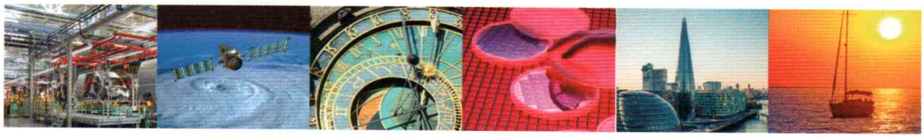
2 Review exercise



(E) **1** Find the equation of the line which passes through the points $A(-2, 8)$ and $B(4, 6)$, in the form $ax + by + c = 0$. **(3)**

← Section 5.2

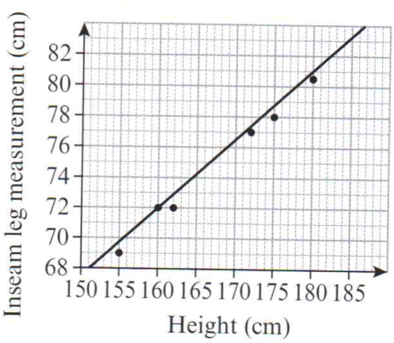
(E) **2** The line l passes through the point $(9, -4)$ and has gradient $\frac{1}{3}$. Find an equation for l, in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

← Section 5.2

(E/P) **3** The points $A(0, 3)$, $B(k, 5)$ and $C(10, 2k)$, where k is a constant, lie on the same straight line. Find the two possible values of k. **(5)**

← Section 5.1

(E/P) **4** The scatter graph shows the height, h cm, and inseam leg measurement, l cm, of six adults. A line of best fit has been added to the scatter graph.



a Use two points on the scatter graph to calculate the gradient of the line. **(2)**

b Use your answer to part **a** to write a linear model relating height and inseam in the form $l = kh$, where k is a constant to be found. **(1)**

c Comment on the validity of your model for small values of h. **(1)**

← Section 5.5

(E) **5** The line l_1 has equation $y = 3x - 6$.
The line l_2 is perpendicular to l_1 and passes through the point $(6, 2)$.

a Find an equation for l_2 in the form $y = mx + c$, where m and c are constants **(3)**

The lines l_1 and l_2 intersect at the point C.

b Use algebra to find the coordinates of C. **(2)**

The lines l_1 and l_2 cross the x-axis at the points A and B respectively.

c Calculate the exact area of triangle ABC. **(4)**

← Sections 5.3, 5.4

(E) **6** The lines $y = 2x$ and $5y + x - 33 = 0$ intersect at the point P. Find the distance of the point from the origin O, giving your answer as a surd in its simplest form. **(4)**

← Sections 5.2, 5.4

(E/P) **7** The perpendicular bisector of the line segment joining $(5, 8)$ and $(7, -4)$ crosses the x-axis at the point Q. Find the coordinates of Q. **(4)**

← Section 6.1

(E) **8** The circle C has centre $(-3, 8)$ and passes through the point $(0, 9)$. Find an equation for C. **(4)**

← Section 6.2

(E/P) **9** **a** Show that $x^2 + y^2 - 6x + 2y - 10 = 0$ can be written in the form $(x - a)^2 + (y - b)^2 = r^2$, where a, b and r are numbers to be found. **(2)**

b Hence write down the centre and radius of the circle with equation $x^2 + y^2 - 6x + 2y - 10 = 0$. **(2)**

← Section 6.2

E/P **10** The line $3x + y = 14$ intersects the circle $(x - 2)^2 + (y - 3)^2 = 5$ at the points A and B.

a Find the coordinates of A and B. **(4)**

b Determine the length of the chord AB. **(2)**

← Section 6.3

E/P **11** The line with equation $y = 3x - 2$ does not intersect the circle with centre (0, 0) and radius r. Find the range of possible values of r. **(8)**

← Section 6.3

E/P **12** The circle C has centre (1, 5) and passes through the point $P(4, -2)$. Find:

a an equation for the circle C. **(4)**

b an equation for the tangent to the circle at P. **(3)**

← Section 6.4

E/P **13** The points $A(2, 1)$, $B(6, 5)$ and $C(8, 3)$ lie on a circle.

a Show that $\angle ABC = 90°$. **(2)**

b Deduce a geometrical property of the line segment AC. **(1)**

c Hence find the equation of the circle. **(4)**

← Section 6.5

E/P **14** $\dfrac{2x^2 + 20x + 42}{224x + 4x^2 - 4x^3} = \dfrac{x + a}{bx(x + c)}$

where a, b and c are constants. Work out the values of a, b and c. **(4)**

← Section 7.1

E **15** **a** Show that $(2x - 1)$ is a factor of $2x^3 - 7x^2 - 17x + 10$. **(2)**

b Factorise $2x^3 - 7x^2 - 17x + 10$ completely. **(4)**

c Hence, or otherwise, sketch the graph of $y = 2x^3 - 7x^2 - 17x + 10$, labelling any intersections with the coordinate axes clearly. **(2)**

← Section 7.3

E/P **16** $f(x) = 3x^3 + x^2 - 38x + c$

Given that $f(3) = 0$,

a find the value of c, **(2)**

b factorise $f(x)$ completely, **(4)**

← Section 7.3

E **17** $g(x) = x^3 - 13x + 12$

a Use the factor theorem to show that $(x - 3)$ is a factor of $g(x)$. **(2)**

b Factorise $g(x)$ completely. **(4)**

← Section 7.3

E/P **18** **a** It is claimed that the following inequality is true for all real numbers a and b. Use a counter-example to show that the claim is false:

$$a^2 + b^2 < (a + b)^2$$ **(2)**

b Specify conditions on a and b that make this inequality true. Prove your result. **(4)**

← Section 7.5

E/P **19** **a** Use proof by exhaustion to prove that for all prime numbers p, $3 < p < 20$, p^2 is one greater than a multiple of 24. **(2)**

b Find a counterexample that disproves the statement 'All numbers which are one greater than a multiple of 24 are the squares of prime numbers.' **(2)**

← Sections 7.5

E/P **20** **a** Show that $x^2 + y^2 - 10x - 8y + 32 = 0$ can be written in the form $(x - a)^2 + (y - b)^2 = r^2$, where a, b and r are numbers to be found. **(2)**

b Circle C has equation $x^2 + y^2 - 10x - 8y + 32 = 0$ and circle D has equation $x^2 + y^2 = 9$. Calculate the distance between the centre of circle C and the centre of circle D. **(3)**

c Using your answer to part **b**, or otherwise, prove that circles C and D do not touch. **(2)**

← Sections 6.4, 7.5

E **21** **a** Expand $(1 - 2x)^{10}$ in ascending powers of x up to and including the term in x^3. **(3)**

b Use your answer to part **a** to evaluate $(0.98)^{10}$ correct to 3 decimal places. **(1)**

← Section 8.5

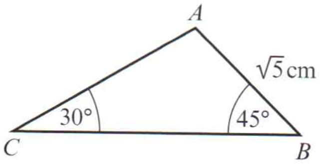
E/P **22** If x is so small that terms of x^3 and higher can be ignored,
$(2 - x)(1 + 2x)^5 \approx a + bx + cx^2$.
Find the values of the constants a, b and c. **(5)**

← Section 8.4

E/P **23** The coefficient of x in the binomial expansion of $(2 - 4x)^q$, where q is a positive integer, is $-32q$. Find the value of q. **(4)**

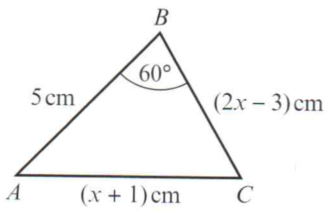
← Section 8.4

E **24** The diagram shows triangle ABC, with $AB = \sqrt{5}$ cm, $\angle ABC = 45°$ and $\angle BCA = 30°$. Find the exact length of AC. **(3)**



← Section 9.2

E/P **25** The diagram shows triangle ABC, with $AB = 5$ cm, $BC = (2x - 3)$ cm, $CA = (x + 1)$ cm and $\angle ABC = 60°$.



a Show that x satisfies the equation $x^2 - 8x + 16 = 0$. **(3)**

b Find the value of x. **(1)**

c Calculate the area of the triangle, giving your answer to 3 significant figures. **(2)**

← Section 9.4

E/P **26** Ship B is 8 km, on a bearing of 030°, from ship A.
Ship C is 12 km, on a bearing of 140°, from ship B.

a Calculate the distance of ship C from ship A. **(4)**

b Calculate the bearing of ship C from ship A. **(3)**

← Section 9.4

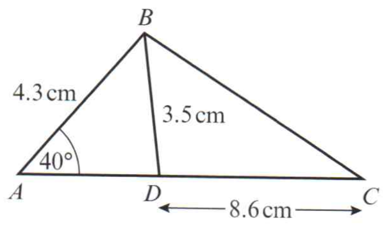
E/P **27** The triangle ABC has vertices $A(-2, 4)$, $B(6, 10)$ and $C(16, 10)$.

a Prove that ABC is an isosceles triangle. **(2)**

b Calculate the size of $\angle ABC$. **(3)**

← Sections 5.4, 9.4

E/P **28** The diagram shows ΔABC.
Given that angle ADB is acute, calculate the area of ΔABC. **(6)**



← Section 9.4

E/P **29** The circle C has centre (5, 2) and radius 5. The points $X(1, -1)$, $Y(10, 2)$ and $Z(8, k)$ lie on the circle, where k is a positive integer.

a Write down the equation of the circle. **(2)**

b Calculate the value of k. **(1)**

c Show that $\cos \angle XYZ = \frac{\sqrt{2}}{10}$ **(5)**

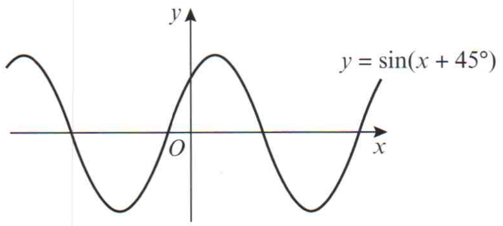
← Sections 6.2, 9.4

E **30** **a** On the same set of axes, in the interval $0 \leqslant x \leqslant 360°$, sketch the graphs of $y = \tan(x - 90°)$ and $y = \sin x$. Label clearly any points at which the graphs cross the coordinate axes. **(5)**

b Hence write down the number of solutions of the equation $\tan(x - 90°) = \sin x$ in the interval $0 \leqslant x \leqslant 360°$. **(1)**

← Section 9.6

E **31** The graph shows the curve $y = \sin(x + 45°)$, $-360° \leqslant x \leqslant 360°$.



a Write down the coordinates of each point where the curve crosses the x-axis. **(2)**

b Write down the coordinates of the point where the curve crosses the y-axis. **(1)**

← Section 9.6

E/P **32** A pyramid has four triangular faces and a square base. All the edges of the pyramid are the same length, s cm. Show that the total surface area of the pyramid is $(\sqrt{3} + 1)s^2$ cm^2. **(3)**

← Sections 9.4, 10.2

E **33** **a** Given that $\sin\theta = \cos\theta$, find the value of $\tan\theta$. **(1)**

b Find the values of θ in the interval $0 \leqslant \theta < 360°$ for which $\sin\theta = \cos\theta$. **(2)**

← Sections 10.3, 10.4

E **34** Find all the values of x in the interval $0 \leqslant x < 360°$ for which $3\tan^2 x = 1$. **(4)**

← Section 10.4

E **35** Find all the values of θ in the interval $0 \leqslant \theta < 360°$ for which $2\sin(\theta - 30°) = \sqrt{3}$. **(4)**

← Section 10.5

E **36** **a** Show that the equation $2\cos^2 x = 4 - 5\sin x$ may be written as $2\sin^2 x - 5\sin x + 2 = 0$. **(2)**

b Hence solve, for $0 \leqslant x < 360°$, the equation $2\cos^2 x = 4 - 5\sin x$. **(4)**

E **37** Find all of the solutions in the interval $0 \leqslant x < 360°$ of $2\tan^2 x - 4 = 5\tan x$ giving each solution, in degrees, to one decimal place. **(6)**

← Section 10.6

E **38** Find all of the solutions in the interval $0 \leqslant x < 360°$ of $5\sin^2 x = 6(1 - \cos x)$ giving each solution, in degrees, to one decimal place. **(7)**

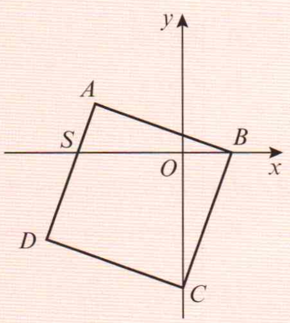
← Section 10.6

E/P **39** Prove that $\cos^2 x(\tan^2 x + 1) = 1$ for all values of x where $\cos x$ and $\tan x$ are defined. **(4)**

← Sections 7.4, 10.3

Challenge

1 The diagram shows a square $ABCD$ on a set of coordinate axes. The square intersects the x-axis at the points B and S, and the equation of the line which passes through B and C is $y = 3x - 12$.



a Calculate the area of the square.

b Find the coordinates of S.

← Sections 5.2, 5.4

2 Prove that the circle $(x + 4)^2 + (y - 5)^2 = 8^2$ lies completely inside the circle $x^2 + y^2 + 8x - 10y = 59$.

← Sections 1.5, 6.2

3 Prove that for all positive integers n and k, $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$.

← Sections 7.4, 8.2

4 Solve for $0° \leqslant x \leqslant 360°$ the equation $2\sin^3 x - \sin x + 1 = \cos^2 x$.

← Section 10.6