$ maximum

4 a



b



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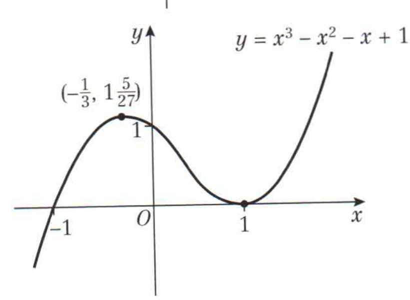

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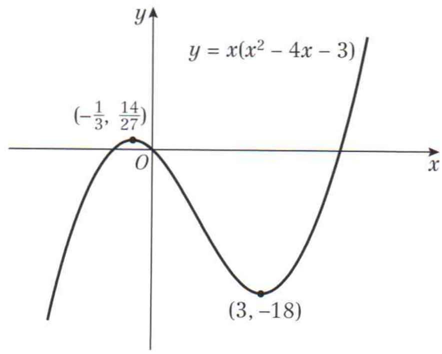
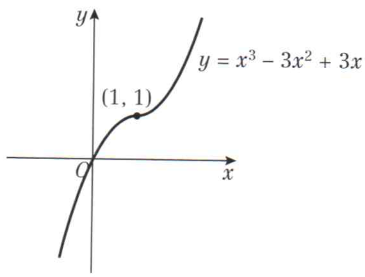


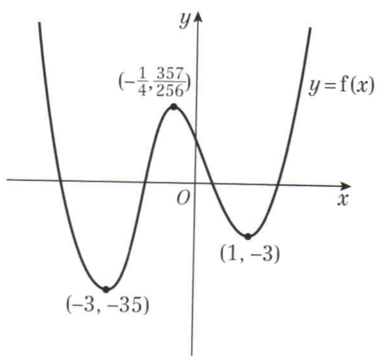
5 (1, 1) inflection (gradient is positive either side of point)



6 Maximum value is 27; $f(x) \leqslant 27$

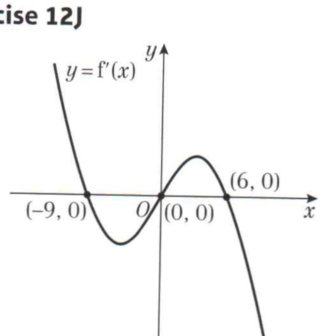
7 **a** (1, −3): minimum, (−3, −35): minimum, $\left(-\frac{1}{4}, \frac{357}{256}\right)$: maximum

b

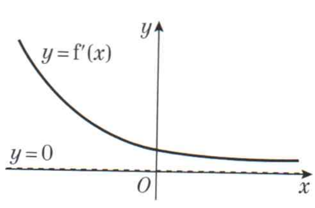


Exercise 12J

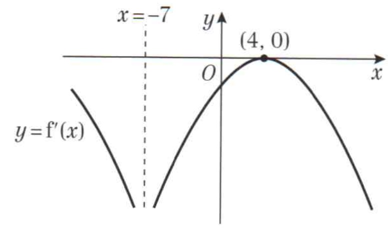
1 **a**



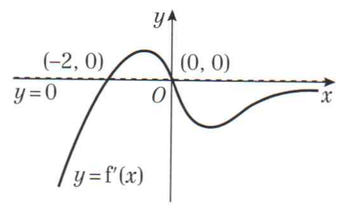
b



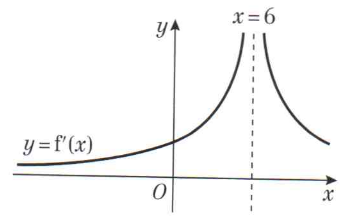
c



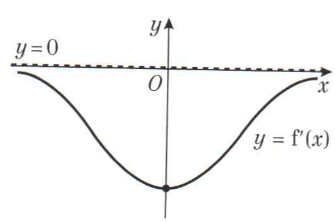
d



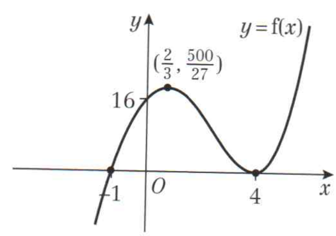
e



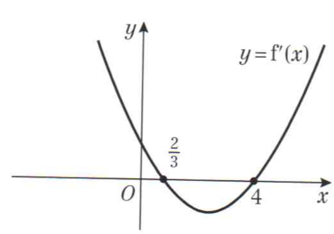
f



2 **a**



b



c $f(x) = x^3 - 7x^2 + 8x + 16$
$f'(x) = 3x^2 - 14x + 8 = (3x - 2)(x - 4)$

d (4, 0), $\left(\frac{2}{3}, 0\right)$ and (0, 8)

Exercise 12K

1 $2t - 3$ 2 2π 3 $-\frac{4}{3}$

4 $48\pi\ \text{cm}^2$ per cm 5 $18\ \text{m s}^{-1}$

6 a Let x = width of garden.
$x + 2y = 80 \Rightarrow x = 80 - 2y$
Area $A = xy = y(80 - 2y)$

b $20\ \text{m} \times 40\ \text{m}$, $800\ \text{m}^2$

7 a $2\pi r^2 + 2\pi rh = 600\pi \Rightarrow h = \frac{300 - r^2}{r}$

$V = \pi r^2 h = \pi r(300 - r^2) = 300\pi r - \pi r^3$

b $2000\pi\ \text{cm}^3$

8 a Let θ = angle of sector.

$\pi r^2 \times \frac{\theta}{360} = 100 \Rightarrow \theta = \frac{36\,000}{\pi r^2}$

$P = 2r + 2\pi r \times \frac{\theta}{360} = 2r + \frac{200\pi r}{\pi r^2}$

$= 2r + \frac{200}{r}$

Area $< \pi r^2$, so $\pi r^2 > 100$

$\therefore r > \sqrt{\frac{100}{\pi}}$

b 40 cm

9 a Let h = height of rectangle.
$P = \pi r + 2r + 2h = 40 \Rightarrow 2h = 40 - 2r - \pi r$

$A = \frac{\pi}{2}r^2 + 2rh = \frac{\pi}{2}r^2 + r(40 - 2r - \pi r)$

$= 40r - 2r^2 - \frac{\pi}{2}r^2$

b $\frac{800}{4 + \pi}\ \text{cm}^2$

10 a $18x + 14y = 1512 \Rightarrow y = \frac{1512 - 18x}{14}$

$A = 12xy = 12x\left(\frac{1512 - 18x}{14}\right)$

$= 1296x - \frac{108x^2}{7}$

b $27\,216\ \text{mm}^2$

Mixed exercise 12

1 $f'(x) = \lim_{h \to 0} \frac{10(x + h)^2 - 10x^2}{h} = \lim_{h \to 0} \frac{20xh + 10h^2}{h}$

$= \lim_{h \to 0} (20x + 10h) = 20x$

2 a y-coordinate of $B = (\delta x)^3 + 3(\delta x)^2 + 6\delta x + 4$

Gradient $= \frac{((\delta x)^3 + 3(\delta x)^2 + 6\delta x + 4) - 4}{(1 + \delta x) - 1}$

$= \frac{(\delta x)^3 + 3(\delta x)^2 + 6\delta x}{(\delta x)} = (\delta x)^2 + 3\delta x + 6$

b 6

3 $4, 11\frac{3}{4}, 17\frac{25}{27}$

4 $2, 2\frac{2}{3}$

5 $(2, -13)$ and $(-2, 15)$

6 a $1 - \frac{9}{x^2}$ b $x = \pm 3$

7 $\frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$

8 a $\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$ b $(4, 16)$

9 a $x + x^{\frac{3}{2}} - x^{-\frac{1}{2}} - 1$ b $1 + \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ c $4\frac{1}{16}$

10 $6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2}$

11 $a = 1, b = -4, c = 5$

12 a $3x^2 - 10x + 5$

b i $\frac{1}{3}$ ii $y = 2x - 7$ iii $\frac{7}{2}\sqrt{5}$

13 $y = 9x - 4$ and $9y + x = 128$

14 a $\left(\frac{4}{5}, -\frac{2}{5}\right)$ b $\frac{1}{5}$

15 P is $(0, -1)$, $\frac{dy}{dx} = 3x^2 - 4x - 4$

Gradient at $P = -4$, so L is $y = -4x - 1$.
$-4x - 1 = x^3 - 2x^2 - 4x - 1 \Rightarrow x^2(x - 2) = 0$
$x = 2 \Rightarrow y = -9$, so Q is $(2, -9)$
Distance $PQ = \sqrt{(2 - 0)^2 + (-9 - (-1))^2} = \sqrt{68} = 2\sqrt{17}$

16 a $x = 4, y = 20$

b $\frac{d^2y}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}} + 96x^{-3}$

At $x = 4$, $\frac{d^2y}{dx^2} = \frac{15}{8} > 0$

(4, 20) is a local minimum.

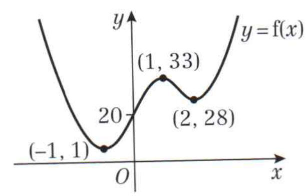
17 $(1, -11)$ and $\left(\frac{7}{3}, -\frac{329}{27}\right)$

18 a $7\frac{31}{32}$

b $f'(x) = \left(x - \frac{1}{x}\right)^2 \geqslant 0$ for all values of x bigger than 0

19 (1, 4)

20 a (1, 33) maximum, (2, 28) and (–1, 1) minimum

b


21 a $\frac{250}{x^2} - 2x$ b $(5, 125)$

22 a $P\left(x, 5 - \frac{1}{2}x^2\right)$

$OP^2 = (x - 0)^2 + \left(5 - \frac{1}{2}x^2 - 0\right)^2$

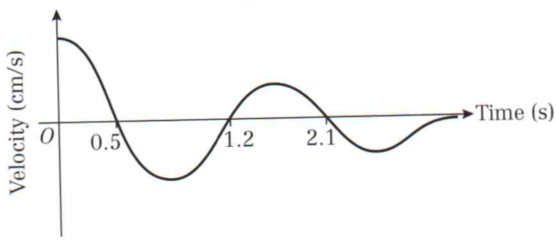
$= \frac{1}{4}x^4 - 4x^2 + 25$

b $x = \pm 2\sqrt{2}$ or $x = 0$

c When $x = \pm 2\sqrt{2}$, $f''(x) \geqslant 0$ so minimum
When $x = \pm 2\sqrt{2}$, $OP^2 = 9$ so $OP = 3$

23 a $3 + 5(3) + 3^2 - 3^3 = 0$ therefore C on curve

b A is $(-1, 0)$; B is $\left(\frac{5}{3}, 9\frac{13}{27}\right)$

24


25 $\frac{10}{3}, \frac{2300\pi}{27}$

26 $\frac{dA}{dx} = 4\pi x - \frac{2000}{x^2}$

$\frac{dA}{dx} = 0$: $4\pi x = \frac{2000}{x^2} \to x^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$

27 a $y = 1 - \frac{x}{2} - \frac{\pi x}{4}$

b $R = xy + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$

$= x\left(1 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8}$

$= x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$

$= \frac{x}{8}(8 - 4x - \pi x)$

c $\frac{2}{4+\pi}\,\text{m}^2$ $(0.280\,\text{m}^2)$

28 a $SA = \pi x^2 + 2\pi x + \pi x^2 + 2\pi x h = 80\pi$

$h = \frac{40 - x - x^2}{x}$

$V = \pi x^2 h = \pi x^2\left(\frac{40 - x - x^2}{x}\right)$

$= \pi(40x - x^2 - x^3)$

b $\frac{10}{3}$ **c** $\frac{\mathrm{d}^2V}{\mathrm{d}x^2} < 0 \therefore$ maximum

d $\frac{2300\pi}{27}$ **e** $22\frac{2}{9}\%$

29 a Length of short sides $= \frac{x}{\sqrt{2}}$

Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2}\left(\frac{x^2}{2}\right) = \frac{1}{4}x^2\,\text{m}^2$

b Let l be length of EF.

$\frac{1}{4}x^2 l = 4000 \Rightarrow l = \frac{16\,000}{x^2}$

$S = 2\left(\frac{1}{4}x^2\right) + \frac{2xl}{\sqrt{2}}$

$= \frac{1}{2}x^2 + \frac{32\,000x}{\sqrt{2}x^2} = \frac{x^2}{2} + \frac{16\,000\sqrt{2}}{x}$

c $x = 20\sqrt{2}$, $S = 1200\,\text{m}^2$ **d** $\frac{\mathrm{d}^2S}{\mathrm{d}x^2} > 0$

Challenge

a $x^7 + 7x^6h + 21x^5h^2 + 35x^4h^3$

b $\frac{\mathrm{d}}{\mathrm{d}x}(x^7) = \lim_{h\to 0}\frac{(x+h)^7 - x^7}{h} = \lim_{h\to 0}\frac{7x^6h + 21x^5h^2 + 35x^4h^3}{h}$

$= \lim_{h\to 0}(7x^6 + 21x^5h + 35x^4h^2) = 7x^6$

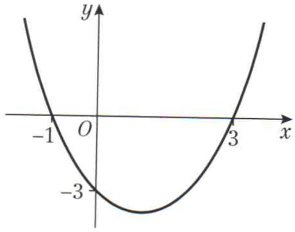
CHAPTER 13

Prior knowledge check

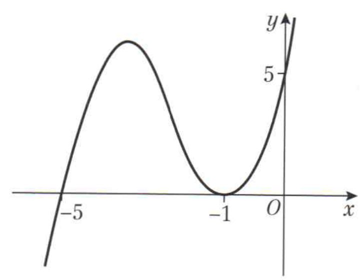
1 a $x^{\frac{5}{2}}$ **b** $2x^{\frac{3}{2}}$ **c** $x^{\frac{5}{2}} - \sqrt{x}$ **d** $x^{-\frac{3}{2}} + 4x$

2 a $6x^2 + 3$ **b** $x - 1$ **c** $3x^2 + 2x$ **d** $-\frac{1}{x^2} - 3x^2$

3 a



b



Exercise 13A

1 a $y = \frac{1}{6}x^6 + c$ **b** $y = 2x^5 + c$
c $y = x^{-1} + c$ **d** $y = 2x^{-2} + c$
e $y = \frac{3}{5}x^{\frac{5}{3}} + c$ **f** $y = \frac{8}{3}x^{\frac{3}{2}} + c$
g $y = -\frac{2}{7}x^7 + c$ **h** $y = 2x^{\frac{1}{2}} + c$
i $y = -10x^{-\frac{1}{2}} + c$ **j** $y = \frac{9}{2}x^{\frac{4}{3}} + c$
k $y = 3x^{12} + c$ **l** $y = 2x^{-7} + c$
m $y = -9x^{\frac{1}{3}} + c$ **n** $y = -5x + c$
o $y = 3x^2 + c$ **p** $y = \frac{10}{3}x^{0.6} + c$

2 a $y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$ **b** $y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$
c $y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$ **d** $y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$
e $y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$ **f** $y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c$

3 a $\mathrm{f}(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$ **b** $\mathrm{f}(x) = x^6 - x^{-6} + x^{-\frac{1}{6}} + c$
c $\mathrm{f}(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$ **d** $\mathrm{f}(x) = 2x^5 - 4x^{-2} + c$
e $\mathrm{f}(x) = 3x^{\frac{2}{3}} - 6x^{-\frac{2}{3}} + c$
f $\mathrm{f}(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$

4 $y = \frac{4x^3}{3} + 6x^2 + 9x + c$

5 $\mathrm{f}(x) = -3x^{-1} + 4x^{\frac{3}{2}} + \frac{x^2}{2} - 4x + c$

Challenge

$y = -\frac{12}{7x^{\frac{7}{2}}} - \frac{4}{5x^{\frac{5}{2}}} + \frac{3}{2x^2} + \frac{1}{x} + c$

Exercise 13B

1 a $\frac{x^4}{4} + c$ **b** $\frac{x^8}{8} + c$
c $-x^{-3} + c$ **d** $\frac{5x^3}{3} + c$

2 a $\frac{1}{5}x^5 + \frac{1}{2}x^4 + c$ **b** $\frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$
c $2x^{\frac{5}{2}} - x^3 + c$

3 a $-4x^{-1} + 6x^{\frac{1}{2}} + c$ **b** $-6x^{-1} - \frac{2}{3}x^{\frac{3}{2}} + c$
c $-4x^{-\frac{1}{2}} + \frac{x^3}{3} - 2x^{\frac{1}{2}} + c$

4 a $x^4 + x^{-3} + rx + c$ **b** $\frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$
c $\frac{px^5}{5} + 2qx - 3x^{-1} + c$

5 a $t^3 + t^{-1} + c$ **b** $\frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$
c $\frac{p}{4}t^4 + q^2t + pr^3t + c$

6 a $x^2 - \frac{3}{x} + c$ **b** $\frac{4}{3}x^3 + 6x^2 + 9x + c$
c $\frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$

7 **a** $\frac{1}{3}x^3 + 2x - \frac{1}{x} + c$ **b** $\frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$
c $2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$

8 **a** $\frac{3}{5}x^{\frac{5}{3}} - \frac{2}{x^2} + c$ **b** $-\frac{1}{x^2} - \frac{1}{x} + 3x + c$
c $\frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$ **d** $\frac{8}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$
e $3x + 2x^{\frac{1}{2}} + 2x^3 + c$ **f** $\frac{2}{5}x^{\frac{5}{2}} + 3x^2 + 6x^{\frac{3}{2}} + c$

9 **a** $-\frac{A}{x} - 3x + c$ **b** $\frac{2}{3}\sqrt{P}\,x^{\frac{3}{2}} - \frac{1}{x^2} + c$
c $-\frac{p}{x} + \frac{2qx^{\frac{3}{2}}}{3} + rx + c$

10 $-\frac{6}{x} + \frac{8x^{\frac{3}{2}}}{3} - \frac{3x^2}{2} + 2x + c$

11 $2x^4 + 3x^2 - 6x^{\frac{1}{2}} + c$

12 **a** $(2 + 5\sqrt{x})^2 = 4 + 10\sqrt{x} + 10\sqrt{x} + 25x = 4 + 20\sqrt{x} + 25x$
b $4x + \frac{40x^{\frac{3}{2}}}{3} + \frac{25x^2}{2} + c$

13 $\frac{x^6}{2} - 8x^{\frac{1}{2}} + c$

14 $p = -4$, $q = -2.5$

15 **a** $1024 - 5120x + 11\,520x^2$
b $1024x - 2560x^2 + 3840x^3 + c$

Exercise 13C

1 **a** $y = x^3 + x^2 - 2$ **b** $y = x^4 - \frac{1}{x^2} + 3x + 1$
c $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + \frac{1}{3}$ **d** $y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$
e $y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$ **f** $y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$

2 $f(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$

3 $y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$

4 $f(x) = 3x^3 + 2x^2 - 3x - 2$

5 $y = 6x^{\frac{1}{2}} - \frac{4x^{\frac{5}{2}}}{5} + \frac{118}{5}$

6 **a** $p = \frac{1}{2}$, $q = 1$ **b** $y = 4x^{\frac{3}{2}} + \frac{5x^2}{2} - \frac{421}{2}$

7 **a** $f(t) = 10t - \frac{5t^2}{2}$ **b** $7\frac{1}{2}$

8 **a** $f(t) = -4.9t^2 + 35$ **b** 23.975 m
c 35 m **d** 2.67 seconds
e e.g. the ground is flat

Challenge

1 $f_2(x) = \frac{x^3}{3}$; $f_4(x) = \frac{x^4}{12}$ **b** $\frac{2x^{n+1}}{(n+1)!}$

2 $f_2(x) = x + 1$; $f_3(x) = \frac{1}{2}x^2 + x + 1$; $f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

Exercise 13D

1 **a** $152\frac{1}{4}$ **b** $48\frac{2}{5}$ **c** $5\frac{1}{3}$ **d** 2
2 **a** $5\frac{1}{4}$ **b** 10 **c** $11\frac{5}{6}$ **d** $60\frac{1}{2}$
3 **a** $16\frac{2}{3}$ **b** $46\frac{1}{2}$ **c** $\frac{11}{14}$ **d** $2\frac{1}{2}$
4 $A = -7$ or 4
5 28
6 $-8 + 8\sqrt{3}$
7 $k = \frac{25}{4}$
8 450 m

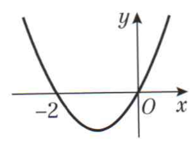
Challenge
$k = 2$

Exercise 13E

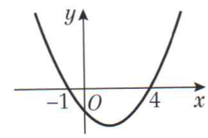
1 **a** 22 **b** $36\frac{2}{3}$ **c** $48\frac{8}{15}$ **d** 6
2 4 **3** 6 **4** $10\frac{2}{3}$
5 $21\frac{1}{3}$ **6** $\frac{4}{81}$ **7** $k = 2$
8 **a** $(-1, 0)$ and $(3, 0)$ **b** $10\frac{2}{3}$
9 $1\frac{1}{3}$

Exercise 13F

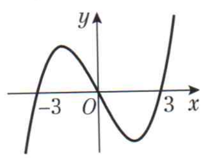
1 **a** $1\frac{1}{3}$



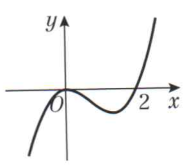
b $20\frac{5}{6}$



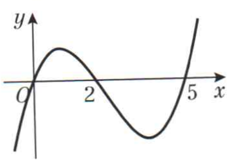
c $40\frac{1}{2}$



d $1\frac{1}{3}$



e $21\frac{1}{12}$



2 **a** $(-3, 0)$ and $(2, 0)$ **b** $21\frac{1}{12}$

3 **a** $f(-3) = 0$
b $f(x) = (x + 3)(-x^2 + 7x - 10)$
c $f(x) = (x + 3)(x - 5)(2 - x)$
d $(-3, 0)$ $(2, 0)$ $(5, 0)$
e $143\frac{5}{6}$

Challenge

1 **a** $4\frac{1}{2}$ **b** 9 **c** $\frac{9a}{2}$ **d** $4\frac{1}{2}$ **e** $\frac{9}{2a}$

2 **a** B has x-coordinate 1
$\int_0^1 (x^3 + x^2 - 2x)\,dx = \left[\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right]_0^1$
$= \frac{1}{4} + \frac{1}{3} - 1 = -\frac{5}{12}$
So area under x-axis is $\frac{5}{12}$
Area above x-axis is
$\left(\frac{1}{4}0^4 + \frac{1}{3}0^3 - 0^2\right) - \left(\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right) = \frac{5}{12}$
So the x-coordinate of a satisfies
$3x^4 + 4x^3 - 12x^2 + 5 = 0$
Then use the factor theorem twice to get
$(x - 1)^2(3x^2 + 10x + 5) = 0$

b A has coordinates $\left(\frac{-5+\sqrt{10}}{3}, \frac{-80+37\sqrt{10}}{27}\right)$

The roots at 1 correspond to point B.

The root $\frac{-5-\sqrt{10}}{3}$ gives a point on the curve to the left of -2 below the x-axis, so cannot be A.

Exercise 13G

1 **a** $A(-2, 6)$, $B(2, 6)$ **b** $10\frac{2}{3}$
2 **a** $A(1, 3)$, $B(3, 3)$ **b** $1\frac{1}{3}$
3 $6\frac{2}{3}$
4 4.5
5 **a** $(2, 12)$ **b** $13\frac{1}{3}$
6 **a** $20\frac{5}{6}$ **b** $17\frac{1}{6}$
7 **a, b** Substitute into equation for y
c $y = x - 4$ **d** $8\frac{3}{5}$
8 $3\frac{3}{8}$
9 **a** Substitute $x = 4$ into both equations
b 7.2
10 **a** $21\frac{1}{3}$ **b** $2\frac{5}{9}$
11 **a** $(-1, 11)$ and $(3, 7)$ **b** $21\frac{1}{3}$

Mixed exercise 13

1 **a** $\frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x + c$ **b** $\frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} + c$
2 $\frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + \frac{1}{6}$
3 **a** $2x^4 - 2x^3 + 5x + c$ **b** $2x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$
4 $\frac{4}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$
5 $x = \frac{1}{3}t^3 + t^2 + t - 8\frac{2}{3}$; $x = 12\frac{1}{3}$
6 **a** $A = 6$, $B = 9$ **b** $\frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$
7 **a** $\frac{9}{2}x^{-\frac{1}{2}} - 8x^{-\frac{3}{2}}$ **b** $6x^{\frac{3}{2}} + 32x^{\frac{1}{2}} - 24x + c$
8 $a = 4$, $b = -3.5$
9 25.9 m
10 **a** $f(t) = 5t + t^2$ **b** 7.8 seconds
11 **a** $-1, 3$ **b** $10\frac{2}{3}$
12 **a** $-\frac{2x^{\frac{3}{2}}}{3} + 5x - 8\sqrt{x} + c$ **b** $\frac{7}{3}$
13 **a** $(3, 0)$ **b** $(1, 4)$ **c** $6\frac{3}{4}$
14 **a** $\frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$ **b** $2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$ **c** $A = 6$, $B = -2$
15 **a** $\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{-\frac{1}{2}}(4 - x)$
b $(4, 16)$ **c** 133 (3 sf)
16 **a** $(6, 12)$ **b** $13\frac{1}{3}$
17 **a** $A(1, 0)$, $B(5, 0)$, $C(6, 5)$ **b** $10\frac{1}{6}$
18 **a** $q = -2$ **b** $C(6, 17)$ **c** $1\frac{1}{3}$
19 $-\frac{9}{x} - \frac{16x^{\frac{3}{2}}}{3} + 2x^2 - 5x + c$
20 $A = -6$ or 1
21 **a** $f'(x) = \frac{(2 - x^2)(4 - 4x^2 + x^4)}{x^2} = 8x^{-2} - 12 + 6x^2 - x^4$
b $f''(x) = -16x^{-3} + 12x - 4x^3$
c $f(x) = -\frac{8}{x} - 12x + 2x^3 - \frac{x^5}{5} - \frac{47}{5}$
22 **a** $(-3, 0)$ and $(\frac{1}{2}, 0)$ **b** $14\frac{7}{24}$
23 **a** $(-\frac{3}{2}, 0)$ and $(4, 0)$ **b** $55\frac{11}{24}$
24 **a** -2 and 3 **b** $21\frac{1}{12}$

Challenge

$10\frac{5}{12}$

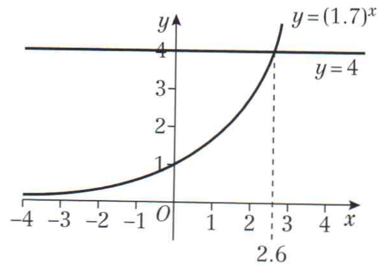
CHAPTER 14

Prior knowledge check

1 **a** 125 **b** $\frac{1}{3}$ **c** 32 **d** 49 **e** 1
2 **a** 6^6 **b** y^{21} **c** 2^6 **d** x^4
3 gradient 1.5, intercept 4.1

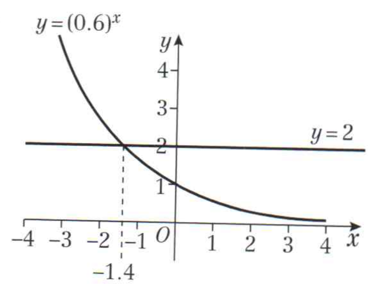
Exercise 14A

1 **a**



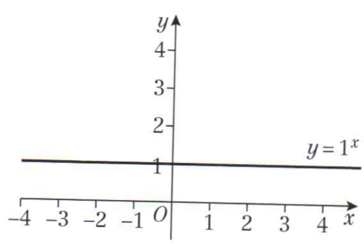
b $x \approx 2.6$

2 **a**



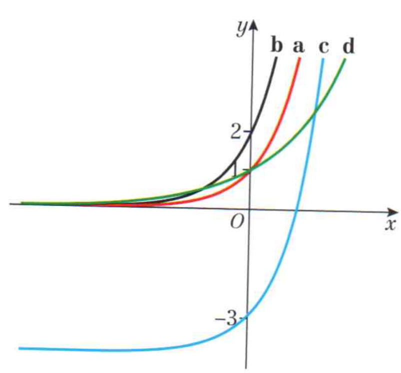
b $x \approx -1.4$

3



4 **a** True, because $a^0 = 1$ whenever a is non-zero.
b False, for example when $a = \frac{1}{2}$
c True, because when a is positive, $a^x > 0$ for all values of x

5



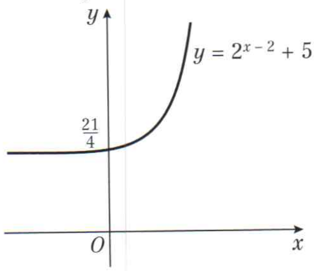
Asymptotes **a** $y = 0$ **b** $y = 0$ **c** $y = -4$ **d** $y = 0$

6 $k = 3, a = 2$

7 **a** As x increases, y decreases

b $p = 1.2, q = 0.2$

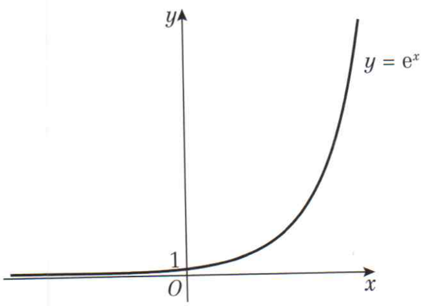
Challenge



Exercise 14B

1 **a** 2.718 28 **b** 54.598 15 **c** 0.000 04 **d** 1.221 40

2 **a**

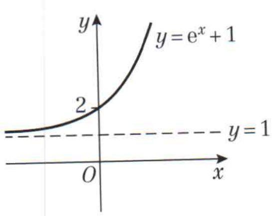


b Student's own answers

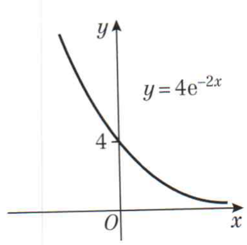
c $e = 2.71828...$

$e^3 = 20.08553...$

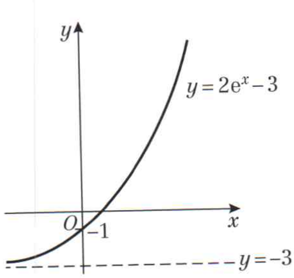
3 **a**



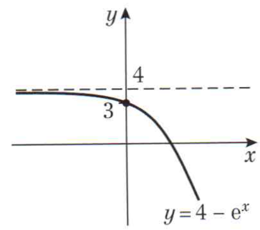
b



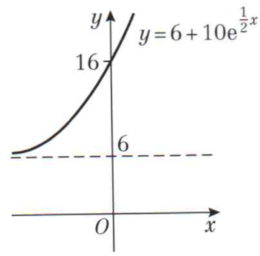
c



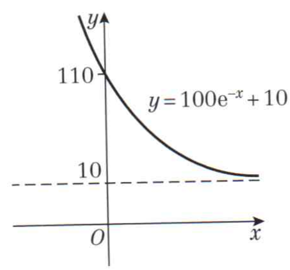
d



e



f

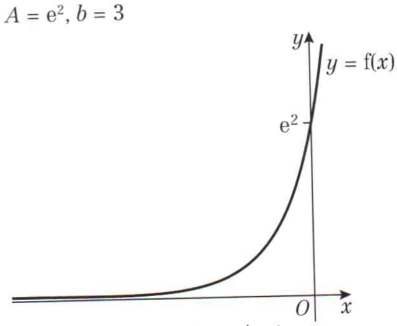


4 **a** $A = 1, C = 5$, b is positive

b $A = 4, C = 0$, b is negative

c $A = 6, C = 2$, b is positive

5 $A = e^2, b = 3$



6 **a** $6e^{6x}$ **b** $-\frac{1}{3}e^{-\frac{1}{3}x}$ **c** $14e^{2x}$

d $2e^{0.4x}$ **e** $3e^{3x} + 2e^x$ **f** $2e^{2x} + e^x$

7 **a** $3e^6$ **b** 3 **c** $3e^{-1.5}$

8 $f'(x) = 0.2e^{0.2x}$

The gradient of the tangent when $x = 5$ is

$f'(5) = 0.2e^1 = 0.2e$.

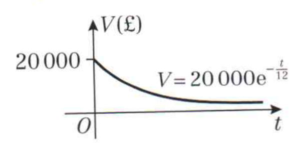
The equation of the tangent is therefore $y = (0.2e)x + c$.

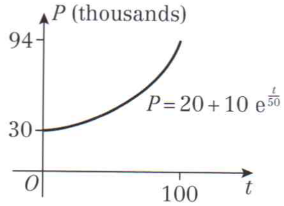
At $(5, e)$, $e = 0.2e \times 5 + c$, so $c = 0$ and when $x = 0, y = 0$.

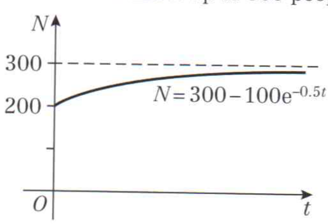
Exercise 14C

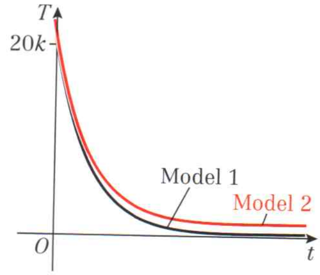
1 **a** £20 000 **b** £14 331

c



2 a 30 000 b 38 221
c


d Model predicts population of the country to be over 200 million, this is highly unlikely and by 2500 new factors are likely to affect population growth. Model not valid for predictions that far into the future.
3 a 200
b Disease will infect up to 300 people.
c


4 a i 15 rabbits ii 132 rabbits
b The initial number of rabbits
c $\frac{dR}{dm} = 2.4\,e^{0.2m}$
When $m = 6$, $\frac{dR}{dm} = 7.97 \approx 8$
d The rabbits may begin to run out of food or space
5 a 0.565 bars
b $\frac{dp}{dh} = -0.13e^{-0.13h} = -0.13p$, $k = -0.13$
c The atmospheric pressure decreases exponentially as the altitude increases
d 12%
6 a Model 1: £15 733
Model 2: £15 723 Similar results
b Model 1: £1814
Model 2: £2484 Model 2 predicts a larger value
c


d In Model 2 the tractor will always be worth at least £1000. This could be the value of the tractor as scrap metal.

Exercise 14D

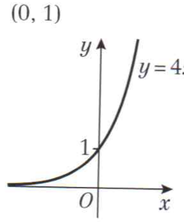
1 a $\log_4 256 = 4$ b $\log_3 \frac{1}{9} = -2$
c $\log_{10} 1\,000\,000 = 6$ d $\log_{11} 11 = 1$
e $\log_{0.2} 0.008 = 3$
2 a $2^4 = 16$ b $5^2 = 25$
c $9^{\frac{1}{2}} = 3$ d $5^{-1} = 0.2$
e $10^5 = 100\,000$
3 a 3 b 2 c 7 d 1
e 6 f $\frac{1}{2}$ g −1 h −2
i 10 j −2
4 a 625 b 9 c 7 d 9
e 20 f 2
5 a 2.475 b 2.173 c 3.009 d 1.099
6 a $5 = \log_2 32 < \log_2 50 < \log_2 64 = 6$
b 5.644
7 a i 1 ii 1 iii 1 b $a^1 \equiv a$
8 a i 0 ii 0 iii 0 b $a^0 \equiv 1$

Exercise 14E

1 a $\log_2 21$ b $\log_2 9$ c $\log_5 80$
d $\log_6\left(\frac{64}{81}\right)$ e $\log_{10} 120$
2 a $\log_2 8 = 3$ b $\log_6 36 = 2$ c $\log_{12} 144 = 2$
d $\log_8 2 = \frac{1}{3}$ e $\log_{10} \frac{1}{10} = -1$
3 a $3\log_a x + 4\log_a y + \log_a z$
b $5\log_a x - 2\log_a y$
c $2 + 2\log_a x$
d $\log_a x - \frac{1}{2}\log_a y - \log_a z$
e $\frac{1}{2} + \frac{1}{2}\log_a x$
4 a $\frac{4}{3}$ b $\frac{1}{18}$ c $\sqrt{30}$ d 2
5 a $\log_3(x + 1) - 2\log_3(x - 1) = 1$
$\log_3\left(\frac{x + 1}{(x - 1)^2}\right) = 1$
$\frac{x + 1}{(x - 1)^2} = 3$
$x + 1 = 3(x-1)^2$
$x + 1 = 3(x^2 - 2x + 1)$
$3x^2 - 7x + 2 = 0$
b $x = 2$
6 $a = 9, b = 4$

Challenge
$\log_a x = m$ and $\log_a y = n$
$x = a^m$ and $y = a^n$
$x \div y = a^m \div a^n = a^{m-n}$
$\log_a\left(\frac{x}{y}\right) = m - n = \log_a x - \log_a y$

Exercise 14F

1 a 6.23 b 2.10 c 0.431
d 1.66 e −3.22 f 1.31
g 1.25 h −1.73
2 a 0, 2.32 b 1.26, 2.18 c 1.21
d 0.631 e 0.565, 0.712 f 0
g 2 h −1
3 a 5.92 b 3.2
4 a (0, 1)


b $\frac{1}{2}, \frac{3}{2}$
5 a 0.7565 b 7.9248 c 0.2966

Exercise 14G

1 **a** $\ln 6$ **b** $\frac{1}{2}\ln 11$ **c** $3 - \ln 20$
d $\frac{1}{4}\ln\left(\frac{1}{3}\right)$ **e** $\frac{1}{2}\ln 3 - 3$ **f** $5 - \ln 19$

2 **a** e^2 **b** $\frac{e}{4}$ **c** $\frac{1}{2}e^4 - \frac{3}{2}$
d $\frac{1}{6}(e^{\frac{5}{2}} + 2)$ **e** $18 - e^{\frac{1}{2}}$ **f** 2, 5

3 **a** $\ln 2, \ln 6$ **b** $\frac{1}{2}\ln 2, 0$ **c** e^3, e^{-5}
d $\ln 4, 0$ **e** $\ln 5, \ln\left(\frac{1}{3}\right)$ **f** e^6, e^{-2}

4 $\ln 3, 2\ln 2$

5 **a** $\frac{1}{8}(e^2 + 3)$ **b** $\frac{1}{5}(\ln 3 + 40)$ **c** $\frac{1}{5}\ln 7, 0$
d e^3, e^{-1}

6 $\dfrac{1 + \ln 5}{4 + \ln 3}$

7 **a** The initial concentration of the drug in mg/l
b 4.91 mg/l
c $3 = 6e^{-\frac{t}{10}}$
$\frac{1}{2} = e^{-\frac{t}{10}}$
$\ln\left(\frac{1}{2}\right) = -\frac{t}{10}$
$t = -10\ln\left(\frac{1}{2}\right) = 6.931\ldots$ = 6 hours 56 minutes

8 **a** $(0, 3 + \ln 4)$ **b** $(4 - e^{-3}, 0)$

9 **a** $V = 27\,000e^{-0.0811t}$
b According to the model, when $t = 8$, $V = 14\,100$ (3 s.f.) so model is reliable.

10 **a** $P = 7.6 + 0.225t$ **b** $P = 7.6e^{0.0233t}$
c When $t = 50$, linear model predicts 18.85 million people, and exponential model predicts 24.4 million people. Exponential model is best supported by the given fact.

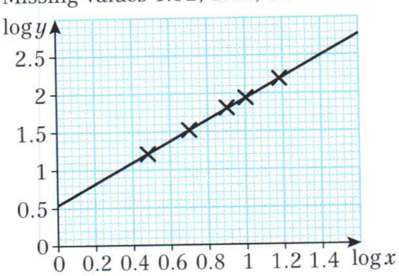
Challenge
As $y = 2$ is an asymptote, $C = 2$.
Substituting (0, 5) gives $5 = Ae^0 + 2$, so A is 3.
Substituting (6, 10) gives $10 = 3e^{6B} + 2$.
Rearranging this gives $B = \frac{1}{6}\ln\left(\frac{8}{3}\right)$.

Exercise 14H

1 **a** $\log S = \log(4 \times 7^x)$
$\log S = \log 4 + \log 7^x$
$\log S = \log 4 + x\log 7$
b gradient $\log 7$, intercept $\log 4$

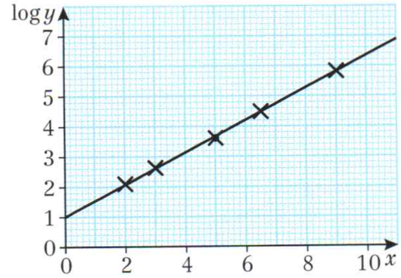
2 **a** $\log A = \log(6x^4)$
$\log A = \log 6 + \log x^4$
$\log A = \log 6 + 4\log x$
b gradient 4, intercept $\log 6$

3 **a** Missing values 1.52, 1.81, 1.94
b



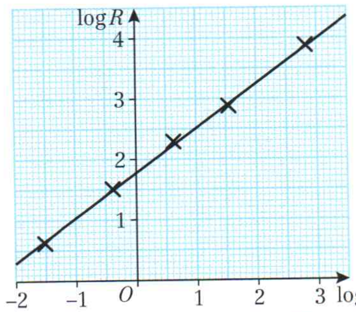
c Approximately $a = 3.5$, $n = 1.4$

4 **a** Missing values 2.63, 3.61, 4.49, 5.82
b



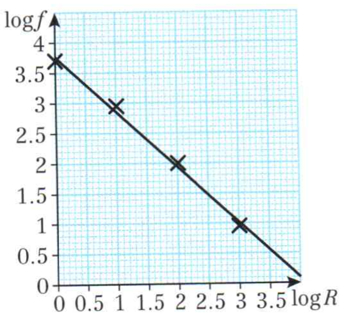
c Approximately $b = 3.4$, $a = 10.4$

5 **a** Missing values −0.39, 0.62, 1.54, 2.81
b



c Approximately $a = 60$, $b = 0.75$
d Approximately 1,600 kcal per day (2 s.f.)

6 **a** Missing values 2.94, 1.96, 0.95
b



c Approximately $A = 5800$, $b = -0.9$
d Approximately 690 times

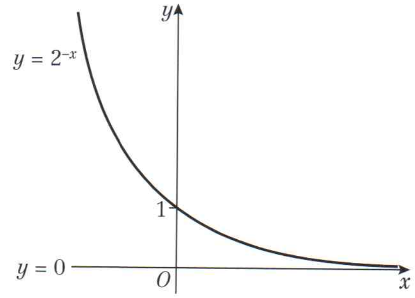
7 **a** $\log N = 0.095t + 1.6$
b $a = 40$, $b = 1.2$
c The initial number of sick people
d 9500 people. After 30 days people may start to recover, or the disease may stop spreading as quickly.

8 **a** $\log A = 2\log w - 0.1049$
b $q = 2$, $p = 0.7854$
c Circles: p is approximately one quarter π, and the width is twice the radius, so $A = \frac{\pi}{4}w^2 = \frac{\pi}{4}(2r)^2 = \pi r^2$.

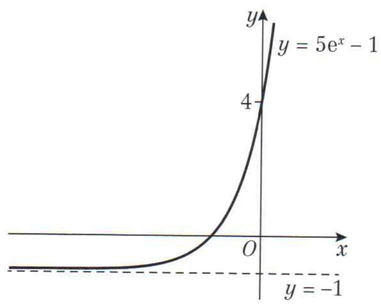
Challenge
$y = 5.8 \times 0.9^x$

Mixed exercise 14

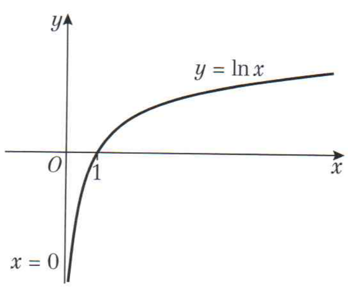
1 a



b



c



2 a $2\log_a p + \log_a q$ **b** $\log_a p = 4$, $\log_a q = 1$

3 a $\frac{1}{4}p$ **b** $\frac{3}{4}p + 1$

4 a 2.26 **b** 1.27 **c** 7.02

5 a $4^x - 2^{x+1} - 15 = 0$

$2^{2x} - 2 \times 2^x - 15 = 0$

$(2^x)^2 - 2 \times 2^x - 15 = 0$

$u^2 - 2u - 15 = 0$

b 2.32

6 $x = 6$

7 a $-e^{-x}$ **b** $11e^{11x}$ **c** $30e^{5x}$

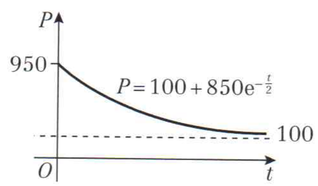
8 a $\dfrac{e^8 + 5}{2}$ **b** $\dfrac{\ln 5}{4}$ **c** $-\frac{1}{2}\ln 14$ **d** $\dfrac{3 + \sqrt{13}}{2}$

e 0 **f** $\dfrac{e^4}{2}$

9 a £950 **b** £290

c After 4.28 years **d** £100

e



f A good model. The computer will always be worth something

10 a $y = \left(\dfrac{2}{\ln 4}\right)x$

b (0, 0) satisfies the equation of the line.

c 2.43

11 a We cannot go backwards in time

b 75°C

c 5 minutes

d The exponential term will always be positive, so the overall temperature will be greater than 20°C.

12 a $V = aS^b$

$\log V = \log(aS^b)$

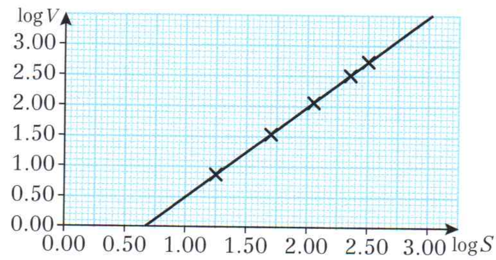
$\log V = \log a + \log(S^b)$

$\log V = \log a + b\log S$

b

$\log S$	1.26	1.70	2.05	2.35	2.50
$\log V$	0.86	1.53	2.05	2.49	2.72

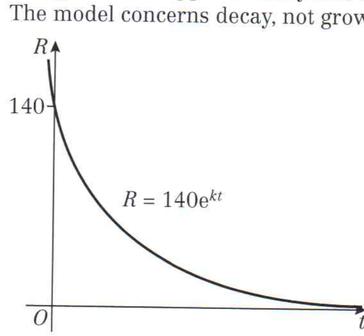
c



d The gradient is approximately 1.5; $a \approx 0.09$

13 a The model concerns decay, not growth

b



c $70 = 140e^{30k}$

$\frac{1}{2} = e^{30k}$

$\ln\left(\frac{1}{2}\right) = 30k$

$k = \frac{1}{30}\ln\left(\frac{1}{2}\right)$

$k = -\frac{1}{30}\ln(2)$, so $c = -\frac{1}{30}$

14 a 6.4 million views

b $\dfrac{dV}{dx} = 0.4e^{0.4x}$

c 9.42×10^{16} new views per day

d This is too big, so the model is not valid after 100 days

15 a 4.2

b i 1.12×10^{25} dyne cm

ii 3.55×10^{26} dyne cm

c divide **b ii** by **b i**

16 a They exponentiated the two terms on LHS separately rather than combining them first.

b $x = 2 + 2\sqrt{2}$

17 $x = 2$, $x = \log_3 2$ or 0.631 (3 s.f.)

Challenge

a $y = 9^x = 3^{2x}$, $\log_3(y) = 2x$
b $y = 9^x$, $x = \log_9 y$, $\log_3 y = 2\log_9 y = \log_9 y^2$
c $x = -\frac{1}{3}$ or $x = -2$

Review exercise 3

1 -4.5
2 $\sqrt{7}$
3 **a** All equal to $\sqrt{145}$
b $(x-1)^2 + (y+3)^2 = 145$
4 **a** $-2\mathbf{i} - 8\mathbf{j}$
b $|\overrightarrow{AB}| = |\overrightarrow{AC}| = \sqrt{85}$
c $|\overrightarrow{BC}| = \sqrt{68}$

$\cos\angle ABC = \dfrac{85 + 68 - 85}{2 \times \sqrt{85} \times \sqrt{68}} = \dfrac{1}{\sqrt{5}}$

5 12
6 **a** 5N **b** 7
7 $m = 50\sqrt{3} + 30$, $n = 50$
8 **a** $\sqrt{(-75)^2 + 180^2} = 195 > 150 = \sqrt{90^2 + 120^2}$
b Boat A: 6.5 m/s; Boat B: 5 m/s; Both boats arrive at the same time – it is a tie.
9 $\lim\limits_{h\to 0} \dfrac{5(x+h)^2 - 5x^2}{h}$

$= \lim\limits_{h\to 0} \dfrac{5x^2 + 10xh + 5h^2 - 5x^2}{h}$

$= \lim\limits_{h\to 0} \dfrac{10xh + 5h^2}{h}$

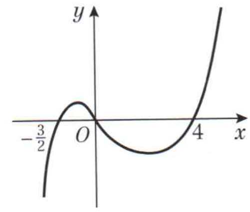
$= \lim\limits_{h\to 0} 10x + 5h$

$= 10x$

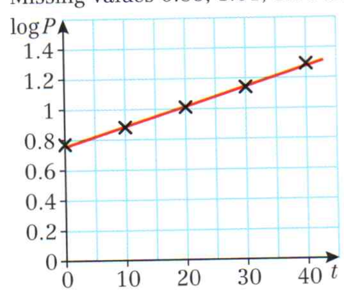
10 $\dfrac{dy}{dx} = 12x^2 + x^{-\frac{1}{2}}$
11 **a** $\dfrac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$
b Substitute $x = 4$ into equation for C
c Gradient of tangent $= -3$ so gradient of normal $= \frac{1}{3}$
Substitute (4, 8) into $y = \frac{1}{3}x + c$
Rearrange $y = \frac{1}{3}x + \frac{20}{3}$
d $PQ = 8\sqrt{10}$
12 **a** $\dfrac{dy}{dx} = 8x - 5x^{-2}$, at P this is 3
b $y = 3x + 5$
c $k = -\frac{5}{3}$
13 **a** $P = 2, Q = 9, R = 4$
b $3x^{\frac{1}{2}} + \frac{9}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}$
c When $x = 1$, $f'(x) = 5\frac{1}{2}$, gradient of $2y = 11x + 3$ is $5\frac{1}{2}$, so it is parallel with tangent
14 $f'(x) = 3x^2 - 24x + 48 = 3(x-4)^2 \geqslant 0$
15 **a** $A(1,0)$ and $B(2,0)$
b $(\sqrt{2}, 2\sqrt{2} - 3)$
16 **a** $V = \pi r^2 h = 128\pi$, so $h = \dfrac{128}{r^2}$

$S = 2\pi rh + 2\pi r^2 = \dfrac{256\pi}{r} + 2\pi r^2$

b 96π cm^2
17 **a** $\dfrac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$
b $\dfrac{d^2y}{dx^2} = 6 - x^{-\frac{3}{2}}$
c $x^3 + \frac{8}{3}x^{\frac{3}{2}} + c$
18 **a** $2x^3 - 5x^2 - 12x$
b $2x^3 - 5x^2 - 12x$
$= x(2x^2 - 5x - 12)$
$= x(2x+3)(x-4)$
c



19 $6\frac{3}{4}$
20 4
21 **a** $-x^4 + 3x^2 + 4 = (-x^2 + 4)(x^2 + 1)$; $x^2 + 1 = 0$ has no real solutions; so solutions are $A(-2, 0)$, $B(2, 0)$
b 19.2
22 $4\frac{1}{2}$
23 **a** $P(-1, 4)$, $Q(2, 1)$
b 4.5
24 **a** $k = -1$, $A(0, 2)$
b ln 3
25 **a** 425 °C **b** 7.49 minutes **c** 1.64 °C/minute
d The temperature can never go below 25 °C, so cannot reach 20 °C.
e $T = 410e^{-0.05t} + 15$, $t \geqslant 0$
26 **a** -0.179 **b** $x = 15$
27 **a** $x = 1.55$ **b** $x = 4$ or $x = \frac{1}{2}$
28 **a** $\log_p 2$ **b** 0.125
29 **a** $x = 2$ **b** $x = \ln 3$ or $x = \ln 1 = 0$
30 **a** Missing values 0.88, 1.01, 1.14 and 1.29
b



c $P = ab^t$
$\log P = \log(ab^t) = \log a + t\log b$
This is a linear relationship. The gradient is $\log b$ and the intercept is $\log a$.
d $a = 5.8$, $b = 1.0$
31 **a** $\log 2x - \log y = \log(x+y)$

$\log\dfrac{2x}{y} = \log(x+y)$

$\dfrac{2x}{y} = x + y$

$2x = xy + y^2$

$2x - xy = y^2$

$x(2 - y) = y^2$

$x = \dfrac{y^2}{2-y}$

b $0 < y < 2$
$y > 0$ given. $x > 0$ also given, and $y^2 > 0$, so $2 - y$ must be > 0. Hence $y < 2$. Note strict inequality because denominator cannot be 0.

Challenge

1 a 0
b 1

2 a $f'(-3) = f'(2) = 0$, so $f'(x) = k(x + 3)(x - 2)$ $= k(x^2 + x - 6)$; there are no other factors as $f(x)$ is cubic
b $2x^3 + 3x^2 - 36x - 5$

3 51.2

4 a $f(0) = 0^3 - k(0) + 1 = 1$; $g(0) = e^{2(0)} = 1$; $P(0, 1)$
b $\frac{1}{2}$

Exam-style practice

1 a $\frac{1}{3}$ **b** $5\sqrt{2}$

2 $y = \frac{2}{3}x + \frac{8}{3}$

3 a error 1: $= -\frac{3}{\sqrt{x}} = -3x^{-\frac{1}{2}}$, not $-3x^{\frac{1}{2}}$

error 2: $\left[\frac{x^5}{5} - 2x^{\frac{3}{2}} + 2x\right]_1^2 = \left(\frac{32}{5} - 2\sqrt{8} + 4\right) - \left(\frac{1}{5} - 2 + 2\right)$

not $\left(\frac{1}{5} - 2 + 2\right) - \left(\frac{32}{5} - 2\sqrt{8} + 4\right)$

b 5.71 (3 s.f.)

4 $x = 30°, 90°, 150°$

5 a $2x^2(x + 3)$
b $2x^2(x + 3) = 980 \Rightarrow 2x^3 + 6x^2 - 980 = 0$ $\Rightarrow x^3 + 3x^2 - 490 = 0$
c For $f(x) = x^3 + 3x^2 - 490 = 0$, $f(7) = 0$, so $x - 7$ is a factor of $f(x)$ and $x = 7$ is a solution.
d Equation becomes $(x - 7)(x^2 + 10x + 70) = 0$
Quadratic has discriminant $10^2 - 4 \times 1 \times 70 = -180$
So the quadratic has no real roots, and the equation has no more real solutions.

6 $x + 15y + 106 = 0$

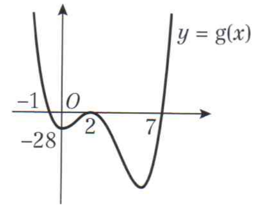
7 a $\log_{10} P = 0.01t + 2$
b 100, initial population
c 1.023
d Accept answers from 195 to 200

8 $1 + \cos^4 x - \sin^4 x \equiv 1 + (\cos^2 x + \sin^2 x)(\cos^2 - \sin^2 x)$
$\equiv (1 - \sin^2 x) + \cos^2 x \equiv 2\cos^2 x$

9 Magnitude $= \sqrt{29}$, angle $= 112°$ (3 s.f.)

10 a 56.5° (3 s.f.) **b** £49.63

11 a



b −4, −1, 4

12 $x = 3$ or $-\frac{5}{3}$

13 a $1 - 15x + 90x^2$
b 0.859
c Greater: The next term will be subtracting from this, and future positive terms will be smaller.

14 a $f(x) = \int(x^{-\frac{3}{2}} - 1 - x^{-2})dx = -2x^{-\frac{1}{2}} - x + x^{-1} + c$
$= -\frac{x^2 + 2\sqrt{x} - 1}{x} + c$
b $c = \frac{5}{3} + \frac{2}{3}\sqrt{3}$

15 a $5^2 + 1^2 - 4 \times 5 + 6 \times 1 = 12$, so $(5, 1)$ lies on C.
Centre $= (2, -3)$, radius $= 5$
b $y = -\frac{3}{4}x + \frac{19}{4}$
c $12\frac{15}{32}$

Online Full worked solutions are available in SolutionBank.

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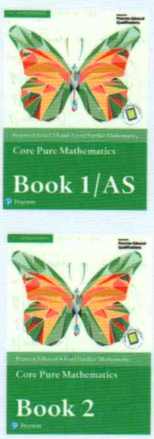

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