3 Review exercise

E **1** The vector $9\mathbf{i} + q\mathbf{j}$ is parallel to the vector $2\mathbf{i} - \mathbf{j}$. Find the value of the constant q. **(2)**

← Section 11.2

E/P **2** Given that $|5\mathbf{i} - k\mathbf{j}| = |2k\mathbf{i} + 2\mathbf{j}|$, find the exact value of the positive constant k. **(4)**

← Section 11.3

E/P **3** Given the four points $X(9, 6)$, $Y(13, -2)$, $Z(0, -15)$, and $C(1, -3)$,

a Show that $|\overrightarrow{CX}| = |\overrightarrow{CY}| = |\overrightarrow{CZ}|$. **(3)**

b Using your answer to part **a** or otherwise, find the equation of the circle which passes through the points X, Y and Z. **(3)**

← Sections 6.2, 11.4

E/P **4** In the triangle ABC, $\overrightarrow{AB} = 9\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{AC} = 7\mathbf{i} - 6\mathbf{j}$.

a Find $\overrightarrow{BC}$. **(2)**

b Prove that the triangle ABC is isosceles. **(3)**

c Show that $\cos \angle ABC = \frac{1}{\sqrt{5}}$ **(4)**

← Sections 9.1, 11.5

E/P **5** The vectors **a**, **b** and **c** are given as $\mathbf{a} = \begin{pmatrix} 8 \\ 23 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -15 \\ x \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -13 \\ 2 \end{pmatrix}$, where x is an integer. Given that $\mathbf{a} + \mathbf{b}$ is parallel to $\mathbf{b} - \mathbf{c}$, find the value of x. **(4)**

← Section 11.2

E **6** Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle.

$\mathbf{F}_1 = 2\mathbf{i} - 5\mathbf{j}$ newtons

$\mathbf{F}_2 = \mathbf{i} + \mathbf{j}$ newtons

The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$.

a Calculate the magnitude of $\mathbf{R}$ in newtons. **(3)**

A third force, $\mathbf{F}_3$ begins to act on the particle, where $\mathbf{F}_3 = k\mathbf{j}$ newtons and k is a positive constant. The new resultant force is given by $\mathbf{R}_{new} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$.

b Given that the angle between the line of action of $\mathbf{R}_{new}$ and the vector $\mathbf{i}$ is 45 degrees, find the value of k. **(3)**

← Section 11.6

E/P **7** A helicopter takes off from its starting position O and travels 100 km on a bearing of 060°. It then travels 30 km due east before landing at point A. Given that the position vector of A relative to O is $(m\mathbf{i} + n\mathbf{j})$ km, find the exact values of m and n. **(4)**

← Sections 10.2, 11.6

E/P **8** At the very end of a race, Boat A has a position vector of $(-65\mathbf{i} + 180\mathbf{j})$ m and Boat B has a position vector of $(100\mathbf{i} + 120\mathbf{j})$ m. The finish line has a position vector of $10\mathbf{i}$ km.

a Show that Boat B is closer to the finish line than Boat A. **(2)**

Boat A is travelling at a constant velocity of $(2.5\mathbf{i} - 6\mathbf{j})$ m/s and Boat B is travelling at a constant velocity of $(-3\mathbf{i} - 4\mathbf{j})$ m/s.

b Calculate the speed of each boat. Hence, or otherwise, determine the result of the race. **(4)**

← Section 11.6

E/P **9** Prove, from first principles, that the derivative of $5x^2$ is $10x$. **(4)**

← Section 12.2

10 Given that $y = 4x^3 - 1 + 2x^{\frac{1}{2}}, x > 0$, find $\frac{dy}{dx}$. **(2)**

← Section 12.5

11 The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, x > 0$.

a Find an expression for $\frac{dy}{dx}$ **(2)**

b Show that the point $P(4, 8)$ lies on C. **(1)**

c Show that an equation of the normal to C at point P is $3y = x + 20$. **(2)**

The normal to C at P cuts the x-axis at point Q.

d Find the length PQ, giving your answer in simplified surd form. **(2)**

← Section 12.6

12 The curve C has equation $y = 4x^2 + \frac{5 - x}{x}, x \neq 0$. The point P on C has x-coordinate 1.

a Show that the value of $\frac{dy}{dx}$ at P is 3. **(3)**

b Find an equation of the tangent to C at P. **(3)**

This tangent meets the x-axis at the point $(k, 0)$.

c Find the value of k. **(1)**

← Section 12.6

13 $f(x) = \frac{(2x + 1)(x + 4)}{\sqrt{x}}, \quad x > 0$.

a Show that $f(x)$ can be written in the form $Px^{\frac{3}{2}} + Qx^{\frac{1}{2}} + Rx^{-\frac{1}{2}}$, stating the values of the constants P, Q and R. **(2)**

b Find $f'(x)$. **(3)**

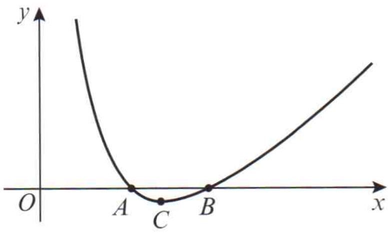
c A curve has equation $y = f(x)$. Show that the tangent to the curve at the point where $x = 1$ is parallel to the line with equation $2y = 11x + 3$. **(3)**

← Section 12.6

14 Prove that the function $f(x) = x^3 - 12x^2 + 48x$ is increasing for all $x \in \mathbb{R}$. **(3)**

← Section 12.7

E/P **15** The diagram shows part of the curve with equation $y = x + \frac{2}{x} - 3$. The curve crosses the x-axis at A and B and the point C is the minimum point of the curve.



a Find the coordinates of A and B. **(2)**

b Find the exact coordinates of C, giving your answers in surd form. **(4)**

← Section 12.9

E/P **16** A company makes solid cylinders of variable radius r cm and constant volume 128π cm^3.

a Show that the surface area of the cylinder is given by $S = \frac{256\pi}{r} + 2\pi r^2$. **(2)**

b Find the minimum value for the surface area of the cylinder. **(4)**

← Section 12.11

E **17** Given that $y = 3x^2 + 4\sqrt{x}, x > 0$, find

a $\frac{dy}{dx}$ **(2)**

b $\frac{d^2y}{dx^2}$ **(2)**

c $\int y\,dx$ **(3)**

← Sections 12.8, 13.2

E **18** The curve C with equation $y = f(x)$ passes through the point $(5, 65)$.
Given that $f'(x) = 6x^2 - 10x - 12$,

a use integration to find $f(x)$ **(3)**

b hence show that $f(x) = x(2x + 3)(x - 4)$ **(2)**

c sketch C, showing the coordinates of the points where C crosses the x-axis. **(3)**

← Sections 4.1, 13.3

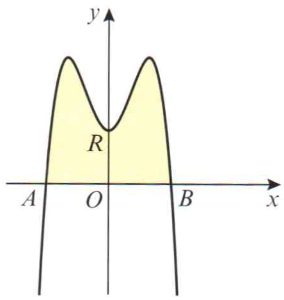
19 Use calculus to evaluate $\int_1^8 (x^{\frac{1}{3}} - x^{-\frac{1}{3}})\,dx$.

← Section 13.4

E/P **20** Given that $\int_0^6 (x^2 - kx)\,dx = 0$, find the value of the constant k. **(3)**

← Section 13.4

E/P **21** The diagram shows a section of the curve with equation $y = -x^4 + 3x^2 + 4$. The curve intersects the x-axis at points A and B. The finite region R, which is shown shaded, is bounded by the curve and the x-axis.

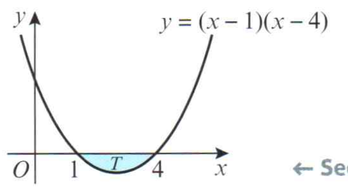


a Show that the equation $-x^4 + 3x^2 + 4 = 0$ only has two solutions, and hence or otherwise find the coordinates of A and B. **(3)**

b Find the area of the region R. **(4)**

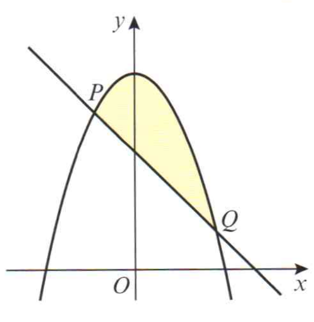
← Sections 4.2, 13.5

E **22** The diagram shows the shaded region T which is bounded by the curve $y = (x - 1)(x - 4)$ and the x-axis.
Find the area of the shaded region T. **(4)**



← Section 13.6

E/P **23** The diagram shows the curve with equation $y = 5 - x^2$ and the line with equation $y = 3 - x$. The curve and the line intersect at the points P and Q.

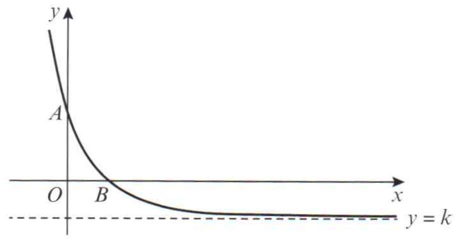


a Find the coordinates of P and Q. **(3)**

b Find the area of the finite region between PQ and the curve. **(6)**

← Section 13.7

E **24** The graph of the function $f(x) = 3e^{-x} - 1$, $x \in \mathbb{R}$, has an asymptote $y = k$, and crosses the x and y axes at A and B respectively, as shown in the diagram.



a Write down the value of k and the y-coordinate of A. **(2)**

b Find the exact value of the x-coordinate of B, giving your answer as simply as possible. **(2)**

← Sections 14.2, 14.7

E/P **25** A heated metal ball S is dropped into a liquid. As S cools, its temperature, T°C, t minutes after it enters the liquid, is modelled by

$T = 400e^{-0.05t} + 25, \quad t \geqslant 0.$

a Find the temperature of S as it enters the liquid. **(1)**

b Find how long S is in the liquid before its temperature drops to 300 °C. Give your answer to 3 significant figures. **(3)**

c Find the rate, $\frac{dT}{dt}$, in °C per minute to 3 significant figures, at which the temperature of S is decreasing at the instant $t = 50$. **(3)**

d With reference to the equation given above, explain why the temperature of S can never drop to 20 °C. **(2)**

The above model is found to be initially accurate, but the minimum temperature of S after a long period of time is found to be 15 °C.

e Use this information to suggest a refinement to the original equation. **(2)**

← Sections 14.3, 14.7

E **26** **a** Find, to 3 significant figures, the value of x for which $5^x = 0.75$. **(2)**

b Solve the equation $2\log_5 x - \log_5 3x = 1$ **(3)**

← Sections 14.5, 14.6

E **27** **a** Solve $3^{2x-1} = 10$, giving your answer to 3 significant figures. **(3)**

b Solve $\log_2 x + \log_2(9 - 2x) = 2$ **(3)**

← Sections 14.5, 14.6

P **28** **a** Express $\log_p 12 - (\frac{1}{2}\log_p 9 + \frac{1}{3}\log_p 8)$ as a single logarithm to base p. **(3)**

b Find the value of x in $\log_4 x = -1.5$. **(2)**

← Sections 14.4, 14.5

P **29** Find the exact solutions to the equations

a $\ln x + \ln 3 = \ln 6$ **(2)**

b $e^x + 3e^{-x} = 4$ **(4)**

← Section 14.7

30 The table below shows the population of Angola between 1970 and 2010.

Year	Population, P (millions)
1970	5.93
1980	7.64
1990	10.33
2000	13.92
2010	19.55

This data can be modelled using an exponential function of the form $P = ab^t$, where t is the time in years since 1970 and a and b are constants.

a Copy and complete the table below, giving your answers to 2 decimal places. **(1)**

Time in years since 1970, t	$\log P$
0	0.77
10	
20	
30	
40	

b Plot a graph of $\log P$ against t using the values from your table and draw in a line of best fit. **(2)**

c By rearranging $P = ab^t$, explain how the graph you have just drawn supports the assumed model. **(3)**

d Use your graph to estimate the values of a and b to two significant figures. **(4)**

← Section 14.8

E/P **31** Given that $x, y > 0$ satisfy the equation $\log 2 + \log x = \log y + \log(x + y)$,

a show that $x = \dfrac{y^2}{2 - y}$ **(3)**

b state the full restriction on the value of y, and justify your answer. **(2)**

Challenge

1 The position vector of a moving object is given by $(\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$, where $0 \leqslant \theta \leqslant 90°$.

a Find the value of θ when the object has a bearing of 090° from the origin.

b Calculate the magnitude of the position vector. ← Sections 10.2, 10.3, 11.3, 11.4

2 The graph of the cubic function $y = f(x)$ has turning points at $(-3, 76)$ and $(2, -49)$.

a Show that $f'(x) = k(x^2 + x - 6)$, where k is a constant.

b Express $f(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are real constants to be found. ← Sections 12.9, 13.3

3 Given that $\int_0^9 f(x)\,dx = 24.2$, state the value of $\int_0^9 (f(x) + 3)\,dx$. ← Sections 4.5, 13.5

4 The functions f and g are defined as $f(x) = x^3 - kx + 1$, where k is a constant, and $g(x) = e^{2x}$, $x \in \mathbb{R}$. The graphs of $y = f(x)$ and $y = g(x)$ intersect at the point P, where $x = 0$.

a Confirm that $f(0) = g(0)$ and hence state the coordinates of P.

b Given that the tangents to the graphs at P are perpendicular, find the value of k.

← Sections 5.3, 14.3