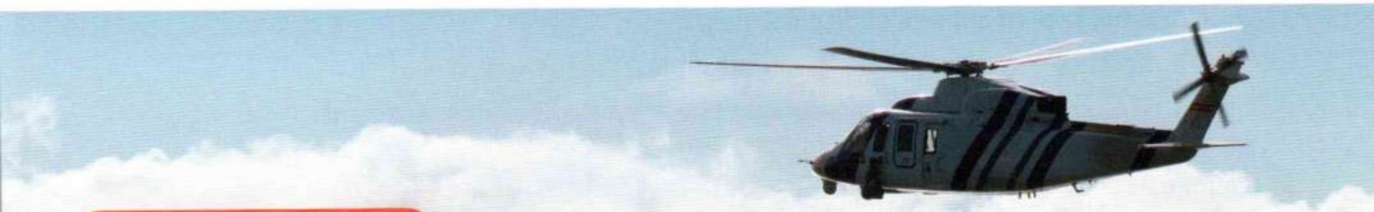
10 Forces and motion

Objectives

After completing this chapter you should be able to:

- Draw force diagrams and calculate resultant forces → pages 157–159
- Understand and use Newton's first law → pages 157–159
- Calculate resultant forces by adding vectors → pages 160–162
- Understand and use Newton's second law, $F = ma$ → pages 162–166
- Apply Newton's second law to vector forces and acceleration → pages 166–169
- Understand and use Newton's third law → pages 169–172
- Solve problems involving connected particles → pages 169–177



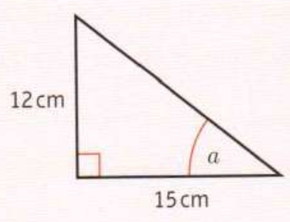
Prior knowledge check

1 Calculate:

a $(2\mathbf{i} + \mathbf{j}) + (3\mathbf{i} - 4\mathbf{j})$ **b** $(-\mathbf{i} + 3\mathbf{j}) - (3\mathbf{i} - \mathbf{j})$

← Section 8.4

2 The diagram shows a right-angled triangle.



Work out:

a the length of the hypotenuse

b the size of the angle a.

Give your answers correct to 1 d.p. ← GCSE Mathematics

3 A car starts from rest and accelerates constantly at 1.5 m s^{-2}.

a Work out the velocity of the car after 12 seconds.

After 12 seconds, the driver brakes, causing the car to decelerate at a constant rate of 1 m s^{-2}.

b Calculate the distance the car travels from the instant the driver brakes until the car comes to rest.

← Chapter 9

The weight of an air–sea rescue crew man is balanced by the tension in the cable. By modelling the forces in this situation, you can calculate how strong the cable needs to be.

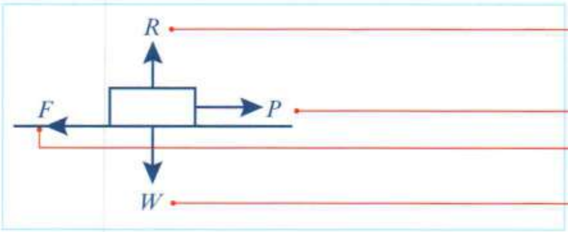
→ Exercise 10A Q5

10.1 Force diagrams

A force diagram is a diagram showing all the forces acting on an object. Each force is shown as an arrow pointing in the direction in which the force acts. Force diagrams are used to model problems involving forces.

Example 1

A block of weight W is being pulled to the right by a force, P, across a rough horizontal plane. Draw a force diagram to show all the forces acting on the block.



R is the normal reaction of the rough horizontal plane on the block. ← Section 8.3

P is the force pulling the block.

F is the resistance due to friction between the block and the plane.

W is the weight of the block.

When the forces acting upon an object are balanced, the object is said to be in equilibrium.

- **Newton's first law of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object.**

Watch out Constant velocity means that neither the speed nor the direction is changing.

When there is more than one force acting on an object you can resolve the forces in a certain direction to find the resultant force in that direction. The direction you are resolving in becomes the positive direction. You add forces acting in this direction and subtract forces acting in the opposite direction.

In your answers, you can use the letter R, together with an arrow, R(↑), to indicate the direction in which you are resolving the forces.

In this section you will only resolve forces that are horizontal or vertical.

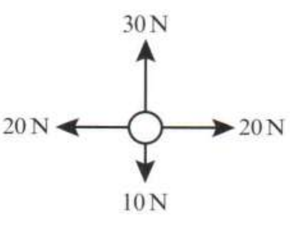
- **A resultant force acting on an object will cause the object to accelerate in the same direction as the resultant force.**

Example 2

The diagram shows the forces acting on a particle.

a Draw a force diagram to represent the resultant force.

b Describe the motion of the particle.



a 

b The particle is accelerating upwards.

R(→): 20 – 20 = 0 so the horizontal forces are balanced.

R(↑): 30 – 10 = 20 so the resultant force is 20 N upwards.

Exercise 10A

1 A box is at rest on a horizontal table. Draw a force diagram to show all the forces acting on the box.

2 A trapeze bar is suspended motionless from the ceiling by two ropes. Draw a force diagram to show the forces acting on the ropes and the trapeze bar.

3 Ignoring air resistance, draw a diagram to show the forces acting on an apple as it falls from a tree.

4 A car's engine applies a force parallel to the surface of a horizontal road that causes the car to move with constant velocity. Considering the resistance to motion, draw a diagram to show the forces acting on the car.

5 An air–sea rescue crew member is suspended motionless from a helicopter. Ignoring air resistance, show all the forces acting on him.

(P) 6 A satellite orbits the Earth at constant speed. State, with a reason, whether any resultant force is acting on the satellite.

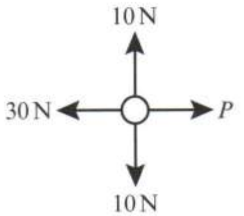
Problem-solving

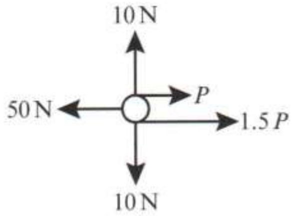
Consider the velocity of the satellite as it orbits the Earth.

7 A particle of weight 5 N sits at rest on a horizontal plane. State the value of the normal reaction acting on the particle.

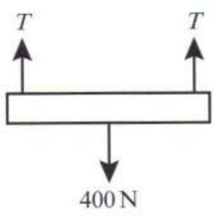
8 Given that each of the particles is stationary, work out the value of P:

a 

b 

c 

9 A hoist lifts a platform vertically at constant velocity as shown in the diagram.

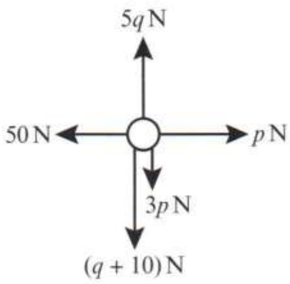


a Ignoring air resistance, work out the tension, T in each rope.

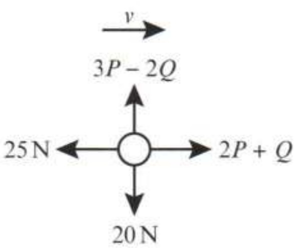
The tension in each rope is reduced by 50 N.

b Describe the resulting motion of the platform.

10 The diagram shows a particle acted on by a set of forces. Given that the particle is at rest, find the value of p and the value of q.



(P) **11** Given that the particle in this diagram is moving with constant velocity, v, find the values of P and Q.



Problem-solving

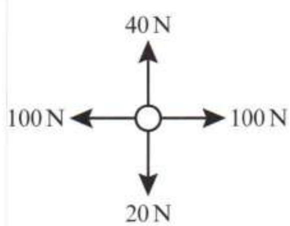
Set up two simultaneous equations.

12 Each diagram shows the forces acting on a particle.

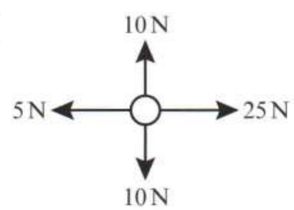
i Work out the size and direction of the resultant force.

ii Describe the motion of the particle.

a



b



13 A truck is moving along a horizontal level road. The truck's engine provides a forward thrust of 10 000 N. The total resistance is modelled as a constant force of magnitude 1600 N.

a Modelling the truck as a particle, draw a force diagram to show the forces acting on the truck.

b Calculate the resultant force acting on the truck.

(P) **14** A car is moving along a horizontal level road. The car's engine provides a constant driving force. The motion of the car is opposed by a constant resistance.

a Modelling the car as a particle, draw a force diagram to show the forces acting on the car.

b Given that the resultant force acting on the car is 4200 N in the direction of motion, and that the magnitude of the driving force is eight times the magnitude of the resistance force, calculate the magnitude of the resistance.

Problem-solving

Use algebra to describe the relationship between the driving force and the resistance.

10.2 Forces as vectors

You can write forces as vectors using **i–j** notation or as column vectors.

- **You can find the resultant of two or more forces given as vectors by adding the vectors.**

Links If two forces $(p\mathbf{i} + q\mathbf{j})$ N and $(r\mathbf{i} + s\mathbf{j})$ N are acting on a particle, the resultant force will be $((p + r)\mathbf{i} + (q + s)\mathbf{j})$ N. ← Pure Year 1, Section 11.2

When a particle is in equilibrium the resultant vector force will be $0\mathbf{i} + 0\mathbf{j}$.

Example 3

The forces $2\mathbf{i} + 3\mathbf{j}$, $4\mathbf{i} - \mathbf{j}$, $-3\mathbf{i} + 2\mathbf{j}$ and $a\mathbf{i} + b\mathbf{j}$ act on an object which is in equilibrium. Find the values of a and b.

$(2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - \mathbf{j}) + (-3\mathbf{i} + 2\mathbf{j}) + (a\mathbf{i} + b\mathbf{j}) = 0$

$(2 + 4 - 3 + a)\mathbf{i} + (3 - 1 + 2 + b)\mathbf{j} = 0$

$\Rightarrow \quad 3 + a = 0 \quad$ and $\quad 4 + b = 0$

$\Rightarrow \quad a = -3 \quad$ and $\quad b = -4$

If an object is in equilibrium then the resultant force will be zero.

You can consider the **i** and **j** components separately because they are perpendicular.

Example 4

In this question **i** represents the unit vector due east, and **j** represents the unit vector due north. A particle begins at rest at the origin. It is acted on by three forces $(2\mathbf{i} + \mathbf{j})$ N, $(3\mathbf{i} - 2\mathbf{j})$ N and $(-\mathbf{i} + 4\mathbf{j})$ N.

a Find the resultant force in the form $p\mathbf{i} + q\mathbf{j}$.

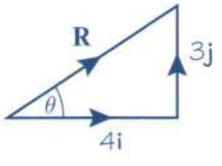
b Work out the magnitude and bearing of the resultant force.

c Describe the motion of the particle.

a $(2\mathbf{i} + \mathbf{j}) + (3\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} + 4\mathbf{j}) = 4\mathbf{i} + 3\mathbf{j}$

Add together the **i**-components and the **j**-components.

b



$\mathbf{R} = (4\mathbf{i} + 3\mathbf{j})$ N

Therefore the magnitude of $\mathbf{R}$ is given by

$|\mathbf{R}| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ N

$\tan\theta = \frac{3}{4}$

$\theta = 36.9°$ (1 d.p.)

Bearing $= 90° - 36.9° = 053.1°$

c The particle accelerates in the direction of the resultant force.

Notation The unit vector **i** is usually taken to be due east or the positive x-direction. The unit vector **j** is usually taken to be due north or the positive y-direction. Questions involving finding bearings will often specify this.

Use Pythagoras' theorem to find the magnitude of the resultant.

Use $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Bearings are measured clockwise from north so subtract θ from 90°.

Exercise 10B

1 In each part of the question a particle is acted upon by the forces given. Work out the resultant force acting on the particle.

a $(-\mathbf{i} + 3\mathbf{j})$ N and $(4\mathbf{i} - \mathbf{j})$ N

b $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ N and $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$ N

c $(\mathbf{i} + \mathbf{j})$ N, $(5\mathbf{i} - 3\mathbf{j})$ N and $(-2\mathbf{i} - \mathbf{j})$ N

d $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ N, $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ N and $\begin{pmatrix} -2 \\ -7 \end{pmatrix}$ N

Notation $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ N is the same as $(5\mathbf{i} + 3\mathbf{j})$ N.

2 An object is in equilibrium at O under the action of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$. Find $\mathbf{F}_3$ in these cases.

a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$ and $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$

b $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ and $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$

(P) 3 The forces $\begin{pmatrix} a \\ 2b \end{pmatrix}$ N, $\begin{pmatrix} -2a \\ -b \end{pmatrix}$ N and $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ N act on an object which is in equilibrium.

Find the values of a and b.

4 For each force find:

i the magnitude of the force

ii the angle the force makes with $\mathbf{i}$

a $(3\mathbf{i} + 4\mathbf{j})$ N

b $(5\mathbf{i} - \mathbf{j})$ N

c $(-2\mathbf{i} + 3\mathbf{j})$ N

d $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ N

5 In this question, $\mathbf{i}$ represents the unit vector due east, and $\mathbf{j}$ represents the unit vector due north. A particle is acted upon by forces of:

a $(-2\mathbf{i} + \mathbf{j})$ N, $(5\mathbf{i} + 2\mathbf{j})$ N and $(-\mathbf{i} - 4\mathbf{j})$ N

b $(-2\mathbf{i} + \mathbf{j})$ N, $(2\mathbf{i} - 3\mathbf{j})$ N and $(3\mathbf{i} + 6\mathbf{j})$ N

Work out:

i the resultant vector

ii the magnitude of the resultant vector

iii the bearing of the resultant vector.

(P) 6 The forces $(a\mathbf{i} - b\mathbf{j})$ N, $(b\mathbf{i} + a\mathbf{j})$ N and $(-4\mathbf{i} - 2\mathbf{j})$ N act on an object which is in equilibrium. Find the values of a and b.

Problem-solving

Use the $\mathbf{i}$ components and the $\mathbf{j}$ components to set up and solve two simultaneous equations.

(P) 7 The forces $(2a\mathbf{i} + 2b\mathbf{j})$ N, $(-5b\mathbf{i} + 3a\mathbf{j})$ N and $(-11\mathbf{i} - 7\mathbf{j})$ N act on an object which is in equilibrium. Find the values of a and b.

8 Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle. $\mathbf{F}_1 = (-3\mathbf{i} + 7\mathbf{j})$ N, $\mathbf{F}_2 = (\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})$ N.

a Given that this particle is in equilibrium, determine the value of p and the value of q.

The resultant of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.

b Calculate, in N, the magnitude of $\mathbf{R}$.

c Calculate, to the nearest degree, the angle between the line of action of $\mathbf{R}$ and the vector $\mathbf{j}$.

E/P **9** A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by $\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j})$ N and $\mathbf{F}_2 = (a\mathbf{i} + 2a\mathbf{j})$ N, where a is a positive constant.

a Find the angle between $\mathbf{F}_2$ and $\mathbf{i}$. **(2 marks)**

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.

b Given that $\mathbf{R}$ is parallel to $13\mathbf{i} + 10\mathbf{j}$, find the value of a. **(4 marks)**

E **10** Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by the vectors $\mathbf{F}_1 = \begin{pmatrix}-7\\-4\end{pmatrix}$ N, $\mathbf{F}_2 = \begin{pmatrix}4\\2\end{pmatrix}$ N and $\mathbf{F}_3 = \begin{pmatrix}a\\b\end{pmatrix}$ N, where a and b are constants.

Given that P is in equilibrium,

a find the value of a and the value of b. **(3 marks)**

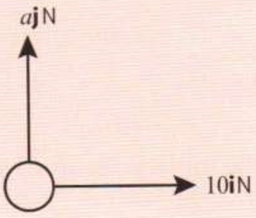
b The force $\mathbf{F}_1$ is now removed. The resultant of $\mathbf{F}_2$ and $\mathbf{F}_3$ is $\mathbf{R}$. Find:

i the magnitude of $\mathbf{R}$ **(2 marks)**

ii the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with the horizontal. **(3 marks)**

Challenge

An object is acted upon by a horizontal force of 10**i** N and a vertical force a**j** N as shown in the diagram. The resultant of the two forces acts in the direction 60° to the horizontal. Work out the value of a and the magnitude of the resultant force.



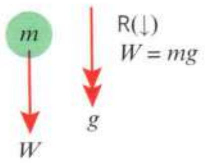
10.3 Forces and acceleration

A non-zero resultant force that acts on a particle will cause the particle to accelerate in the direction of the resultant force.

- **Newton's second law of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced: $F = ma$.**

A force of 1 N will accelerate a mass of 1 kg at a rate of 1 m s^{-2}. If a force F N acts on a particle of mass m kg causing it to accelerate at a m s^{-2}, the **equation of motion** for the particle is $\boldsymbol{F = ma}$.

Gravity is the force between any object and the Earth. The force due to gravity acting on an object is called the **weight** of the object, and it acts vertically downwards. A body falling freely experiences an acceleration of $g = 9.8$ m s^{-2}. Using the relationship $F = ma$ you can write the equation of motion for a body of mass m kg with weight W N.



- $\boldsymbol{W = mg}$

Example 5

Find the acceleration when a particle of mass 1.5 kg is acted on by a resultant force of 6 N.

$F = ma$

$6 = 1.5a$

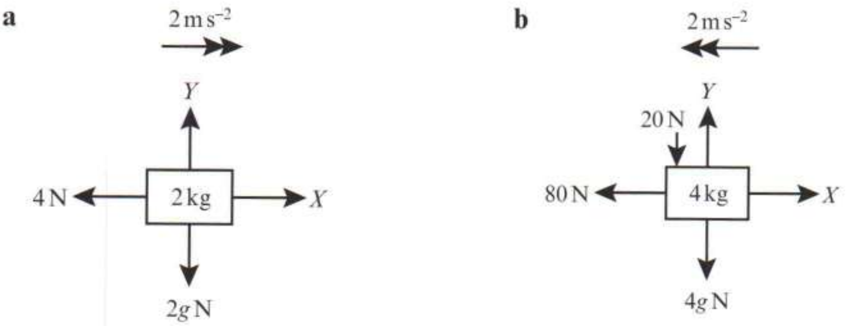
$a = 4$

The acceleration is 4 m s^{-2}.

Substitute the values you know and solve the equation to find a.

Example 6

In each of these diagrams the body is accelerating as shown. Find the magnitudes of the unknown forces X and Y.



a R(→), $X - 4 = 2 \times 2$

$X = 8\text{ N}$

R(↑), $Y - 2g = 2 \times 0$

$Y = 2g$

$= 19.6\text{ N}$

b R(←), $80 - X = 4 \times 2$

$X = 72\text{ N}$

R(↑), $Y - 20 - 4g = 4 \times 0$

$Y = 20 + (4 \times 9.8)$

$= 59.2\text{ N}$ (3 s.f.)

R(→), means that you are finding the resultant force in the horizontal direction, in the direction of the arrow. The arrow is in the positive direction.

This resultant force causes an acceleration of 2 m s^{-2}. Use $F = ma$.

It is usually easier to take the positive direction as the direction of the acceleration.

There is no vertical acceleration, so $a = 0$.

Example 7

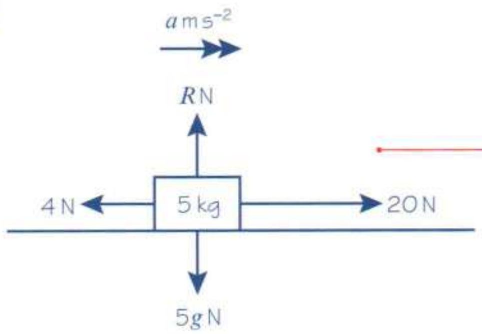
A body of mass 5 kg is pulled along a rough horizontal table by a horizontal force of magnitude 20 N against a constant friction force of magnitude 4 N. Given that the body is initially at rest, find:

a the acceleration of the body

b the distance travelled by the body in the first 4 seconds

c the magnitude of the normal reaction between the body and the table.

a



Draw a diagram showing all the forces and the acceleration.

R(→), $20 - 4 = 5a$

$$a = \frac{16}{5} = 3.2$$

The body accelerates at 3.2 m s^{-2}.

Resolve horizontally, taking the positive direction as the direction of the acceleration, and write down an equation of motion for the body.

b $s = ut + \frac{1}{2}at^2$

Since the acceleration is constant.

$s = (0 \times 4) + \frac{1}{2} \times 3.2 \times 4^2$

$= 25.6$

Substitute in the values.

The body moves a distance of 25.6 m.

c R(↑), $R - 5g = 5 \times 0 = 0$

$$R = 5g = 5 \times 9.8 = 49\text{ N}$$

The normal reaction has magnitude 49 N.

Resolve vertically. Since the body is moving horizontally $a = 0$, so the right-hand side of the equation of motion is 0.

Exercise 10C

1 Find the acceleration when a particle of mass 400 kg is acted on by a resultant force of 120 N.

2 Find the weight in newtons of a particle of mass 4 kg.

3 An object moving on a rough surface experiences a constant frictional force of 30 N which decelerates it at a rate of 1.2 m s^{-2}. Find the mass of the object.

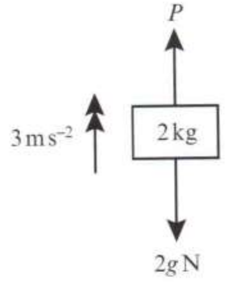
(P) **4** An astronaut weighs 735 N on the earth and 120 N on the moon. Work out the value of acceleration due to gravity on the moon.

Problem-solving

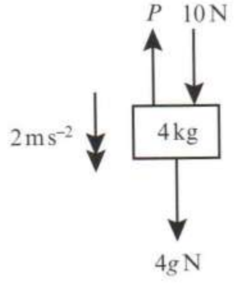
Start by finding the mass of the astronaut.

5 In each scenario, the forces acting on the body cause it to accelerate as shown. Find the magnitude of the unknown force.

a

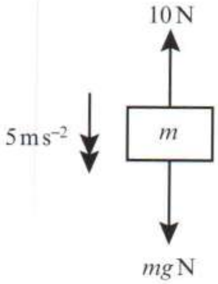


b

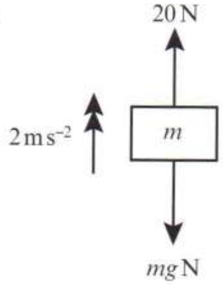


6 In each situation, the forces acting on the body cause it to accelerate as shown. In each case find the mass of the body, m.

a

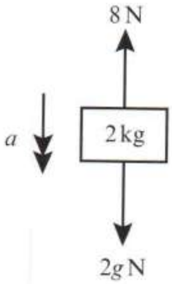


b

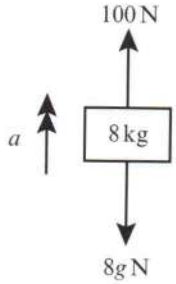


7 In each situation, the forces acting on the body cause it to accelerate as shown with magnitude $a\,\text{m s}^{-2}$. In each case find the value of a.

a



b



8 A force of 10 N acts upon a particle of mass 3 kg causing it to accelerate at $2\,\text{m s}^{-2}$ along a rough horizontal plane. Calculate the value of the force due to friction.

Problem-solving

Draw a force diagram showing all the forces acting on the particle.

9 A lift of mass 500 kg is lowered or raised by means of a metal cable attached to its top. The lift contains passengers whose total mass is 300 kg. The lift starts from rest and accelerates at a constant rate, reaching a speed of $3\,\text{m s}^{-1}$ after moving a distance of 5 m. Find:

a the acceleration of the lift **(3 marks)**

b the tension in the cable if the lift is moving vertically downwards **(2 marks)**

c the tension in the cable if the lift is moving vertically upwards. **(2 marks)**

Hint Use $v^2 = u^2 + 2as$.

10 A trolley of mass 50 kg is pulled from rest in a straight line along a horizontal path by means of a horizontal rope attached to its front end. The trolley accelerates at a constant rate and after 2 s its speed is $1\,\text{m s}^{-1}$. As it moves, the trolley experiences a resistance to motion of magnitude 20 N. Find:

a the acceleration of the trolley **(3 marks)**

b the tension in the rope. **(2 marks)**

11 The engine of a van of mass 400 kg cuts out when it is moving along a straight horizontal road with speed $16\,\text{m s}^{-1}$. The van comes to rest without the brakes being applied.

In a model of the situation it is assumed that the van is subject to a resistive force which has constant magnitude of 200 N.

a Find how long it takes the van to stop. **(3 marks)**

b Find how far the van travels before it stops. **(2 marks)**

c Comment on the suitability of the modelling assumption. **(1 mark)**

Challenge

A small stone of mass 400 g is projected vertically upwards from the bottom of a pond full of water with speed 10 m s^{-1}. As the stone moves through the water it experiences a constant resistance of magnitude 3 N. Assuming that the stone does not reach the surface of the pond, find:

a the greatest height above the bottom of the pond that the stone reaches

b the speed of the stone as it hits the bottom of the pond on its return

c the total time taken for the stone to return to its initial position on the bottom of the pond.

10.4 Motion in 2 dimensions

- **You can use F = *m*a to solve problems involving vector forces acting on particles.**

Notation In this version of the equation of motion, **F** and **a** are vectors. You can write acceleration as a 2D vector in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-2} or $\begin{pmatrix} p \\ q \end{pmatrix}$ m s^{-2}.

Example 8

In this question **i** represents the unit vector due east, and **j** represents the unit vector due north.

A resultant force of $(3\mathbf{i} + 8\mathbf{j})$ N acts upon a particle of mass 0.5 kg.

a Find the acceleration of the particle in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-2}.

b Find the magnitude and bearing of the acceleration of the particle.

a $\mathbf{F} = m\mathbf{a}$

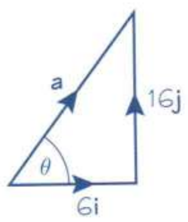
$(3\mathbf{i} + 8\mathbf{j}) = 0.5 \times \mathbf{a}$

$\mathbf{a} = (6\mathbf{i} + 16\mathbf{j})$ m s^{-2}

Write the vector equation of motion.

To divide $(3\mathbf{i} + 8\mathbf{j})$ by 0.5, you divide each component by 0.5.

b



Draw a diagram to represent the acceleration vector.

$|\mathbf{a}| = \sqrt{6^2 + 16^2} = 2\sqrt{73}$ N

$= 17.1$ m s^{-2} (1 d.p.)

Use Pythagoras' theorem to work out the magnitude of the acceleration vector.

$\tan\theta = \frac{16}{6}$ so $\theta = 69.4°$ (1 d.p.)

So the bearing of the acceleration is

$90° - 69.4° = 020.6°$

Remember bearings are always measured clockwise from north.

Example 9

Forces $\mathbf{F}_1 = (2\mathbf{i} + 4\mathbf{j})$ N, $\mathbf{F}_2 = (-5\mathbf{i} + 4\mathbf{j})$ N, and $\mathbf{F}_3 = (6\mathbf{i} - 5\mathbf{j})$ N act on a particle of mass 3 kg. Find the acceleration of the particle.

Resultant force

$= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ — Add the vectors to find the resultant force.

$= (2\mathbf{i} + 4\mathbf{j}) + (-5\mathbf{i} + 4\mathbf{j}) + (6\mathbf{i} - 5\mathbf{j})$

$= 3\mathbf{i} + 3\mathbf{j}$

$3\mathbf{i} + 3\mathbf{j} = 3\mathbf{a} \Rightarrow \mathbf{a} = (\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$ — Use $\mathbf{F} = m\mathbf{a}$.

Example 10

A boat is modelled as a particle of mass 60 kg being acted on by three forces:

$$\mathbf{F}_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix}\text{N} \qquad \mathbf{F}_2 = \begin{pmatrix} 10p \\ 20q \end{pmatrix}\text{N} \qquad \mathbf{F}_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix}\text{N}$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}\text{m s}^{-2}$, find the values of p and q.

Resultant force

$= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= \begin{pmatrix} 80 \\ 50 \end{pmatrix} + \begin{pmatrix} 10p \\ 20q \end{pmatrix} + \begin{pmatrix} -75 \\ 100 \end{pmatrix}$

$= \begin{pmatrix} 5 + 10p \\ 150 + 20q \end{pmatrix}\text{N}$ — Find the resultant force acting on the boat in terms of p and q.

$\mathbf{F} = m\mathbf{a}$

$\begin{pmatrix} 5 + 10p \\ 150 + 20q \end{pmatrix} = 60 \times \begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix} = \begin{pmatrix} 48 \\ -90 \end{pmatrix}$ — Use $\mathbf{F} = m\mathbf{a}$. Remember that you need to multiply each component in the acceleration by 60.

So $5 + 10p = 48 \Rightarrow p = 4.3$

and $150 + 20q = -90 \Rightarrow q = -12$ — Solve separate equations for each component to find the values of p and q.

Exercise 10D

In all the questions in this exercise **i** represents the unit vector due east, and **j** represents the unit vector due north.

1 A resultant force of $(\mathbf{i} + 4\mathbf{j})$ N acts upon a particle of mass 2 kg.

a Find the acceleration of the particle in the form $(p\mathbf{i} + q\mathbf{j})\,\text{m s}^{-2}$.

b Find the magnitude and bearing of the acceleration of the particle.

2 A resultant force of $(4\mathbf{i} + 3\mathbf{j})$ N acts on a particle of mass m kg causing it to accelerate at $(20\mathbf{i} + 15\mathbf{j})\,\text{m s}^{-2}$. Work out the mass of the particle.

3 A particle of mass 3 kg is acted on by a force **F**. Given that the particle accelerates at $(7\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$:

a find an expression for **F** in the form $(p\mathbf{i} + q\mathbf{j})$ N

b find the magnitude and bearing of **F**.

4 Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle of mass m. Find the acceleration of the particle, $\mathbf{a}\,\text{m s}^{-2}$, given that:

a $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$ N, $\mathbf{F}_2 = (-3\mathbf{i} + \mathbf{j})$ N, $m = 0.25$ kg

b $\mathbf{F}_1 = (3\mathbf{i} - 4\mathbf{j})$ N, $\mathbf{F}_2 = (2\mathbf{i} + 3\mathbf{j})$ N, $m = 6$ kg

c $\mathbf{F}_1 = (-40\mathbf{i} - 20\mathbf{j})$ N, $\mathbf{F}_2 = (25\mathbf{i} + 10\mathbf{j})$ N, $m = 15$ kg

d $\mathbf{F}_1 = 4\mathbf{j}$ N, $\mathbf{F}_2 = (-2\mathbf{i} + 5\mathbf{j})$ N, $m = 1.5$ kg

Notation You are asked to find the acceleration as a vector, **a**. You can give your answer as a column vector or using **i–j** notation.

5 A particle of mass 8 kg is at rest. It is acted on by three forces, $\mathbf{F}_1 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ N, $\mathbf{F}_2 = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ N and $\mathbf{F}_3 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ N.

a Find the magnitude and direction of the acceleration of the particle, $\mathbf{a}\,\text{m s}^{-2}$.

b Find the time taken for the particle to travel a distance of 20 m.

Hint Use $s = ut + \frac{1}{2}at^2$ with $s = 20$ and $u = 0$.

E/P 6 Two forces, $(2\mathbf{i} + 3\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P. The resultant of the two forces is **R**. Given that **R** acts in a direction which is parallel to the vector $(-\mathbf{i} + 4\mathbf{j})$, show that $4p + q + 11 = 0$. **(4 marks)**

Problem-solving You can write **R** in the form $(-k\mathbf{i} + 4k\mathbf{j})$ N for some constant k.

E 7 A particle of mass 4 kg starts from rest and is acted upon by a force **R** of $(6\mathbf{i} + b\mathbf{j})$ N. **R** acts on a bearing of 045°.

a Find the value of b. **(1 mark)**

b Calculate the magnitude of **R**. **(2 marks)**

c Work out the magnitude of the acceleration of the particle. **(2 marks)**

d Find the total distance travelled by the particle during the first 5 seconds of its motion. **(3 marks)**

P 8 Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle. $\mathbf{F}_1 = (-3\mathbf{i} + 7\mathbf{j})$ N, $\mathbf{F}_2 = (\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})$ N.

a Given that this particle is in equilibrium, determine the value of p and the value of q.

Force $\mathbf{F}_2$ is removed.

b Given that in the first 10 seconds of its motion the particle travels a distance of 12 m, find the exact mass of the particle in kg.

P 9 A particle of mass m kg is acted upon by forces of $(5\mathbf{i} + 6\mathbf{j})$ N, $(2\mathbf{i} - 2\mathbf{j})$ N and $(-\mathbf{i} - 4\mathbf{j})$ N causing it to accelerate at $7\,\text{m s}^{-2}$. Work out the mass of the particle. Give your answer correct to 2 d.p.

10 Two forces, $\begin{pmatrix}2\\5\end{pmatrix}$ N and $\begin{pmatrix}p\\q\end{pmatrix}$ N, act on a particle P of mass m kg. The resultant of the two forces is **R**.

a Given that **R** acts in a direction which is parallel to the vector $\begin{pmatrix}1\\-2\end{pmatrix}$, show that $2p + q + 9 = 0$. **(4 marks)**

b Given also that $p = 1$ and that P moves with an acceleration of magnitude $15\sqrt{5}$ m s^{-2}, find the value of m. **(7 marks)**

Challenge

A particle of mass 0.5 kg is acted on by two forces:

$\mathbf{F}_1 = -4\mathbf{i}$ N $\qquad \mathbf{F}_2 = (k\mathbf{i} + 2k\mathbf{j})$ N

where k is a positive constant.

Given that the particle is accelerating at a rate of $8\sqrt{17}$ m s^{-2}, find the value of k.

10.5 Connected particles

If a system involves the motion of more than one particle, the particles may be considered separately. However, if all parts of the system are moving in the **same straight line**, then you can also treat the whole system as a single particle.

- **You can solve problems involving connected particles by considering the particles separately or, if they are moving in the same straight line, as a single particle.**

Watch out Particles need to remain in contact, or be connected by an inextensible rod or string to be considered as a single particle.

Example 11

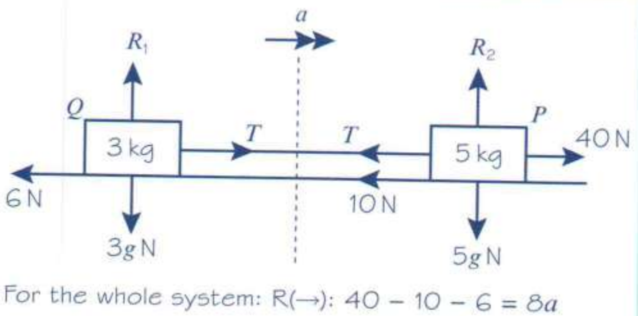
Two particles, P and Q, of masses 5 kg and 3 kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40 N along a rough horizontal plane. Particle P experiences a frictional force of 10 N and particle Q experiences a frictional force of 6 N.

a Find the acceleration of the particles.

b Find the tension in the string.

c Explain how the modelling assumptions that the string is light and inextensible have been used.

a



For the whole system: $R(\rightarrow)$: $40 - 10 - 6 = 8a$

$8a = 24$

$a = 3$ m s^{-2}

Problem-solving

In part **a**, by considering the system as a single particle you eliminate the need to find the tension in the string. Otherwise you would need to set up two simultaneous equations involving a and T.

In part **b** the particles need to be considered separately to find the tension in the string.

b For P: $R(\rightarrow)$: $40 - T - 10 = 5 \times 3$

$T = 15\text{ N}$

You could also have chosen particle Q to find the tension. Check to see that it gives the same answer.

c Inextensible $\Rightarrow$ acceleration of masses is the same.

light $\Rightarrow$ tension is the same throughout the length of the string and the mass of the string is negligible.

- **Newton's third law states that for every action there is an equal and opposite reaction.**

Newton's third law means that when two bodies A and B are in contact, if body A exerts a force on body B, then body B exerts a force on body A that is equal in magnitude and acts in the opposite direction.

Example 12

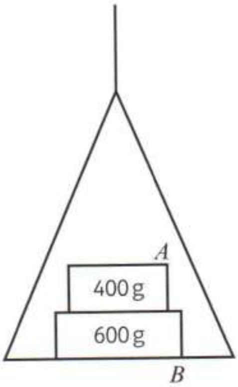
A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400 g and the mass of B is 600 g. A rests on top of B, as shown in the diagram.

The scale-pan is raised vertically, using the string, with acceleration 0.5 m s^{-2}.

a Find the tension in the string.

b Find the force exerted on mass B by mass A.

c Find the force exerted on mass B by the scale-pan.



a For the whole system:

You can use this since all parts of the system are moving in the same straight line.

$R(\uparrow)$: $T - 0.4g - 0.6g = (0.4 + 0.6)a$

Note that you must convert 400 g to 0.4 kg and 600 g to 0.6 kg.

So, $T - g = 1 \times 0.5$

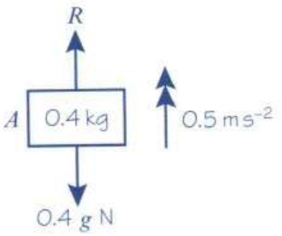
$a = 0.5$

$T = 10.3\text{ N}$

Simplify.

The tension in the string is 10 N (2 s.f.)

b



For A only:

$R(\uparrow)$: $R - 0.4g = 0.4 \times 0.5$

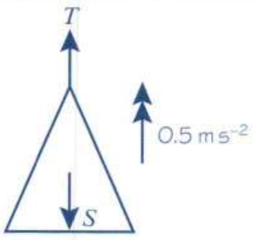
Find the force exerted on A by B and then use Newton's 3rd law to say that the force exerted on B by A will have the same magnitude but is in the opposite direction.

$R = 4.12\text{ N}$

So the force exerted on B by A is 4.1 N (2 s.f.) downwards.

You have used $g = 9.8\text{ m s}^{-2}$ so give your final answer correct to two significant figures.

c



For scale-pan only:

$R(\uparrow)$ $T - S = 0 \times 0.5$

$= 0$

So, $T = S = 10.3$ N

So, the force exerted on B by the scale-pan is 10 N (2 s.f.) upwards.

Problem-solving

It's easier to find the force exerted on the scale-pan by B and then use Newton's 3rd law to say that the force exerted on B by the scale-pan has the same magnitude but is in the opposite direction.

The scale-pan is light, i.e. its mass is 0.

From part **a**.

Use Newton's 3rd law.

Exercise 10E

1 Two particles P and Q of masses 8 kg and 2 kg respectively, are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of magnitude F is applied to P in a direction away from Q and when the string is taut the particles move with acceleration $0.4\,\text{m s}^{-2}$..

a Find the value of F.

b Find the tension in the string.

c Explain how the modelling assumptions that the string is light and inextensible are used.

Hint For part **b** consider P on its own.

2 Two particles P and Q of masses 20 kg and m kg are connected by a light inextensible rod. The particles lie on a smooth horizontal plane. A horizontal force of 60 N is applied to Q in a direction towards P, causing the particles to move with acceleration $2\,\text{m s}^{-2}$.

a Find the mass, m, of Q.

b Find the thrust in the rod

3 Two particles P and Q of masses 7 kg and 8 kg are connected by a light inextensible string. The particles are on a smooth horizontal plane. A horizontal force of 30 N is applied to Q in a direction away from P. When the string is taut the particles move with acceleration, $a\,\text{m s}^{-2}$.

a Find the acceleration, a, of the system.

b Find the tension in the string.

4 Two boxes A and B of masses 110 kg and 190 kg sit on the floor of a lift of mass 1700 kg. Box A rests on top of box B. The lift is supported by a light inextensible cable and is descending with constant acceleration $1.8\,\text{m s}^{-2}$.

a Find the tension in the cable.

b Find the force exerted by box B

i on box A **ii** on the floor of the lift.

(P) **5** A lorry of mass m kg is towing a trailer of mass $3m$ kg along a straight horizontal road. The lorry and trailer are connected by a light inextensible tow-bar. The lorry exerts a driving force of 50 000 N causing the lorry and trailer to accelerate at $5\,\text{m}\,\text{s}^{-2}$. The lorry and trailer experience resistances of 4000 N and 10 000 N respectively.

a Find the mass of the lorry and hence the mass of the trailer.

b Find the tension in the tow-bar.

c Explain how the modelling assumptions that the tow-bar is light and inextensible affect your calculations.

(E) **6** Two particles A and B of masses 10 kg and 5 kg respectively are connected by a light inextensible string. Particle B hangs directly below particle A. A force of 180 N is applied vertically upwards causing the particles to accelerate.

a Find the magnitude of the acceleration. **(3 marks)**

b Find the tension in the string. **(2 marks)**

(E/P) **7** Two particles A and B of masses 6 kg and m kg respectively are connected by a light inextensible string. Particle B hangs directly below particle A. A force of 118 N is applied vertically upwards causing the particles to accelerate at $2\,\text{m}\,\text{s}^{-2}$.

a Find the mass, m, of particle B. **(3 marks)**

b Find the tension in the string. **(2 marks)**

(E/P) **8** A train engine of mass 6400 kg is pulling a carriage of mass 1600 kg along a straight horizontal railway track. The engine is connected to the carriage by a shunt which is parallel to the direction of motion of the coupling. The shunt is modelled as a light rod. The engine provides a constant driving force of 12 000 N. The resistances to the motion of the engine and the carriage are modelled as constant forces of magnitude R N and 2000 N respectively.

Given that the acceleration of the engine and the carriage is $0.5\,\text{m}\,\text{s}^{-2}$:

a find the value of R **(3 marks)**

b show that the tension in the shunt is 2800 N. **(2 marks)**

(E) **9** A car of mass 900 kg pulls a trailer of mass 300 kg along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 200 N and 100 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N.

a Show that the acceleration of the car and trailer is $0.75\,\text{m}\,\text{s}^{-2}$. **(2 marks)**

b Find the magnitude of the tension in the tow-bar. **(3 marks)**

The car is moving along the road when the driver sees a set of traffic lights have turned red. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate.

c Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N, find the value of F. **(7 marks)**

10.6 Pulleys

In this section you will see how to model systems of connected particles involving pulleys. The problems you answer will assume that particles are connected by a light, inextensible string, which passes over a **smooth pulley**. This means that the tension in the string will be the same **on both sides** of the pulley. The parts of the string each side of the pulley will be either horizontal or vertical.

Watch out You cannot treat a system involving a pulley as a single particle. This is because the particles are moving in different directions.

Example 13

Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

a **i** Write down an equation of motion for P.

ii Write down an equation of motion for Q.

b Find the acceleration of each mass.

c Find the tension in the string.

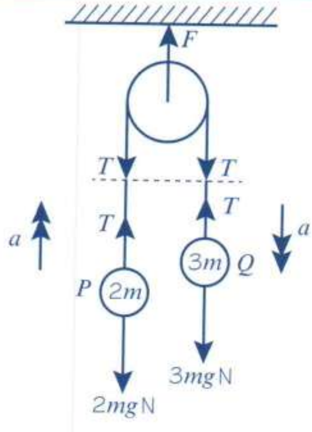
d Find the force exerted on the pulley by the string.

e Find the distance moved by Q in the first 4 s, assuming that P does not reach the pulley.

f State how you have used the fact that the pulley is smooth in your calculations.

Problem-solving Resolve vertically for both P and Q. This will give you simultaneous equations involving the tension T and the acceleration a which can then be solved.

a



Draw a diagram showing all the forces acting on each mass and the pulley, and the acceleration.

i For P, R(↑): $T - 2mg = 2ma$ (1)

ii For Q, R(↓): $3mg - T = 3ma$ (2)

Now resolve for each mass separately, in the direction of its acceleration.

b Adding equations (1) and (2):

$3mg - \cancel{T} + \cancel{T} - 2mg = 3ma + 2ma$ — Add the equations to eliminate T.

$\cancel{m}g = 5\cancel{m}a$ — Simplifying.

$\frac{1}{5}g = a$

The acceleration of each mass is $\frac{1}{5}g$. — You could also give your final answer as 2.0 m s^{-2} (2 s.f.).

c From (1): $T - 2mg = 2m \times \frac{1}{5}g$

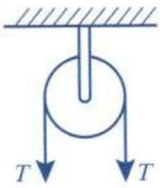
Substitute for a.

$$T = \frac{12mg}{5}\text{ N}$$

Collect terms.

The tension in the string is $\frac{12mg}{5}$ N.

d



The force exerted on the pulley by the string is $2T$ N downwards or $\frac{24mg}{5}$ N.

e $u = 0$, $a = \frac{1}{5}g$, $t = 4$, $s = ?$

$s = ut + \frac{1}{2}at^2$

Since a is a constant we can use any of the formulae for constant acceleration.

$= 0 + \frac{1}{2} \times 1.96 \times 4^2$

$= 15.68$ m

$= 15.7$ m (3 s.f.)

Q moves through a distance of 15.7 m (3 s.f.)

f The tension in the string is the same at P as at Q.

Example 14

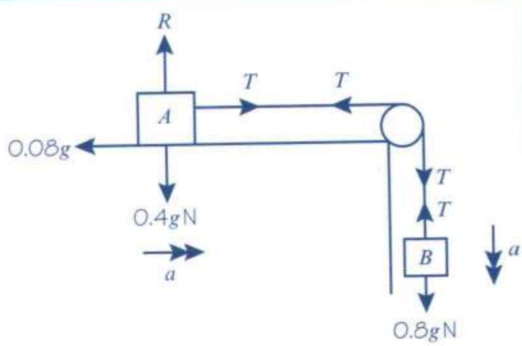
Two particles A and B of masses 0.4 kg and 0.8 kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5 m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5 m above horizontal ground. A frictional force of magnitude $0.08g$ opposes the motion of particle A. The system is released from rest. Find:

a the acceleration of the system

b the time taken for B to reach the ground

c the total distance travelled by A before it first comes to rest.

a



Problem-solving

Draw a diagram showing all the forces and the accelerations. The pulley is smooth so the tension in the string is the same on each side of the pulley.

For A only: $R(\rightarrow)$, $T - 0.08g = 0.4a$ (1)

For B only: $R(\downarrow)$, $0.8g - T = 0.8a$ (2)

> Write equations of motion for each of A and B separately.

Add (1) and (2):

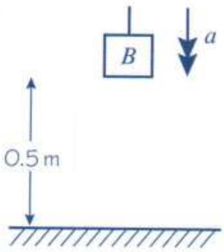
> To eliminate the T terms.

$$0.8g - \cancel{T} + \cancel{T} - 0.08g = 0.8a + 0.4a$$
$$0.72g = 1.2a$$
$$0.6g = a$$

The acceleration of the system is $0.6g$

> You could also give your answer as 5.9 m s^{-2} (2 s.f.).

b



$u = 0$, $s = 0.5$,
$a = 5.88$, $t = ?$

> Use an unrounded value of the acceleration.

$s = ut + \frac{1}{2}at^2$

> The acceleration is constant.

$0.5 = 0 + \frac{1}{2} \times 5.88 \times t^2$

$t = 0.41$ (2 s.f.)

The time taken for B to hit the ground is 0.41 s (2 s.f.)

c Find the speed of B when it hits the ground.

$u = 0$, $a = 5.88$, $t = 0.412\,39$, $v = ?$

> Use an unrounded value for t.

$v = u + at$

$v_B = 0 + 5.88 \times 0.412\,39 = 2.424\,87$ m s^{-1}.

> Using surds, $v_B = \sqrt{\frac{3g}{5}}$

Speed of A on the table is $2.424\,87$ m s^{-1}.

> Since the string is inextensible.

Once B hits the ground the string will go slack and A will begin to decelerate as it slides against the friction on the table.

From (1): $-0.08g = 0.4a'$

> Put $T = 0$ in equation (1) as string is now slack.

$a' = -0.2g$

> This is the new acceleration of A along the table.

$u_A = 2.424\,87$, $v = 0$, $a' = -0.2g$, $s = ?$

$v^2 = u^2 + 2as$

$0^2 = 2.424\,87^2 - 0.4gs$

$s = 1.5$ m (2 s.f.)

A slides a further 1.5 m along the table before it comes to rest.

$\therefore$ Total distance moved by A is $0.5 + 1.5 = 2.0$ m (2 s.f.)

Exercise 10F

1 Two particles A and B of masses 4 kg and 3 kg respectively are connected by a light inextensible string which passes over a small smooth fixed pulley. The particles are released from rest with the string taut.

a Find the tension in the string.

When A has travelled a distance of 2 m it strikes the ground and immediately comes to rest.

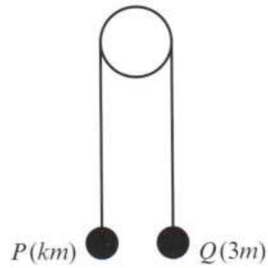
b Find the speed of A when it hits the ground.

c Assuming that B does not hit the pulley, find the greatest height that B reaches above its initial position.

Problem-solving

After A hits the ground B behaves like a particle moving freely under gravity.

(E/P) **2** Two particles P and Q have masses km and $3m$ respectively, where $k < 3$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with P and Q at the same height above a horizontal plane, as shown in the diagram. The system is released from rest. After release, Q descends with acceleration $\frac{1}{3}g$.



a Calculate the tension in the string as Q descends. **(3 marks)**

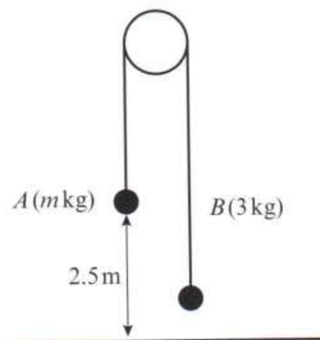
b Show that $k = 1.5$ **(3 marks)**

c State how you have used the information that the pulley is smooth. **(1 mark)**

After descending for 1.8 s, the particle Q reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between P and the pulley is such that, in the subsequent motion, P does not reach the pulley.

d Show that the greatest height, in metres, reached by P above the plane is $1.26g$. **(7 marks)**

(E/P) **3** Two particles A and B have masses m kg and 3 kg respectively, where $m > 3$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially A is 2.5 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in the figure. After A has been descending for 1.25 s, it strikes the ground. Particle A reaches the ground before B has reached the pulley.



a Show that the acceleration of B as it ascends is 3.2 m s^{-2}. **(3 marks)**

b Find the tension in the string as A descends. **(3 marks)**

c Show that $m = \frac{65}{11}$. **(4 marks)**

d State how you have used the information that the string is inextensible. **(1 mark)**

When A strikes the ground it does not rebound and the string becomes slack. Particle B then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

e Find the time between the instant when A strikes the ground and the instant when the string becomes taut again. **(6 marks)**

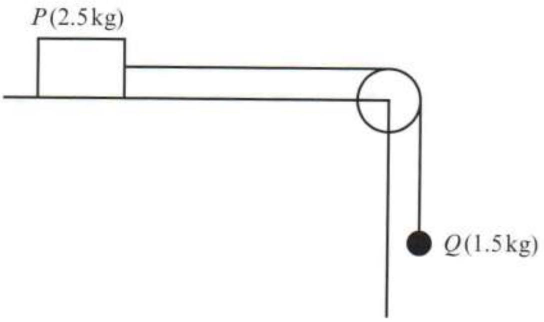
4 Two particles A and B of masses 5 kg and 3 kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table and the string passes over a small smooth pulley which is fixed at the edge of the table. Particle B hangs freely. The friction between A and the table is 24.5 N. The system is released from rest. Find:

a the acceleration of the system

b the tension in the string

c the magnitude of the force exerted on the pulley by the string.

5 A box P of mass 2.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley fixed at the edge of the table. The other end of the string is attached to a sphere Q of mass 1.5 kg which hangs freely below the pulley. The magnitude of the frictional force between P and the table is k N. The system is released from rest with the string taut. After release, Q descends a distance of 0.8 m in 0.75 s.



a Modelling P and Q as particles:

i calculate the acceleration of Q **(3 marks)**

ii show that the tension in the string is 10.4 N (to 3 s.f.) **(4 marks)**

iii find the value of k. **(3 marks)**

b State how in your calculations you have used the information that the string is inextensible. **(1 mark)**

Mixed exercise 10

1 A motorcycle of mass 200 kg is moving along a level road. The motorcycle's engine provides a forward thrust of 1000 N. The total resistance is modelled as a constant force of magnitude 600 N.

a Modelling the motorcycle as a particle, draw a force diagram to show the forces acting on the motorcycle.

b Calculate the acceleration of the motorcycle.

2 A man of mass 86 kg is standing in a lift which is moving upwards with constant acceleration $2\,\text{m}\,\text{s}^{-2}$. Find the magnitude and direction of the force that the man is exerting on the floor of the lift.

3 A car of mass 800 kg is travelling along a straight horizontal road. A constant retarding force of F N reduces the speed of the car from $18\,\text{m}\,\text{s}^{-1}$ to $12\,\text{m}\,\text{s}^{-1}$ in 2.4 s. Calculate:

a the value of F

b the distance moved by the car in these 2.4 s.

4 A block of mass 0.8 kg is pushed along a rough horizontal floor by a constant horizontal force of magnitude 7 N. The speed of the block increases from $2\,\text{m}\,\text{s}^{-1}$ to $4\,\text{m}\,\text{s}^{-1}$ in a distance of 4.8 m. Calculate:

a the magnitude of the acceleration of the block **(3 marks)**

b the magnitude of the frictional force between the block and the floor. **(3 marks)**

(P) **5** A car of mass 1200 kg is moving along a level road. The car's engine provides a constant driving force. The motion of the car is opposed by a constant resistance.

Given that car is accelerating at $2\,\text{m}\,\text{s}^{-2}$, and that the magnitude of the driving force is three times the magnitude of the resistance force, show that the magnitude of the driving force is 3600 N.

6 Forces of $(3\mathbf{i} + 2\mathbf{j})$ N and $(4\mathbf{i} - \mathbf{j})$ N act on a particle of mass 0.25 kg. Find the acceleration of the particle.

(P) **7** Forces of $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ N, $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ N and $\begin{pmatrix} a \\ -2b \end{pmatrix}$ N act on a particle of mass 2 kg causing it to accelerate at $\begin{pmatrix} 3 \\ 2 \end{pmatrix}\text{m}\,\text{s}^{-2}$. Find the values of a and b.

8 A sled of mass 2 kg is initially at rest on a horizontal plane. It is acted upon by a force of $(2\mathbf{i} + 4\mathbf{j})$ N for 3 seconds. Giving your answers in surd form,

a find the magnitude of acceleration

b find the distance travelled in the 3 seconds.

(P) **9** In this question **i** and **j** represent the unit vectors east and north respectively.

The forces $(3a\mathbf{i} + 4b\mathbf{j})$ N, $(5b\mathbf{i} + 2a\mathbf{j})$ N and $(-15\mathbf{i} - 18\mathbf{j})$ N act on a particle of mass 2 kg which is in equilibrium.

a Find the values of a and b.

b The force $(-15\mathbf{i} - 18\mathbf{j})$ N is removed. Work out:

i the magnitude and direction of the resulting acceleration of the particle

ii the distance travelled by the particle in the first 3 seconds of its motion.

(E/P) **10** A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find:

a the acceleration of the car **(3 marks)**

b the tension in the tow-rope. **(3 marks)**

When the car and trailer are moving at $12\,\text{m}\,\text{s}^{-1}$, the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged:

c find the distance moved by the car in the first 4 s after the tow-rope breaks. **(6 marks)**

d State how you have used the modelling assumption that the tow-rope is inextensible. **(1 mark)**

(E/P) **11** A train of mass 2500 kg pushes a carriage of mass 1100 kg along a straight horizontal track. The engine is connected to the carriage by a shunt which is parallel to the direction of motion of the coupling. The horizontal resistances to motion of the train and the carriage have magnitudes R N and 500 N respectively. The engine of the train produces a constant horizontal driving force of magnitude 8000 N that causes the train and carriage to accelerate at $1.75\,\text{m}\,\text{s}^{-2}$.

a Show that the resistance to motion R acting on the train is 1200 N. **(2 marks)**

b Find the magnitude of the compression in the shunt. **(3 marks)**

The train must stop at the next station so the driver reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the train of magnitude 2000 N causing the engine and carriage to decelerate.

c Given that the resistances to motion are unchanged, find the magnitude of the tension in the shunt. Give your answer correct to 2 s.f. **(7 marks)**

P **12** Particles P and Q of masses $2m$ kg and m kg respectively are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. They both hang at a distance of 2 m above horizontal ground. The system is released from rest.

a Find the magnitude of the acceleration of the system.

b Find the speed of P as it hits the ground.

Given that particle Q does not reach the pulley:

c find the greatest height that Q reaches above the ground.

d State how you have used in your calculation:

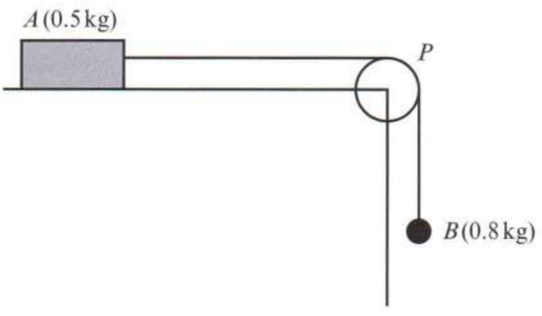
i the fact that the string is inextensible **ii** the fact that the pulley is smooth.

P **13** Two particles have masses 3 kg and m kg, where $m < 3$. They are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The particles are held in position with the string taut and the hanging parts of the string vertical, as shown. The particles are then released from rest. The initial acceleration of each particle has magnitude $\frac{3}{7}g$. Find:

a the tension in the string immediately after the particles are released **(3 marks)**

b the value of m. **(3 marks)**

P **14** A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in the figure. The resistance to motion of A from the rough table has a constant magnitude F N. The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Modelling A and B as particles, calculate:



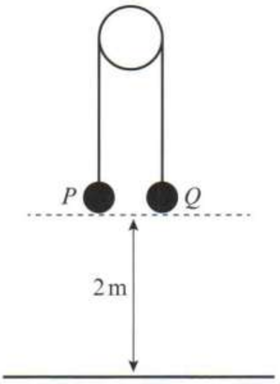
a the acceleration of B **(3 marks)**

b the tension in the string **(4 marks)**

c the value of F. **(3 marks)**

d State how in your calculations you have used the information that the string is inextensible. **(1 mark)**

E/P **15** Two particles P and Q have masses 0.5 kg and 0.4 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 2 m above the floor, as shown. The particles are released from rest and in the subsequent motion Q does not reach the pulley.



a **i** Write down an equation of motion for P. **(2 marks)**

ii Write down an equation of motion for Q. **(2 marks)**

b Find the tension in the string immediately after the particles are released. **(2 marks)**

c Find the acceleration of A immediately after the particles are released. **(2 marks)**

When the particles have been moving for 0.2 s, the string breaks.

d Find the further time that elapses until Q hits the floor. **(9 marks)**

Challenge

In this question **i** and **j** are the unit vectors east and north respectively.

Two boats start from rest at different points on the south bank of a river. The current in the river provides a constant force of magnitude 3**i** N on both boats.

The motor on boat A provides a thrust of (−7**i** + 2**j**) N and the motor on boat B provides a thrust of (k**i** + **j**) N. Given that the boats are accelerating in perpendicular directions, find the value of k.

Summary of key points

1. **Newton's first law** of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object.
2. A **resultant** force acting on an object will cause the object to **accelerate in the same direction** as the resultant force.
3. You can find the **resultant** of two or more forces given as vectors by adding the vectors.
4. **Newton's second law** of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced: $F = ma$.
5. $W = mg$
6. You can use $\mathbf{F} = m\mathbf{a}$ to solve problems involving vector forces acting on particles.
7. You can solve problems involving connected particles by considering the particles separately or, if they are moving in the same straight line, as a single particle.
8. **Newton's third law** states that for every action there is an equal and opposite reaction.