

# Exam-style practice

## Mathematics

### AS Level

### Paper 1: Pure Mathematics

Time: 2 hours

You must have: Mathematical Formulae and Statistical Tables, Calculator

- 1 a Given that  $4 = 64^n$ , find the value of  $n$ . (1)  
b Write  $\sqrt{50}$  in the form  $k\sqrt{2}$  where  $k$  is an integer to be determined. (1)
- 2 Find the equation of the line parallel to  $2x - 3y + 4 = 0$  that passes through the point (5, 6). Give your answer in the form  $y = ax + b$  where  $a$  and  $b$  are rational numbers. (3)

- 3 A student is asked to evaluate the integral  $\int_1^2 \left( x^4 - \frac{3}{\sqrt{x}} + 2 \right) dx$

The student's working is shown below

$$\begin{aligned} \int_1^2 \left( x^4 - \frac{3}{\sqrt{x}} + 2 \right) dx &= \int_1^2 (x^4 - 3x^{\frac{1}{2}} + 2) dx \\ &= \left[ \frac{x^5}{5} - 2x^{\frac{3}{2}} + 2x \right]_1^2 \\ &= \left( \frac{1}{5} - 2 + 2 \right) - \left( \frac{32}{5} - 2\sqrt{8} + 4 \right) \\ &= -4.54 \text{ (3 s.f.)} \end{aligned}$$

- a Identify two errors made by the student. (2)  
b Evaluate the definite integral, giving your answer correct to 3 significant figures. (2)
- 4 Find all the solutions in the interval  $0 \leq x \leq 180^\circ$  of  $2\sin^2(2x) - \cos(2x) - 1 = 0$  giving each solution in degrees. (7)

- 5 A rectangular box has sides measuring  $x$  cm,  $x + 3$  cm and  $2x$  cm.

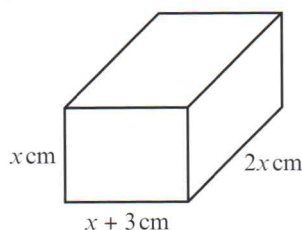


Figure 1

- a Write down an expression for the volume of the box. (1)

Given that the volume of the box is  $980 \text{ cm}^3$ ,

- b Show that  $x^3 + 3x^2 - 490 = 0$ . (2)

- c Show that  $x = 7$  is a solution to this equation. (1)

- d Prove that the equation has no other real solutions. (4)

6  $f(x) = x^3 - 5x^2 - 2 + \frac{1}{x^2}$

The point  $P$  with  $x$ -coordinate  $-1$  lies on the curve  $y = f(x)$ . Find the equation of the normal to the curve at  $P$ , giving your answer in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are positive integers. (7)

- 7 The population,  $P$ , of a colony of endangered Caledonian owl-nightjars can be modelled by the equation  $P = ab^t$  where  $a$  and  $b$  are constants and  $t$  is the time, in months, since the population was first recorded.

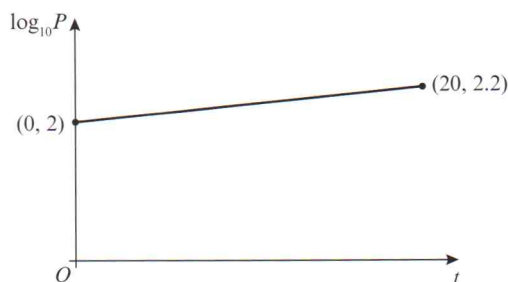


Figure 2

The line  $l$  shown in figure 2 shows the relationship between  $t$  and  $\log_{10} P$  for the population over a period of 20 years.

- a Write down an equation of line  $l$ . (3)

- b Work out the value of  $a$  and interpret this value in the context of the model. (3)

- c Work out the value of  $b$ , giving your answer correct to 3 decimal places. (2)

- d Find the population predicted by the model when  $t = 30$ . (1)

- 8 Prove that  $1 + \cos^4 x - \sin^4 x \equiv 2\cos^2 x$ . (4)

- 9 Relative to a fixed origin, point  $A$  has position vector  $6\mathbf{i} - 3\mathbf{j}$  and point  $B$  has position vector  $4\mathbf{i} + 2\mathbf{j}$ .

Find the magnitude of the vector  $\overrightarrow{AB}$  and the angle it makes with the unit vector  $\mathbf{i}$ . (5)

- 10 A triangular lawn  $ABC$  is shown in figure 3:

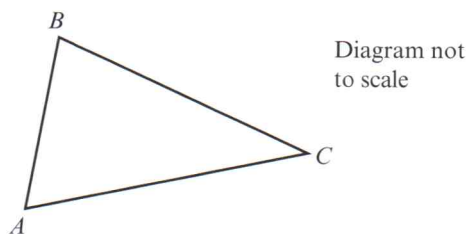


Figure 3

Given that  $AB = 7.5$  m,  $BC = 10.6$  m and  $AC = 12.7$  m,

- a Find angle  $BAC$ . (3)

Grass seed costs £1.25 per square metre.

- b Find the cost of seeding the whole lawn. (5)

11  $g(x) = (x - 2)^2(x + 1)(x - 7)$

- a Sketch the curve  $y = g(x)$ , showing the coordinates of any points where the curve meets or cuts the coordinate axes. (4)

- b Write down the roots of the equation  $g(x + 3) = 0$ . (1)

12 Given that  $9^{2x} = 27^{x^2-5}$ , find the possible values of  $x$ . (6)

13  $f(x) = (1 - 3x)^5$

- a Expand  $f(x)$ , in ascending powers of  $x$ , up to the term in  $x^2$ . Give each term in its simplest form. (3)

- b Hence find an approximate value for  $0.97^5$ . (2)

- c State, with a reason, whether your approximation is greater or smaller than the true value. (2)

14  $f'(x) = \frac{\sqrt{x} - x^2 - 1}{x^2}$ ,  $x > 0$

- a Show that  $f(x)$  can be written as  $f(x) = -\frac{x^2 + 2\sqrt{x} - 1}{x} + c$  where  $c$  is a constant. (5)

Given that  $f(x)$  passes through the point  $(3, -1)$ ,

- b find the value of  $c$ . Give your answer in the form  $p + q\sqrt{r}$  where  $p$ ,  $q$  and  $r$  are rational numbers to be found. (4)

15 A circle,  $C$ , has equation  $x^2 + y^2 - 4x + 6y = 12$

- a Show that the point  $A(5, 1)$  lies on  $C$  and find the centre and radius of the circle. (5)

- b Find the equation of the tangent to  $C$  at point  $A$ . Give your answer in the form  $y = ax + b$  where  $a$  and  $b$  are rational numbers. (4)

- c The curve  $y = x^2 - 2$  intersects this tangent at points  $P$  and  $Q$ . Given that  $O$  is the origin, find, as a fraction in simplest form, the exact area of the triangle  $POQ$ . (7)