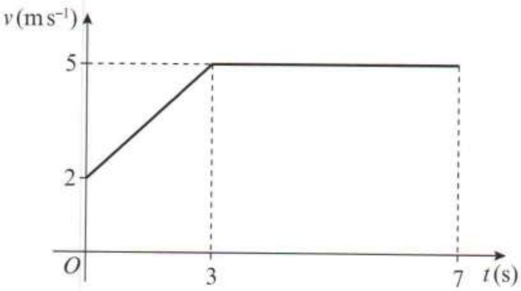
Review exercise 2

(E) **1**



The figure shows the velocity–time graph of a cyclist moving on a straight road over a 7 s period. The sections of the graph from $t = 0$ to $t = 3$, and from $t = 3$ to $t = 7$, are straight lines, The section from $t = 3$ to $t = 7$ is parallel to the t-axis.

State what can be deduced about the motion of the cyclist from the fact that:

a the graph from $t = 0$ to $t = 3$ is a straight line **(1)**

b the graph from $t = 3$ to $t = 7$ is parallel to the t-axis. **(1)**

c Find the distance travelled by the cyclist during this 7 s period. **(4)**

← Section 9.2

2 A train stops at two stations 7.5 km apart. Between the stations it takes 75 s to accelerate uniformly to a speed 24 m s^{-1}, then travels at this speed for a time T seconds before decelerating uniformly for the final 0.6 km.

a Draw a velocity–time graph to illustrate this journey. **(3)**

Hence, or otherwise, find:

b the deceleration of the train during the final 0.6 km **(3)**

c the value of T **(5)**

d the total time for the journey. **(4)**

← Sections 9.2, 9.3

(E/P) **3** An electric train starts from rest at a station A and moves along a straight level track. The train accelerates uniformly at 0.4 m s^{-2} to a speed of 16 m s^{-1}. The speed is then maintained for a distance of 2000 m. Finally the train retards uniformly for 20 s before coming to rest at a station B. For this journey from A to B,

a find the total time taken **(5)**

b find the distance from A to B **(5)**

c sketch the displacement–time graph, showing clearly the shape of the graph for each stage of the journey. **(3)**

← Sections 9.1, 9.3

(E) **4** A small ball is projected vertically upwards from a point A. The greatest height reached by the ball is 40 m above A. Calculate:

a the speed of projection **(3)**

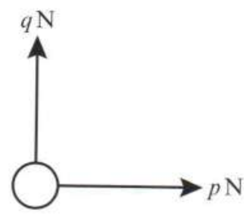
b the time between the instant that the ball is projected and the instant it returns to A. **(3)**

← Sections 9.4, 9.5

(E/P) **5** A ball is projected vertically upwards and takes 3 seconds to reach its highest point. At time t seconds, the ball is 39.2 m above its point of projection. Find the possible values of t. **(5)**

← Sections 9.4, 9.5

(E/P) **6** A light object is acted upon by a horizontal force of p N and a vertical force of q N as shown in the diagram.



The resultant of the two forces has a magnitude of $\sqrt{40}$ N which acts in the direction of 30° to the horizontal. Calculate the value of p and the value of q.

← Sections 8.4, 10.1, 10.2

(E) **7** A car of mass 750 kg, moving along a level straight road, has its speed reduced from $25\,\text{m s}^{-1}$ to $15\,\text{m s}^{-1}$ by brakes which produce a constant retarding force of 2250 N. Calculate the distance travelled by the car as its speed is reduced from $25\,\text{m s}^{-2}$ to $15\,\text{m s}^{-1}$. **(5)**

← Sections 9.3, 10.3

(E) **8** An engine of mass 25 tonnes pulls a truck of mass 10 tonnes along a railway line. The resistances to the motion of the engine and the truck are modelled as constant and of magnitude 50 N per tonne. When the train is travelling horizontally, the tractive force exerted by the engine is 26 kN. Modelling the engine and the truck as particles and the coupling between the engine and the truck as a light horizontal rod calculate:

a the acceleration of the engine and the truck **(4)**

b the tension in the coupling. **(3)**

c State how in your calculations you have used the information that

i the engine and the truck are particles

ii the coupling is a light horizontal rod. **(2)**

← Sections 8.1, 8.2, 10.3, 10.5

(E/P) **9** A ball is projected vertically upwards with a speed $u\,\text{m s}^{-1}$ from a point A, which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above A.

a Show that $u = 22.4$. **(3)**

The ball reaches the ground T seconds after it has been projected from A.

b Find, to three significant figures, the value of T. **(3)**

The ground is soft and the ball sinks 2.5 cm into the ground before coming to rest. The mass of the ball is 0.6 kg. The ground is assumed to exert a constant resistive force of magnitude F newtons.

c Find, to three significant figures, the value of F. **(4)**

d Sketch a velocity–time graph for the entire motion of the ball, showing the values of t at any points where the graph intercepts the horizontal axis. **(4)**

e State one physical factor which could be taken into account to make the model used in this question more realistic. **(1)**

← Sections 8.1, 8.2, 9.5, 10.3, 10.4

(E) **10** A particle A, of mass 0.8 kg, resting on a smooth horizontal table, is connected to a particle B, of mass 0.6 kg, which is 1 m from the ground, by a light inextensible string passing over a small smooth pulley at the edge of the table. The particle A is more than 1 m from the edge of the table. The system is released from rest with the horizontal part of the string perpendicular to the edge of the table, the hanging parts vertical and the string taut.

Calculate:

a the acceleration of A **(5)**

b the tension in the string **(1)**

c the speed of B when it hits the ground **(3)**

d the time taken for B to reach the ground. **(3)**

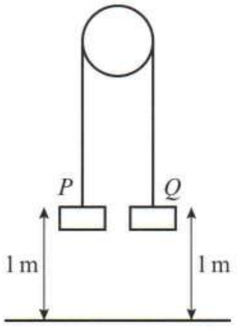
e The string in this question is described as being 'light'.

i Write down what you understand by this description.

ii State how you have used the fact that the string is light in your answer to parts **a** and **b**. **(2)**

← Sections 8.1, 8.2, 9.5, 10.4, 10.6

(E/P) **11** Two particles P and Q have mass 0.6 kg and 0.2 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor. The particles are released from rest and in the subsequent motion Q does not reach the pulley.



a Find the tension in the string immediately after the particles are released. **(6)**

b Find the acceleration of P immediately after the particles are released. **(2)**

When the particles have been moving for 0.4 s, the string breaks.

c Find the further time that elapses until P hits the floor. **(9)**

d State how in your calculations you have used the information that the string is inextensible. **(1)**

← Sections 9.5, 10.4, 10.6

E **12** A trailer of mass 600 kg is attached to a car of mass 900 kg by means of a light inextensible tow-bar. The car tows the trailer along a horizontal road. The resistances to motion of the car and trailer are 300 N and 150 N respectively.

a Given that the acceleration of the car and trailer is $0.4\,\text{m}\,\text{s}^{-2}$, calculate:

i the tractive force exerted by the engine of the car

ii the tension in the tow-bar. **(6)**

b Given that the magnitude of the force in the tow-bar must not exceed 1650 N, calculate the greatest possible deceleration of the car. **(3)**

← Sections 10.1, 10.3, 10.5

E **13** A boy sits on a box in a lift. The mass of the boy is 45 kg, the mass of the box is 20 kg and the mass of the lift is 1050 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of $2\,\text{m}\,\text{s}^{-2}$. By modelling the cable as being light and inextensible, find:

a the tension in the cable **(3)**

b the magnitude of the force exerted on the box by the boy **(3)**

c the magnitude of the force exerted on the box by the lift. **(3)**

← Sections 8.1, 8.2, 10.1, 10.4, 10.5

E/P **14** Two forces $\mathbf{F}_1 = (2\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})$ N, where λ and μ are scalars, act on a particle. The resultant of the two forces is $\mathbf{R}$, where $\mathbf{R}$ is parallel to the vector $\mathbf{i} + 2\mathbf{j}$.

a Find, to the nearest degree, the acute angle between the line of action of $\mathbf{R}$ and the vector $\mathbf{i}$. **(2)**

b Show that $2\lambda - \mu + 1 = 0$. **(5)**

Given that the direction of $\mathbf{F}_2$ is parallel to $\mathbf{j}$,

c find, to three significant figures the magnitude of $\mathbf{R}$. **(4)**

← Sections 8.4, 10.2

E/P **15** A force $\mathbf{R}$ acts on a particle, where $\mathbf{R} = (7\mathbf{i} + 16\mathbf{j})$ N.

Calculate:

a the magnitude of $\mathbf{R}$, giving your answers to one decimal place **(2)**

b the angle between the line of action of $\mathbf{R}$ and $\mathbf{i}$, giving your answer to the nearest degree. **(2)**

The force $\mathbf{R}$ is the resultant of two forces $\mathbf{P}$ and $\mathbf{Q}$. The line of action of $\mathbf{P}$ is parallel to the vector $(\mathbf{i} + 4\mathbf{j})$ and the line of action of $\mathbf{Q}$ is parallel to the vector $(\mathbf{i} + \mathbf{j})$.

c Determine the forces $\mathbf{P}$ and $\mathbf{Q}$ expressing each in terms of $\mathbf{i}$ and $\mathbf{j}$. **(6)**

← Sections 8.4, 10.2

E/P **16** A particle P moves on the x-axis. At time t seconds, its acceleration is $(5 - 2t)\,\text{m s}^{-2}$, measured in the direction of x increasing. When $t = 0$, its velocity is $6\,\text{m s}^{-1}$ measured in the direction of x increasing. Find the time when P is instantaneously at rest in the subsequent motion. **(5)**

← Sections 11.1, 11.4

E/P **17** At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is $v\,\text{m s}^{-1}$, where $v = 6t - 2t^2$. Find:

a the maximum value of v **(4)**

b the time taken for P to return to O. **(5)**

← Sections 11.1, 11.2, 11.3, 11.4

E/P **18** A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(3t^2 - 8t + 5)\,\text{m s}^{-1}$. When $t = 0$, P is 12 m from the origin O. Find:

a the values of t when P is instantaneously at rest **(3)**

b the acceleration of P when $t = 4$ **(3)**

c the total distance travelled by P in the third second. **(4)**

← Sections 11.1, 11.2, 11.3, 11.4

E **19** A particle moves in a straight line and at time t seconds has velocity $v\,\text{m s}^{-1}$, where

$v = 6t - 2t^{\frac{3}{2}}, t \geqslant 0$

a Find an expression for the acceleration of the particle at time t. **(2)**

When $t = 0$, the particle is at the origin.

b Find an expression for the displacement of the particle from the origin at time t. **(4)**

← Sections 11.1, 11.2, 11.3, 11.4

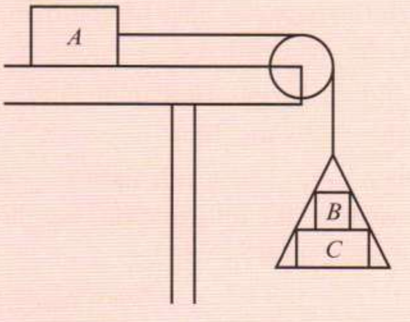
Challenge

1 A tram starts from rest at station A and accelerates uniformly for t_1 seconds covering a distance of 1750 m. It then travels at a constant speed $v\,\text{m s}^{-1}$ for t_2 seconds covering a distance of 17 500 m. The tram then decelerates for t_3 seconds and comes to rest at station B. Given that the total time for the journey is 7 minutes and $3t_1 = 4t_3$, find t_1, t_2 and t_3 and the distance between station A and station B.

← Section 9.3

2 *[In this question use $g = 10\,\text{m s}^{-2}$]*

One end of a light inextensible string is attached to a block A of mass 5 kg. The block A is held at rest on a rough horizontal table. The motion of the block is subject to a resistance of 2 N. The string lies parallel to the table and passes over a smooth light pulley which is fixed at the top of the table. The other end of the string is attached to a light scale pan which carries two blocks B and C, with block B on top of block C as shown. The mass of block B is 5 kg and the mass of block C is 10 kg.



The scale pan hangs at rest and the system is released from rest. By modelling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find:

a the acceleration of the scale pan

b the tension in the string

c the magnitude of the force exerted on block B by block C

d the magnitude of the force exerted on the pulley by the string.

e State how in your calculations you have used the information that the string is inextensible.

← Sections 10.5, 10.6