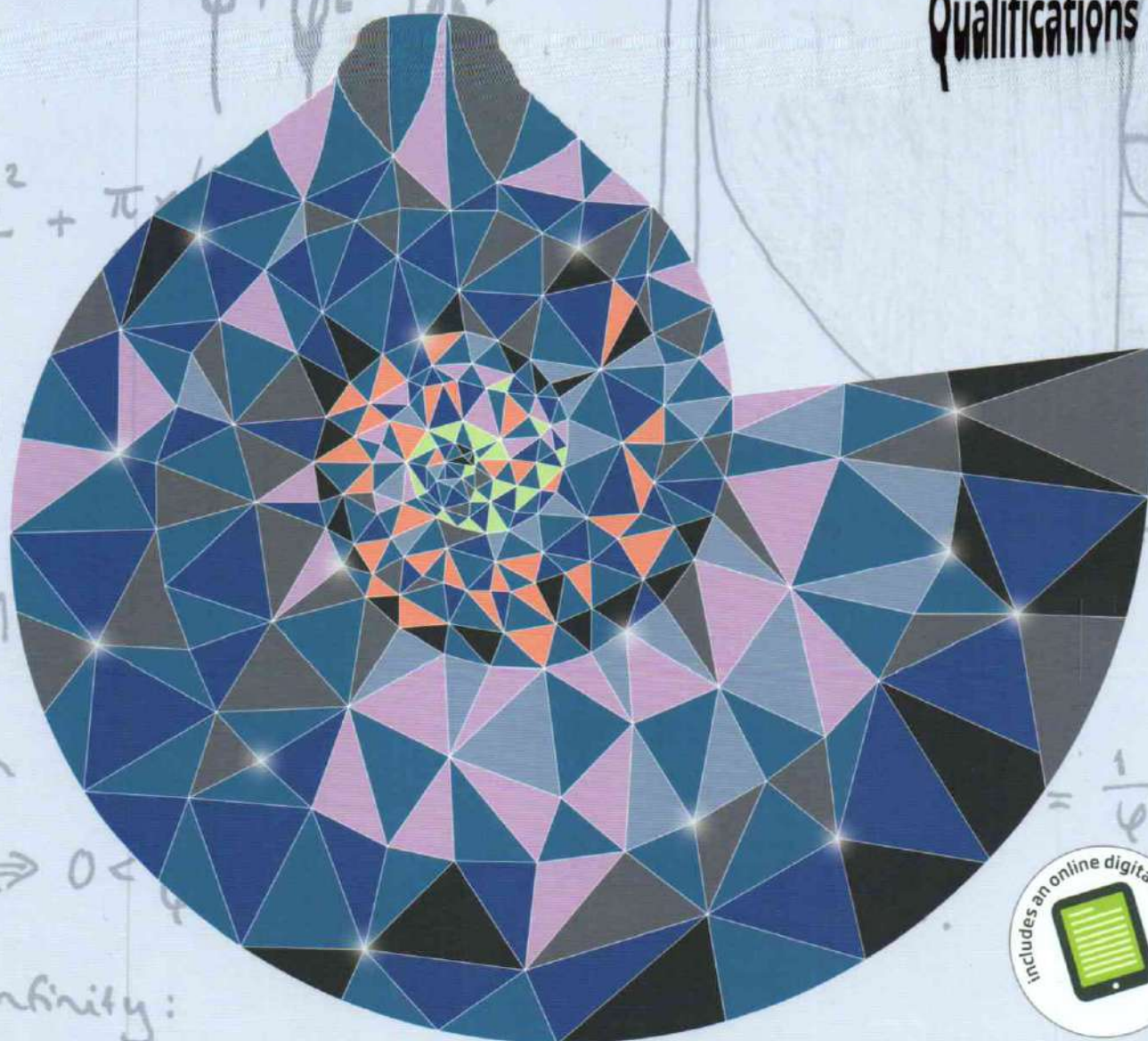


11 - 19 PROGRESSION

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Pure Mathematics

Year 2

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Pure Mathematics Year 2

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Overarching themes

The following three overarching themes have been fully integrated throughout the Pearson Edexcel AS and A level Mathematics series, so they can be applied alongside your learning and practice.

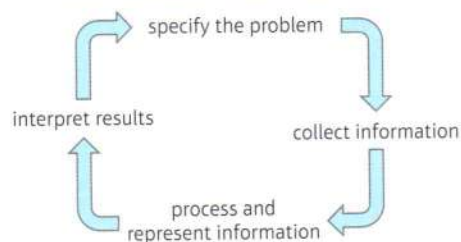
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- Rigorous and consistent approach throughout
- Notation boxes explain key mathematical language and symbols
- Dedicated sections on mathematical proof explain key principles and strategies
- Opportunities to critique arguments and justify methods

2. Mathematical problem solving

- Hundreds of problem-solving questions, fully integrated into the main exercises
- Problem-solving boxes provide tips and strategies
- Structured and unstructured questions to build confidence
- Challenge boxes provide extra stretch

The Mathematical Problem-solving cycle



3. Mathematical modelling

- Dedicated modelling sections in relevant topics provide plenty of practice where you need it
- Examples and exercises include qualitative questions that allow you to interpret answers in the context of the model
- Dedicated chapter in Statistics & Mechanics Year 1/AS explains the principles of modelling in mechanics

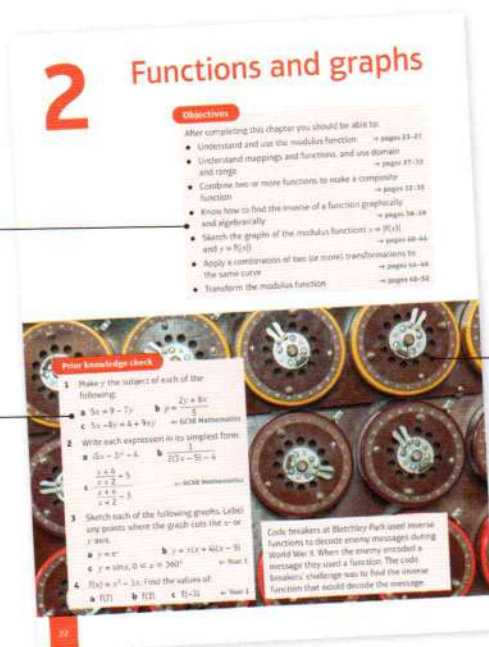
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Each chapter starts with a list of objectives

The Prior knowledge check helps make sure you are ready to start the chapter



The real world applications of the maths you are about to learn are highlighted at the start of the chapter with links to relevant questions in the chapter

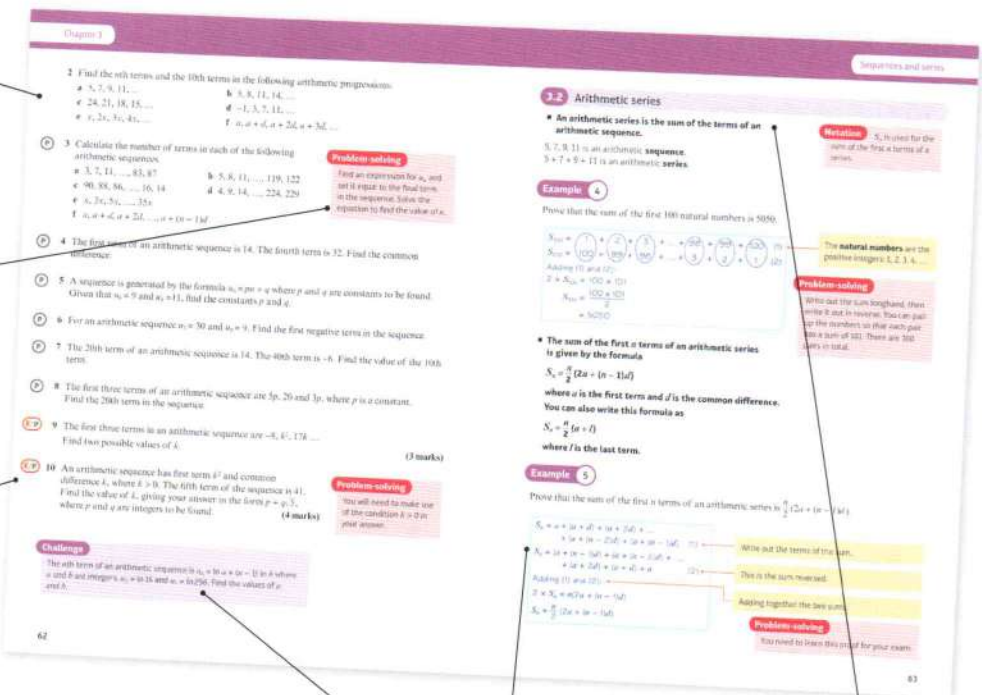
Exercise questions are carefully graded so they increase in difficulty and gradually bring you up to exam standard

Problem-solving boxes provide hints, tips and strategies, and Watch out boxes highlight areas where students often lose marks in their exams

Exercises are packed with exam-style questions to ensure you are ready for the exams

Exam-style questions are flagged with **E**

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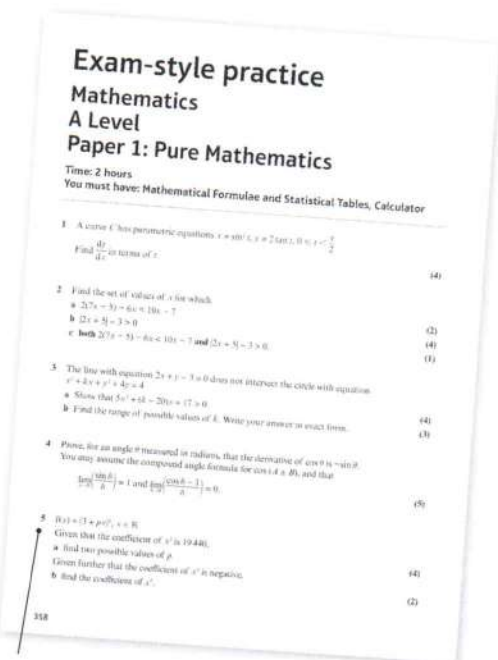
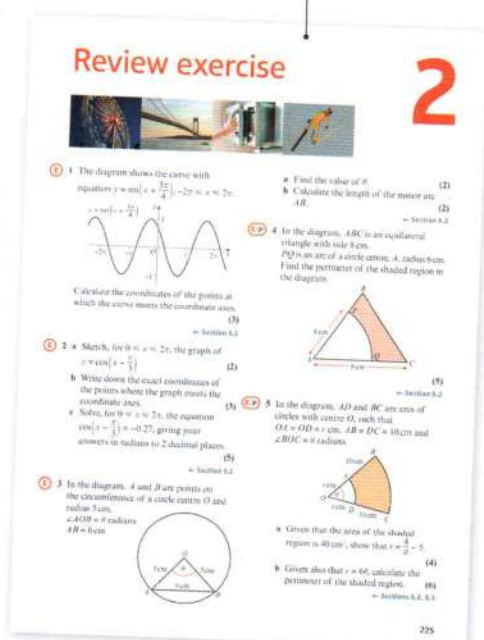
Each chapter ends with a Mixed exercise and a Summary of key points

Challenge boxes give you a chance to tackle some more difficult questions

Step-by-step worked examples focus on the key types of questions you'll need to tackle

Each section begins with explanation and key learning points

Every few chapters a Review exercise helps you consolidate your learning with lots of exam-style questions



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Pure Mathematics Year 1/AS

SolutionBank

Differentiation 12A

1 a Examples of estimates of gradients:

Gradient of tangent at $x = -1$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{-1 - (-0.5)}$$

$$= \frac{0}{-0.5} = 0$$

Gradient of tangent at $x = 0$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{0 - (-0.5)}$$

$$= \frac{0}{0.5} = 0$$

Gradient of tangent at $x = 1$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{1 - 0.5}$$

$$= \frac{0}{0.5} = 0$$

Gradient of tangent at $x = 2$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{2 - 1}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 3$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{3 - 2}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 4$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{4 - 3}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 5$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{5 - 4}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 6$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{6 - 5}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 7$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{7 - 6}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 8$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{8 - 7}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 9$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{9 - 8}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 10$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{10 - 9}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 11$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{11 - 10}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 12$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{12 - 11}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 13$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{13 - 12}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 14$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{14 - 13}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 15$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{15 - 14}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 16$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{16 - 15}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 17$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{17 - 16}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 18$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{18 - 17}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 19$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{19 - 18}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 20$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{20 - 19}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 21$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{21 - 20}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 22$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{22 - 21}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 23$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{23 - 22}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 24$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{24 - 23}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 25$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{25 - 24}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 26$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{26 - 25}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 27$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{27 - 26}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 28$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{28 - 27}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 29$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{29 - 28}$$

$$= \frac{0}{1} = 0$$

Gradient of tangent at $x = 30$ is

$$\frac{f_2 - f_1}{x_2 - x_1} = \frac{1 - 1}{30 - 29}$$

$$= \frac{0}{1} = 0$$

Pure Mathematics Year 1/AS

SolutionBank

Differentiation, Mixed Exercise 12

1 $f(x) = 10x^2$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{10(x+h)^2 - 10x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{10(x^2 + 2xh + h^2) - 10x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{10x^2 + 20xh + 10h^2 - 10x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{20xh + 10h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(20x + 10h)}{h}$$

$$= \lim_{h \rightarrow 0} (20x + 10h)$$

$$= 20x + 10(0) = 20x$$

$$\text{So } f'(x) = 20x$$

As $h \rightarrow 0$, $20h + 10h \rightarrow 20x$

So $f'(x) = 20x$

2 a A has coordinates (1, 4).

The y-coordinate of B is

$$(1 + 3x^2 + 3x) = 4 + 3$$

$$= 1^3 + 3(1) + 3(1) = 4 + 3 = 7$$

$$= (1x)^3 + 3(1x)^2 + 3(1x) = 4 + 3 = 7$$

$$\text{Gradient of AB} = \frac{7 - 4}{1 - 1} = \frac{3}{0}$$

$$= \frac{3}{0} = \infty$$

$$= \frac{3}{0} = \infty$$

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$$= \frac{3}{0} = \infty$$

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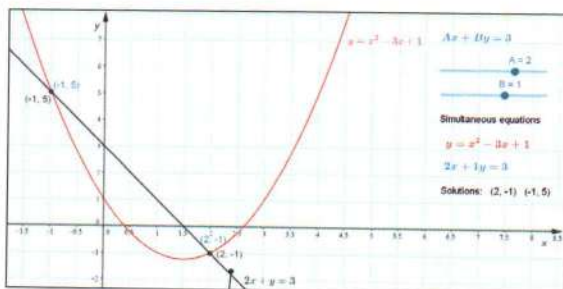
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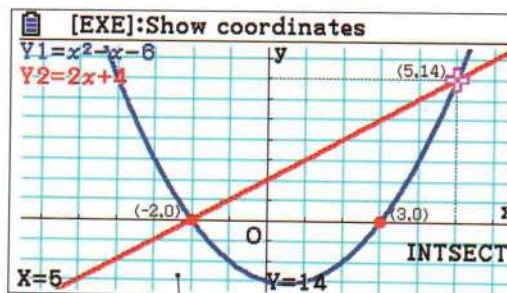
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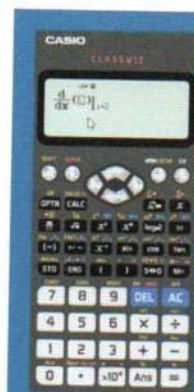
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